Marine Hydrodynamics
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Lecture - 7
Application of Equation of Motion (contd.)

Up to this class on marine hydrodynamics. In the last class, we have talked about the some of the application of the Kelvin's, particularly we have talked several application of the equation of the motion, and today again we will continue the same. now before going to going, let me just point out few of the, few characteristics of the irrotational motion, and these are very essential, because one of the basic advantage, or basic advantage that in case of irrotational motion outflow is. In the fluid region the fluid satisfy the Laplace equation phi is Laplace equation, because of the flow because of the potential type. Because of the flow is a potential type, almost all characteristics potential function, velocity potential phi is satisfying the fluid region, and that is one of the advantage. And here, I will just mention few of them, in the context of the fluid flip problems. So the first one in this region will be.

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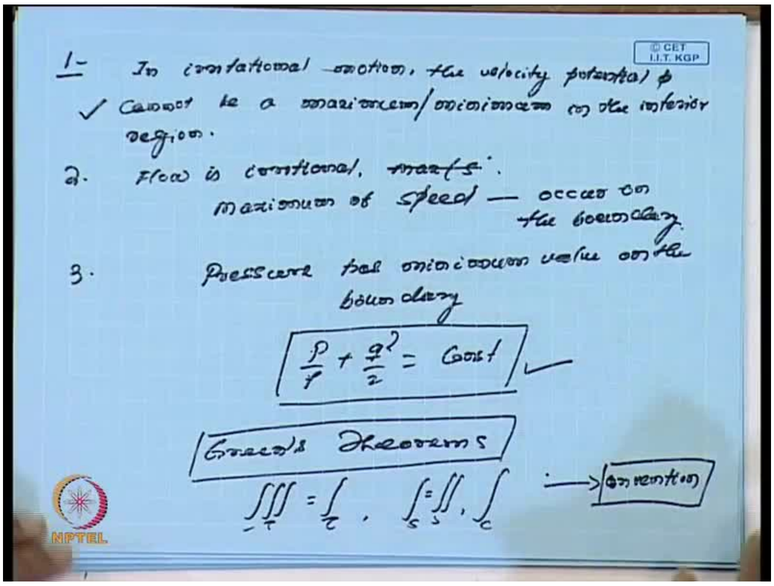

In irrotational motion, the velocity potential phi, cannot be a maximum, or a minimum. It cannot act an, cannot be a maximum or minimum in the interior region. The second one, when the motion itself when flow is a rotational, now a large in the contest of fluid flow problem. Inflow is a rotational, pruproline in close to durational the maximum value of speed, maximum of speed, or I will say of speed, maximum value speed. It has to be occur notstribide will occur on the boundary. on the third one I will say, the pressure, has its minimum value; of course, this is a, if one is two, because we know that if we say the body forces are neglected, and p by rho plus q square by 2 is equal to constant. The motion is rotational, and jested motion. so if we say that the, speed is maximum, because p by rho plus q square by 2 is constant, in speed is if in this case speed is maximum, automatically pressure has to be minimum. So this is obvious from here, but, I leave it to you. We will confined this in most of the text book, on potential role, or even if basic differential equation, partial differential equation books, where we will talk about velocity potential, these two results are very. and this is also a classical result in the theory of Laplace equation; that when the boundary is the maximum minimum value of the potential value of a, potential function, will only occur on the boundary, it cannot tab, and in fact, the flow singular if anything is there, that will be only on the boundary also.

It cannot be in the interior of this, this is a very, because one of the nice characteristic, nice, nicety of the Laplace equation, is that, it is a functional is differential function, and continuous function. So inside the domain will not have any difficulty, because the flow is continuous. The velocity potential is continuously differentiable. On the other hand of the problem comes, when it comes to the boundary, because it carries any similarity, in the flow that has to be always on the boundary, so that is one of the. So always in many situations, we always apply this, when it comes to flow, particularly potential flow problems. many times you apply lot of, many characteristics of this line integral, surface integral, volume integral, because the greens theorem ,which is famous integration theorem on greens theorem on converting line integral to surface integral, are surface integral, line integral to surface integral or surface integral to volume integral,

are based on greens theorem there in the Lapalace equation there is a very important role it is a main thing in the Lapalace equation in this case so for potential flow purulence weather it is a today we are looking at marine hydrodynamics problem when you feel where potential function when is it potential comes into picture the green employ is very major role in fact it was told that perhaps before working on physical scientist whether it is a look when it waves are in a continue mechanism one cannot ignore the name of the greens because of his theorem and because every now and then one utilize moment one look into the problem in potential clue or associated to flux execution when this theorem on us to apply and the in fact in the marine hydrodynamics field last class of prevalence where would depend we assume the close potential.

There occurrence may apply often to convert the a from a complex to when a three dimensional domain to two dimensional domain and again from a two dimensional domain surface to a one dimensional line and which makes our life easily because computers not we use save a lot of time and often this is one of the main result of greens theorem which we use in various problems of flue mechanism and mathematical physics so and I when point out here that, often I use the this symbol, these three symbols s and c. I always mean that when it is a over a volume, even if I put s into integral, this is equal to, actually it is the volume integral. See on the other hand, when I look at s I always mean s surface integral.

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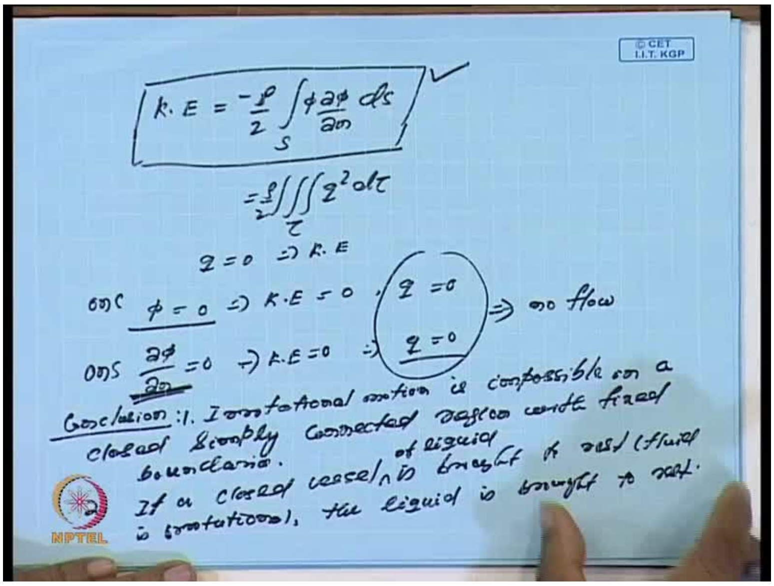

That means its double integral. Whereas, either put as a line integral r call it along a c. So this is a usual convention, that I am using in this lecture, which is often used in several books, convince, this convention I will be using. I am using on less other unless I specify, I will be using this three conventional. Although I use a similar integral, but, I mean here at triple integral. Here although I use a singular integral, I mean as a double integral, because it is a surface integral, and same thing happened in case of a line integral. So to this understanding, now I will go back to the potential energy; particular energy that is. in the last lesson we were talking about definition of the kinetic energy, and in the definition I have already told you, that the kinetic energy k associate to the potential flow is given by rho by 2 integral over s pi del by pi by del and d s. this is the definition of kinetic energy. Now this I will go to Kelvin's result, which is known as. Some more results on the this. Now what will happen, this is my basic definition of kinetic energy. What will happen in these, if this is also can be written as rho by 2, this is q square deta. Now if I say q is 0. If q is 0; that means kinetic energy is 0.

For the, which I will say, if pi 0, then also kinetic energy is 0, and kinetic energy 0 means; that means q is also 0. So there is no flow, and that to where pi is 0, pi is 0 on the boundary. Similarly, if I said del pi del n is 0. If I say on s, on s if del pi by del n is 0. Then may kinetic energy is also 0, which implies by q is 0 also, that means there is no flow, is implies there is no flow, but, you conclude irrotational motion conclusion. Irrotational motion is impossible in a close simple connected region, with fixed boundary. The second conclusion we say, this is one; if a closed version is brought to rest, and of course, we have seen the, vessel is half fluid and fluid is irrotational, then also I will say they are liquid, if a closed vessel up, rather I will say of liquid. A closed vessel of liquid is brought to rest as single fluid is irrotational, then the liquid is also brought to rest. So that means if the, when I say this becomes on fix boundary, the boundary is bromine, then I say del pi by del n boundary is not moving, it is a fix boundary. So once the boundary is fixed, the velocity is fixed, happens a fluid is brought to rest, and that is only possible, only when the close irrotational, so these are two conclusion; that is obvious for this definition of the kinetic energy, or a irrotational fluid flow. Now I will go to Kelvin's minimum energy theorem.

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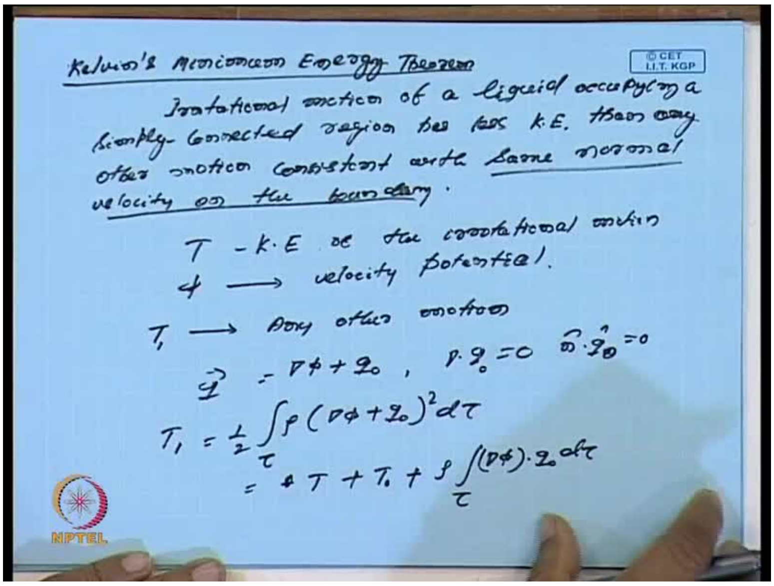

Let us start with Kelvin's minimum energy theorm. It says that the irrotational motion of liquid, irrotational motion of a liquid occupying a simple connected region, has less kinetic energy than any other motion consistent with the, with same normal velocity on the boundary, so this is. And we say that, suppose let it T with the kinetic energy of the associated with the, more operational rotational kinetic energy of the, irrotational motion of a fluid. Let phi be the corresponding velocity potential and let T 1 be the kinetic energy associate to its any other motion. Then what will happen, my q bar will be in the first pending velocity q bar will be, del phi plus q naught, where del dot q naught is 0, because q is incompressible and n hat that q n, because this is also 0, because it say that any of the motion constant to this same normal velocity on the boundary. So q n that is q 0 this is 0, than what will happen to t one kinetic energy associated with these, this will be half integral labor of tow in to dot phi plus q naught square theta. If I expand this, I can get half, I can get this gives me, the kinetic energy associated with, emesis potential phi; that is t plus double of potential associated to dethain the velocity t naught; that is t naught plus what it will give me, plus I will have rho integral labor of tow grid pi dot q naught into q bar deta.

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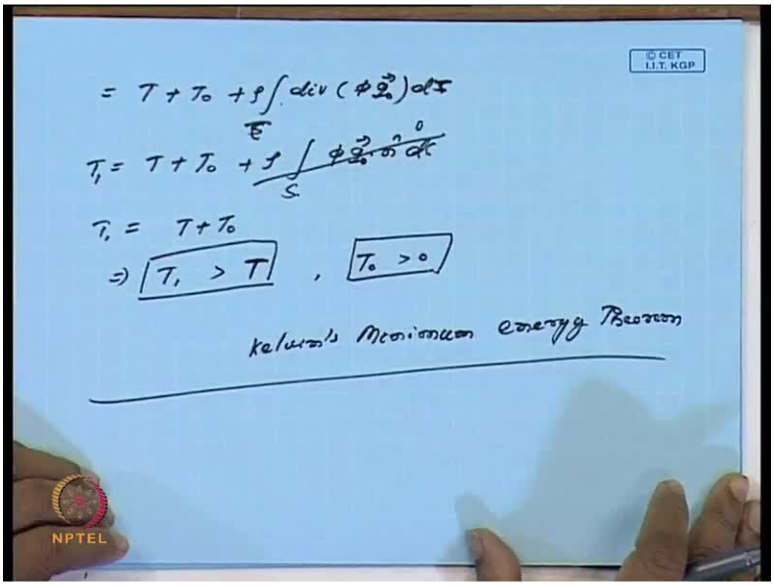

And just now I have used the result, is related to the velocity potential, may be used an identity. The identity says, if I utilize this results, which I can always write related to q bar dot pi, dot pi is a the velocity potential. You can always write this is equal to t plus t naught plus integral of power of s rho times divergent of pi of q bar d s. sorry This is tou, this is a deta. And this one again if I use it, then I always write it a t plus t naught plus rho. This one I can always wrote as s into pi q 0 bar dot n hat d s d s is a, and this will give me. So this is my t one; that is kinetic energy of any other motion. So this is a quantity, if this is a surface then q bar dot n that is 0, this quantity will give me 0, and that will give me t plus t naught.

So which implies t one is, because is greater than t. Let us for the t naught is greater than 0, because that is a flow possible kinetic energy has to be, greater than 0, so t 1 is greater than t, this is what. I mean if I have any other motion, the same region having same normal velocity on the boundary, than the t 1 will, t will have the less kinetic energy then the motion, that is exist in the fluid. So this is what the, what I say this is called Kelvin's minimum energy theorem. So this is some of the one of the very important characteristics of the kinetic energy. In fact, this definition of kinetic energy, and this characteristic we use, when it comes to what our problem, particularly up to always we comes, it comes to calculate the energy associated with the flow. When it comes to the energy, basically we have two energy, basically potential energy and kinetic energy, so in that case we use this definition of, because our in what ways most of the time we were in the flow is in compressively irrotational, so use the definition of kinetic energy, in terms of the velocity potential. And so it will come to this later, in a big way when you come to what were problems. Now with this background, I will have, I will come to very classical problem, and that is propastic circular cylinder, and while from this pluplast circular cylinder.

I will come to deal mores paradox; that means you have a fluid, when the flow is in compressible, and flow is in compressible and we say, then how you have the drag force acting on a body 0, and this was. This remains as a paradox for a long time, until the theory of viscous flow was well developed, and this remain as a paradox for a long time; that is known as a dealomus paradox, because you trust. The paradox says that the in a uniform flow if a cylinder flow fast a cylinder, than the total drag force is 0, and this was a pardox for a long time. Let us come to see that how the, when you have. Will consider a new uniform flow fast as cylinder, and then we will derive that how the drag force, the drag force will be 0.

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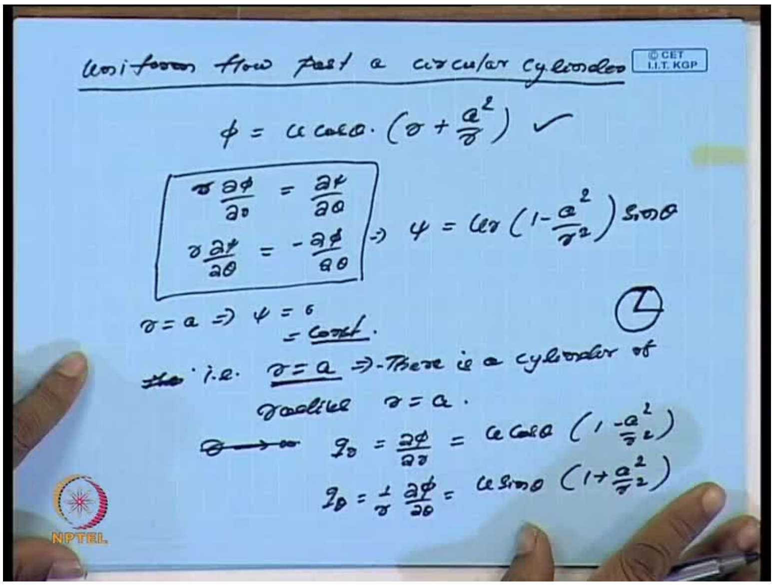

Now let us come to that uniform flow past a circular cylinder. This is where one of the interesting problem, which is many times has a very important role, when it is. Suppose I consider the velocity potential pi is equal to u cos theta into r plus a square by r. Now if I apply question, and suppose we have given this pi, but, what will happen to this flow here, if I have this is the pi. Then we have to find what is string function, and to find the string function, we have to go to the (( )) equation. Particularly in the cylindrical polar could net. In one of my past lectures, I have already told you, what exactly the relations are del pi by del r is equal to del si by del theta, and r del by del si by del theta is equal to minus del phi by del theta. So if we know pi we can always get si. So by using this characteristics, we can always react this, we will get our si as. I am not going to the detail, this is u r 1 minus a square by r square sin theta, even if I have this, you can get. Now what will happen, where r is the, where cylinder. This is the radius of the circle, and uniform cylinder, which is along this z axis, whose axis is along the z axis. Now if si is this what will happen r is equal to a. if r equal to a pi is equal to 0, which I sa constant this means the renapins is means that is r is equal to a if either radius of this circular cylinder, then what r is equal to this is 0, means this will be these are all it convey, because si is equal to constant, and this there is no flow after this, so you can always consider. There is a cylinder, the flow, there is a cylinder of the radius r is equal to a. Now what will happen when r tends to infinity. If r tends to infinity, then what will happen to by flow. What will happen to del si by, I will come to this. Now what will happen. If we are going to this, let us calculate what is q r velocity the radial lines. So q r is a del phi by del r and phi by del r. This is u co theta 1 minus a square by r square, and what is q theta that tangent velocity; that is 1 by r del del pi by del theta and that is again u sin theta 1 plus a square by r square. So this itself shows that when altering to infinity, this might will be 0, this will be 0, and then you have q r is cos theta q theta is u sin theta and r is tending to infinity.

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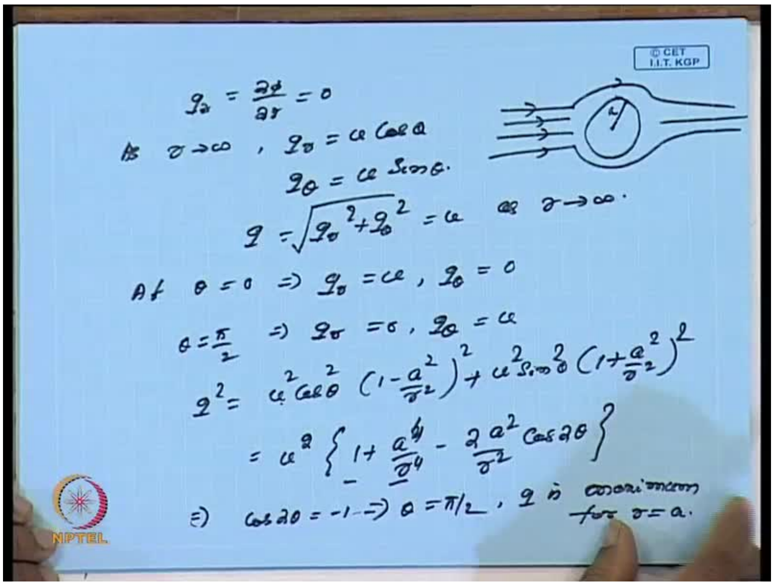

Now what will happen to q z again here, because I am considering two tends flow, so it will be del pi by del r that is 0 here. So as r tends to infinity, we have q r is u cos theta and q theta is u sin theta. Then what will happen to q q r square plus q theta square root over that gives us u as r tends to infinity. Now further I will just look at two things, what will happen if theta is equal to. And here all this say consider when you have a cylinder of radius a. Now at theta is equal to 0, what happen, you have q r equal to u and q theta equal to 0. And on the other end theta is equal to pi by two implies q r is 0 q theta theta is pi by t so q r is 0 q theta is u. Now what will happen to q square, because I am in given that general definition of q; that is u cos theta 1 minus a square by r square; that is your q r square plus you have u square sin square theta 1 plus a square by r square. This is my q r, this is my q theta square, this is q r square and that gives me.

This will give me, if you expand it, this will give us u square 1 plus, because sin square theta plus cos square theta and that will give 1 plus a forth by r forth minus 2 a square by r square cos 2 theta, because you will see that u square u square sin square theta plus four square theta is 1. Then you have to look at these term a forth by r forth u square cos square theta, this is a forth pi of u square sin square theta, so that will give you this one, and then where you get minus 2 u r 2 u square a square 1 by r square. Here it will give you cos square theta, here it will give you sin square theta, so cos square theta sin square theta minus cos square theta, and that will give you minus cos 2 theta. So if this is the case when we got q square is this, and it gives us q square is. What will happen when it is a general expression of q theta, so it gives when theta is a total f 1 to cos theta minus 1. If cos theta if cos theta 2 theta is minus 1 which implies theta is equal to pi by 2 and q square will be in that case theta is pi by 2 and this pi by 2 theta is minus 1. So in that case, this will be minus minus plus ,so you have q is maximum. So here r is equal to a, maximum for r is equal to a. So what theta is equal to pi by 2 r is equal to a, the speed becomes maximum. So basically this flow will be looking like this.

You have a plain flow, you have a circular cylinder like this. This is cylinder of circle radius a, this is r region; that is fluid is flowing, this is a digit cylinder is kept here, and in that case what will happen. Now we have already seen that, u square we have seen that q square is 2 u square to u square.

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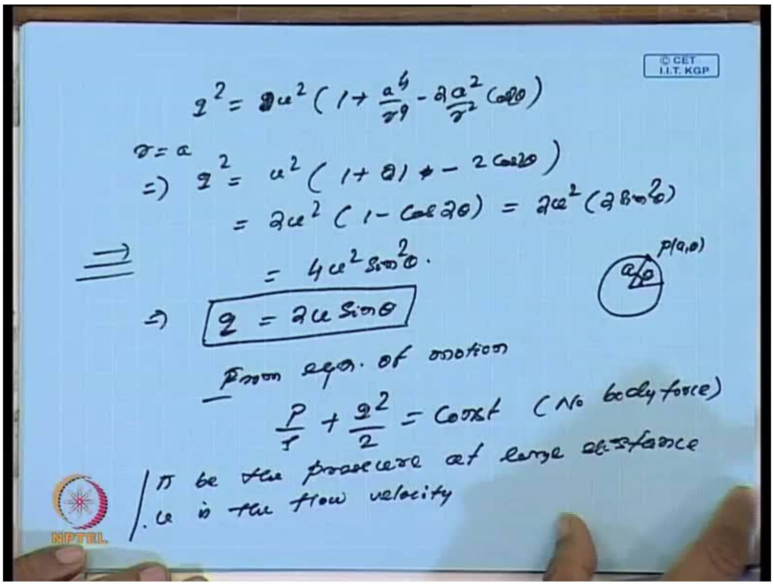

For other I will say u square into 1 plus a forth by r forth minus 2 a square by r square cos theta, and what will happen when r is equal to a, if r is equal to a that will be q square will be u square into 1 plus r is equal to a means on the surface of the cylinder. This is 2 and this is again 1 plus minus we have 2 cos theta 2 cos 2 theta, actually cos 2 theta. So this is 2 u square 1 minus square cos 2 theta 2 u square to 2 sin square theta, which is a minus 4 u square sin square theta, which implies me q will be 2 u sin theta. And if in the cylinder, if a is the radius of the cylinder, and theta is the angel, at any point on the cylinder at any point on the cylinder, p is any point on the cylinder, and that is a theta, then my speed is given by this, the flow. If I am to conclude the pressure than I have to apply, so this is the speed I got, I got the flow characteristics by calculate the pressure at any point, I need to apply I. Seems body processor neglected we have to memories it from equation motion.

Here p square by p pi over plus q square by 2 is equal to constant, where a no body force, no body force is negated then this will be. Now if say that, when the flow is uniform flow, which is flowing to infinity. If I say pi is the pi be the pressure at infinity, pressure at large distance in the horizontal lines, large distance, when that we say, because the flow is uniform. So let me say that already I know that u is the flow velocity.

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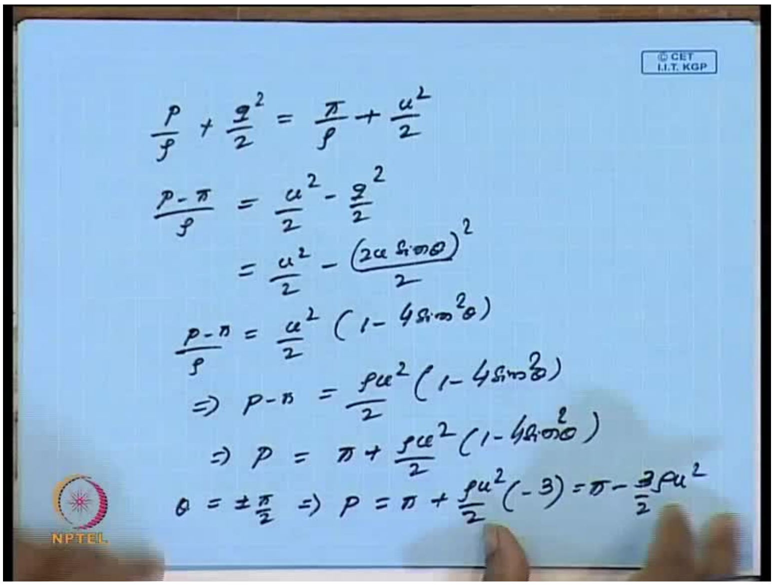

If you use the flow velocity, then I will have p by a at any point on this in the p by rho, if I want to conclude the flow plus q square by 2 is equal to pi by rho, because I just equate the flow on the cylinder to any point u square by 2, and already I know the cylinder, so my p by rho, or rather p minus pi by rho is equal to u square by 2 minus q square by 2. And on the cylinder when they circle, on the surface of the cylinder my q is 2 u sin theta. So I have u square by 2 minus q square that is 2 u sin theta square by 2 and that will give me. So who gives me u square by 2 1 minus 4 sin square theta; that is my p minus pi by rho, which implies p minus pi is rho u square by 2 into 1 minus 4 sin square theta.

This is. So this p is the pressure at any point on the cylinder, pi is the pressure at infinity fluid pressure, and this is u is the speed at infinity. So which implies my p is pi plus rho u square by 2 to 1 minus 4 sin theta sin square theta. Now what will happen when theta is plus minus pi by 2, theta is plus minus pi by 2, if theta is plus minus pi by 2 sin square theta will give me 1. It implies my p will be pi minus pi plus rho u square by 2 into minus, this is minus 3 which is pi minus 3 by 2 rho u square.

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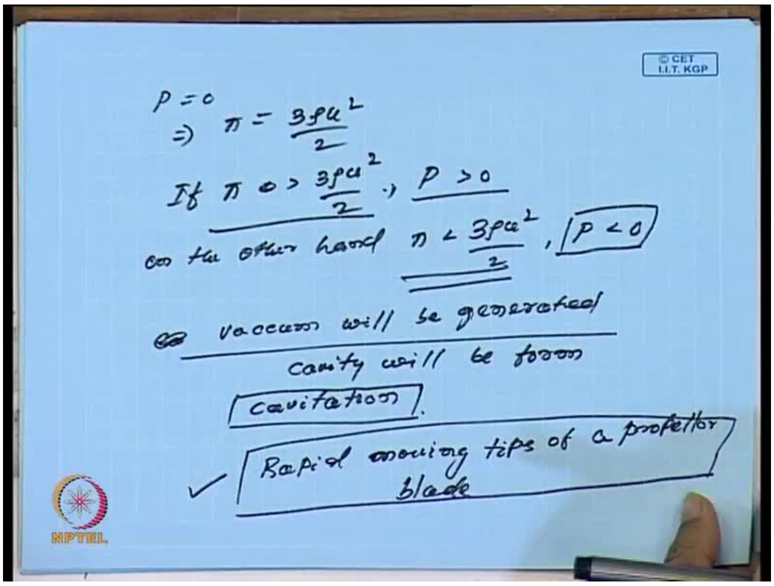

So what will happen, so that means is the pressure has to be positive, the 0 pressure, when p is 0, p is 0 means pi equal to 3 rho u square by 2. If pi is less than. So infigual pi has to be greater than 3 rho u square by 2. If this is greater than 0, this is greater than this, this holds then only my p is greater than 0, on the surface of the cylinder. On the other hand, if pi is less than 3 rho u square by 2 then p will in negative. So that means, under this certain condition p becomes negative, and in that situation cavity, because pressure becomes negative, and then what will happen, vacuum will be generated, and pressure comes negative. Vacuum will be generated, and there not be any fluid there and then. And once vacuum is generated, that will form a cavity on the boundary to the surface, the cavity will be formed. This process, this process I want to call this as cavitation. When there is negative pressure, cavity will form and this process we call this as cavity. In fact this is a very. This problem occurs in the rapid moving tips, this is a, this cavitation the formation of the cavity is very common, in case of rapid moving aircraft, tips of a propeller blade, how much often taken place in this situation. Now is this understanding on cavity, now we know already the pressure. If we know the pressure, than what you can do you can calculate always the force, because when I have derived this equation of motion, I have told that from pressure you can always calculate the force. Now what will be the forces acting on this cylinder.

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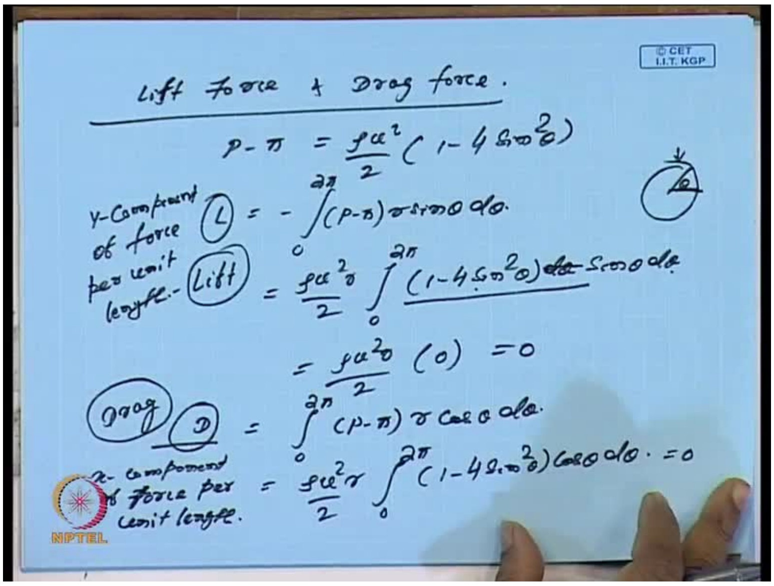

That means you have two process one is lift force, and another one is drag force, when have to calculate the lift force. So we will calculate the lift and drag force. We all know by p minus pi is equal to rho u square by 2 1 minus 4 sin square theta; that is on the cylinder. The pressure on the cylinder, where use the velocity at infinity and pi is the pressure at infinity. Now what will l, l will happen minus 0 to 2 pi, because we have calculating in the circular boundary of the cylinder, so this start from theta theta can vary from 0 to 2 pi and then this p minus pi r sin theta d theta, and then that will give us a negative sign wave taking, because we are the lift is always in the, we are calculating the force on this direction, because otherwise it in the after direction, so that is why the negative sign come. So that will give us rho u square r by 2 into 0 to 2 pi 1 minus 4 sin square theta d theta. And in fact if you calculate this, you can find there is a r sin theta, sorry sin theta d theta 1 minus 4 sin square theta into r is gone sin theta d theta. So this can easily seen in that 1 minus 4 sin square theta into sin theta d theta into 2 pi can easily be seen that this is 0, so the lift force is there.

In a similar manner this is the force that is in the multiple relation. in a similar will calculate drag force; that is d then it will be 0 to 2 pi p minus pi that is r cos theta d theta, and that will again give you us rho u square by 2 r 0 to 2 pi 1 minus 4 sin square theta into cos square theta d theta, and again it can be seen that this integral is 0, it gives the total. So we have seen that both the x component of the post for nuactlet, is the x component of the force, force per unit length, and this force is called the drag force, and this is the y component of the force, and this is called the lift force. This is the lift this is the drag. In fact we have found, here both the lift and the drag force, both are becoming 0, and once these are 0, that means, the lift and the drag force both are 0, and which is quit contradictory, because the it has been observed, that the lift and drag force, the lift and drag when there is a fluid is flowing, and there is a cylinder, and then we are finding that the lift force is, the drag force is 0, which is very contradictory, but, contradictory in the since that how can, because there is a procedure acting on a body, where as the total force is becoming 0, so this was initially predicted by D Alembert.

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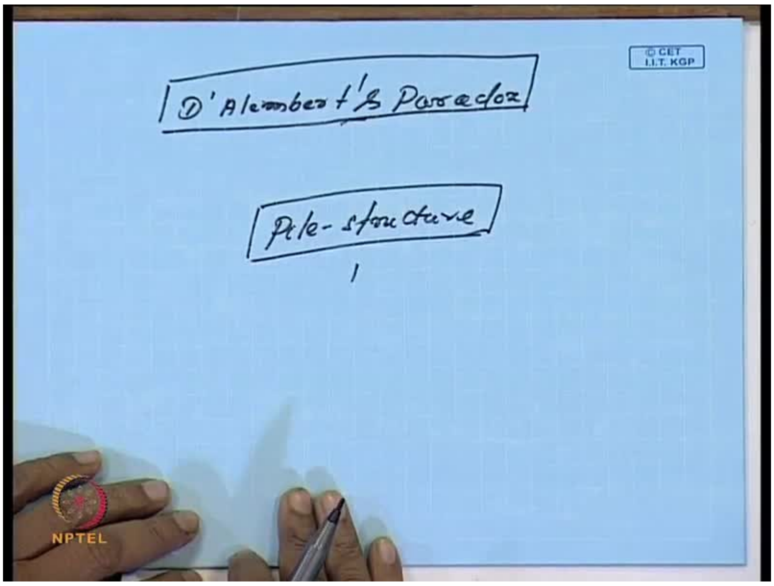

When we have a flow in uniform stream, when a body is in a uniform stream, the total drag force particular on the body is some must in the uniform stream, the total drag force is 0, and that prediction is a paradox, and that is known as D Alembert paradox. In fact this, remains, because in the early days potential flow theory, was so well developed, and that time discussed major problems of application, where based on the potential flow theory, assuming the flow is irrotational, and but, this phalasi, because our predication, and but, at the same time, the experimental or model test results. They varies that here that is showing, even if one can say from the once intuition also, that how can the force will be 0 when we have fluid is acting on a body and you said the force is 0. So that paradox remains, until the theory of our miscues fluid ,was well developed, and after that it was proved that it was because the viscosity is neglected, and in this case viscous force has neglected, because of that it was showing the drag force is 0, and the lift force is 0.

And on the other hand, if you include the; that is one of the criteria I will say that, where the irrotational flow has failed here, to predict the total force on a body. So that is your, but, this remains as a paradox, and this gives a good understanding about the irrotational flow, will not hold good for a all kinds of problems.

Of course, this irrotational flow, when the words for a last class of the problem in fluid mechanics, and other branches. In fact when we come to look at the motion of c p in the water, or look at the, an airplane moving in the sky. In both the cases just surrounding the body, the forces are considered as viscose, and there we talked about the theory of founder, but, away from the body, the forces are again considered potential type, because when the space is large, from particularly the ocean away from the body, particularly the ship, the flow is taken as irrotational. Similarly, in case of aircraft, away from the aircraft the flow is considered as irrotational in nature. Particularly potential flow theory works very well, and similarly, when you look at the, force calculation on a pile type of a structure, in option engineering. If pile structure, here the drag force calculation on the lift force, we calculate that drag portion on the lift force, where the viscosity plays a significant role.

On the other hand, when you look at the calculation of forces, on a large of the structure, then the drag and lift force does not play much so of a role. Sorry the drag force, basically the viscosity is negligible, and in that case we always have seen the flow is potential, so we will come to that afterwards. That today we have talked about some of the application of these equation motion, and then we will in the next few class we spend several cases of the two dimensional flow, and the reactions that we; that flow is irrotational in which, and we will try to understand several flow characteristics. We discuss several flows, and understand there flow characteristics, that will be that will be helpful in understanding last class of flow, and there assumption of the inflow is a irrotational and fluid is in compressive, and in we seen. So today we will stop here, and we will see in the next class.

Thank you all.