Marine Hydrodynamics
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Lecture - 09
Two Dimensional Flows (Contd.)

Welcome to a ninth lecture in the series. Today, in the last class, we are talking about several 2 types of two dimensional flows. And the last flow, we consider is the source and sinks. What happen in case of a source? In case of a source, we have seen that the fluid immerse out from a point. And there we have seen that the at far distance when the radiation becomes very large there is no flow whereas on the other hand if we say that what happen near the when we are the radius becomes very small. When the radius becomes small, that means r tend to 0 we see that fluid speed will be extremely loss, infinitely loss. On the other hand, and here the flow is always in the radial outer direction and it is symmetric, it goes out in a symmetric manner.

On the other hand, if you look at a sink, here the fluid it will absorb all the fluid and again it receives them in a radial direction. So, that is the basic characteristics of a source and then that of a sink. Now, we will understand various flows in this part of the lecture first one analyze simple flows. And then see what will happen if that is a source are a sink inside the flow, how this flow will be disturbed due to the presence of sources and sinks whether the flow part will remain same or there will be a different pattern, because we have seen that. So, the sources and sinks are the kind the kind of similarities comes to enters to the flow pattern.

So, when we so it, it expected that the flow behavior will change. Let us in this lecture, we will highlight some of the examples where the... There are two types of flows and combined together and what is the new flow that is developed. And particularly, we will think of those which are regular and also flow which are of singular nature. Another point we have seen the two flows; one is a source, the other is the (( )) basically, what I say, but what I will say that vertex flow, vertex motion. So, you have seen that in case of particular in case of a circulation. In the case of a source, our flow, which are losing the radial direction, but what happen in case of a circulation? So, I will come to that in detail in this course.

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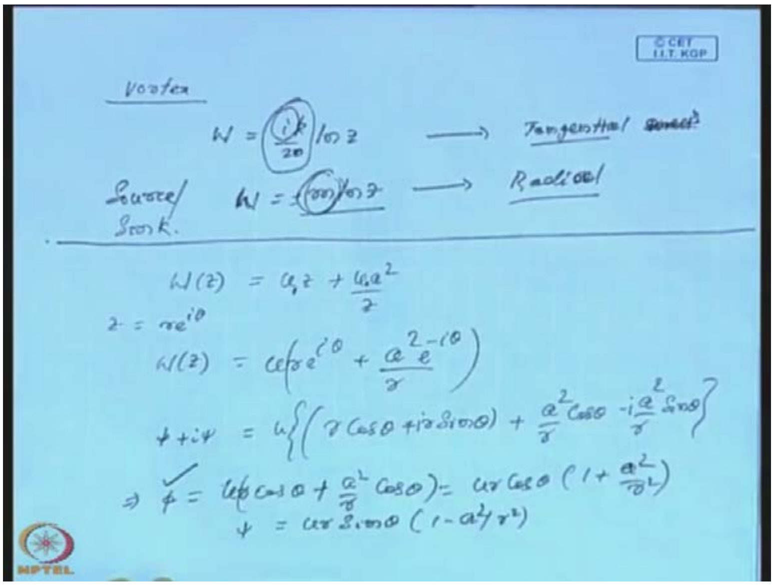

So, we have you know that in case of a vortex. The flow is always, in case of a vortex or W as i k by 2 pi log z, on the other hand in case of a source s n W is m log z. Just look at this in the first case, the flow is in the tangential direction particularly, it is in the tangential direction. On the other hand flow is tangential rather in this case it is a radial. So, see just to look at this here this i how it is effecting, how the flow patterns is changing, just because of this a imaginary number. But both are very complex in nature and some kind of flow similarity, because at z is equal to 0, we have seen that here it is log z, here it is log z or only because of this constants, the flow pattern is changing. That is basic one of the basic differential it is tangential where at here it is radial whether it is a source or a sink in case of a vortex, and in case of a source.

So, this two are very flows which are looks very similar in nature, but only because of this constant here this is serial number this becomes I has a quantity so because of this flow characteristic in this. Now, this I will now go to a different type of flow particularly; this is another very interesting example like flow, because we have talked about singular uniform flow. And once we have talked about the uniform flow. And also in the last class, just and then as an application of the Bernoulli’s equation, we have already known, we already know that flow past a cylinder uniform, flow past a cylinder let us analyze the same problem in the contest of a complex velocity potential.

Now, let me look at W z as u z plus u a square by z if u z is W z u z plus u a square by z. And instead of u z I call this as u naught z plus u naught a square by z then what will happen W z if z is equal to r a to the power i theta then W z equal to u r e to the power i theta plus a square by r by minus i theta. And this also I can write it as u we can always write it as r cos theta plus I r sin theta plus a square by r cos theta minus a square by r sin theta I. And this gives me which implies I will get my pi, because W is nothing but 5 plus i psi if I equate the real and imaginary part. So, I will get phi is u r u times r cos theta then plus a square by r cos theta again. So, I can call this as u r cos theta into 1 plus u r cos theta into 1 plus a square by r. In a similar manner I can easily get psi as u r sin theta 1 minus a square by r it is r square I must write r square here. So, this, this is the velocity potential this is the stream function.

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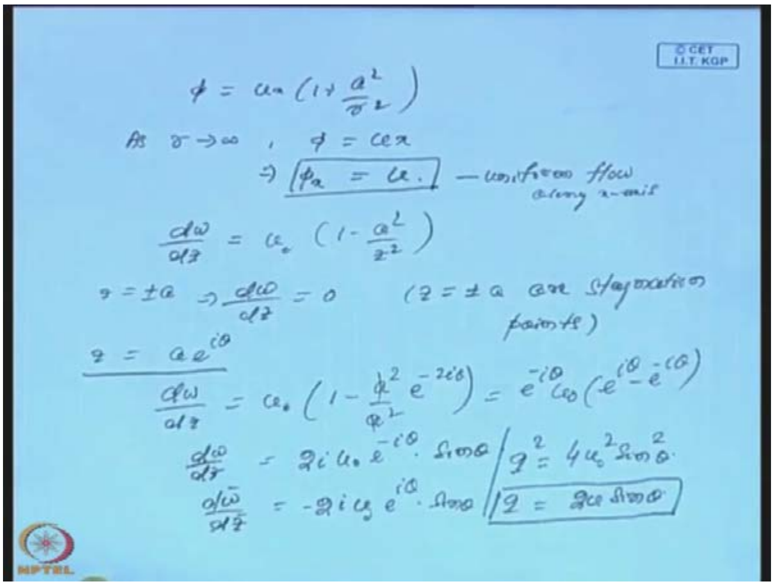

Now, which also can be written as the phi also can be written as u x because x is r cos theta 1 plus a square by r square and so as r tends to infinity where phi becomes u x. And which implies my phi x is equal to u and phi x is u means velocity in the x direction in the Cartesian coordinate. And once phi x is u that means there is a uniform flow that represents a uniform flow along x axis in the x direction. Now, what will happen to d w by d z? If you look at d w by d z, it will be u naught into 1 minus a square by z square and at z is equal to what will happen to at z is equal to plus minus a. If z is equal to plus minus a from here itself, we can get it d w by d z is 0. So that means the points z is equal to and minus a they are are stagnation point they are called stagnation.

Now, when if you look at in a different manner if z is equal to r suppose if z is a e to the power i theta if z is a e to the power i theta what will happen to d w by d z? We should be u u naught 1 minus a square by a square into minus 2 i theta which I can always write it as because this will be 1. So, that will give me e to the power i theta e to the power minus i theta into u naught. So, I will have e to the power i theta minus e to the power minus i theta. And that will give me 2 i cos theta plus i sin theta cos theta minus i sin theta, so 2 i u naught e to the power minus i theta into sin theta. So, again if I will say d w bar by d z bar then that will give me minus 2 i u e to the power i theta into sin theta.

And from these two this is my d w by d z so my q if I multiply this 2 that will give me q square that will give you 4 u naught square into sin square theta. And which gives q is equal to 2 u sin theta that means if you have a cylinder and the boundary of the cylinder q is if I have a cylinder of circular of a radiation or a cylinder of. Because I am dealing with a two dimensional flow if I have a cylinder one cross section of the cylinder considering. So, if on the boundary of the cylinder z is equal to e to the power i theta my q becomes this. And if I if I remember that what happen in case of a uniform flow past, a cylinder when we have worked out, we have seen that on the boundary of the cylinder that q is equal to 2 u sin theta.

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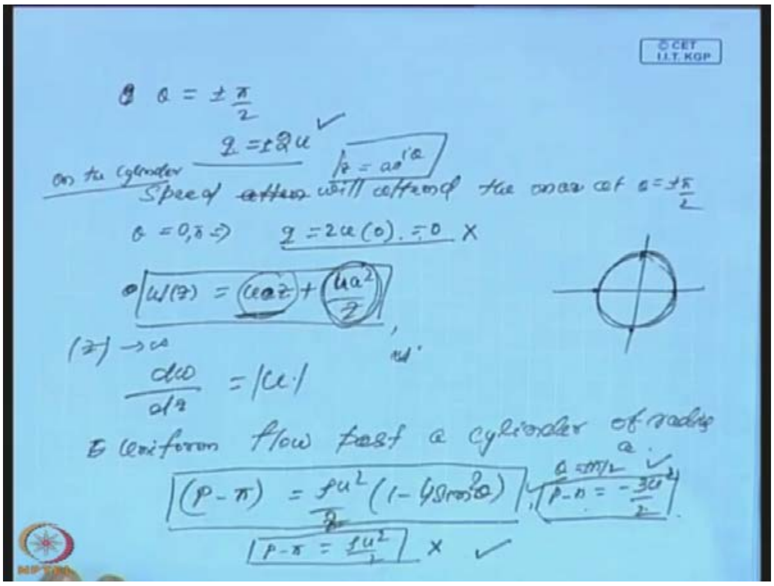

Now again what will happen q when theta becomes cross minus pi by 2 then q will be 2 u plus minus 2 u so whether it is. And that will show that the speed will be attending that means if I look at the speed that will attend, attend the maximum on the boundary of the cylinder attend the maximum at theta is equal to plus minus pi by 2. Again what will happen when theta is on the cylinder, because remember this is on the cylinder, because I am talking z is because I am taking z is equal to e to the power i theta this is a one of the point on the cylinder if the cylinder is of radius a.

Now, what will happen if theta is 0? If theta is 0 then I have again easily see, because q is a 2 u sin theta and 2 u into 0. That will give you 0 so the speed is 0 then theta is 0 and again at pi also we can see this will also 0. So, that means from the surface of the cylinder if I have a flow and here if these, is 0 at this point. And this point I have no flow whereas, at this point and this point my flow is maximum. So, that and again if you look at because we have already seen our W z u u z plus u a square by z so even at z tends to infinity, we have seen that d w by d z is equal to u. And that shows that it is a uniform flow are far filled. And if you look at this, we can always say that the W z given by this will represent the flow uniform flow past a cylinder past a cylinder of radius a.

Another thing is here u a in from the application of Bernoulli's equation if pi is the pressure at infinity. Then p minus pi rho u square by 2 into 1 minus 4 sin square theta here this result, we have derived in the by using the Bernoulli's equation when we did the application of the equation of motion you can recall this is a very important result. Now, if I relate this one at theta is equal to 0, because at theta is equal to plus minus pi by 2 q is the plus minus 2 u and what will happen here theta is if theta becomes cross minus 5 by 2? Then this will give me p minus pi gives me because this will be pi by 2 means this will be 1 so minus 3 e square by 2. And then on the other hand when theta is equal to 0, that is theta is equal to plus minus pi by 2. So, here we have seen that the speed is maximum, but p minus pi we can easily see that this is one of the minimum.

Again when theta is equal to 0 and pi we have seen q is equal to 0, but at that point what will happen to the pressure? So, p minus pi if theta is equal to 0, so this will give us rho u square by 2, because theta is 0 or pi this will give us 0 and so that means here it is. So, this quantity will be one of the maximum. So, what I I want to say that that when the, because we know that in a potential flow when speed is maximum pressure will be minimum at the same point. On the other hand when speed is minimum the pressure is maximum.

So, this is we have one of our result, we have already talked about. So, we have seen that in case of a cylinder that this is exactly what is happening that the speed is maximum at these 2 points. And here at the same point the pressure is attending the minimum. On the other hand when the speed is a attending 0 that means at that same point, the z is maximum so speed and pressure they are inter related. Because from Bernoulli's equation we know that pressure head plus velocity head is constant when nobody force is available.

So, that is what and again we will not go to the details of the force calculation, because we have seen that we have the leap and drag force both are 0 in this case and I am not going to again repeat the same thing. So, this is another example and here there are 2 flows; one is u a z, another is u a square by z. If we analyze the flow separately W z is u z, this part will will represent a flow uniform flow which is moves which is parallel to the x axis at a speed u. Whereas, u a square by z by z if you analyze this flow; this flow actually if there is a, the water is a still water, the fluid is at rest then u a square by z would represent as if a cylinder is of radius a which moving at a speed u in the fluid.

So, on the other hand when you take the 2 fluids into a two velocity potential into account, you merge the 2 things together then the flow is a, is a flow past a uniform flow past a cylinder. So, this is the difference between the two. Now, I will, with this I will go to a very another nice example, where that will give us that is a impact that is very important in for the in the navigation channel upon.

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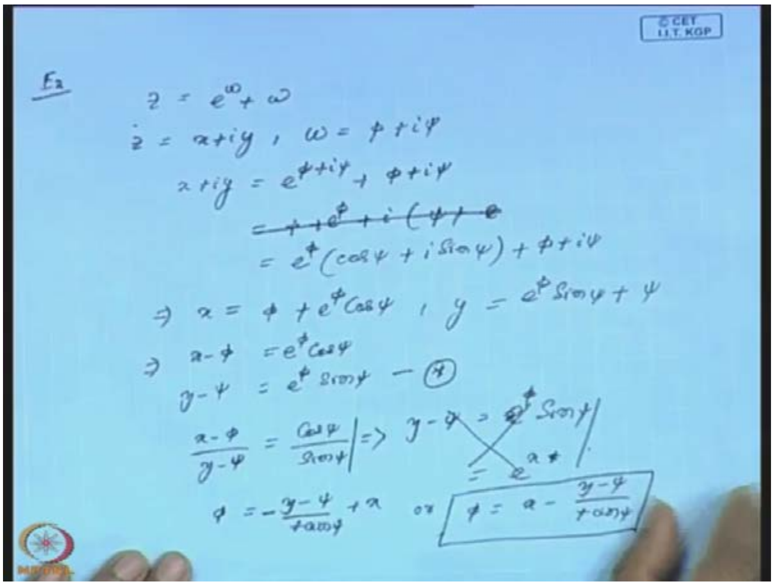

This is one of the very important problem in the olden days, particular to to path, it will determine the path of a channel. So, suppose look at z is equal to e to the power of w. This is a very, very, very interesting way e to the power w plus w where z is equal to x plus I y and when w is equal to phi plus I psi. So, how to go for it, we will do the same thing I will put it, because when it comes to one of the interesting fact in complex function theory application that always we can separate the real and imaginary parts. And afterwards, we will try to simplify the problem.

So, e to the w means e to the phi plus i psi plus phi plus i psi And if you do that that becomes phi plus e to the power phi plus i times psi plus e to the power psi no this will be e to the power phi into cos psi plus i sin psi plus phi plus I psi. And if we again simplify this then our x will be phi plus e to power phi cos psi and our y becomes e to the power phi sin psi plus sin. So, if I simplify further these two, my x minus phi will be e to the power phi cos psi and has to be very little careful y y minus psi will give me e to the power pi sin psi. Now, what will happen here tan psi if I divided by 2 x minus phi by y minus I a very interesting example that is cos psi by sin psi.

Then if you substitute first and that I call this as then from these we can obtain easily y minus I is y minus I is equal to e to the power phi sin psi e to the power phi sin psi. Then a half y minus psi e to the power phi cosine psi and instead of phi I can call it e to the power x plus x rather let me do a little jugglery here this part I will leave it here we have x minus phi. So, I always can get phi is equal to, from here I will get can get phi is equal to y minus psi y minus psi by tan psi y minus psi tan psi that is minus plus x. Or I will call it phi is equal to x minus y minus psi by tan psi I am telling you the algebra is a little complex here. Now, if I substitute for phi in this expression star substitute for phi in star then what I will get if I substitute here? I will get in star.

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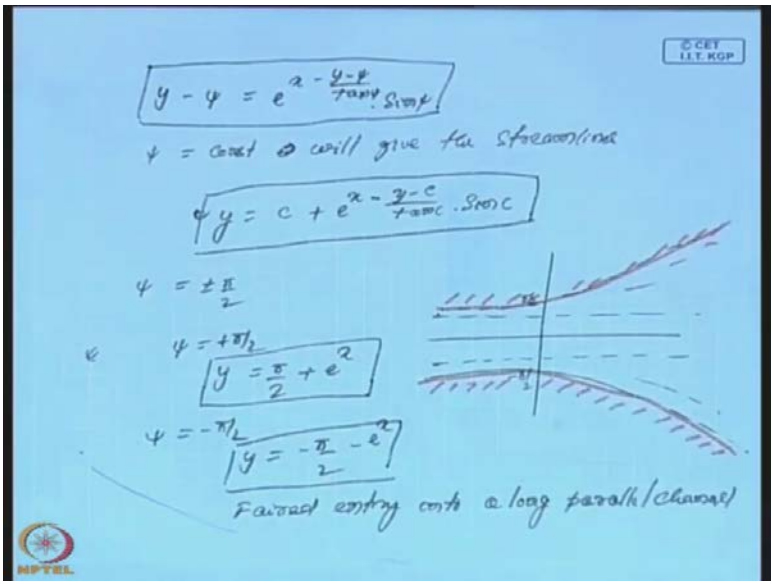

And I will get y minus psi is equal to e to the power x minus y minus psi by tan psi into sin psi. Now, I have eliminated from this I exactly completely there is no psi in this so this is a functioning psi. Now, if I say psi is equal to constant if I say phi is equal to psi is equal to constant then what I will get? Then I will get the streamlines psi is equal to constant will give me, will give the streamlines that means this equation when size equal to constant will give me the streamline. So, in case so I can also write if I say that means I can always write y is equal to c plus e to the power x minus y minus c by tan c into sin c. So, that is the equation of the streamline for each constant each constant c I will get a streamline.

Now, in particular if I put psi is equal to plus minus pi by 2, let us say psi is equal to if psi is equal to plus pi by 2, what will happen my y will be pi by 2 plus e to the power x psi is pi by 2 psi is pi by 2. How much will be, give me this quantity? That will give me 0, because and then then I will get pi by 2 plus e to the power x. On the other hand if I take psi is equal to minus pi by 2 then I will get y is minus pi by 2 minus e to the power x so that means. Now, let me draw this 2 lines; this is a streamline this is a streamline if I draw this we see another x y x is we draw it when you say these are the lines minus pi by 2 this is pi by 2 then what will happen my flow this will give me a flow this will give me a flow like this So, that means e j this is often called a faired entry into a long parallel channel right, because if you look at this as the ocean.

So, these line particularly this line is a streamline; this line is also a streamline if I look at the deep ocean if these 2 lines are the streamlines then there is no there will not be any flow occurs this line. So, always I can say if I use this as a channel for the navigational purpose there will not be any flow across this. So, I can easily call this as a channel without putting any obstacle break water or seawall or any kinds of things. And able to make that there will not be any flow from this side to the channel and in the processes I can call this as a I can use this for my navigational purpose.

Because this is the deep sea when the seas will enter to the channel then it can be easily used, because there will not be any disturbances from outside. And this area I can always utilize as my flow, because there will not be flow any along this lines. So, that is why we call this as a faired entry into a long parallel channel this is one of the very interesting problem but one has to be very careful while working out this problem. Now, I will go to a let me assemble a problem which is of course, little different. So, in a second problem what I will do I will look into a again a problem of a channel, which also can be represented as the flow that one I call it as if a convergent divergent convergent divergent channel.

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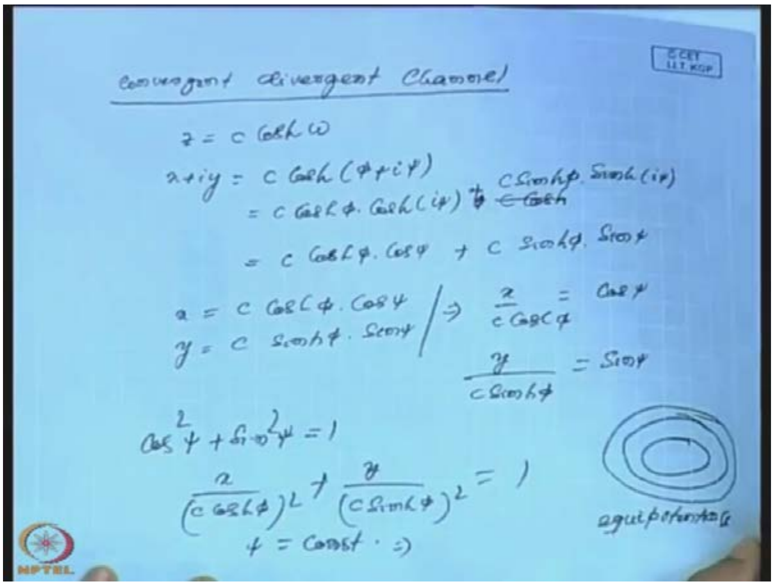

So, what is this? Let us look at this function, suppose I say z is equal to c cos hyperbola is w so of z is equal to x plus I y where problems are very interesting actually c cos hyperbolic phi plus i psi. And now, we have if you expand this; this is equal to c cos hyperbolic phi I will utilize the characteristics of this cos hyperbolic i psi plus c cos, c sin hyperbolic c sin hyperbolic phi into sin hyperbolic i psi. And this will give me, which implies which will give me c cos hyperbolic phi actually w minus psi so cos hyperbolic phi plus minus psi cos cos hyperbolic phi into cos psi cos c sin hyperbolic phi into sin psi.

So, which gives me x is equal to c cos hyperbolic phi into cos sin and my y will give me c sin hyperbolic phi into sin psi. And from this 2 I can always get if I say x by c cos hyperbolic phi equal to cos psi and y by c sin hyperbolic phi will give me sin psi. If I say I utilize the property of the trigonometric function cos square psi plus sin square psi equal to 1. And that gives me that gives me x by c cos hyperbolic phi square plus y by c sin hyperbolic phi square equal to 1 so. And this is then if I put 5 is equal to constant which implies, because this becomes a constant this becomes a constant. So, that will give me ellipse, so my equipotentials are ellipse these are my equipotentials. On the other hand on the other hand if I put the other way like if I put.

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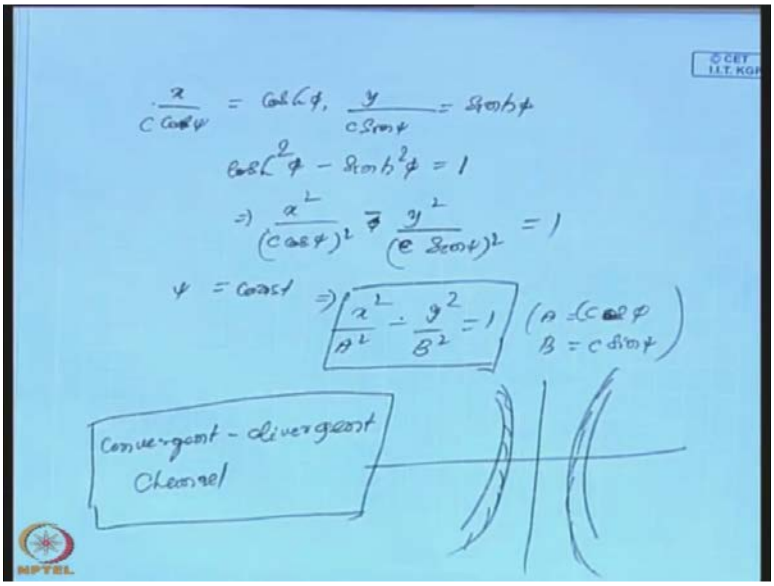

My x by c x by cos psi is equal to cos hyperbolic phi and y by c sin psi as psi hyperbolic psi sin hyperbolic phi. And I will utilize the property cos hyperbolic square phi minus sin hyperbolic square phi equal to 1. Then this will give me x by c cos psi x square by c c cos psi square plus y square or rather minus y square by c sin psi square is equal to 1. Then I put psi is equal to constant if psi is equal to constant that means. And then I can call it x square by a square minus y square by b square equal to 1 where a is c cos psi cos psi and b is equal to c sin psi. Now, what thus this represents? These are the streamlines for each constant c will get a streamline. And this streamlines are nothing but hyperbolas.

So, I can call this that is why it is a flow called as a converging convergent divergent channel. I can always fix my constant and I can use there is no flow along this. And then we use this as a channel, because this path I can always use this as a channel. And because of this often this we call this as a flow in a convergent divergent channel I have a small error here in my previous. So, if I go back to this one there is a x square and there is a y square this is the one of the error is here. So, this can be easily corrected, thank you.

So, this is a flow in a convergent divergent channel, see this 2 examples in fact when in olden days when there was no way to calculate 100s of years back when the potential theory was being developed. And this complex function theory was used for analyzing various flow problems, in fact this 2 problems as given us a very made in environment to find a particular to develop a navigation channel. People used to spend, because the how to understand the flow characteristics, because once the understanding the streamlines are clear and one could understand one we had a good understanding that there is no flow across a streamline. And that concept was used in many physical problems particularly to understand the flow characteristics or if there is a streamline.

In fact the streamline bodies like if you look at air craft or a ship they are all they are all streamline bodies and because if the becomes a streamline. But there is no fluid which will cross the streamline and in the process, we will have less load. Because the, it will be flowing as if it is flowing along with the fluid and there is a it will behave as if there is solid line. If you just because along the streamline a streamlines we can consider this as a solid line as if there is no flow which will cross this. And this concept was in fact, it has helped in the design of several marine vehicles and also ray burn vehicles.

And even if today, we always design a ship or a air craft all this structures are always streamline bodies. So, the understanding of stream lines is very, very important when you look into problems related to aerodynamics or problems related to marine hydrodynamics. Now, with this understanding I have to go to two problems as I have told that what will happen I have already talked about source, I have already talked about the uniform flow and the flow past a cylinder. Now, what will happen? Because what will happen if there is a singularity in a flow particularly.

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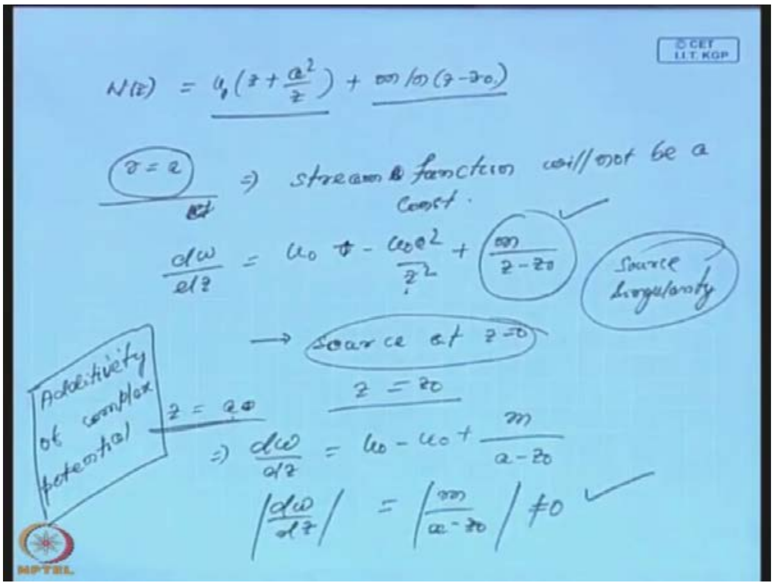

Let us look at W z which is of this wall u into z plus a square by z plus m log 7 z naught. Because now, if you look at this part this is the uniform flow past a cylinder of radius a and the speed of the uniform flow. The uniform flow of the speed is u naught whereas, this is the source of strength n look at a z is equal to z naught. Now, what will happen whether this flow will represent when you add this whether all the three characteristics will remain or not? Now, if I look at if it has to be suppose what will happen now z is equal to r is equal to a at r is equal to a this W z will not be a constant. Then what will happen to the stream at r is equal to a? It can be easily seen that the stream stream function will not be a constant. Then if you look at d w by d z and d w by d z will give us u 0 plus or minus u 0 a square by z plus m by z minus z naught.

So, when z is equal to 0 at z is equal to z naught, this will represent as source on the other hand at z is equal to 0, w by e e square z square at the same time when z is equal to 0 this will tend to because z is 0 means this term will tend to infinity. And then this will not represent a flow source z will be at names like a source and z is equal to 0. And that of course, at z is equal to z naught this will again behave like a source. On the other hand if I put z is equal to a e to the power i theta z equals to a and z is equal to d w by d z u naught minus, this will give me minus u naught plus m by a minus z naught. So, this is because it has to a, then what will happen to d w by d z mod? We have a cylinder of radius a, because the speed will be 0 well at z equal to a. But in this case d w by d z modulus will be m by a minus z naught and this is not equal to 0.

So, that source that this will not represent the individual flow characteristics, we suggest that because I have 2 things here I have a source which behaves like a singularity. So, if you have a, we have a singularity in the flow and we have a boundary. Then the additive characteristics are the, I will say the additive property of the additivety of complex velocity potential will not complex, potential will not work out. That means if I have 2 different flows I cannot say that whether the combined flow will represent the, by individual flow patterns together. However, there are examples where you can say that this additivety will hold good. But this is an example, where this additivety characteristics will not hold good that is what I want to say. Now I will take another example suppose.

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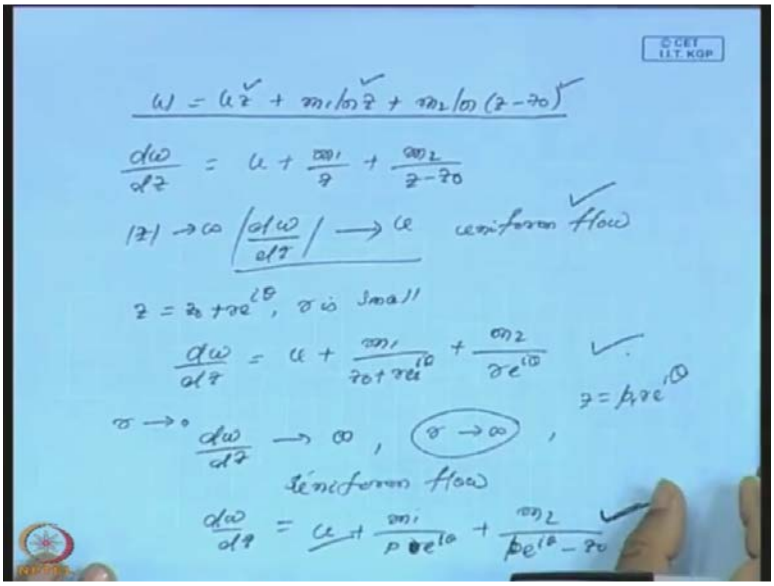

I take w is equal to u z plus m 1 log z plus m 2 log z minus m naught here there is no cylinder, because as if I have sources this is one source; this is one source; this is one source then what does it represent? So, we can always see if I say my d w by d z, this will give me u plus m 1 by z plus m 2 by z minus z naught as mod z tends to infinity as mod z will turn to infinity d w by d z mod will tend to u. So, that is the uniform flow it will represent in uniform flow, because at infinity in the flow again what will happen if I take if I put z is equal to z naught plus r a to the power of i theta? Then where r is a small then I will get my d w by d z will give me u plus m 1 by z naught plus r e to the power i theta plus m 2 by r e to the power i theta.

And again if I take r tends to 0 that means my d w by d z, because this will be behaves like a source this point only d w, because this r tends to 0 means this will tend to r tends to 0. So, this will turn to infinity so d w by d z, once this is tends to infinity; this is also tends to infinity again as r tends to infinity as tends to infinity. And this term will tend to 0 and this term also tend to 0 and in the process I will have d w by d z again will be u. So, I will have a uniform flow in a similar way I can also take suppose I take z is equal to p naught r a to the power i theta. That I can easily say that I will my d w by d z again will be u plus m 1 by p r e to the power i theta plus m 2 by z e e to the power i theta are the p this is a z is p into not r minus z naught.

And here again when p is p is large p is large means again these 2 terms will tend to 0 and will have a uniform speed. Whereas, when p tends to 0, this will give us infinity and again this will represent a the motion of a source of strength m 1. So, in the three, in fact, we analyze the 3 cases; the first case, we have seen that we can have a uniform flow. In the second class, we have seen that the characteristics of a source are of strength m 2 located at z is equal to z naught this characteristics also remains. And then the fact is we have taken as if the source of strength m 1 is located at z is equal to 0 and the 3 characteristics of the flow they arrange that the. So, the addition of the 3 flows does not affect the flow characteristics.

So, in this case we can always say that the flow exist. On the other hand in the previous examples, we have seen because we have introduced a cylinder. And because of introducing the cylinder, we have introduced a new one ready to the flow. And because of that the additive property was it was impossible to ensure that additive characteristics hold good. So, this is a there are so in general we can say that particularly when there is a, we all know that when there is no source and singularity no source are sink in a flow. The flow will be regular, but the moment we have a flow. And if there are source and sink without any other boundaries then we can say that any number of source we can add to the.

And it will remain the characteristics of the source will remain and we can individual flow characteristics will remain. On the other hand if we introduce a boundary then there is a chance that the flow characteristics will be disturbed and this is a. And all these things are very clear from these two dimensional analysis of the flows. And again and again the complicity the complicity of the flow pattern is has been simplified as there is simplified by introducing these complex function theory particularly the complex velocity potential w.

So, in the next class, we will again introduce some of the more complex problems and understand how this complex function theory is helping us in serving more problems. Apart from that we will then go to the, conform on mapping which is one of the very important mapping particularly, function functionally relationship which will transform from a very. Suppose, we are in a very complex plan from a very complex plane we can always transfer this to a simple plane. And then analyze the flow in the simple plane and again go back to our original plane. So, where that conformal that will come help us in solving again a class of problem.

So, basically we can several two dimensional problems we can analyze by using the complex function theory some of its application. And in the next couple of classes, I will we will spend full time on this complex function theory. And analyze various flow patterns particularly, we will emphasize on two dimensional flow assuming that the similarity remains flow characteristics will not be disturbed by such assumptions. And it has been also been verified from model test and even if observations that yes we can assume for a last class of problem, the two dimensional assumption is a valid assumption. With this I will remain today.

Thank you.