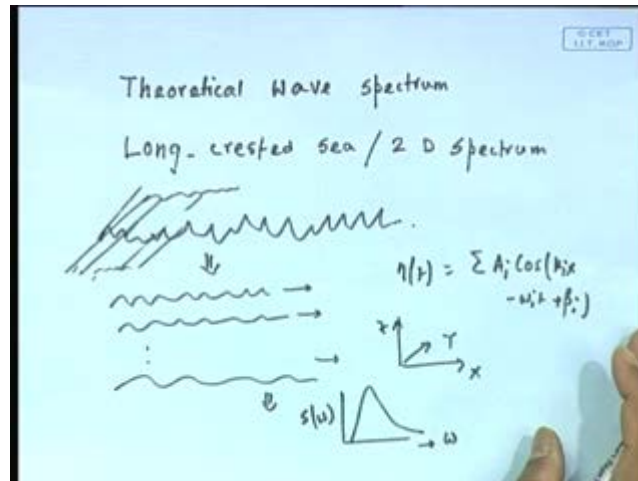


**Seakeeping and Manoeuvring**  
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**Lecture No. # 13**  
**Theoretical Wave Spectrum**

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See today we are going to talk on what is called theoretical wave spectrum, but before I go to this, in last class where we described the concept of spectrum, I have missed out one important point what is called as long-crested or 2 D spectrum. See what we did, if you recall, we said that we are standing somewhere, we have an irregular record, right. We said this is broken down to number of waves.

The point is that, what we have actually not stated one I wrote here was something like that. We said that  $\eta$  is equal to sum of you know something like  $A_i \cos(k_i x - \omega_i t + \beta_i)$ . I now look at this expression. What does it tell? Which direction is this regular wave moving? Well, this tells me that waves are moving in x direction. What it means is that when I took a signal here, I assumed that it is composed of all the regular waves moving in the same direction.

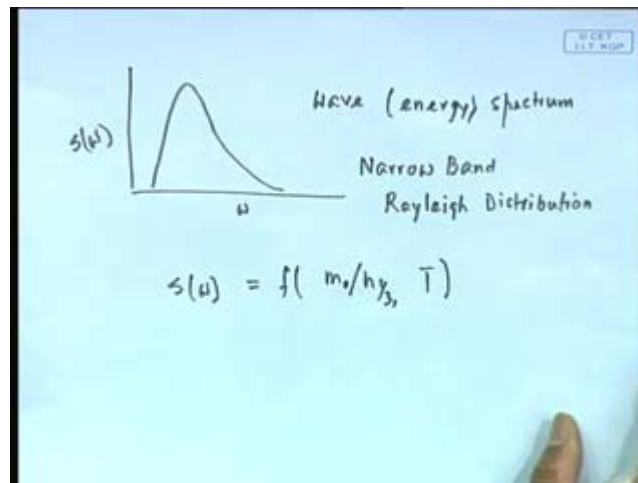
So, what would happen if I stood **if I stood** here? What would happen? I will find this crest line you know right hand left hand side to be infinitely long, which means that if I look at that I will find this wave like you know like fully long crested because there is no variation in this side. See if I were to, in other words, if I were to cut a cross section you know if I were to call this to be z, x and y, if I were to take a z x plane anywhere, I would

see the same picture. Now, which means what we are talking is that what we have presumed is that this irregular record is 2 Dimensional. Exactly, it is unidirectional. So, you can call it see this one long crested. This is the crest line. Crest line is the one that is going along y y axis. It is infinitely long. The entire wave is propagating in one direction. So, it is unidirectional as you said.

In another way of looking at it is that the wave fluid motions are all containing xz plane. So, therefore, it is two-dimensional. So, this spectrum is known as what we have, what we know this we have come down to a spectrum. This is what is called a 2 D spectrum or this description; this way of describing sea is known as long crested sea which I should have mentioned yesterday. Remember one thing. What we have done? We have got only this record. See in analyzing it, I went to a sea. I put my stick some measuring equipment and I only found out this record how much  $\eta$  t with respect to time is. Rest is my analysis.

Now, if I presume that all the waves are coming from one direction x and broke it down, then I make a spectrum. Then, I call that a two-dimensional spectrum long or this c is known as long crested sea. This is what we have done, but as you know if you actually go on a sea, you would not find the crest line to be infinitely long. Both cycles give you a standardized shape. The crest on your right hand side and left hand side is not uniformly long. That description would be obtained if I were to add waves not only of all lengths in one direction, but coming from all directions also, which we will discuss later on. That is what is called short crested or 3 D spectrum. So, I just wanted to mention today as on now that what we talked is a 2 D spectrum so far which presumed automatically that the crests are long. So, we refer to them as either long crested sea or a 2 D spectrum, **all right**.

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As I said, we will come back to the 3 D spectrum description afterwards. Now, we have mentioned yesterday that ultimately, you end up getting a graph. We call it wave spectrum. Actually, you can call, some people use the word wave energy spectrum, but you know we actually by default do not use the word energy. You know we say wave spectrum, but it implies wave energy spectrum. We have said also that if you went around the oceans, if you collected data for ages which people have done, the shape of the spectrum turns out to resemble a narrow band, Rayleigh spectrum. Well, narrow band I did not mention, but it is narrow band means there is more it is contained within that broad band. It would be actually what you mean by you know spread over long distance.

Now, the question comes, the second question, right. I have collected data for last hundred years at various locations. I know it is like this, but as I was concluding in last lecture, my aim is essentially predictive. I want to know how I would describe the sea for operation tomorrow. Let us say I have a Trans Atlantic voyage. I would like to know or I have an off shore structure located in a Gulf of Mexico or in North Sea. Obviously, I would like to know what kind of wave it is going to encounter. Not, what is encountered already because I have not designed it.

So, therefore, I want a description to this end. What has happened now? Sea state comes later you see. You see sea steps will come much later. What happens is we actually have to come up with a shape by a formula. Now, this formula  $s(\omega)$  if it is a narrow band spectrum, we know the shape of the formula in terms of its area under the spectrum or in terms of some characteristic height parameter or some height and period parameter. The question is that people have investigated this and numbers of formulas have been

proposed, which fits the data in specific locations. For example, you have collected a data and say north sea for may be hundred years or if not hundred, for a long time, then you keep fitting and then, you find out that yes, it fits the particular formula. Another place, another formula like that number of such formulas has evolved.

Those formulas are actually called theoretical wave spectrum. So, there will be, this will be function of, it can be function of well essentially  $m$  zero, but  $m$  zero can be written in terms of  $H$  one-third and there can be function of  $T$ , some  $T$  characteristic. It can be you know  $T$  p t zero. I will come back to that some characteristic time period. Never mind this for the time being, let me just show some formula. Then, you will realize what we are talking.

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The slide contains the following handwritten text and a graph:

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left[-\beta \left(\frac{g}{U\omega}\right)^4\right]$$

with  $\alpha = 8.1 \times 10^{-3}$ ,  $\beta = 0.74$   
 $U = \text{Wind Speed.}$

The graph shows a plot of  $S(\omega)$  versus  $\omega$ . The curve is a bell-shaped peak that starts at the origin, rises to a maximum, and then decays towards the x-axis. The x-axis is labeled  $\omega$ .

Pierson-Moskowitz spectrum  
 Beaufort Scale  
 → wind speed

See the very first formula that was proposed had looked like that. One of the very earliest one, it says alpha g square by omega 5 exponential because it has to look like that exponential minus beta g by u omega 4 with alpha. There was a formula proposed at very beginning, long back I believe. It might have been about forty years back or. So, like that you see here alpha and beta are constant given by these numbers.

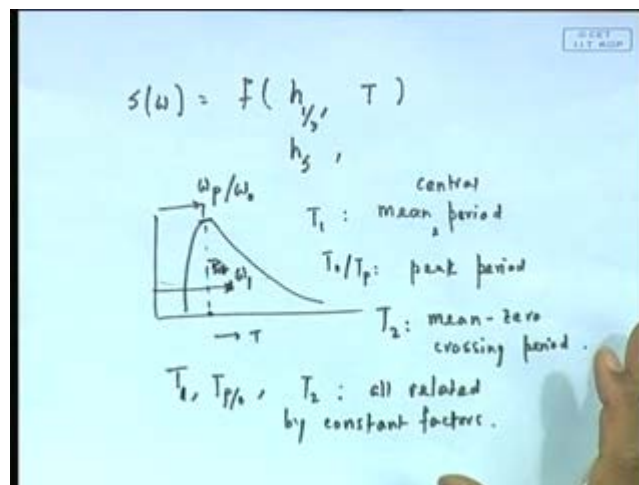
The unknown here of course is  $u$  here which is actually wind speed. Here, the formula was given in terms of wind speed omega is a variable. So, what it means is that if I want to find the formula, how omega versus  $s$  omega. All I have to do is to calculate for different omegas, say 0.1, 0.1, 1.1, 2 etcetera. For a given  $u$ , say for a given  $u$ , you calculate that you will end up getting a shape like that. Another  $u$  you will end up getting a shape like that. Obviously, it is depended on only single parameter  $u$  here which is

wind speed. See earlier formula evolved taking the shape to be a function of the well, shape and size to be function of wind speed because it was thought that you know larger the wind blows, more energy will go in etcetera. This is actually called the original form of Pearson Moskowitz spectrum.

However, this wind speed that was parameter in aerospace people, you know there is a so-called Beaufort scale. They see when we talk in practical engineering, people like to put some kind of a scale like very strong wind storm condition etcetera. You know wind speed can be let us say 1 knot to 100 knots or even more, but then you kind of quantify. If it more than say 100 to 120 knots, you say it is like a severe storm etcetera.

So, there was a scale which the aeronautical people always use called Beaufort scale you know and that was essentially based on wind speed, but however, original formula also had a functional dependence on wind speed, but subsequently, we people who are in marine side or oceanography side started thinking why wind speed. It should be some measure of H one-third or some measure of height and period only height. See this is a single parameter give only one wind. You will get a shape where the shape is that is not decided. You know if I give wind speed say, 20 knots, it is here. No matter what you do, 30 knots, it is fixed, but then modified formulas started coming, where people stated saying that I will write the formula in sum in terms of either H one-third or H one-third and some particular period T.

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That is later formula are of the formula are of the form of something like h. Now, normally you see here I will just write, first write h and t. What does h tells me? H tells

me the area under that it can be  $H_{1/3}$ ,  $h_{avg}$ ,  $H_{1/10}$  etcetera. So, I read that because that will give me the severity in a measure of severity. Now, peak as I said earlier people begin to use this one third more or significant.

Well, one third is same as I can call it by default significant not with respect to  $t$ . There are three or so,  $T$  is there one.  $T$  is what we have said yesterday mean period in a spectrum. The mean period that is  $T$ . Well, you can call it by this we call it by  $T_1$ . I believe yesterday, right.  $T_1$  mean actually we can put mean central period, but you can also have a peak period. This is where the peak value occurs. So, this would be there. Of course, this actually, this is not well this is not  $T$ , but this is a measure of  $T$  because this is actually  $\omega$ .

So, let me call this  $\omega_1$  and this one, I will call it  $\omega_2$  or  $\omega_p$  or  $\omega_0$ . Remember  $T$  is inverse of that  $2\pi$  by  $\omega_p$  or  $T = 2\pi/\omega_0$ . So,  $T_0$  or some people call it  $T_p$  is a peak period. Also, you can have  $T_2$  which is the mean 0 crossing period. My point of saying here is that essentially, I need a factor designating the area under which tells me how high the waves are in some sense. So, this can be any of the  $h$  parameter and typically, one use  $H_{1/3}$  with respect to where it is located, this side. You need to tell one part of typical period or frequency. Now, in height there is no debate. All formulas are using  $h$  significant  $H_{1/3}$ , but period there have been formulas which uses either the mean central period or peak period or mean 0 crossing period, but the beauty is that all these  $t$ 's are related by a factor constant factor.

For example, we will see later on that  $t_0$  and  $T_1$  is given by some constant factor etcetera. We will see that later on that all these  $T_s$ ,  $T_1$ ,  $T_p$  or  $T_0$ ,  $T_2$  or by now, there are formulas that evolve which uses  $H_{1/3}$  and  $T_1$  or  $H_{1/3}$  and  $T_p$ . In other words are same spectrum people have represented in different formulas.

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Bretshneider Wave Spectra :

$$S(\omega) = \frac{173 \cdot H_{1/3}^2}{T_1^4} \omega^{-5} \left\{ \frac{-692 - 42}{T_1^4} \omega \right\}$$
$$T_1 = 1.086 T_2, \text{ or } T_1 = 0.772 T_p$$

ISSC      ITTC

So, I am going to talk about two or three formulas which are most used. Here, you see in our literature, one is what first I will write in terms of this which is an oldest spectrum, in fact two parameters older. All this I will let me first write it down. Most of the spectrum basically what happened became named after the person who has developed it you know. See this is Bretshneider. This formula looks like that all of them will have same exponential and  $T_1$  which is the mean central period related by this thing by or this is one. In fact, this is, this can be also called to be modified form of p m spectrum.

You will find out that the p m spectrum in the original form was written in terms of  $u$ . So, here the  $u$  is replaced by what is known as two parameters spectrum, that is  $H_{1/3}$  and  $T_1$ . That means, you have to know given  $H_{1/3}$ , two meter,  $T_1$  one-tenth second, you will get a shape. If you give something else, you get another shape etcetera. This is 1.

Now, I will tell you another one which is modified p m spectrum. This also a sense of modified p m. There are minor differences between various formulas proposed. What has happened see going back again little of history. Remember if you go historically ships were ships period much more off shore structures. You know there are ships around much earlier than off shore structure and there was a need for finding out ship motion. So, this formula aimed at actual application to the ship and this formula is probably 1940s. So, subsequently or may be little later around that time 1950s or so. That is this straight formula somewhat similar to this in the shape and all. However, instead of  $u$ , we want to use now  $H_{1/3}$  here.

Then came like our story in resistance ITTC and ISSC committees. Then, you know there are two ISSC is let me just tell you briefly is International Ship Structure Congress and ITTC, of course you know Conference. Now, waves were important from loading point of view. How much of share force unloaded gives and unload. So, ships structure congress were interested to actually freeze on a formula that uniformly can be used by ship people.

The other thing is that remember ship is, most ocean going ships are geographically operating in certain area. You may have a ship taking sugar from Calcutta going to Singapore, unload, take something, go to Russia etcetera. So, mostly the ships are covering all over the ocean. So, they wanted a formula that represent the wave condition on an average sense for the entire globe. Then, they came up with a formula which is called ISSC formula which adapted by ITTC which is also called modified p m formula, which is what I will write.

The reason I am saying that this because by default, when somebody does not give you any spectrum, we use that mostly for our design purpose ITTC recommended spectrum, just as we are using ITTC recommended formula for our c f in resistance calculation. So, that formula is it looks; well once again the same formula can be expressed in various ways. So, I will write one way and then, another way we will write separately. So, this is given s.

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The image shows a whiteboard with handwritten mathematical formulas for the Modified P.M. Spectrum. The formulas are as follows:

$$\frac{S(\omega)}{H_{1/3}^2 T_1} = \frac{0.11}{\pi} \left( \frac{\omega T_1}{2\pi} \right)^{-5} \exp \left[ -0.44 \left( \frac{\omega T_1}{2\pi} \right)^{-4} \right]$$

$T_1 = \text{mean control period} = 2\pi m_0 / m_1$   
 $T_m = \text{mean wave period} = 2\pi \sqrt{\frac{m_0}{m_2}}$   
 $T_1 = 1.026 T_m$   
 $T_{0.1\%} = 1.408 T_m$

A hand-drawn circle around the text "Modified P.M. Spectrum" is labeled "Modified P.M. Spectrum". A hand-drawn bracket groups the last three equations with the label "a.k.".

Well, this is while let me write it down first. Oh no, this is written as in a different form because it is non-dimensional z. You know this expression, this is a meter square, second



meter square second and I at the end of the class, I am going to ask you to do some assignment or problem. See the formula looks same. If you look at these, you will find out. If you look at these  $2\omega$  minus  $5\omega$  minus  $5$ , here comes  $\omega$  minus  $4$ . Here,  $T_{14}$  is same as  $T_{1-4}$ . So, these two are same. It is a constant that change.

You know if you look at that, there is a similarity of that. See, this is also  $T_{1-4}$ , this is also  $T_{1-4}$  into  $\omega$  minus  $4$ , same as these two. What is different? This constant here,  $H$  one-third square comes here. So, if you look at that the formula is basically same formula in a different way with slight modification. May be there  $T_{1-4}$  of course, we said is basically mean central period. That we know. Let me write it again. Let me also write it down. This is  $2\pi$ . Complete it.

$T_2$  is as I mentioned before is the well, no in this case, not I will write it  $t_m$ . It is a mean period, mean wave period. Another one we can actually write mean wave period, just mean. We take all the waves and just average period, basically not central period which is given by  $2\pi$  etcetera you know like that, but this is same as  $T_{1-4}$  is  $1.060$  etcetera and  $T_0$  is this is not important. You know what I want to say only in this one is that all the  $T$ 's are connected. It was given in terms of  $1/T$  like  $T_{1-4}$ . If you do not want it, you can change it in terms of  $T_p$  or something like that. It is just the constants will change. That is all. So, it is not a big deal. This is what is called we call it modified  $p_m$  spectrum. So, this is the one that typically we use. This is the one that typically we tend to use for ship. This or even the previous one also you could use, not a big deal. If you plot them, the changes are not much high, much different.

I will get back to this in a minute, but there is one more spectrum, that is well, this is not very important. We all should know. Let me put it this way. We are treating a subject of how the ship behaves in waves and so, we are on the receiving end. My owner tells us look my ship is going to operate from a to b. I ask him what kind of wave conditions are there. Either he gives me a wave condition or a spectrum by a formula. Then, I use that formula. If he does not give, he says, no you choose yourself. Then, we will use this one. So, what I mean that this is what do you use is really typically depends on the person who is asking to do it because suppose, he is doing, let us say a vessel which only goes to a Calcutta visage Madras and Port Blair. So, we may want to know a particular spectrum that fits the data mostly for Bay of Bengal only.

Then, I have to ask him give me the formula. If he cannot give me, then if you choose, I will use this ITTC recommended formula. You get the point now? So, remember at this point, I have only a formula and will only talk about how the ship looks like etcetera in a minute. So, this is what is called a two parameter formula because I need to find out, I need to get two inputs to plot this which is what I will ask you to do at the end of the class like plot these ships for various combinations of H one-third and T.

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The image shows handwritten notes on a whiteboard titled "JONSWAP spectrum". The formula for the spectrum is given as:

$$S(\omega) = \frac{320 H_s^2}{T_p^4} \omega^{-5} \exp\left\{-\frac{1950 - \omega^2}{T_p^4} \omega^2\right\} \gamma^A$$

The peakedness factor is defined as:

$$\gamma = 3.3 \text{ (peakedness factor)}$$

The exponent A is defined as:

$$A = \exp\left\{-\left(\frac{\omega}{\omega_p} - 1\right)^2\right\}$$

The peak frequency  $\omega_p$  is defined as:

$$\omega_p = 2\pi/T_p, \quad T_p = \text{peak - Pz}$$

The damping factor  $\sigma$  is defined as:

$$\sigma = \begin{cases} 0.07 & \text{if } \omega < \omega_p \\ 0.09 & \text{if } \omega > \omega_p \end{cases}$$

So, that you can idea how the ship looks like, but before I do that, there is one more that is most important, that is what we call the second one is called Jonswap. Well, second one means in these two, second one I also address neither and the other one Jonswap spectrum. Now, this is interesting you know. This implies joint North Sea wave project. What has happened is that when in North Sea you know North Sea on the north east of Scotland, may be south west of Norway etcetera.

Finland, that area, there is a huge oil reserve and at one time, there was a largest number of wild in platforms. Now, when you have an oil platform sitting there, it is not a ship going from a to b to c and all that. What is point of designing it for waves which is average? You want to have wave only at that, that is at that location that is 0.1, 0.2 is that north sea happens to be always more rough in an average sense always north sea. It is a well known fact.

You know the waves are of a different characteristic. There are larger waves etcetera. So, this people took a project joint north sea, huge project and at the end after several as they came up this spectrum called Jonswap spectrum. It has become historically very

important and that is why, we are talking about it. It is used very extensively for North Sea. Of course, you use it, but the form of that is used even for other geographically specific locations. So, it looks like that. In fact, it is a modification of p m spectrum. It will look like again the two ways of writing this. One way of writing was like that in terms of T p peak period; this is written in terms of peak period. Here this is p. Again you will see the form has to be always same that exponential sum constant by a period power of 4 into omega power of minus 4.

See if you look at that, you will find out that this is actually similar thing. You know T p omega minus 4, T p 4. If you actually multiply that, it will become omega minus 1 which is T p. So, H one-third into sum t unit of that you will see that unit of that meter square second because T power of 4 into omega minus 5 gives you what omega minus 1 which is equal to T p. So, like that and here also is the same thing, but here the most important part is that they have a factor called gamma power of a which is called peachiness factor which is what I will know. So, gamma is given as 3.3. You know is called peachiness factor a is defined as exponential. This is very, any text book will give you actually this one same peak frequency, something like that. So, this was what a Jonswap spectrum is.

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JONSWAP (Alternative rep.)

$$S(\omega) = 155 \frac{H_y^2}{T_1^4 \omega^5} \exp\left(\frac{-944}{T_1^4 \omega^4}\right) (3.3)^\gamma$$

$$\gamma = \exp\left[-\left(\frac{0.191 \omega T_1 - 1}{\sqrt{2} \sigma}\right)^2\right]$$

$$\sigma = 0.07 \quad \omega \leq 5.24/T_1$$

$$= 0.09 \quad \omega > 5.24/T_1$$

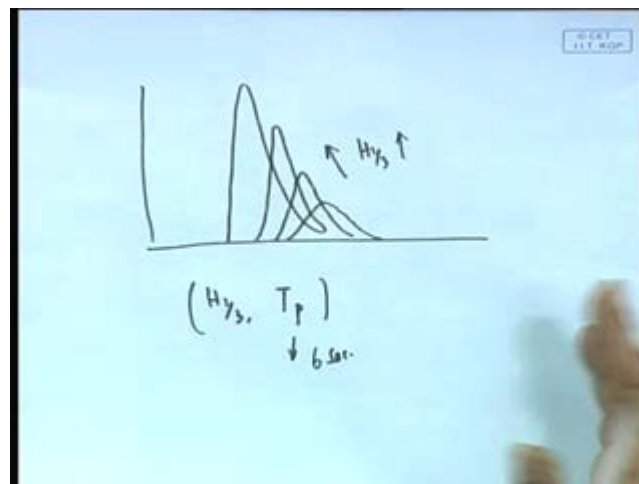
$$T_1 = 0.834 T_0, = 1.073 T_2 \text{ etc.}$$

You know I wrote one way of the formula. The same thing can be represented in terms of T p, in terms of T 1. The other way I will also write that one alternative representation. So, here s omega will look like 155. This is actually same thing. I have a point of saying this. Actually, y is same as a here, but you just have written with a different, just same as this thing and this is just a relation.

See why I mentioned, why I wrote that? See my main reason of writing that is like this. Same spectrum you can represent by different formula. You know if you put the numbers and write a computer program, you are going to get the same result. It is just that in one place you are putting  $T_1$  and the other place  $T_p$ . So, it is a same thing. You can rewrite my point of telling is that depending on which book you open. Who need you may find out the formula looks different, but actually they are all same thing. That is what my point is. So, there do not look for a formula. That looks in all across the book same.

They represent the same graph, but they look different because somebody use another two parameters, somebody use one another this parameter, etcetera. So, now, the point is having said that now we look at this. Here let me go back. First the shape part of it. One by one from, say modified p m spectrum, I will go back to this modified p m spectrum part again just keeping that in mind how will it look like.

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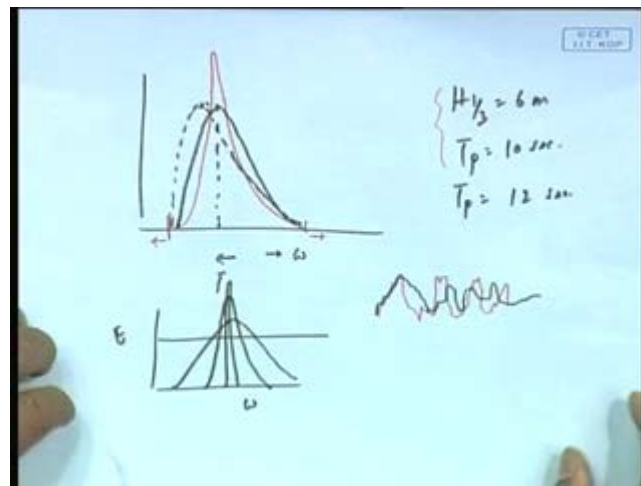


You will find out and this is what you will be doing and I will tell you the actual numbers if you use  $H$  one-third, say for given  $T_p$ , you increase  $H$  one-third, you will find out that the shape goes up. Of course, what you do is that well, here we have used  $H$  one-third and say, certain  $T_p$ . Now, we have to understand one thing. Let me take the peak period only can I have say  $T_p$  of some numbers, say six second and  $H$  one-third, 2, 3, 4, 5, 6,8 meters.

In other words, can I have this uncorrelated with this? The answer is no because see what does  $T_p$  means.  $T_p$  means the period which means the wave length. Now, I cannot have very short wave length with very high wave height because naturally high length has got

some relation. Well, I may not be exactly able to tell that my  $h$  by  $\lambda$  is constant or something, but I can of course tell that I cannot even arbitrarily high  $h$  for a given  $T_p$ . Otherwise, what I mean that for a given  $h$ . Let us say for a given  $h$  of 6 meter, my  $T_p$  may be varying between say 8 to 10 second, but not 4 to 20 seconds. There is some slight variation that we will talk about that later on. Therefore, to plot these  $h$ , see here this plot this one is for  $T_p$  equal to say 6 second, this for 8 second, this is for 10 second and this for 12 second.

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Now, you could let us say as an example, now my point is that if I look other way round. Suppose I take took a graph where I want to see  $H$  one-third is say 6 meter and  $T_p$  is equal to 10 second, which means I am going to get a shape like that. Now, if another one with  $T_p$  equal to say 12 second, what will I get? Tell me. We will be getting a shape which has a same area because my  $H$  one-third is same, but my peak period is supposed to have got well. This is my period increasing; this is my  $\omega$ , right. So, it is supposed to have got shifted.

So, you will find that my shape would become something different slightly, but it will look something like that may be. So, there is some variation with  $T_p$  that you will find out if I do that, but my point of course, is that you cannot have this for  $T_p$  arbitrarily small. There is a range of  $T_p$ , but typically; obviously,  $H$  one-third gives you the area,  $T_p$  gives you the peak period. If I increase  $T_p$ , peak period will shift the location, will shift. So, you will end up getting this shape, but now the important point. Let us say for the same combination. This if I were to put Jonswap's spectrum. You know what you will find? You will find that the Jonswap's spectrum looks something like this.

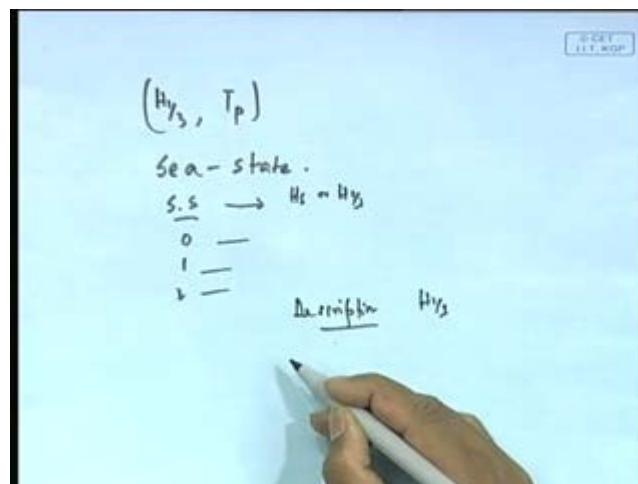
What does the spectrum tell me? Remember if I give a formula for a given spectrum, say I say  $H \propto \frac{1}{\lambda^3}$  so and so that's this one. So, I got a shape. What I find out? I find out that there are practically no waves below this length. There are practically no waves above this length, also below this length, above this length or rather I know. I also know where the maximum energy is, what the difference between these two shapes is. Main thing is that in Jonswap, you will find that you end up getting much more numbers of waves of one frequency, whereas this is more spread.

See let me give an example of what is called as white noise. You know suppose a spectrum is like this. Some frequency versus some energy. Some energy, what does it mean? It is actually called white noise which means all the frequency components exist equal amount. Actually, if you see our white light, you know all the seven frequency that is of same amount. That is why it is called white noise. Also you can call it sometime this is a very broad band spectrum means all frequency exist. Now, when I have this, it is a narrow band spectrum, but this is even more narrow. This is even more narrow and if you actually have only one frequency, it will be a straight line. So, this is what it means is that therefore, relatively speaking, Jonswap's spectrum tells me that it is far more narrowed and then, there are much more waves for a given in a record which is of one type only. In other words, if I were to see in a spectrum form, it may look more like.

There is a dominant one form, whereas other one will look much more spread, you know like if you break it down, you will find more or less more wide distribution of all the frequencies equally, whereas, if you break it down, you will find much more number of this frequency. Now, you see why it is important. Suppose, an offshore structure which happens to behave badly in this frequency, then you have it. So, you see the characteristics are different. It is very important to understand when you have a spread spectrum which is an average sense which means that I have actually almost all kind of frequencies more equally evenly distributed, but this one is much more what is called peaky. That is why this term was called peakiness factor. You know this is a typical characteristic that happens in North Sea. That means, if in a geographical location, if a wind blows at certain speed, certain energy goes inside. It tends to excite some wavelength. Only most, not all and from design point of view, you know if I have to for example, represent this spectrum by one unit frequency, then I obviously will take the peak one.

Well, I can say that if this body does not be badly in this peak frequency, maybe it is quite, but in a sea case of course, it is you know in the p m's case, it is more wide. So, this is what one thing that I wanted to tell. Now comes the question how do I use it? Before I go to your assignment problem, let me talk about how do I use this kind of information. What H one-third should I use, what T p I should use? This is if I tell you that use H one-third six meters, T p ten seconds. I know what a spectrum is. That is one side that is I have already told you that use this parameter. So, you know this spectrum, but there is another dilemma that we will have, that is I have to design a ship and I like to know what spectrum I should use. May be before that I will tell you one more characteristic.

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Now, you see we have got this H one-third, T p. The shape now again H one-third, you put say 3.95 meter, T p 11.2 second I get a ship. H one-third is a continuous number, but people do not like to talk the condition in this continuous number. So, people use some kind of scaling you know like we use the word rough, very rough in your grey rough. We say you know poor average etcetera or bulb number in like your grade 70-80, b or is it c, I do not know b. 80 to 90 like that.

So, here also people want to say, look we are going to talk in terms of qualitative sums. So, we introduce the word sea state. So, you have then a kind of a description c state one. For example, you would say by a description sea state, one is very calm, water etcetera at the new sea. See if I call sea state, you know like 1, 2, 3, 4, etcetera say 0, 1, 2, then I will relate that to an H 1 significant or H one-third between some values.

For example, according to one, it is not, this is not very uniform thing you know. This say sea state you know like there will be some kind of number. Well, this is, there are some charts there. What would happen is that we have an ocean data. It says in one and description, the H one-third limit say, for example, describe you know very calm water, no white you know like a white cap formation, which means you do not find this white breaking breakers etcetera.

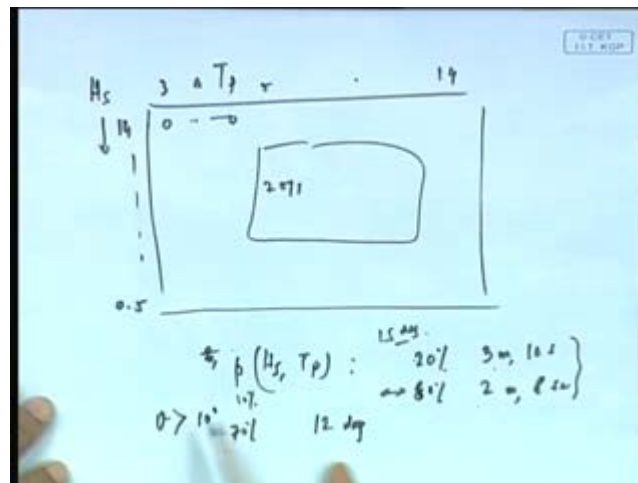
H one-third is between this to this range. T p is between this to this range and we call it sea state to be one or two or three whatever. So, what I am saying that in order to describe people, use some kind of scale sea state, say sea state 4 root may mean perhaps that wave height is between say 2 to 3 meter, significant wave height period between 10 to 11 second etcetera. There is a chart that is not fully frozen that there are WMO, World Meteorological Organization. They also give a chart, we can use that.

So, what happened to our design is something like that, say a naval ship design you are doing. So, let us say one that we are building, say you know like the aircraft carrier. So, there is a requirement show that your design is good, such that my rho do not exceed 10 degrees, more than 5 percent time in sea state 3. What would you do? You will actually go to sea state three, take that as an input spectrum, you go to the chart, and find out for sea state three. What is the average h and t used that put that back in the formula and then, I have got the spectrum for sea state three.

Then, I of course, used my ship for that sea state three. So, there is one side, it deterministic requirement people have say that make sure that your ship can operate up to sea state four and there is let us say, there is a requirement of operating. You can operate provided some parameter, some acceleration does not exceed 0.2g at some place let us say. So, like that one can state that is an owner telling.



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Sometimes, we need a long term statistics to know the percentile. What has happened by long term statistics is that people have collected this data all over the world. For each location, there is the number of time  $h$  significant occurred and  $T_p$  occurred. This is a big chart has been found out. For example, say zero etcetera. What it means is that this  $h$  is this  $T_p$  combination occurs. So, many times you see here, if I were to put some number zero etcetera somewhere, see here let us say 14 meter like that go to 0.5 meter. Somewhere this occurs. This  $T_p$ 's are may be 3, 4, 5 like that to 14 second. So, you write this number say here 2011. Actually, I cannot show this here. What I mean is that there are long term statistics available of  $T_p$  and  $H_1$  that combination. What it means that so many times a wave has been observed in past which has this  $H_1$  one-third, this  $T_p$ .

See suppose, I have collected data for 100 years in some location. I find out here that out, well I have collected data and I analyzed that one million data of which I find out that five data occurred where  $H_1$  one-third is 14 meter,  $T_p$  is 10 second two thousand occurred when  $h_p$  is so and so.  $T_p$  is also basically, this is a joint probability how many times the occurrences are. Earlier one happened in a signal. I have only broken down to one how many times  $H_1$  one-third occurred or  $h_s$  occurred. Now, we are doing a joint working. How many times  $h_s$  and  $T_p$  of this combination occurred? So, this is what is called joint probability. So, from the joint probability, you can find out. Therefore, what is the chance that my  $H_1$  one-third will occur more than certain value and  $T_p$  more than certain value?

So, by that you can statistically combine and figure out how many times my  $h_s$  and  $T_p$  combination occur. So, let us say  $h_s$  and  $T_p$  combination occurs certain you know like

say two into say whatever say certain  $h_s T_p$ , say this combination, this probability you know probability of occurrence of this. So, what you will do? Of course, to that I will try to now determine how much is the rho in this combination, but I know that this occurs only five percent time. So, therefore, I will take 0.5 percent into that number. Then, another  $h_s T_p$  occurs 20 percent time. I find the response, I multiply with that and like that I can find out what is the probability of that response. It is simple addition. I mean I am not going to go through this detail, but what I want to tell you it is a simple addition. Why simple addition? Because you see if given  $H$  one-third, given  $T_p$ , I know the spectrum, but now I am telling another graph, where I can tell how many times is  $H$  one-third  $T_p$  occurs, what is the chance of this  $H$   $T_p$  occurring.

So, I can take all the combination and therefore, find out what is the probability of the final response. It is just by adding. See suppose, in this graph says this part you will find out that the numbers are more. Here you have no record, where  $h_s$  14m occurred with a  $T_p$  of five second. No record at all which means 14 meter significant wave height do not occur with a peak period of three second. Not one, but may be only two times it occurs with six seconds. Like that you get a joint probability distribution.

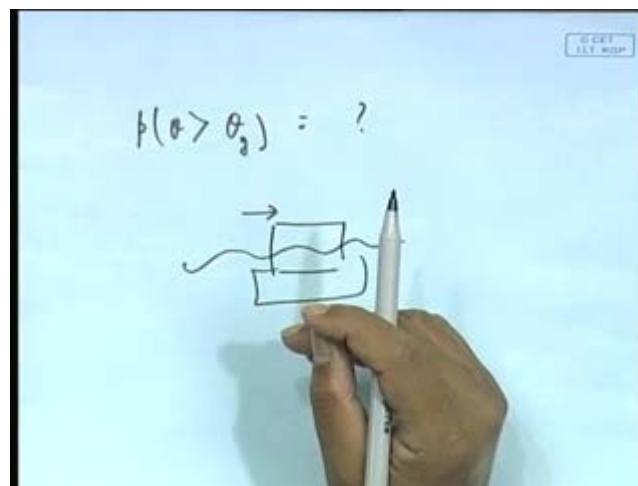
Therefore, meaning is that joint probability will tell me what is the chance of this  $h_s T_p$  occurring and now, I will take all those and find out for all of them response multiply with the probability function. I end up getting what is the probability of that particular response. So, it is very simple. Let me put it the other way round just for simplification. Let us take just two examples. Let us say that I have a record which says 20 percent my  $h_s$  occurs three meters with  $T_p$  equal to say 10 seconds and rest 80 percent of time my  $h_s$  occur two meter  $T_p$  equal to eight second. Just take an example. Actually, it will be much more longer, but let us take an example that let us see that in a particular location, I found out only 20 percent of time. My  $h_s$  as three meter,  $T_p$  ten second and the rest 80 percent time two meter eight second.

Now, I find out for that the response I find that my theta rho becomes 12 degree and I find out here that the rho becomes 15 degree. So, I can always take and then, I can find the probability of that. Also, then 0.8 times, this plus 0.2 times this to get my sin of average in a sense. In fact, it will be not so straight forward. You have to find the probability of design also. See I have probability of this occurring say 20 percent time or rather let me put the other way round then.

Chances of theta exceeding 10 degree I want to find out. I find out that in this sea, it is 70 percent time in this, it is 10 percent time, but this 70 percent is occurring 80 percent of time. So, I have got total chances 0.7 into 0.8 plus 0.1 into 0.2. You understand? So, therefore, what is my chance of theta exceeding 10 degree will become 0.7 70 percent of time, but that phenomenon is occurring 80 percent time. So, 0.7 into 0.8 plus 10 percent of time for phenomenon occurring is 20 percent time. So, point you know 1 into 0.2. So, therefore, you add, you end up getting the so-called total probability. See theta is occurring more than 10 percent time in this sea, two meter eight second sea and it occurring more than 10 degree for 10 percent of time in three meter ten degree ten second wave.

Now, it is occurring 70 percent of time in this wave, but this wave is occurring after total on 80 percent of time. So, I have this into that number. So, this is what the requirement. No, the requirement will be in the design. Ultimately that you see what would happen is like that design will tell me the probability strength. It will tell you that please tell us that your theta does not exceed 10 degree for at least 95 percent of time

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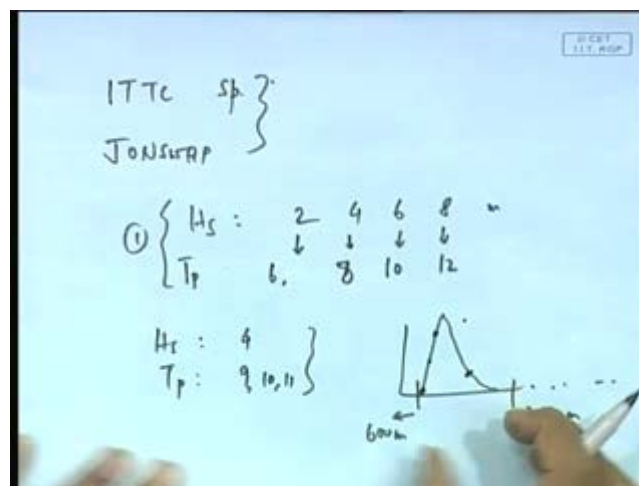
Absolutely. So, therefore, what happen is here you will find out there is a. I have to find out what is my probability of theta exceeding theta given let us say. So, I will find out this now by combining the probability. This is what my requirement is. Now if I find out that this is more, then I will say my ship cannot operate in this condition. Essentially, you are trying to find out operability, let us say let me say the other way round.

Let us say that if the heave is very high, more than three meter, then you cannot dig oil. Now, I want to find out how much of time it is going to be more than three meter. My requirement is that look in a design, you must make sure that 98 percent time you or 90 percent time you must have heave less than three meter because otherwise, I cannot dig oil. Now, I find out that in a design, it was 95 percent time. So, I accept it.

So, I have only 5 percent down time so-called, but it was 80 percent time. Then, you might want to change the design. No, it isn't a designed stage, not after designing when you are evolving the design. This is how you can use the data. See when we use also an off shore structure or something, let us say let us take an off shore structure. There is an off shore structure. There you know you have and waves are coming.

You want to find out how you would design, what is the stress you will take. You must take some kind of wave. So, here we have deterministic in some sense, we can take I am going to take the highest wave that occurs once in 100 years. I will take that wave and I will see whether the ship survives or the body survives. That is one way of looking at it. Otherwise, I will say I will find out what is the chance of my structure failure is H one-third occurring more than so much percent and time and therefore, my stress more than so and so percent time.

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So, therefore, I will find out what is by combining what is a chance of my stress occurring at a certain location more than so and so percent time? As long as it is very small, I will accept that is these are the kind of uses. Anyhow, I will like to drop that and I want to tell you this assignment. See now what I want you to do is that these two

spectrum, this ITTC spectrum and this Jonswap, both of them  $h_s = 2, 4, 6, 8$  meter and  $T_p$  for 2 meter will take 6.

Let me take this way. Only this is seven, may or rather eight. May be this may be ten. This combination that is  $2, 6, 4, 8, 6, 10, 8, 12$ , I want you to calculate the spectrum and plot the spectrum. Then, for one case, this is one case for  $h_s$  equal to 4,  $T_p$  you will take 9, 10, 11 to plot that many reasons for doing that. You know only when you do this plot, then you will understand that what is the kind of waves, what is the omega range because otherwise, what would happen is you will begin this formula and you do not have the time. You will begin this formula, you will find out that omega is minus 4. If you take omega 0, 1, 2, 3, 4, 5, you will find out that you have got one point here, one point here, one point here, then all zero zero point.

By only doing, you will know within what it remains. Then, you will have an understanding that actual ocean waves lie between what range of omega, what range of length. Therefore, what is a peak? You will find out interestingly that no waves occurred normally below 50 meter normally above 600 or so meter. Mostly it is about 100-200 meter. This you can only do if you actually obtain a number because omega if you put 0, it is infinite  $\lambda \omega$ . If you put 0.01, it is something like may be one meter wavelength or whatever I do not know naught or you should do omega into one two pi by omega is two pi. It is  $t$  is you will find out. So, only when you play with a number, you will understand what you need to. Otherwise, you will get one point here, one point here, one point here and all points which is of no meaning.

Next run you would narrow it down, you get two more points. You know that how I should do that, so that I can get all the points here and nothing beyond that. That will give you an idea regarding which range of ocean waves or which range of length, the ocean waves really lie and if you do not do that, you have no practical feel. You know that is very important that you must do it this combination running a program is one second time.

What is important is to find out what range I will do if you just write do omega as one to infinity one to ten thousand. You will just be completely wrong. So, that you will do by narrowing down with that I will end it. So, you do that as an assignment in the next class.

We will go now. It is time for us to put ship in the waves. Now, we know how to describe waves. We now have to put the ship in the waves and with that I will end.