Marine Hydrodynamics

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Lecture - 6
Applications of Equitation of Motion

Welcome here, to the sixth lecture in the series today. And today, we will concentrate on application of the equation of motion. And last class, we have already talked about the Euler equation of motion. And then we have assuming the potential type we derived the Bernoulli's equation of motion then again in case of a steady, in the motion steady then we obtained the equation of motion a very compact form that is the pressure head plus velocity head plus the gravitational head is constant. So, if with this, I will today what we will concentrate that will show an example, we listed the application of this equation of motion.

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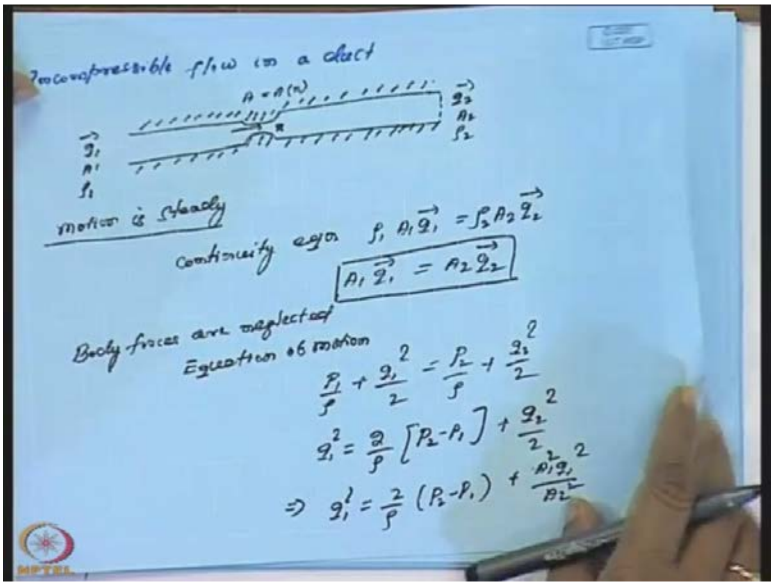

So, I will start with incompressible flow in a duct. So, let us consider a duct that is need not be uniform. So, let, let this side the velocity be q and r and the cross sectional area is A 1, let the density be rho 1. And here, let us say the, the outlet, this is velocity is q 2 bar, the cross sectional area is A 2, the rho 2 is the density. And this is the flow direction, this is along the x and here A is equal to it is only depending on x. That means, so we have the area, we discussed is and hear it is depending on the flow direction along the x axis.

Here, now this point at which again the cross sectional area is A 2 and velocity is q 2 density of the fluid is rho 2. Now, assuming that the flow motion is steady, if we say that motion is steady, hence the motion is steady. Since, it is where already, considering in this inviscid fluid and for a incompressible flow in the duct. We will have some continuity equation, you can write in a compact form that is rho 1 A 1 q 1 bar is rho 2 rho 2 q 2 bar, that is law of conservation of mass. And if I say fluid is homogeneous, I have to insist the density in remains the same then we have A 1 q 1 bar is equal to A 2 q 2 bar.

Further, if you go to assume, if you say body force are neglected then from equation of motion. You will have velocity head, pressure head will be constant equation of motion, because we have assumed the motion. So, we will have P 1 by rho plus q 1 square by 2 is equal to P 2 by rho plus q 2 square by 2. And which gives us q 1 square is equal to 2 by rho, if you do this, this is P 2 minus p 1 plus q 2 square by 2. Because, so which implies q 1 square equal to 2 by rho P 2 minus P 1 plus. Because we will utilize this relation, if you right q 1 square q 2 square is A 1 q 1 bar q 1 square divided by A 2, this is q 1 square. So, A 1 square A 2 square and that is q 1 square and now, once we know q 1 square then what will a that is simplification.

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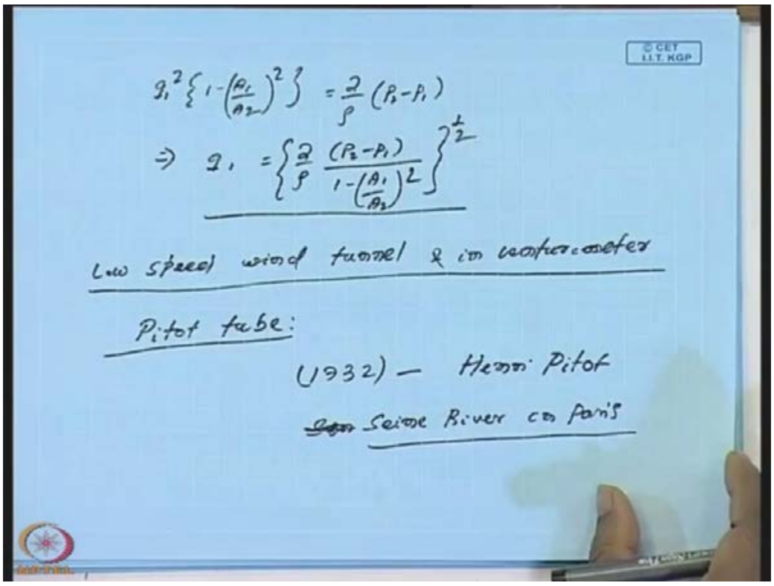

So, which gives us, which gives us q 1 square 1 minus A 1 by A 2, this is equal to 2 by rho P 2 minus P 1, which implies over q 1. The 2 by rho 2 P 2 minus P 1 divided by 1 minus A 1 by A 2 square 2 bar 1 by half. And this gives us; this gives us the flow velocity in the inlet. So, provided if we know the difference of pressure and we know the cross sectional area at the 2 ends then we can always find the inlet by velocity. So, this is what, this concept, this is the; this is the relation for the obtaining the getting, the flow velocity in a duct where the size particular. If the cross sectional area is changing and in fact of this concept; this concept is used in the low speed wind tunnel and in venture meter. In fact this venture meters, they are a devices which find application in many areas of engineering.

For example in the carburetor of a auto mobile engines, there is in venture meter throw which the incoming air is mixed with the fuel. Further in aero deveins, a venture meter is used to measure air speed fluid velocity in a pipe, also can be measured by using this venture meter. There is another classical example of this equation, in pitot tube. On this is again another devices, which is used to major the flow velocity in a stream had been if the to it is also use the flow velocity, a flight fell flight velocities is in the aircrafts. And in fact that this device fast was developed by in 19 1732 by internal one was Henn Pitot and he used this relation to obtain. In fact, he used this relation to measure the velocity of s e Seine river, river in Paris just to use the, this Pitot, he, he developed and he used it to measure the flow velocity of on the river of Seine in Paris.

And this, I am not going to the details of the venture meter and pitot tube. But the concept is here is that you have in giving, giving the difference of pressure, you know and if you have the cross sectional area you know then you can always find the flow. So, this is the way, the flow velocity is measured in tubes or in a stream. By using this concept the details of it, 531venture meter can be found in most of the text book of fluid mechanics. And going to that detail here, now let us as an example, I will show you how a flow velocity is measured. So, suppose I consider venture meter.

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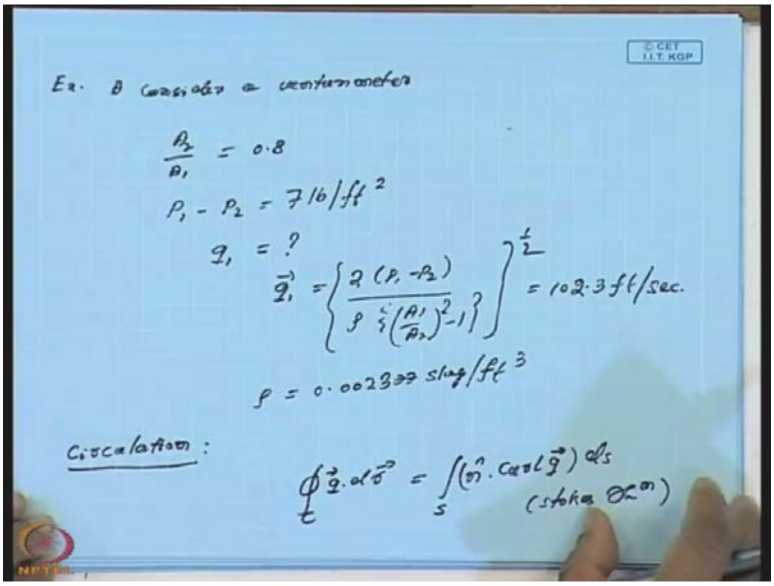

Here, the third example, consider venture meter, consider venture meter, where the throat to an inlet area; that means, A 2 by A 1. This is throat to inlet area is giving by 0.8 and it is mounted in a flow at standard sea level conditions. If the pressure dependence between the inlet and the throat; that means, P 1 minus P 2 that is the pressure, pressure difference between the inlet and the throat that is 7 pound per feet square. Now, what is q 1 then what is the flow velocity at the inlet. So, if you utilize this equation q 1 bar will be 2 into P 1 minus P 2 by rho into A 1, A 1 by A 2 square minus 1, 1 and half to do that then you can find it is substitute for A, A 2 by A 1. And P 1 minus P 2, then you can easily find that this is 1 0 2 1 0 2 pond of 3 feet per second. Of course, here we are using only of rho as 0.002377 slag per feat 3 with this rho, this is what the q 1 gives.

So, this is the way the flow velocity is measured in venture a meter. Now, I will before going proceeding, further I will make a note of some of the things in a rotational motion. So, that I will come a little later, but rather before going to the irrotational motion and are early some of the other application, I will talk about. So, I will be before going to the other application, basically application of equation motion in the conclusion of a circulation in a flow. As you know, by the definition of circulation is gamma is a curve. I have already mentioned, this if c is at most curve, if is c is a closed curve, then the circulation. And the, this curve mix with the fluid consisting of the same particle then the, the circulation around this closed curve c is moving with the fluid is giving by integral approver. A closed curve q bar dot d r bar this is what the circulation.

Hear q bar is the velocity victor and d r bar is the position vector. When a particle, particle in the curve c then if you apply stokes theorem to this, then you can always write this. We can write integral over s, because to the line integral, we can go to surface element integral which is n hat dot curl of q bar d s. So, this is a from a line integral, you go to the surface corresponding surface then that will gives us the, this is by the application of the stokes theorem. Of course, while deriving this, we assume that q r is a, the velocity vector q bar is differentiable at all points. So, with this now, this I will again, I have already again mentioned, what is a basically conservative vector field.

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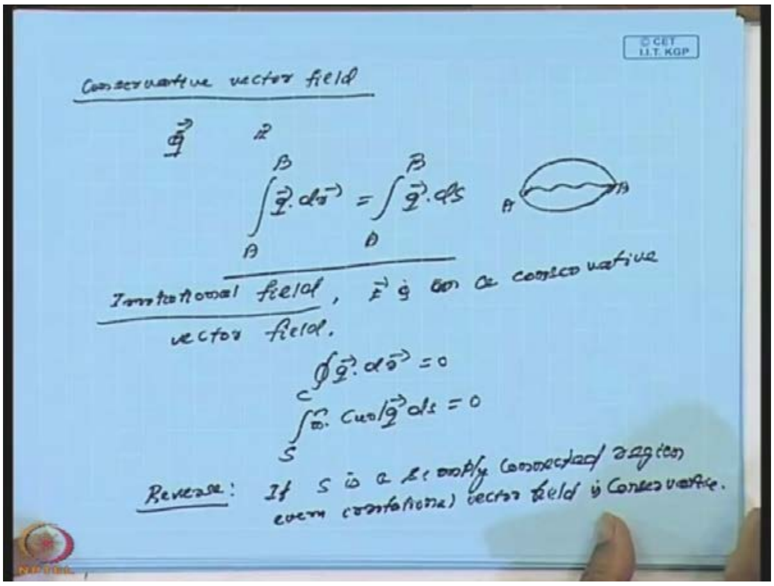

I have already told this conservative vector field, if you took any, let that be a uniform vector field. If F is a uniform vector field or F is a function dependent vector field within the region r then. And let if A and B are 2 points then keep the integral q bar dot d r bar A to B that is the path q bar dot d s. And if this integral is independent of path from A to B then suppose here, here. In fact, I am repeating this, because now, I will come to the particular problem. So, whether from any 2 points A to B, when it removes, if this integral is independent of path of integration then we call this vector field. I will rather call it as q bar is vector field. So, then it will say; will say that this is a conservative vector field that is another way of looking at the same thing. Now, with this; that means, if the vector field is not conservative then you may not have then what will happen then this relation may not hold good.

Now, I will come to a particular example, so if in a conservative vector field and often the conservative vector field are called the irrotational vector field. They are also called, so if the vector field is conservative. If F bar is in a conservative field then we can have, because already, we have q bar dot d r bar is equal to 0. Because q bar can be written a vector based conservative q bar can be written. This can also come, this will come from you can write always this is as S N hat curl of q bar and conservative vector field. So, curl of q bar will be 0 because it is Irrotational vector field. So, curl up the vector field will be 0, that is d s will be 0. Once this is 0, so that gives us q bar dot d r bar a 0 along curl c. Now, I will be proceed, you further the deport, I will just a mention 2 things.

So, as I have mentioned A B D irrotational vector field, every conservative vector field is a irrotational field. On the other hand, the rivers is not always to that is the rivers, either if you feel this irrotational can, I say this is a conservative field it is not always to river’s, if A is, if s is a simple connected region. Then only every rotational field is a conservative, every irrotational vector field is conservative. This also can be put in a different way like there is another way of telling same thing that if F is a uniform vector field throughout a region r then necessary. And sufficient condition for the vector field to be conservative in r is that suppose there exist a differential function. So, if you have a uniformly the differentiable function phi.

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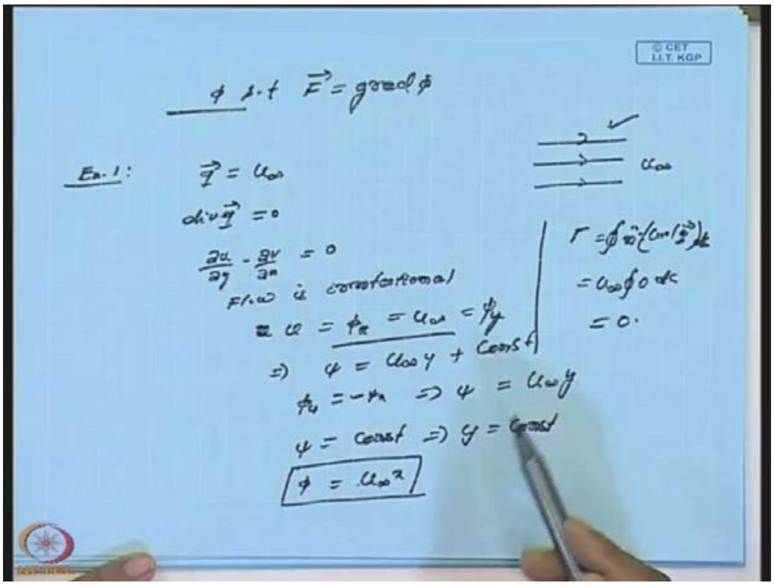

Such that F bar is equal to grad phi, so this is a scalar function. It is a scalar function, but it has to be uniformly differentiable. Now, I will give you an example of air conservative vector field, which is irrotational and vice versa. Let us consider 2 examples here, in the first cases, suppose I will consider the flow, if you are been giving q bar. Example 1; consider flow with a uniform velocity u infinity; u infinity. So make q bar is u infinity then what will happen to this is a constant. So my divergent of q bar divergent of q bar is 0. Since, divergent of q bar is 0, so the fluid is in compressible and a possible fluid flow is possible. Further, we can see that what will happen to del u by del y minus del v by del x. In this case, this is 0, because this is constant this is also, the flow is irrotational.

Now, the question come, if flow is irrotational then we can have a. So, what will be, because q bar is given by the, so you have q bar. So, your u velocity in the along the x axis, you use phi x and that is u which is same as psi y, where psi is the stream function. And that gives us psi is equal to u infinity y plus a constants this can be. And again, if you take phi y is minus psi x that gives us. So, substitute this, this constant will be 0 and that will give us psi is equal to u infinity y. And if psi is equal to constant and we say psi is equal to constant that gives us y is equal to constant. So, these lines are, this is the flow direction and these are the lines at the stream lines. So, this is the, this same from flow, now what will happen to gamma. In this case, if we look at gamma, gamma we can always right integral over C N hat curl of q bar d s and this, this can be written as. And again you have u infinity integral of, because curl of q bar is 0 and so this is 0, so that gives us 0.

And once this is 0, so because here the vector field here, this is a, the function curl of q for the functions. The velocity vector q is a differentiable function and the corresponding since, it differentiable. So, we have u is phi x, so which also gives also. In this case phi is equal to, if you look at this 1 5 is equal to u x, this is all differentiable function. So, this is a differential vector function, the scalar function. So, it is corresponding vector is gradient of phi that is gives us q. And in the process, we see that, because this is a rotational vector, well which is differentiable. So, we are, we are at the from the circulation definition, we say that this is circulation is 0 in a uniform flow. So, this is a very straight forward, now I will go to the example, where I will say that the flow can be irrotational. On the other hand, the circulation will not be 0 and that will. So, that will only when, when the function it is not the reason is not simple connected, so to do that suppose I consider a.

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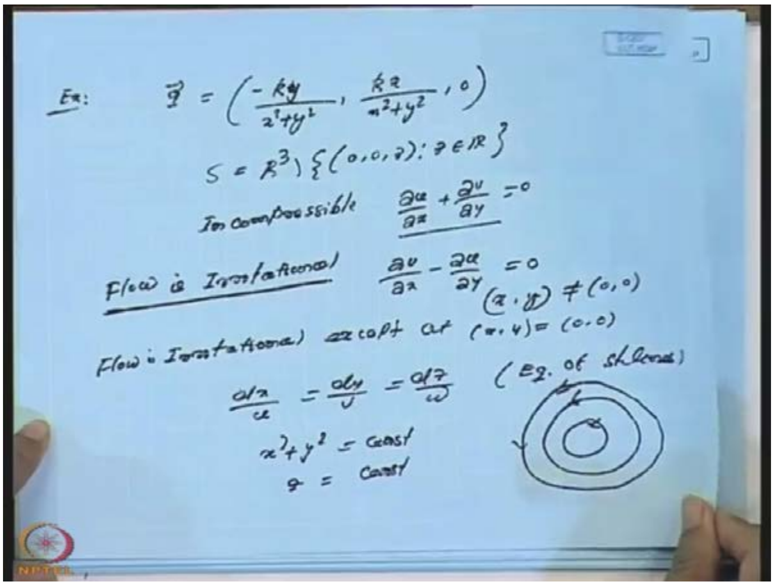

Suppose I take q bar as another example, this example is very interesting one, suppose my q bar is minus k x by x square plus y square and this is, this is k y and this is k x a x square plus y square this is 0. Now, this is let me say my, what will happen to this vector field. When this is in the design S i say that r cube deference 0, 0 z. So, that z belongs to r, now in this case we can easily say that the flow in compressible. So, fluid is in compressible, because you can see that del u by del x plus del v by del y e 0, this will give us. Now, on the other hand this is easily, it can verified, because we have done this example in fact.

Further, we can always see that this flow is irrotational flow. Here also, we can see flow is rotational this flow is irrotational, because we can easily see that del v by del x minus del u by del y and this is equal to 0. In fact this is 0, only for when x y is not equal to 0. Sorry, if x y, because this they are not equal to 0.00 and in that cases flow is rotational. Now; that means, the flow is irrotational except at rotational except at the 0.00 x y is equal to 0.00. Now, what will happen to the streamlines we have dx by u is equal to d y by v d z by w. We have the equation of the streamlines and that will give us, if you substrate for u v.

And all then, you will get in fact flowing a little fast, because I have already worked out this example. And z is equal to constant as I have seen that here the flow is they are all concentric circles center, at the origin all circles with center at the origin. Now, the question comes, but in this case received that this, the flow is irrotational motion is irrotational. But they follow path which is circular that means the practical they follow a circular path. But each particle the particles are not particle motion individual particles they are not they are not rotational. Whereas, the particles from new circular path the, is what we mean, now can we apply the here; we apply here the stokes theorem. No, because as I have said that in the region.

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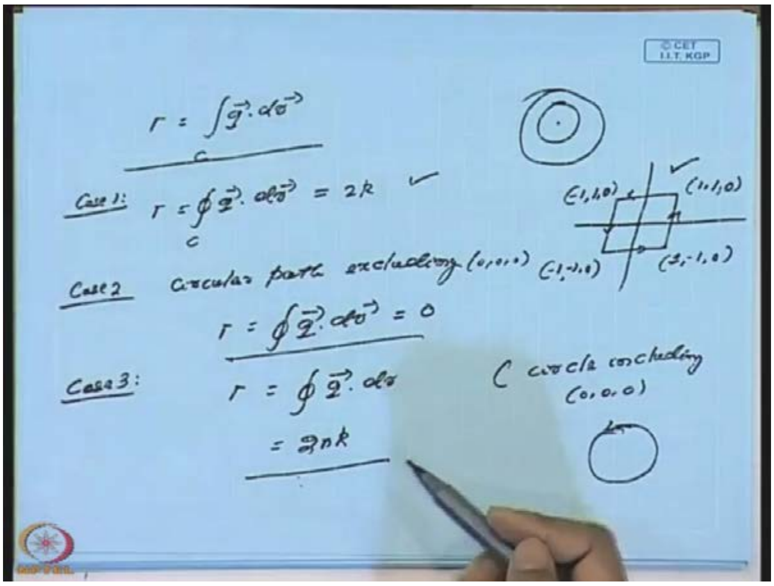

If you take the region any curl, any circular path take look at any circular path. Because this is at 0, 0 this velocity potential will not exist. So, if we look at, we cannot easily directly apply for a circular path that, because it is not differentiable. So, directly you cannot apply stokes theorem. So, if you want to find q bar dot d r bar; that means, the circulation in the flow, we cannot have stokes theorem directly in this case. On the other hand, suppose I take and the points are then that, I will say 1 minus 1 0 1 1 0 and over minus 1 1 0. And then, we have minus 1 minus 1 0. So, this is the origin, these are the 4 points, if I want to find the circulation around this path, the fluid particle follow this path, what is the circulation, And in that case, we can easily see that this gamma is equal to integral over c following this path q bar not d r bar equal to 2 k, this is for this case.

On the other hand, I will take this is case 1, on the other hand F, I go to case 2. Suppose I take a circle except, the origin I take a region circular path excluding 0 0. Then in that case, because if I exclude this region, then I can easily find gamma is q bar d r bar and that will give me 0. Because the function will be differentiable and curl of q will be 0. On the other hand, if we include the 0.00, if I include the 0.00, because at the origin, it will have a indeterminate form or we will have a indeterminate form. So, in that case, we can find a factor in that case. Of course, we cannot apply stokes theorem, but otherwise we can in different way by following the path, we can easily find that integral over q bar the d r bar circle including origin including 0 0 0.

If you calculate this, you will find this is nothing, but 2 pi, in fact you can get this directly also by applying by considering a circular path directly, we can get to and this is. So, in the same circulation, in this case when we are excluding the 0.00, then we are getting 1 reason. Whereas, here we are getting another region and that shows this is contradicting that even, if the flow is irrotational. If the region is not simple, simply connected then we cannot apply, we cannot the field will not be conservative. And in such a case, we cannot apply stokes theorem directly or we cannot apply the circulation. So, here the circulation is varying, because here this point, here this is including it does not include 0, where this is.

So, this is even, if in the same circle, because the, is not a conservative in this conservative in this case, so that is why we have a difference. Now, this I will go to another example is 2 examples are very equivalent. This is one of the very classical example, where we have a, a rotational field which is not conservative. And also you can see that they are the, the usual stokes theorem. We are not able to apply, because the with inside the region, we have a singularity in the flow at particularly q r is curl up of q r omega bar is not dependent origin. Now I will take a another example like suppose, I have been given q bar.

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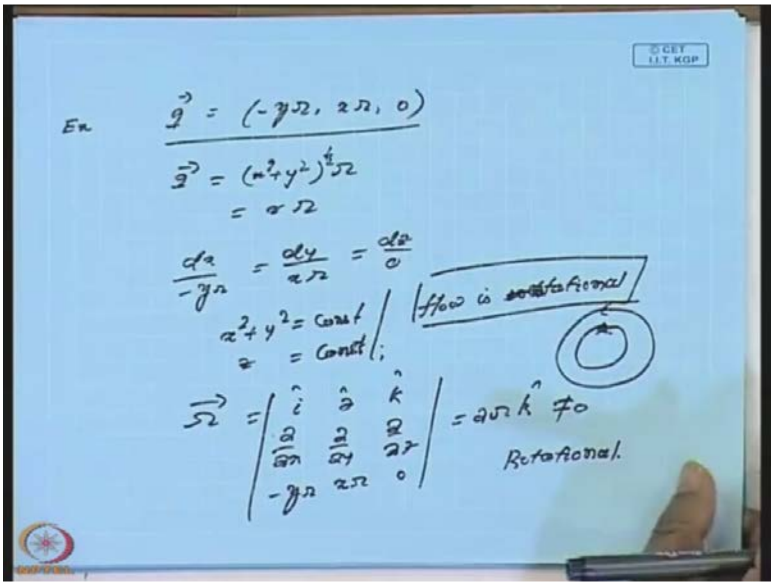

Basically to get our understanding on the circulation, if I say minus y omega and x omega 0, if q bar is given by this. What will happen to the flow, and I just say, so my q bar will be x square plus y square root over into omega. And I can call it r omega, where r is the distance from the particular from the origin and omega is the just scalar quantity. If I; if I look at this d x minus, minus y nu equal to d y by its omega and this is equal to d z by 0. In this case also, in fact I have repeating this example, because here I have a purpose is different and then z is equal to constant. Here also, the flow is they follow a circular path the same example and here also it can be seen that the motion is irrotational flow; is irrotational. And here also the particles they follow a circular path, they also follow a circular path.

And on the other hand sorry, sorry this is flow is irritation. Because if you will find omega bar i j k and del by del x del by del y del by del z minus y omega x omega 0, then this as solutions 2 omega k hat. So, this is not equal to 0, hence flow is rotational, rotational. And once the flow is rotational, and it has a vorticity omega bar, this vorticity vector is 2 omega k hat. Now for this flow, what will happen, if I want to find the circulation? If I want to find the circulation.

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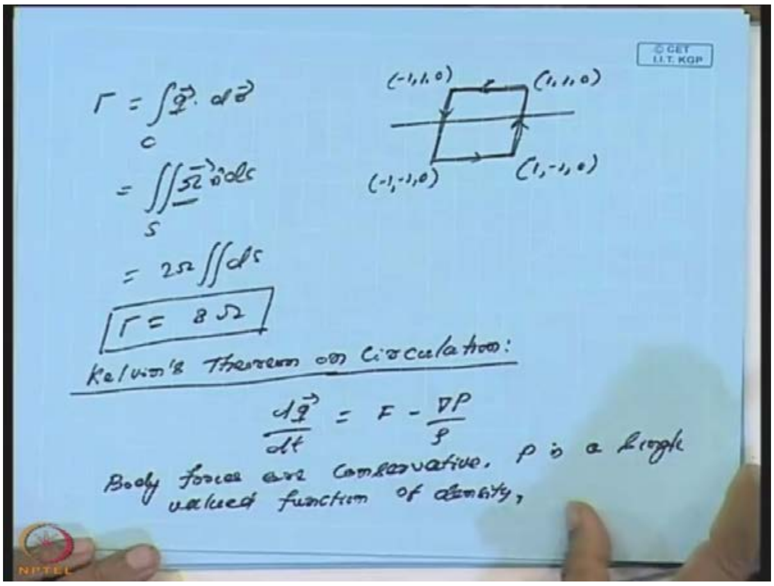

Suppose, again I take a circular, I take a rectangular. This is 1 1 0, this is minus 1 1 0, this is minus 1 minus 1 0, this is 1 minus 1 0. Suppose, this follows, this is the path of the particle and what will happen in this path of gamma that you see q bar not d r bar. And this will give us is also you can write is as integral over s omega bar d s and omega bar d naught and this will give us. Because this is all in the circular, this paths are n hat, this is and, this and gives us omega bar. Already, we know that is 2 omega, this omega is a scalar quantity, this is the curl of vector. Whereas, this omega is a scalar quantity 2 omega integral over d s, and integral over d s means, this is a rectangle of side which is 2. So, that will give us 8 omega, and so here we are getting that circulation gamma is 8 omega. So, here the flow is rotational and we have a circulation. On the other hand, in the previous case, we had a situation where the flow was irrotational. But still circulation was exist and up to different cases which we have.

Now, this background on circulation, I will go to volume to important result that what we call the Kelvin's theorem and circulation Kelvin's theorem on circulation. What is that before coming to the statement of the theorem? I will go to the other equation. Because this is a basic ideas write based on earlier equation. We have d q bar by d t is equal to F minus dot P by 4, what will happen; what will happen. If I assume that the vector field is conservative; that means, if the body forces are conservative body forces are conservative and the precept P is a singular function of velocity. If these 2 thing happen, then I can what I can do that, I can always write that vector field the F Eigen knowledge, because F is in a vector field which is conservative field.

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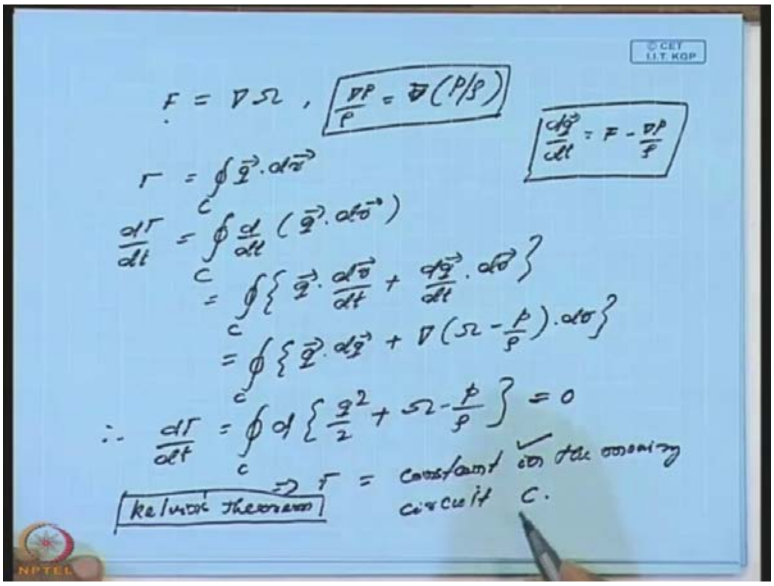

So, if can always write as dot of the scalar quantity dot of in the, when we call this as a scalar and also since we have P i say is a single value function of density, you can write grad P by rho as d f grad of P by rho. And if the flow is in homogeneous particularly, the flow is homogeneous density is constant. This is easy, otherwise also this is an important result, I am not going to the details. But it can be proved, proved by little complex, complex in general for in the flow is in compressible is particularly. When P is a single value function of density, it can proved that basic just assume that.

So, in that case, what will happen to gamma q bar dot d r bar, then what will happen to d gamma by d t. So, we have that gives us the grad of change in circulation and they would give us integral curve c, d by b t q bar dot d r bar on that itself cases integral aver c q bar dot d r by d t plus d q bar by d t dot d alpha. And again this once gives us interval over 5, that is q bar dot d q bar plus d d q by d t and d q by d t. If i look at the equation of motion that is 8 minus grad P by rho, here I know d q bar by d t equal to F minus grad b by rho. And if I just combine every things, I have F is grad omega and again grad P by rho is grad of P by rho. So, you can always write grad of omega minus P by rho dot d r and which I can further simplify, because this at q bar dot d d q bar and this is a grad of omega minus P by rho dot d r. So, you can always write as d of these are deferential of is a deferential of q square by 2 plus omega minus p by rho. And this itself is because, this is an exact deferential and the path j, it is a closed curve c. So, this curve is a closed curve in the fluid.

And since it is a closed curve and this is a deferential. So, this gives me 0; that means, we have, what we have, we have got that are, but d gamma by d t 0 and that impales. So, we have d gamma by d t, this gives; this gives gamma is equal to constant in the moving circuit c. So; that means, when the flow is here for an inviscid fluid in the body processer conservative and the pressure is a single value function of density, then the circulation around a closed curve c moving with the fluid is constant. And this is what, we say these are the Kelvin’s theorem, this is what we call the Kelvin’s theorem, this is Kelvin’s theorem that gamma is constant. When in a mobile circuit c, when the body forces and flow is and we say as well as the function of singular value function of density and this appear value of result.

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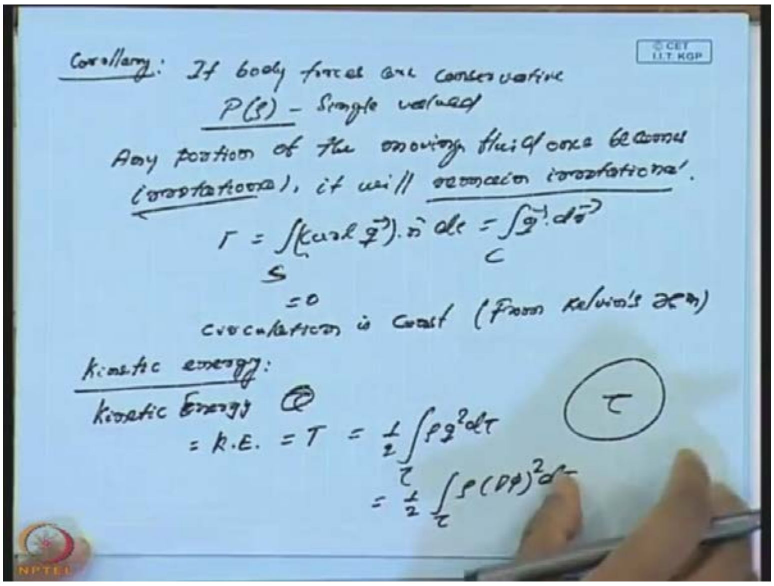

In fact used in many perpendicularly, when you look at flow around particularly, will calculating circulation with this, I can go little, I will just, when conclusion will drop on here will which will call it as a corollary, this corollary is a, corollary is a, if body forces are conservative and the pressure is a single valid function of density. P is a function of 1 is single value then any portion of the moving fluid rather any portion of the moving fluid once becomes a rotational. And if it becomes irrotational, it will continue to be rotational all time, it will remain the rotational once becomes a rotational. It will remain a rotational and this is oblivious, because we have seen that gamma is curl of q bar n hat to d s. And this is because the flow is body forces of conservative the fill is conservative vector fill. So, we have gamma, you can write in this form q which is same as the q bar dot d r bar and so that curl of q bar. Since, the once it becomes a rotational along a curl c s, this is c.

So, once it becomes irrotational curl of q bar will be 0 and that will give you gamma 0. And since we know the just we have proved that from Kelvin's theorem that if the circulation remains constant in a conservative vector field with A here. When pressure is a single valued function of density and here gamma is a 0 circulation is constant since, circulation constant from Kelvin's theorem. So, since to it will remain constant, so whether once the flow remains irrotational, it will remain. So, that is what, so this is another result. Now, if this now with this background, we will go to the kinetic energy associated with a irrotational motion, only just go to the kinetic energy. How we will depend the kinetic energy suppose. Let us think of a reason, tau once you join it the volume tau is the reason volume; basically, tau is the volume.

So, the kinetic energy, if I denote it by t either call it kinetic energy, one time kinetic energy KE which donated by t here the tau and this is t, it is given by 1 by 2 integral about tau rho q square theta. Now, I have q is equal to grad pi, if flow is irrotational, if flow is irrotational q becomes grad pi. So, this equals 1 by 2 into tau grad pi square theta. Now, if I apply, now you have rho grad phi, I know grad phi square, if I just use lagrangian's theorem.

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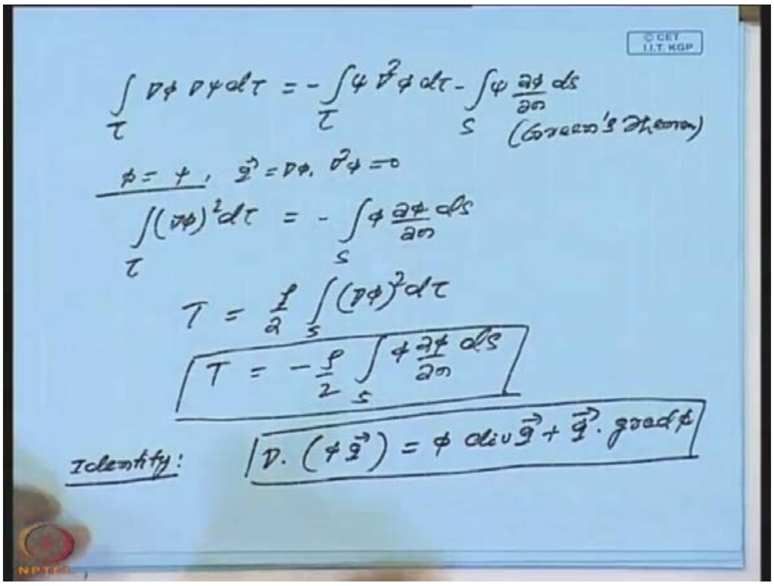

Some lagrangian's theorem, we have integral about tau grad pi that grad psi d tau, this is equal to minus integral over tau psi del square phi theta minus integral over psi del y by del l ds into this is basically lagrangian's theorem this is lagrangian's theorem. And if I put here what will happen? If I put phi is equal to psi if I put pi is equal to psi then I will get integral about tau grad by square theta. This is size equal to phi so this becomes and again my flow is rotational, my flow is irrotational my q bar is equal to grad phi and del square phi is zero. And so in that case so, the second time for a fluid rotational fluid tau grad pi square d tau will be minus integral about s phi del phi by del l. And once this becomes this term from the definition of kinetic energy T that is 1 by 2 rho grad phi square d tau and that becomes 1 by 2 minus rho by 2 integral over s is equals phi by del phi by n d s this is my t kinetic energy this is very very important because and n is the flow in the out turned on normal directions.

Now, before going to conclude this part of the lecture, I will just give you another identity on the vector calculus, that identify says he have divergent of phi dot q phi into q bar phi is scalar q is a vector. And this equal to phi into divergent of q bar plus q bar dot grad phi in fact this is e g. To prove just you expand this one, if you expand the left side, we can arrive at the right side is a very important identity which we will use the next lecture. So, I will conclude this part of a lecture that is what we have done. We have worked out few examples to talk about the application. So, the application of the equation of motion initially, we worked out 2 examples. Related to first, we talked about venture meter phi to tue, the principle behind you, one which this is best these two devices.

And then we talked about circulation worked out few examples in circulation. We defined conservative vector field then we say how, what is the necessary sufficient condition for a conservative vector field to be rotational? And taking that in to account and the Kelvin's and the circulation, we proved the Kelvin's theorem on circulation. And after that, now we will go to just define now the definition of the kinetic energy. In the next lecture, we will talk about some of the results in kinetic energy, with this for today we will stop here.

Thank you.