Marine Hydrodynamics
Prof. Trilochan Sahoo
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture - 05
Equation of Motion (Law of Conservation of Momentum)

Welcome, today is the fifth lecture in the series. And the last 4 lectures, we have already given a brief introduction to marine hydrodynamics, flow descriptions, equation of continuity, which physical mean the conservation of mass and we have not talked very several examples. Now, today lecture, we will concentrate on equation of motion and basically we will whose physical significance each law of conservation of momentum. And when we think of law of conservation of momentum is always Newton's second law of motion comes into our mind. Because that says that rate of change of linear momentum is equal to the total force that is acting on the fluid. In today's lecture discussing in the equation of motion, we will assume that the, it is for inviscid in the non viscous fluid that is emissivity fluid. And hence the equation we will concentrate today on Euler equation of motion.

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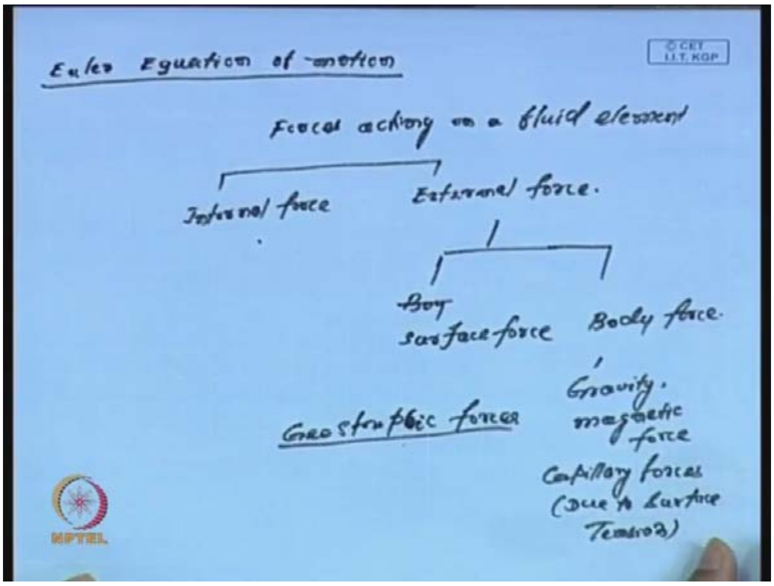

What you call this; this is a Euler equation of motion. So, when we think of law of conservation of this equation of motion particularly the force comes into picture. And when you look at forces acting on a fluid relevant always you look into what are the forces. Let us look into what are the forces acting on a fluid element. When you look at this there will be 2 types of forces which are acting one is the internal force, the other one is the external force. The internal force always they balance each other and their sum is 0 as action has every action has equal and opposite reaction. On the other hand when we think of external forces there are 2 types of external forces that is always well as called the body force. Rather we will say surface force; external forces are surface forces and the other one what you call the body force.

The surface force always act on the boundary on the particle with the fluid, on the other hand body force is external force. And this body force that is always like forces like gravity basically gravitational force of attraction. If the body is in a magnetic field then magnetic force, gravity force, magnetic force and also another other applied forces are capillarity capillary forces that acts on the capillary forces that is due to surface tension, that is due to surface tension. Then we have geo strophic forces have geo strophic forces and that is this is called due to the corollary, corollary acceleration due to the rotation of the earth. Now with this so these are the various forces which acts on a fluid element. And then another thing is what we say that is a conservation of vector field, what are mean by conservation of vector field?

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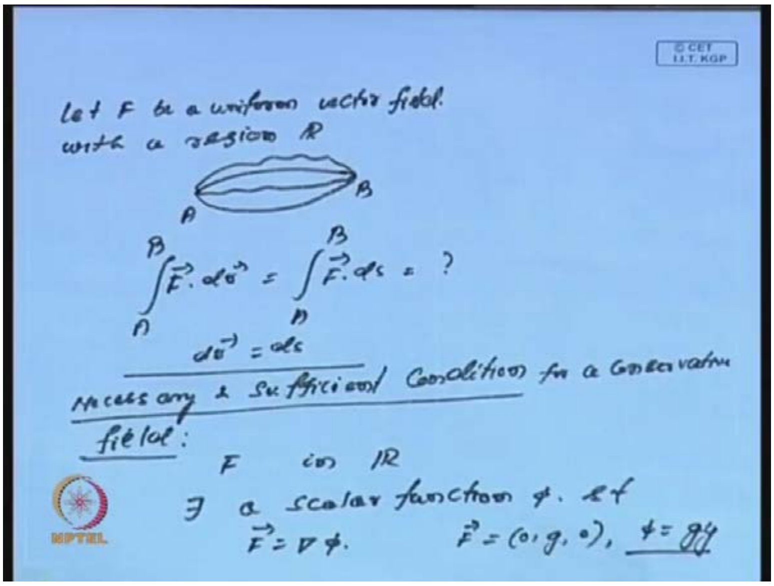

Suppose let, let us consider 2 elements let F be a vector field. Let F be a uniform vector field within a region R and let A be a point here and B is another point here. So what will happen? What is the integral A to B F bar dot d r bar? If this is same as A to B F bar dot d s and this from; this to this, the integral it can be you can think of in many direction, so here what we have taken d r bar is d s. So, if the value of the integral this value of the integral from A to B is independent of path on the integration whether we go from this way, whether we go from this way, whether we go from this way, whether we go from. If it is infinity of path then we say the value of F in r said to constitute a conservative refill, so that is away we define a conservative way.

Now, there is a let us see what is the necessary condition necessary and sufficient condition for a vector field to a conservative, for a conservative vector field? The condition is that as we have said that if F is a uniform vector function, this is l F is a function in a conservative vector field that is in R. Then, then the field F is said to be conservative in R if there exist we can find a scalar function there exist a scalar function phi, such that we can represent grad of F bar equal to grad of phi. If this can be done then we call this as a vector field, as a conservative vector field. One of the best example of a conservative vector field is the gravitational field, if I take F bar a 0 g 0 basically the gravitational field. So, you can always find if I look at the function phi as g y, g is the gravitational force and phi is g y then automatically allocate grad of phi is F bar 0 g 0. So, because of this we always say the gravitational field is a conservative field. Now, before going to derive the equation of motion I have another result to tell you, what is the Gauss theorem?

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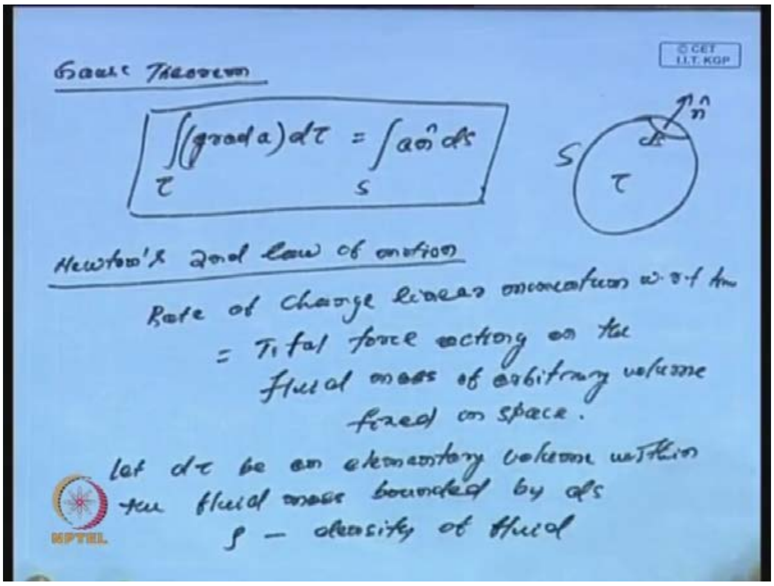

This is the theorem which is it can be easily obtained from the this theorem can be obtained from the strokes this diversions theorem that is it this theorem says tau grad of a d tau is equal to integral over s a n hat d s. Where what is this, let us look at a surface s and let the total volume will close to the surfaces in tau. Then if I have a element d s, if I have an element d s then draw there is a normal you draw here n hat unit number then you have tau the volume. So, grad a, a is a, a is any function scalar function, a is a scalar function so then grad a integral over tau d tau is integral over s a n hat d s. And this is a very important result which will be using in a derivation of the Euler equation.

Now, as I am already mentioned let us come to the Newton's second law of motion this is basically based on this. So, this says as is as I have already mention this is the rate of change of linear momentum with respect to time is equal to the total force. The total force acting on the fluid acting on the fluid mass of arbitrary volume fixed in space. So, this and our law of conservation of momentum, which is this is the law of often we call this as law of conservation of momentum. Now, to derive the equation of motion what we will do let us consider again, suppose we have this tau the total volume and s is the surface then d s is an element. Let d tau be an elementary volume within the fluid mass fluid mass bounded with the surface bounded by d s. And as we say rho is a density of the fluid, rho is the fluid density and let if q bar is the velocity.

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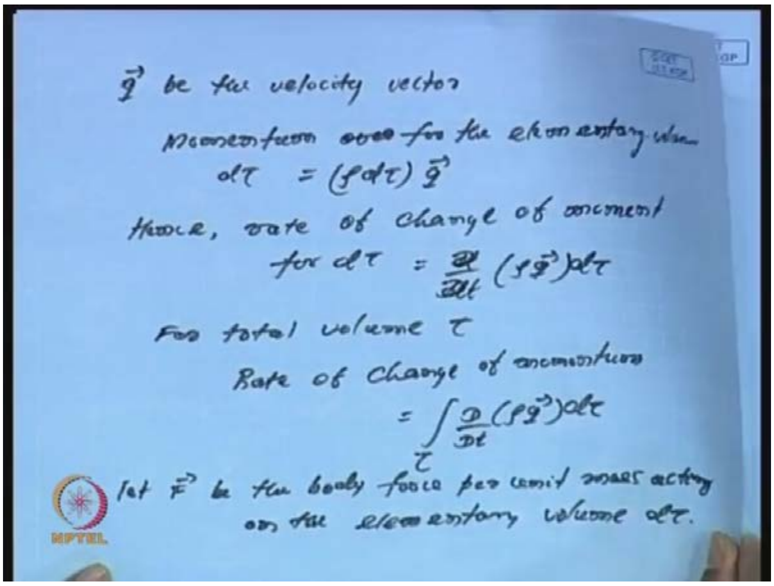

If let us say q bar be the velocity vector, then the rate of change then the momentum for the elementary mass momentum over the elementary, the elementary rather elementary volume d tau would be rho d tau into q bar. So, hence the rate of change of momentum for the elementary volume, d tau which will be d by d t rho q bar d tau. Then if I have, I have my total volume is tau so total volume is tau the total, so the total for total volume tau, tau rate of change of momentum this will be integral out of tau d y. This I can call it this I can take it as d by d t, because rho is a function of x y z t, so always so d by d t rho q bar d tau. Now, let F be the force, let F bar be the force that is the body force, because we have 2 forces acting on a fluid element 1 is the body force, 1 is the surface force F bar with the body force per unit mass acting on the elementary volume d tau.

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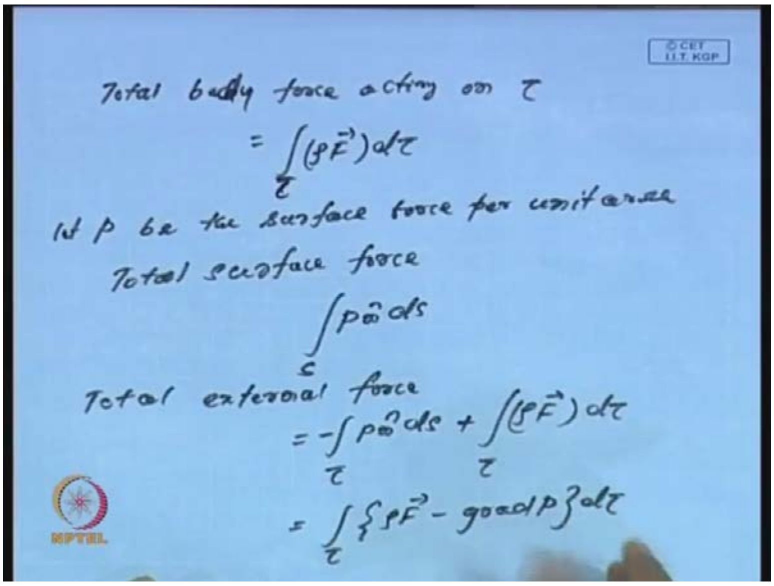

Hence, we have the total body force total body force acting on tau will be integral over tau, because a rho F bar d tau. Because F bar is the body force acting per unit mass, so there is the density so and tau is the total volume. So, rho F bar d tau integral above tau and this integral is a volume integral that is the total body force acting on tau; this is the volume total volume. Now, let us look at the surface force, let p be the surface force per unit area. Hence if you look at that what will be the total surface force, the total surface force will be we have P n hat d s that is on each small element d s and our total surface is s. So, this will be P n hat d s this is the in the outer direction. But the hence the total external force as we have seen that the total force acting is the one is the body force one is the surface force.

Hence the total external force will be the combination of the body force and the surface force. So that is integral over tau p n hat d s and this will be negative sign, because the forces this n hat is in the outer direction and this force is acting in the outer. But where force is acting on the fluid so there will be a negative sign plus, the total body force rho F bar d tau. Now, you apply the gauss theorem that will give us integral over tau that will be give us the rho F bar minus gradient of p d tau we have use the gauss theorem as I have already mentioned. Now, this is the total external force now if I go back to the law of conservation of momentum as I mention that this is should be equal to the rate of change of linear momentum.

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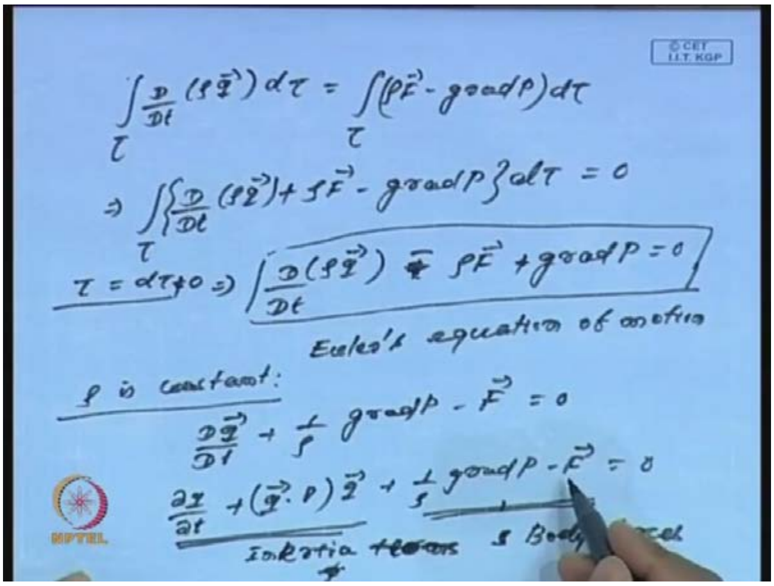

And if I do that rate of change of linear momentum then that will give me integral over tau d by d t rho q bar d tau, which is equal to integral over tau rho F bar minus gradient of p d tau, which gives me integral over tau d by d t rho q bar plus rho F bar minus grad of p d tau will be 0. Now, since tau is arbitrary our tau is arbitrary so you can always say if I say tau is d tau if tau is a d tau. Then what will happen which implies d by d t rho q bar is equal to plus rho F bar minus gradient of p is equal to 0 for F tau is d tau is not equal to 0. Then we have this on this is what I say this is called the Euler equation of motion, Euler equation of motion. So, if rho is constant if I say rho is constant, and then this gives me d by d t q bar plus 1 by rho this is minus, this is minus. So, this will be or this will be right side, the this one sorry there will be minus sign here, and this will be plus 1 by rho grad p minus F bar equal to 0.

So, this becomes F rho is constant then this becomes this and in this I will I can always this also I can always write del q by del t plus q bar dot del as 1 by rho grad p minus F bar is equal to 0. So, this is the inertia term this will call the inertia term, this is the inertia term these are the 2 terms of the inertia terms. This is the surface force rather I will say that these 2 terms that is the body forces. So, inertia terms inertia forces rather I call it inertia so this is this is the inertia force and these are the body forces. And this I can call this as local inertia and this is the convective inertia term; this is called the surface force and this is this is the external force applied force. Particularly this is the body force this is the, this is the body force this is the surface force. So, body force is this term and this term, we call this as the surface force and then this is the local inertia and the convective inertia.

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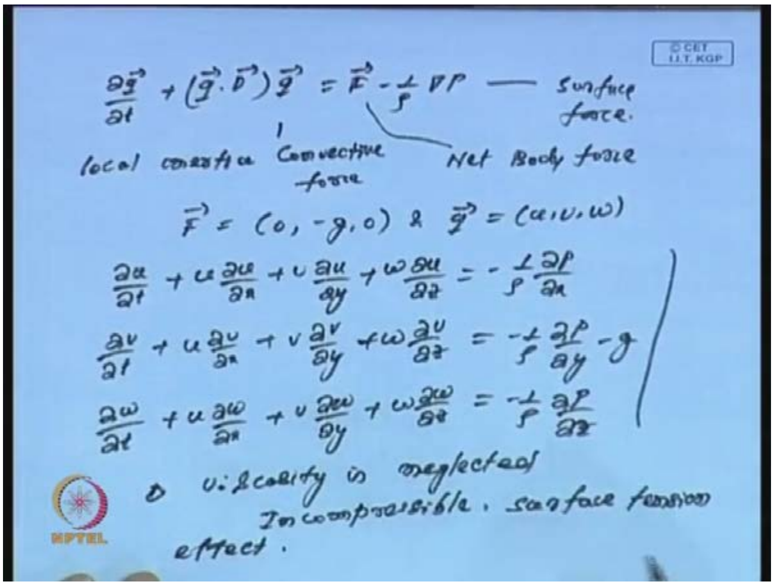

In other words I can always say this I can, I can I will just write it for sake of priority this is del q by del t plus q bar dot del bar into q bar equal to F bar minus 1 by rho grad p. And here in this equation, so we have one is the local inertia this is the local inertia force and that is and this is the convective inertia, convective force. And this is the net I call it net body force and this the surface force. Now, if I take a F bar, because in a gravitational field I can always take F bar is 0 minus g 0.

And if I assume my q bar is equal to u v w, then in the Cartesian coordinate system this Euler equation which will reduce to del u by del t plus u del u by del x plus v del u by del y plus w del u by del z minus 1 by rho del p by del x. And del v by del t these are the y component plus u del v by del x v del v by del y plus w del v by del z minus 1 by rho del p by del y minus g. And then we have del w by del t plus u del w by del x plus v del w by del y plus w del w by del z this is what minus 1 by rho del p by del z. So, this is this 3 equation are the equation of motion in the Cartesian coordinate. Now, in the derivation of this we have neglected the viscosity we have not taken into account of vector viscosity, viscosity is neglected and we have continued the fluid as in compressible.

Because we have taken rho as constant in this so this 2 data assumptions same a, but what will happen and also we are not taken into account surface tension of it. In addition the thermal conductivity, conductivity of the fluid edge also not taken into account. So, this is one of the very ideal situation, and we sometimes called this as the ideal flow; this Euler equation is satisfied to any ideal fluid.

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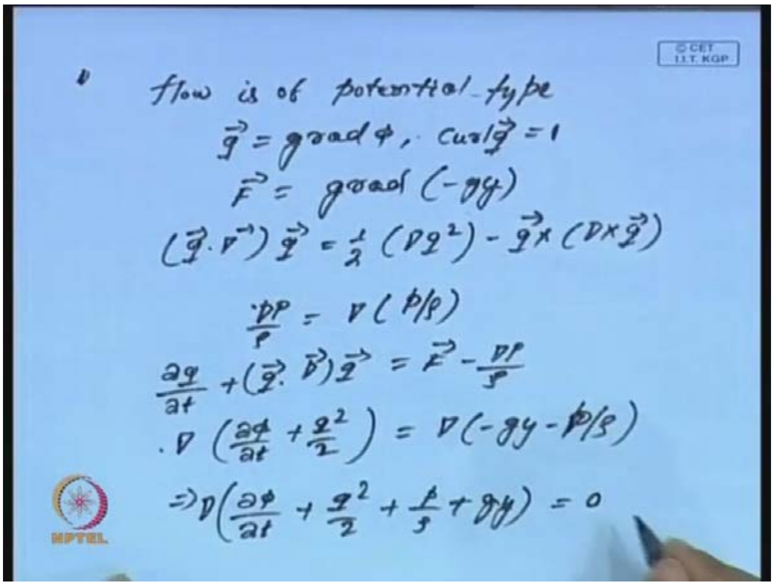

Now, with this if I just say that my flow is potential type. If flow is a potential type, flow is a potential type I will have q bar is equal to grad phi q bar is grad phi and I have curl of q bar is 0. And further since and the gravitational field as I have mentioned that I can write F bar is grad of minus g y. Further I have another relation that q bar dot del bar into q bar we can always write it in the form half of del of q square is the minus q bar cross del cross q bar. Since my flow is irrotational, I have already written curl of q bar is 0. So, I can and again I can always write since my flow is homogenous assuming that flow is homogenous I can always write grad p by rho grad of p by rho.

Under this assumptions I always can write from my equation of continuity, Euler equation of motion del q by del t plus q bar dot del bar q bar is F bar minus grad p by rho. We should take a small p which I can write in this form grad of because I substitute for I will write it as del phi by del t. So, q is grad phi so grad of del phi by del t then plus this also because I am taking this part is 0. So, q dot del q bar is I can write it grad already taken then half of I can call it as q square by 2. And then I have, have also taken which is equal to grad of minus g y minus grad of again taken the bracket so p by rho I can take it. So, which implies I can also tell this as del phi by del t plus q square by 2 plus p by rho plus g y grad of this equal to 0.

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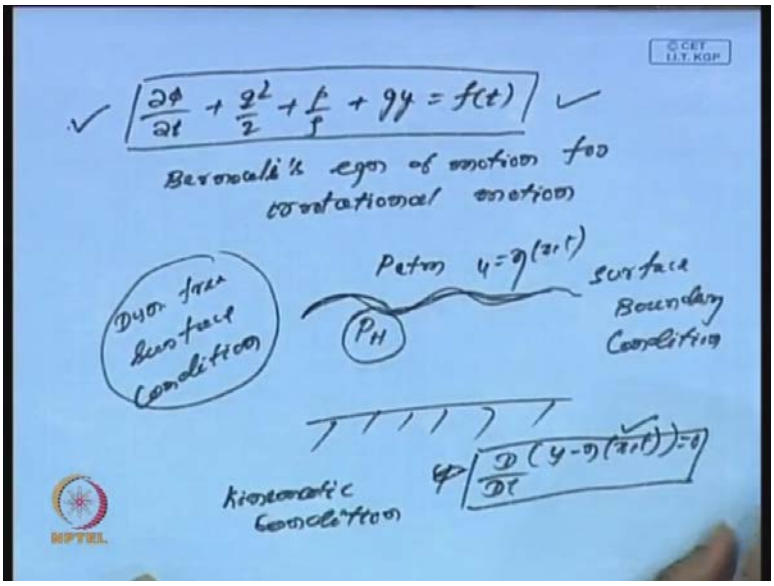

Now, if I from this I can always if I take a gradient if I take the dot predict d r. And integrate it from this I can always get del phi by del t plus q square by 2 plus p by rho plus g y is equal to because gradient I am taking this place derivative. So, it can be a function of t and this function can be arbitrary function. So, this is this which what I call this equation as double Bernoulli's equation of motion, equation of motion for irrotational flow. So, this F is arbitrary now this Bernoulli's equation, it is very important role in fact in many problems of hydro dynamics particularly in the marine environment when you calculate the wave forces on a structure we always take this Bernoulli's equation in account to calculate the to know the hydrodynamic pressure.

And from there we integrate to cover the surface area to get the force acting on a fluid element or acting on the body. In fact, the again when we have a free surface in the water we have a free surface and in that situation also we use the Bernoulli's equation to calculate. Because you have a atmospheric pressure here p atmosphere this side is p h is the hydrodynamic pressure what we do at the free surface, we equate the atmospheric pressure with the hydrodynamic pressure and to get the dynamic free surface condition. On the other hand in the last lecture I was talking about the surface boundary condition, I was talking about the surface boundary condition.

And there we have seen if y is equal to eta if y is equal to eta x t the free surface boundary then we can always say that d by d t y minus eta x t equal to 0. And that gives me the boundary condition another boundary condition that is the surface boundary condition. So, on this water surface of a fluid we have 2 conditions; one is the surface boundary condition we call this as the kinematic condition on the free surface. And this one that is a when we act relate the atmospheric pressure with hydrodynamic pressure on the free surface. Because on the free surface atmospheric pressure is same as the hydrodynamic pressure when the free surface and that gives us dynamic free surface condition.

So, this is one boundary y is equal to eta where 2 conditions are satisfied one is the kinematic condition and one dynamic condition. So, this equation this Bernoulli's equation is very crucial even if in coastal engineering applications when we consider the flow is irrotational and inviscid to calculate the forces on break water. And calculate the surface solution and other physical quantities this potential flow analysis we always apply and for the force calculation always use this Bernoulli's equation. Hence and also in offshore structure offshore structure wave load on offshore structure ship and any other marine structure we use this Bernoulli's equation frequently.

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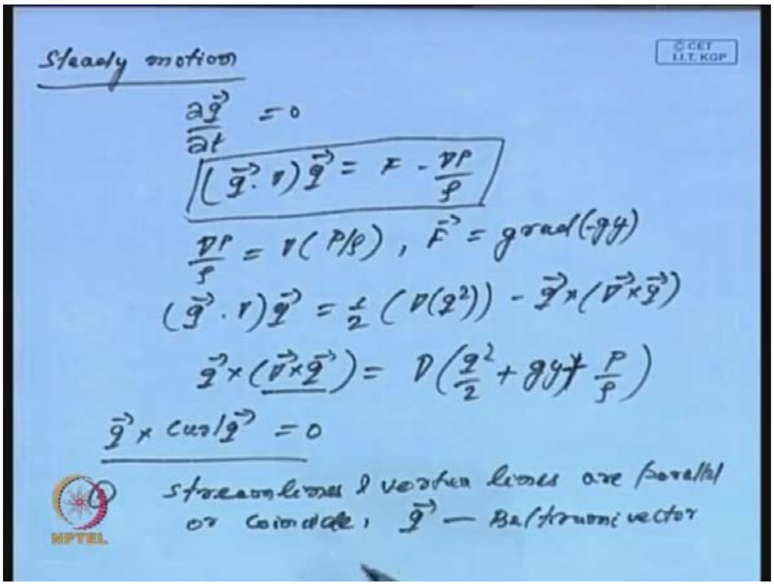

Now, if I in the Euler equation, if I assume that the flow is steady if motion is steady. If motion is steady in that time derivatively you go will have del q bar by del t will be 0.

Once del q bar by del t is 0, then the Euler equation will reduce to q bar dot del into q bar is equal to F minus grad p by rho. And again if we say that the flow field is conservative and we can write this is a fluid is homogeneous. Then we have a we can also write grad p by rho is grad of p by rho and if F bar we can also write it as grad of g y assume the field is electro field or this electro field, the gravitational field is a conservative field. And here conservative field is the gravitational field and again so we can always write and again we utilize the same relation at, at q bar dot del into q bar.

Because half of del q square minus q bar plus del bar plus q bar then we can always rewrite the Euler equation, equation of motion. In case if steady flow as q bar plus del bar plus q bar as del of q square by 2 plus this is minus g y minus g y this will be F minus q is here. So, this is F is minus so once you put this will be q square by 2 plus g y plus p by rho, once we have this, this is a plus sign. Now, there are 2 cases few cases comes in, now if q bar cross this is nothing but curl of q bar. If this is 0, if this is 0 case 1 curl like this that means the stream lines and vertex lines are parallel. In this case your stream lines and vertex lines are parallel or they co inside or co inside if the if they co inside for such motion q bar is called Beltrami vector. So, under this, such under stream lines and vertex lines are parallel are co inside q bar is called as Beltrami vector in that case the left side can be 0.

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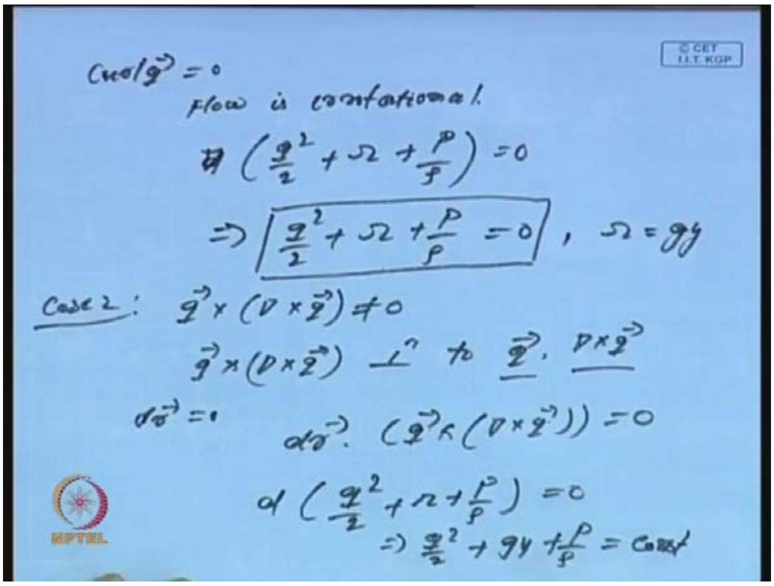

Another case comes when curl of q bar is 0 that means, if curl of q bar is 0 that means flow is irrotational in both the cases this gives us d of q square by 2 plus omega plus p by rho is equal to 0. And which can give us del of this is 0, which give us root take integrate it you can easily get it q square by 2 plus omega plus p by rho is 0. And here this omega is nothing but g y so this is so whether the flow is rotational or irrotational this equation will be satisfied. Another case comes case 2; suppose you have q bar cross del cross q bar is not equal to 0. If this is not equal to 0 that means if this is not equal to 0 this vector q bar cross del cross q bar; this 2 vectors; this vector is perpendicular they are perpendicular to q bar as well as del plus q bar they are perpendicular to this.

And this then and if d r bar is not equal to 0 this is the tangent then d r bar now this is the tangent and they are the perpendicular at the perpendicular to this. So, I can take d r bar dot q bar cross del cross q bar is equal to 0. And once this is 0, so the, from the right side I can from the right side you can always get d of. Because that is also del and when I say q dot d r bar dot del of these that will give me d of q square by 2 plus omega plus p by rho that is 0. And this gives me q square by 2 plus omega, omega itself is g y plus p by rho that is equal to a constant.

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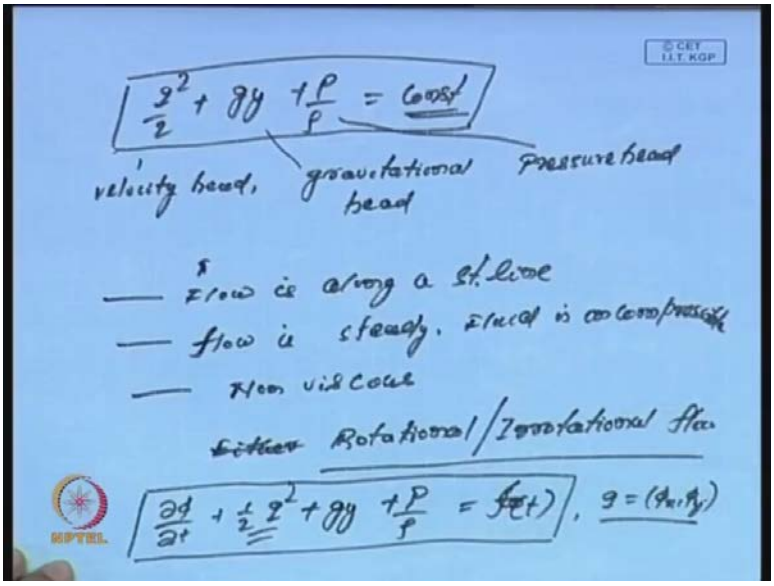

In this case, because unlike the other case in this case the motion is steady so this constant is independent of time, so what does this gives me? This gives me that the combination of 3. So, I have q square by 2 plus omega is g y plus p by rho that is equal to constant and this motion this is independent of time. And this is often I call this as the velocity head this is the velocity head, this is the gravitational head because of gravitational head, and this is the pressure head. So, that means the velocity head plus gravitational head plus, pressure head is constant when the fluid is in steady.

And in this case, we have assumed the flow is along a stream line because we have taken d r is not equal to 0 and flow is steady fluid is also assumed to be in compressible and homogeneous. So, this I will just summarize this when the flow is along a stream line is a stream line. And then the flow is a steady and the fluid is in compressible fluid is in compressible or homogeneous and here we have already said it is inviscid non-viscous. So, then in that situation we have the pressure head, velocity head plus gravitational head plus, pressure head is constant. And here another thing here it comes to mind that in this derivation either the flow is this is suitable for either the flow whether it is a rotational this also holds for both rotational as well as irrotational flow.

So, in both the cases this will hold. So, what we have done today and in this case, we have steady motion when we have on steady motion we have seen the Euler Bernoulli's equation. And that is for unsteady motion and this was this is what I say the Euler equation for the steady motion or sometimes you also call this as the Bernoulli's equation. In this case one of the thing that it is here the fluid can be rotational or irrotational. And in many pipe flows to calculate the pressure always these equations are taken particularly in hydraulic applications flow in a pipe or erase application in civil engineering this equation is taken in to account. On the other hand when it comes to marine hydrodynamics particularly ocean engineering you always emphasize that the motion is always we do not assume the or on steady motion basically we always deal with.

And in that situation we apply the although of course, here we assume that the flow is a irrotational in that case we apply the Bernoulli's equation that is your del phi by del t plus half of q square there we apply that equation del phi by del t plus half of q square plus g y plus p by rho is equal to constant or F of t this is another form. So, and in this case q you can also you can write it as u has the component u that is phi x phi y. So, you can write in terms of phi also this time we can also write in terms of the velocity potential phi. Now, you have already now today we have already derived the 2 things Euler equation of motion, and the then the Bernoulli's equation. And these 2 equations of motion are more different, because in both the cases we have assumed that viscosity of the fluid viscosity is negligible.

And when we will take the fluid viscosity into account the corresponding equation will call them the corresponding equation motion will call this as the navier's stokes equation. And we will come in we will come to that much later and we will walk out few problems particularly how to calculate the pressure and force when the flow is inviscid how to calculate the force and pressure once we know the pressure we can easily calculate the force. So, this is very simple way for inviscid flow we can easily calculate the pressure on a surface, and once we know the pressure we can calculate the force at each and every point or on the body that is. And as I have already mentioned in ocean engineering this equation will play very big role, because wave load calculation is plays a very important role in the design of the marine structures. Basically whether we have today we are concentrated more on various types offshore structures.

Because the various aspect of flow and in ocean major analysis we do particularly when we calculate the wave load on large offshore structures large floating bodies or even from coastal structures. Then we always assume the flow is a in compressible fluid is compressible flow is irrotational. However the complexity there we always face because we have 2 as I have mentioned that we always come across 2 surface, 2 condition on the surface and the both the condition are highly non-linear. And here in fact, the surface is very dynamic although you will deal with a potential flow problem will be assuming that a flow is potential type irrotational motion, so flow is potential type. But still the complexity of the problem remains because on the free surface we have 2 condition; one is a kinematic condition; one is the dynamic condition and these dynamic and kinematic condition will couple them together that becomes more complex. And that brings lot of complexity and this kind of problem; particularly occur in marine hydro dynamics. Even if the flow is potential can the problem complexity remains because of a non-linearity free surface.

And this is how the other problems this is differ there is a significance difference above this branch of hydrodynamics then the other relative branches of fluid mechanics. Here free surface is does not play much role, we look at a pipe flow even if you cannot see the viscosity the complexity of the problem is simpler problems are much easier. Some of the problems are much easier whereas, in this case problems are complex it is because of the free surface condition where we have non-linearity occurs although we solve a Laplace equation. I will come to in detail to those what where problems in few of my lectures, but before have few more things to complete in this because I have to calculate.

So, how to calculate the pressure when a body is moving we have already in our few problems we will work out few more problems. And try to understand how we are calculate the pressure on a structure or when a body is in a fluid how we are calculating the force acting on it. And with these today is lectures I conclude and next lecture, we will again comeback to two dimensional flow. And there, we will emphasize some more problems where the how to calculate the pressure and force. And then we will go to analyze some of the aspect of the problem like source, sink.

And after that we will come back to water sourcing will come back to hydrophole and aero pole section in aero pole theory. And there also, we will come across this calculation of the force that is how the forces are acting, how the force and moment are acting? In fact in many situations, we will see that these inviscid characteristics will play significant role. And in practice it gives a very good result although in many situations we assume that the fluid is non-viscous. But it gives very good results in many cases any way slowly we will go into the details. And today we will conclude this today is lecture here.

Thank you.