Marine Hydrodynamics
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Lecture - 04
Worked Examples on Various Types of Flow

Good afternoon, so today we will it is the 4th lecture in the series and we will be talking about Various Types of a Flow examples. In the last couple of lectures, in the last three lectures rather we have already talked about conservation of mass, then we talked about stream lines, stream function, velocity potentials, rotational flow, potential flow. Now, within this much of information and then we have discussed about relation between the velocity potentials and the string functions.

Now, with this background let us show clear our basics let, so spend some time on working out some more examples and that will give us a very good understanding about the flow characteristics. So, this we can start with this few examples before that I will give you one one of the relation that the stream function satisfies particularly when the flow is irrotational.

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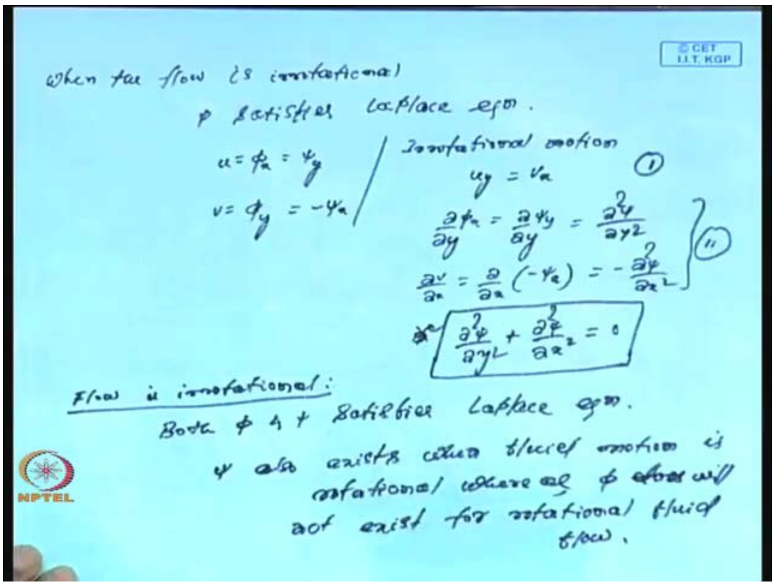

When the flow is irrotational our solution extreme function psi satisfies the Laplace equation satisfy Laplace equation. So, it is very easy we know that we have the phi x is equal to psi y, further we know that we have phi y is minus psi x. If we now for a irrotational flow or a irrotational motion, motion we have we have u y is equal to v x that is in two dimensional, for two dimensional flow. So, if u y is v x, so now, substitute del by del y this is same as del by del y, u is phi x here other I will say u is phi x and v is equal to phi y.

So, del by del y u is phi x, which is same as del by del y psi by and again further we have del by we have v x is del v by del x, del by del x we have v is this minus psi x. So, this is equal to minus del square psi by del x a square and here also this is also equal to del square psi by del y square. Now, from 1 and if we substitute for this 2, in 1 then what will happen from 1 and 2 we will easily get this is e y e y is same as del square psi by del x square sorry del square psi by del x del y square and v x is as as minus del square psi by del x square.

So, this will be plus del square psi by del x square is 0, which is nothing but the Laplace equation. So, this itself is the and this only possible when the flow is irrotational. So, that is very clear that when the flow is irrotational flow is a irrotational we have both phi and psi satisfies Laplace equation. On the other hand on the other hand we have phi psi also exist when fluid is irrotational fluid motion is rotational.

Whereas, whereas phi does not exist phi will not exist for rotational flow rotational fluid flow this is one of the very important characteristic of this a stream lines and stream function. Now, I will a give you another example I will go to another example where we will talk about how to find the flow characteristics.

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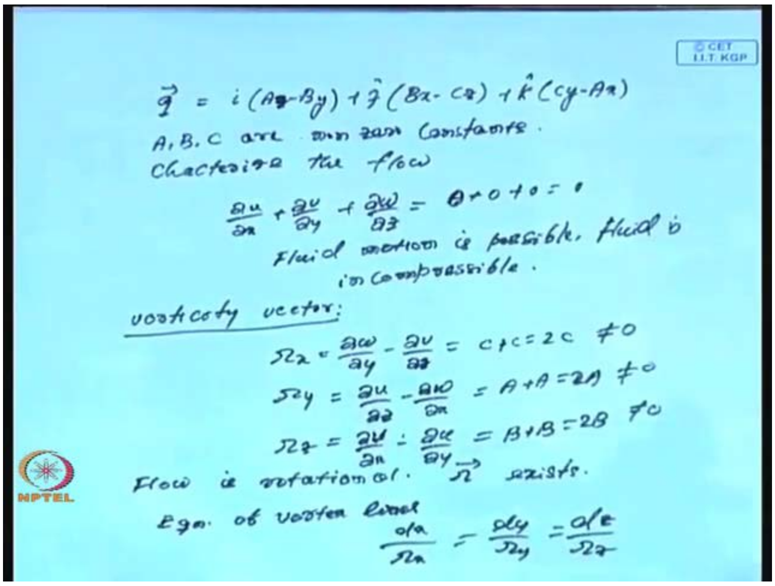

Particularly suppose I have been given a velocity vector q r i hat A x minus B y plus j hat B x minus C z plus k hat into C y minus A x. Then A, B, C are non constants are non zero constants they are non zero constants. So, what will happen to the flow consider characterize the flow characterize the flow. So, the first thing is that to check whether the flow is irrotational, flow fluid is a there is a fluid motion is possible or not. So, we will say that since it is a independent of v v w rho.

So, you have to check that whether del u by del x plus del v by del y plus del w by del z what happened to this, if we do that, so you have del u by del x you have del u by del x u is x minus b. So, it will be del u by del x is A and del v by del y plus 0 del w by del z this is sorry a z minus sorry this is A z minus B y, so this is A z minus B y, so this is 0 plus 0 here also 0 here also 0, so this is 0. So, fluid motion is possible in fluid motion is possible and the flow is in compressible fluid is in compressible in compressible.

Now, again we will check what what will happen to flow, vorticity vector because if we can check that we can find the what are the vorticity vector in this case what are the vorticity vectors vorticity vector omega bar. So, in that case we have omega x is equal to del w by del y minus del v by minus del z del w by del y minus del v by del z. So, del w by del y this is c minus del v by del z plus c this is 2 c and we have omega y is equal to del u by del z minus del w by del x.

So, that will give us A del u by del z del u by del z is A m minus del w del w by del x is minus minus plus A, that is 2 A. And similarly we have omega z is equal to del v by del x minus del u by del y. So, you have del v by del x is B minus minus B that is plus B that is 2 B. So, since omega is omega x omega y omega z is non zero, non zero not equal to 0. So, flow zero flow is rotational in this case the flow is rotational and once the flow is rotational we can have a vorticity vector So, omega bar exists omega bar exist.

And once omega bar exist then we can find what are the vorticity vectors what are the equations of vertex line if you want to find the equation of vortex lines; that means, d x by omega x is equal to d y by omega y d z by omega z.

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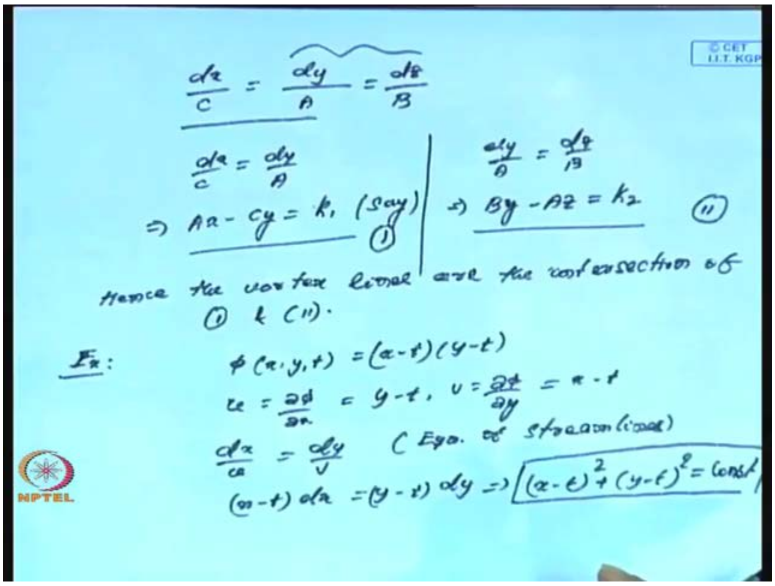

To do that that gives us d x by your omega x is C is d y by omega y, omega y is A and this is d z by omega z omega z is B, this gives us my full length of first two members, then it will give us d x by C is equal to d y by A; that gives us A x minus C y is equal to constant call this constant as k one say. And similarly from the last two equations we can always get we have d y by A is d z by B and which implies A z minus this gives B by minus A z is equal to another constant let this constant be k 2, so one equation is this the other equation is this.

So, the vertex and hence the vertex lines are the hence the vortex lines are the are the intersection of 1 and 2 if I call this as 1 and this I call it as 2 intersection of 1 and 2, so this is the way we will find the vertex lines. Now, I will go to another example this is a very simple example I will concentrate mainly today on various type of example in the velocity potential. Suppose we have been given phi x y t this is x minus t into y minus t, if phi exist then what will happen to u, u is del phi by del x u is del phi by del x.

Now, it will give us y minus t and v is del phi by del y that will give us x minus t. So, so about u v and we can if we will once u v is known then what is the equation of the stream lines that is d x by u is d y by v these are the equation of the stream lines these are the equation of the stream lines. Then what will happen this will give us b b d x x minus t d x y minus t d u, u is y minus t y minus t d y and that gives us x minus t square plus y minus t square is equal to constant.

So, these are the equation of stream lines these are the equation of stream lines. So, and here the centre of this circle is constant for various values of constant, we get a stream line and here the centre is just the time period t. Now, with this understanding I will go to a very one more example on this flow characteristic, to understand the flow characteristic.

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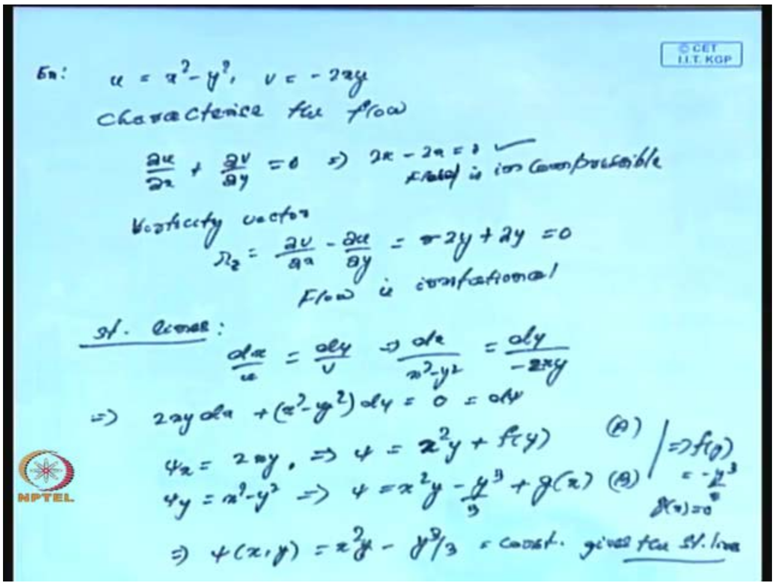

And we just say that suppose the, you have been given the velocity vector u component of the velocity vector u is equal to x square. This is another example u is equal to x square minus y square and v is equal to minus 2 x y, first thing is that characterize the flow, the question comes to characterize the flow. We have to emphasize on few things, one of the thing is that is the first of all whether the flow is in compressible or compressible or whether there is possible fluid motion.

And once the fluid motion exist of a particular nature we need to know whether what are the stream lines, because that will gives us the flow direction. Then once you know the stream lines, then we need to know whether the flow is irrotational and if the flow is rotational, then you find the velocity potential and if the flow is rotational, then better to find out the what is the vorticity vector, because that will give us the direction of the flow in the angular angular motion of the flow, so these are the things.

So, in this case, so to understand this first, when we characterize the flow here the question is itself and first of all we have to say whether there is fluid motion. So, for that we need to know what is the del u by del x plus del v by del y first thing is to check that whether this is a two dimensional motion, the problem is two dimensional in nature. So, flow is in two dimensional so, you have this, so you have del u by del x and that is del u which implies del u by del x this is 2 x and del b by del y minus 2 y minus 2 x is 0.

And since this is 0, so there is a there is fluid motion possible and flow is in compressible that itself. So, that itself clarify that the motion of a incompressible fluid exist there is a flow of a incompressible fluid, incompressible that fluid is incompressible, other we will call it fluid is incompressible. Now, with this, now whether to check whether the flow is what about the vorticity vector, because that will give us the vorticity vector, if you look at the vorticity vector that del v by del x minus del u by del y. And this is equal to del v by del x minus 2 y and minus del u by del y.

So, this is minus minus plus 2 y this is 0, so del v by del so; that means, omega z is 0 0, so it says that flow is irrotational. Once these two things are known that flow is incompressible and irrotational; obviously, the next question is come to now to stream lines, to know the stream lines we have to first look at the equation of the stream lines. So, equation of the stream lines are d x by u d y by v and that gives us d x by x square minus y square is d y by minus 2 x y if this is.

And then so, which simplifies to I will just simplify that will give you 2 x y d x plus x square minus y square d y is equal to 0. Since, we are looking for stream lines, then here if I call this as d psi then what will happen to my psi x, my psi x is 2 x y and my psi y is x square minus y square, if psi x is 2 x which implies means psi I will take two 2 x y. So, this will be x square y plus f of y where x is a arbitrary constant. On the other hand if psi y is x square minus y square, then my psi is equal to this is x square y minus y cube y 3 plus some another function of x, because this is a derivative with respective to y.

So, that will give us another function. So, now, if we compare A and B because a gives an representation of psi B also gives an representation of psi. And here that will give us that from this two we get is f y f y equal to minus y cube by three and also what will be g x, g x is 0 which implies my psi x y is x square y minus y cube by 3 and this psi x y this is psi x y then this is called constant gives us stream line, these are the stream lines.

So, this is better to go by this way, because otherwise if you go by try to directly sum there is a little complexity you will get this answer, but there will be little complex here better go by psi x is 2 x y go for psi and psi y is this which will go for and this will give us the stream line in a very easy manner. Now, with this I will go for now already we have now worked out a few examples, with this now let us concentrate on few one of thing is that what is the boundary surface.

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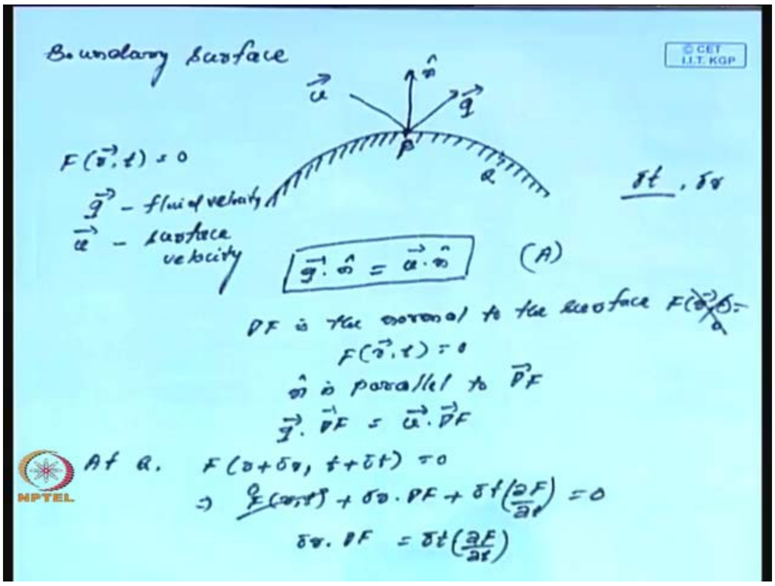

Let us know what is a boundary surface and what are the conditions satisfied on a rigid boundary or a moving boundary. If let me say that this is a boundary surface, so what will happen let me take a point P, so there is I consider the top side is the boundary it is with the fluid. So, let me take a point P here, once I have a point k p, let me draw two things let me call this surface as F is a function of r bar t, where there is fluid is flowing and f is this is a P is a point here.

And f is a F is the boundary surface, this surface I call it this is satisfied F of r bar t where the fluid velocity is q bar and the surface velocity this is the fluid velocity and let u bar is the surface velocity. So, now, at this point, so there will be a fluid velocity that is q bar and let the surface velocity be u bar. So, fluid is moving with velocity q, the surface itself is moving with the velocity u bar.

Let n bar n hat be the unit normal from this point in the outer direction. So, if this as to be a boundary surface if this surface has to be a boundary surface what is the normal component velocity of the fluid that is q bar dot n hat as to be the same as normal component of the velocity of the surface. So, this is the normal component, so that will be u bar dot n hat. So, at a boundary surface this condition has to be satisfied, because this is the normal component of velocity of the fluid particular the point this the normal component of the surface and if now further, F is the surface this surface is F.

So, if I call this surface this is by 0, so what will happen if this is the surface given by F of r bar t is 0, then what is the what is grad F grad f will is the normal to the surface F of r bar t is 0 F of r bar t is 0, if this is the normal to the surface. Now we have n hat is parallel to grad F, because we have n is the unit normal and grad F is normal to the surface, so the both are parallel. So, because of this we can always say from this a from a we can always say that we have q bar dot grad F is same as u bar dot grad f. So, this two has to be same, once this two are same.

Now, let the point because this is a moving surface, so the point let us say that after time delta t the points move towards p moves to q. And here and p moves to q, then the new point at the point q, at q the surface becomes F of r plus del r t plus del t because assuming that there is a shift of the time with the change in time t del t, let there be a change in the position is del r. So, at q again we we have f of because this is the surface.

So, F of r plus del r plus comma t plus del t is 0 and if that is 0, then that itself is gives us F of r t, which is same as rather which implies F of r t plus del r dot grad F plus del t dot del F by del t this is a cross relation dot product this is a multiplication this is 0. Now and already we have F of r t is 0, if F of r t is 0, then this term will give us zero and this is zero. So, we have got del r dot grad F is del t into del F by del t rather put it is not dot product this is just multiplication.

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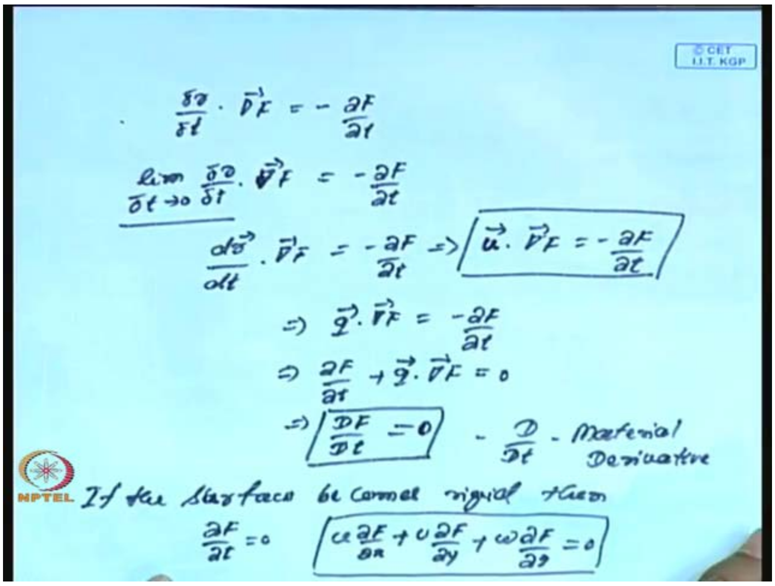

Now, with this now if we go back to this then we have, now take the limit if we look at the limit. So, we have del r by del t, we have del r by del t dot grad F is equal to equal to minus del F by del t and that is, so limit. So, if you take the limit del t tends to 0 del r by del t that is nothing but into grad F that is again same as minus del F by del t. And this is what, look at this, this is nothing but we have already talked about this is d r bar by d t d r bar by d t dot grad of f this is minus del f by del t.

And which is same as and since r bar is a position vector of the surface then we have d r bar by d t give us u bar, we have u bar dot grad of F is minus del f by del t, now this is very important result. So, now, we have already seen that, so which is same as u bar dot grad f is already we have seen in the we have u bar dot grad F can be written as q bar dot grad F. So, which implies q bar dot grad F equal to minus del f by del t, so which implies del f by del t plus q bar dot grad F equal to 0 which nothing but which is same as d f by d t is equal to 0.

That means, if f is the surface it is a boundary surface, then we have on the boundary surface d f by d t zero this is very important result which is always used particularly, when you have a boundary surface. Now, which is same as already d by d t is the where the d by d t, we know this is the material derivative. And again if I simplify this if the surface becomes rigid which gives if the surface is becomes rigid, then we have del f by del t is 0 and this becomes u del f by del x plus v del f by del y plus w del f by del z.

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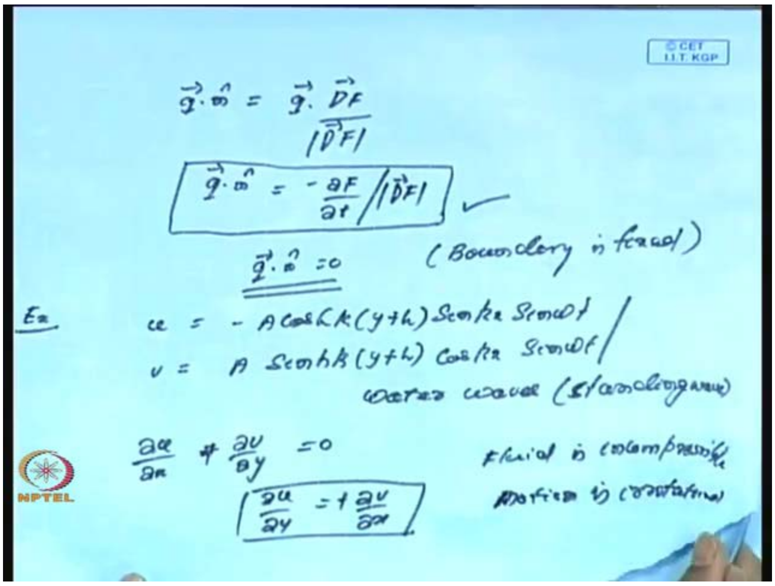

And which gives me hence the normal component of the velocity for the boundary becomes hence q bar n hat becomes q bar dot n hat n hat is grad F by grad F minus and that is and this q dot grad f q dot grad F becomes del f y because d f by d t is 0 q dot grad F becomes minus del f by del t minus del f by del t divided by, this becomes the this is your q bar dot n hat. So, you have two things that is the normal component of velocity on the boundary surface and again we have already shown that q bar on the boundary surface if this is if this 0 then q bar dot n hat is 0 if the boundary is fixed.

So, this is for fixed boundary this is another important result that how to get the boundary surface and if you know the boundary surface and if the boundary is fixed then you have q bar dot n hat is 0 that is... Now, with this I will go to very, very nice example. Now suppose I have been given the velocity vector u is equal to minus A in fact, this kind of problem come in what two ways y plus h sin k x into sin omega t and my v is a sin hyperbolic k into y plus h into cos k x into sin omega t if these are the u v.

So, basically these u v represents these are standing wave in water for a basically standing wave in water particularly in a it is a program related to water waves. Basically it is a standing wave, it can be easily seen that here del u by del x plus del v by del y is equal to 0 it can be easily check this. Further you can find easily that your u v del u by del y equal to minus del v by del x sorry plus del v by del x.

So, here it shows the fluid is incompressible here the fluid is incompressible and again our flow is irrotational motion is irrotational our fluid motion is irrotational. Once these two things are satisfied then we can easily find out that. So, what is our velocity potential u or velocity potential, because we have to find our velocity potential, our stream lines and a, because already flow is rotational, so there is vorticity vector will be 0.

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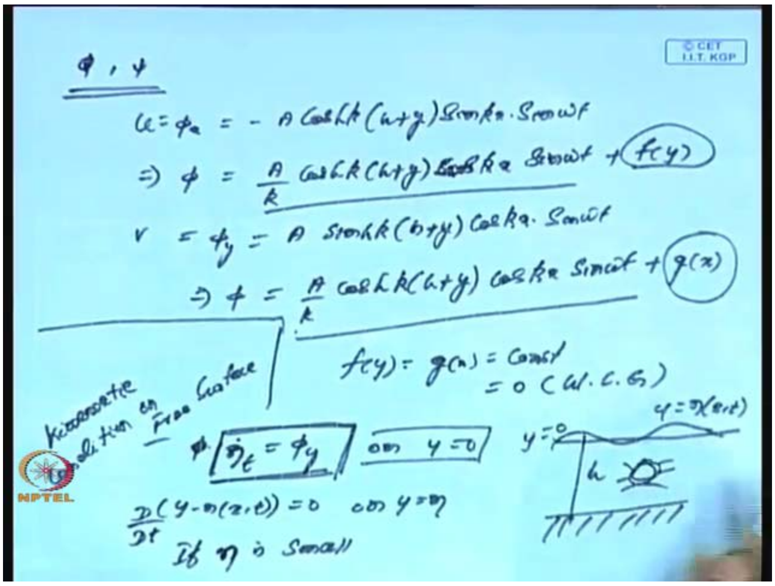

So, we will have to only find what is velocity potential phi and what are the stream lines and accordingly stream functions. So, if we have to find phi we have been already given, we have phi you have been given u is nothing but phi x is minus a cos hyperbolic k into h plus y sin k x sin omega t. And which implies phi is equal to we have phi x a by k a by k cos hyperbolic k into h plus y into cos k x cos k x sin omega t plus f of y again from this is u is equal to phi x.

Further from we have been given v v is equal to phi y which is already given to us that is a sin hyperbolic k into h plus y into cos k x sin omega t. And which implies my phi becomes I have to take the derivative with y becomes a by k since cos hyperbolic k into h plus y into cos k x sin omega t. And which implies plus here it can be constant of g of x because expression for phi this part is same as this part. So, my f y and g x has to be 0 then there has to be a constant is equal to constant on that constant can be taken as 0 without loss of generality.

So, we have got what is phi and once we know phi then easily you can and this very important, because this phi there is a relation in water waves that is phi t, if we phi t plus g t or you can say eta t is phi y that is on y is equal to 0. And where eta is the free surface suppose this is the depth of water, this is what if this is the mean free surface that is y is equal to 0 and here it is the depth of water. And then we can say this is y is equal to eta x t you have just derived that on this surface it can be easily shown that in linear equation d by d t.

Because y is equal to eta y minus eta x t 0, this is the boundary surface, because it is a surface of the fluid. So, on this surface it is 0 just now we have derived boundary surface d by d t this is 0 and that will be that is satisfied on y is equal to eta. And assuming that eta is small, if eta is small if you can say that eta is small, then it can be proved that this condition becomes eta t is equal to phi y is satisfied on y is equal to 0. And that is the again the call the kinematic condition kinematic condition on the free surface on free surface of water.

I will come to this little towards the end when I will come in detail water waves and because what I say that this once we know the velocity potential phi we can know what exactly is our free surface and this free surface because. We know eta t is a phi y already have been given phi y and then we can as if we know what is eta and that is again on y is equal to 0. So, this way knowing the velocity potential we can always come to the know what the free surface is.

Once we know the velocity potential and the free surface then if i have any structure here then I can always calculate the pressure that is happening on this basically the fluid on this structure what is the pressure. So, I can always calculate the pressure, so this is that is why it is very important to know the velocity potential and the stream like the stream functions is important, because it gives us the stream lines will give us the flow direction similarly the velocity potential for potential flow.

Velocity potentials are very important, because this velocity potential gives us that we utilize this to get the free surface elevation. Sometimes we use this even if to calculate the force because we can calculate the pressure by using the Bernoulli's equation I will come to those things in detail later. Because there is a equation for a irrotational flow equation of motion later to Bernoulli's equation and that from there we can calculate the pressure at each point.

And once when we know pressure at each point of the fluid, basically what we call this we will call this as hydrodynamic pressure. And once we know the pressure we can always calculate the force, so so I will come to those equation of motion in my next lectures and once we know this then after knowing the equation of motion Bernoulli's equation and other things then at the towards the end I will come in detail about water waves. So, now, already you have been over of when the fluid is incompressible and the motion is irrotational.

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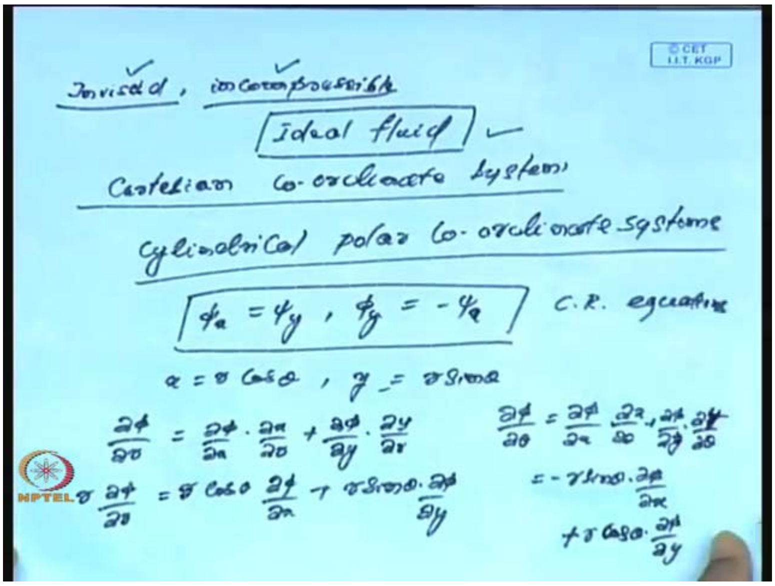

And also another aspect we have not yet emphasized that is when the fluid is inviscid. So, if the fluid is inviscid and incompressible this is another point to be noted I will come to this later because I have not yet talked about viscous fluid. So, when the fluid is inviscid and incompressible we will say that in practice there are very few situations. Where we will come across a fluid which is very 100 percent inviscid or 100 percent incompressible, but for a simplicity of the modeling many fluids we can always under this assumption of in compressibility.

And assume the fluid is inviscid large number of problems can be handled and this is that is why this kind of fluid is called a ideal fluid fluid, where the fluid is in viscid and incompressible we call them as ideal fluid. Now, already, in fact most of the derivation we have done as of date as of now is based on Cartesian coordinate system Cartesian coordinate system without going to detail I will just say what happen if we are looking at polar cylindrical polar coordinate system.

This equation motion I will not go to the details it can be derived from the mini text book you can go through this and independently it can be derived. But, however, I will just say that what will happen suppose we have a very important result that is the relation between phi x is equal to psi y and phi y is equal to minus psi x. Let me say what will happen to these equations what we call the co ceriman equations and that is that is the relation between the stream function and velocity potential.

If what will happen if it is a suppose I say x is r cos theta and my y is r sin theta, if I take this what happened to the flow, what happened to this phi x is psi y and phi y's. So, it can be easily because the derivation what is del phi del r; that means, what is the corresponding forming r theta coordinate if you look at del phi by del r. Basically this is done in this way del phi by del x into del x by del r because plus del phi by del y into del y by del r and that gives us del x by del r, del x by del r is cos theta del x by del r del x by del r is cos theta into del phi by del x.

So, I can always rise r del phi by del r is equal to r cos theta del phi by del x plus del phi by del y del phi by del y del y by del r is sin theta. So, it can be r sin theta into del phi by del y. Similarly if you look at what happened to del phi del theta because I am del phi by del theta, del phi by del theta is same as del phi by del x del phi by del x into del x by del theta plus del phi by del r into del r by del theta. And this gives us del phi by del x del x by del theta, del x by del theta is minus r sin theta into del phi by del x then if I del r by del theta sorry this I this is x this is y so del y by del theta.

So, this is minus r sin theta del phi by del x plus del phi by del y del phi by del y is r cos theta plus r cos theta into del phi by del y. So, once we have now we know that del phi by r del phi by del r is r cos theta del phi by del x plus r sin theta del phi by del y. On the other hand you have del phi by del theta is minus r sin theta del phi by del x into r cos theta plus r cos theta del phi by del y, now in a similar manner what will happen to.

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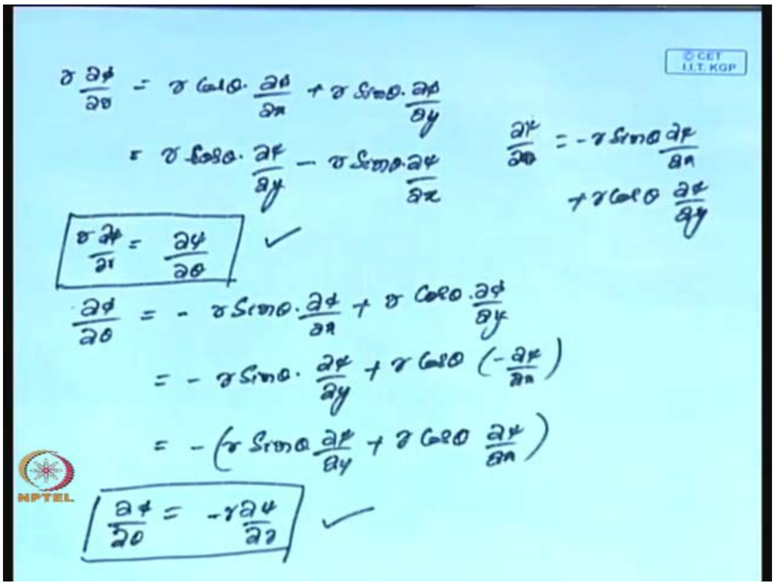

Now, if I go back to r del phi by del r which is nothing but r cos theta del phi by del x plus r sin theta del phi by del y. Now, what I will do del phi by del x is phi x is psi y. So, I can always write r cos theta del psi by del y, then what is then del phi by del y del phi by del y is minus del psi by del x. So, this is minus r sin theta into del psi by del x now if I relate it r cos theta if I go back to my previous place del phi by del theta is minus r sin theta del psi by.

This I can call it as del psi by del theta because in the previous place I have seen del phi by del theta. So, if I replace phi by psi because I know my del psi by del theta del psi by del theta will be minus r sin theta del psi by del x plus r cos theta del psi by del y. So, that will give me r del phi or so, hence I get in a similar manner I can easily get if I proceed in the same manner, I will get it my del phi by del theta I will get minus r sin theta del phi by del x plus r cos theta del phi by del y.

And that will give you minus r sin theta minus r sin theta del phi is del psi by del y plus r cos theta, r cos theta del phi by del y is minus I x minus del psi by del x. And that I can call it minus r minus times r sin theta del psi by del y plus r cos theta del psi by del x and I have if I go back to my del phi by r cos r cos theta del psi by del x plus r sin theta del psi by del y that is nothing but minus del psi by del theta minus r del psi del r. Hence from this two I will get I will get my del phi by del r this is one relation; that means, giving me del phi by del theta is minus r del psi by del r.

And here from here I get which is same as r del phi by del r. So, these two equations are the corresponding equations of the shear equation equation in the cylindrical polar coordinate. In a similar manner I will we can always say that what will happen to the equation of continuity in cylindrical polar coordinate.

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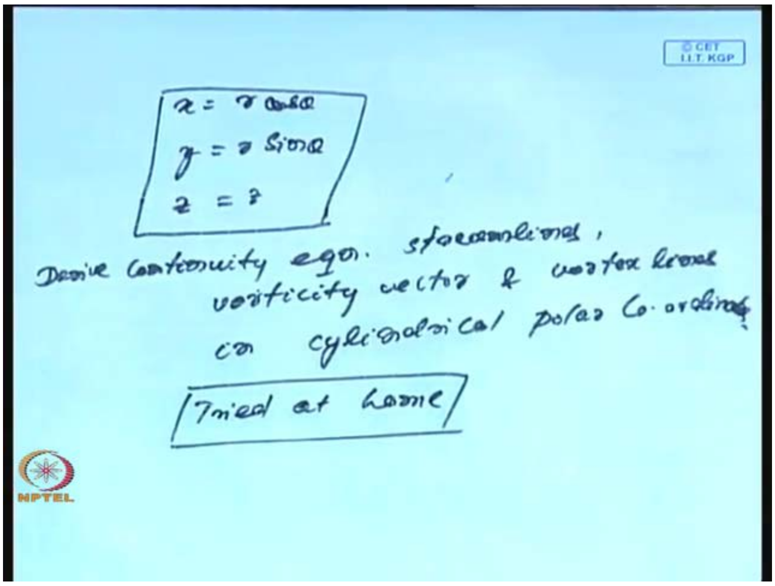

We just take x is equal to in the cylindrical we will x is a r cos theta y is a r sin theta. And z is equal to z if you take this we can similar manner we can always derive what are the continuity equation. Rather these thing I leave it as a homework continuity equation and derive, derive the continuity equation, you can derive the vortex stream lines, vortex line, vorticity vector vorticity vector and still vortex vortex lines in cylindrical polar coordinate.

So, this can be done and I am not going to the details of this this I everyone it is can be tried at home. It is very interesting to do these exercise with these I think I will stop today and next class we will not talk more about this, because we will rather use this in the coming classes we will concentrate on the particularly. In the next class I will talk to we have not yet talked about the motion because forces what are the forces acting on a fluid.

And what about the fluid motion it is something called equation of motion we will come to that and that is based on the law of conservation of momentum. Basically the Newton's second law we will come to this in the next class.

Thank you.