Marine Hydrodynamics
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Lecture - 37
Unsteady Unidirectional Flows (Contd.)

Welcome to you this lecture series on Marine Hydrodynamics. In the last class, we are talking about on steady flows, basically viscous in completion of fluids and flow is laminar, here also the flow is unidirectional, but we can call these as the fully developed flows.

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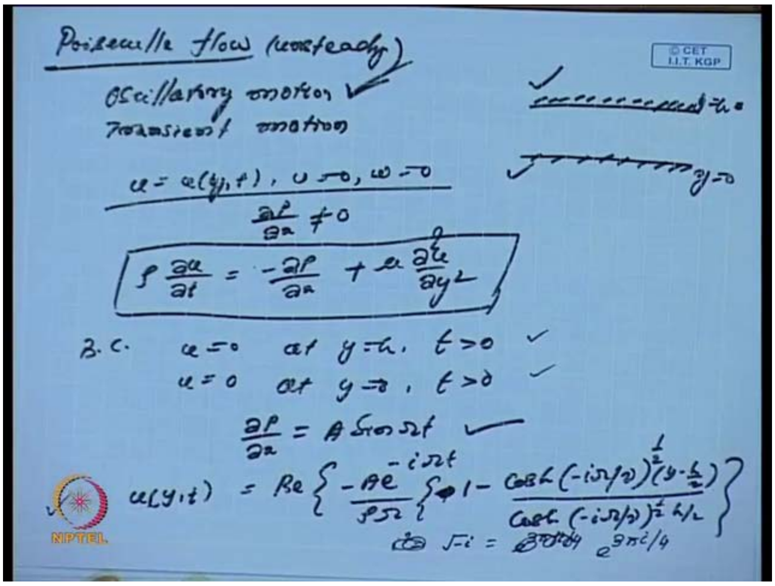

In the last class, we have talked about flow on a single plate as due to the single plate oscillation, either the translation oscillatory motion, a transient motion then we have talked about couette flow both the the case of transient motion as well as oscillatory motion. Today, we will continue the unsteady flow by considering another case that is the what happened to the poiseuille flow, first we consider the poiseuille flow, unsteady poiseuille flow basically, unsteady poiseuille flow.

So, here also will have 2 cases, one is the oscillatory motion and the other is the what will call as oscillatory poiseuille flow or a another is the transient motion, in case of a oscillatory motion, we assume first will concentrate on this. So, in case of a poiseuille flow here, we have non zero pressure gradient have been have 2 plates y is equal to 0, the other is at y is equal to h is the other 2 plates, and here the plates are surface. So, we no slip condition, it is like a channel, we have the no slip condition and we have…

So, the flow is unidirectional, so we have a u is equal to u y t like what we have discussed earlier and we have u is equal to 0, w is equal to 0, again in this case what, we have, we have del p by del x is not equal to 0. And then in the process what we clear, it will get the del u by del t becomes a rho del u by del t minus del p by del x plus del square u by del y square.

This becomes my equation of motion and as then the boundary condition will be get the flow will be I assume the flow is oscillatory. The boundary conditions will be u is equal to 0, but y is equal to h t is greater than 0 and u is equal to again 0, at y is equal to 0, t greater than 0. So, I put the walls there is no slip condition, further we have del p by del x, we assume that del p by del x, in this form or del p by del x is where is the minus will assume minus a call it as A sin omega t, if I assume this, this is oscillator in A.

So, the pressure gradient, it is oscillatory in a as the pressure gradient is oscillatory in nature, which is non zero then what will happen to the flow. One can easily, if I substitute for del p by del x is A a sin omega t then, I will get a u x t u by t rather. It can be obtained u y t will be of this form, we can easily check that, I am minus A e to the power minus i omega t by rho omega plus are into. This is called 1 minus cos hyperbolic minus i omega by n u to the power of half into y minus h by 2 divided by cos hyperbolic minus i omega by n u into power half into h by 2.

And one can easily see that, if you take this one of the solution, then one can easily see that when y is equal to h, this will give me totally, will give me y is equal to s minus this will give me 1, 1 minus 1 will be 0. Further what y is equal to 0, y is 0 means again this will be h by 2, y is 0 in then this will give me 1, in 1 minus 1 will be 0 of this true condition will be satisfied and further.

So, this will be the and again, we have the, we see that this equation is satisfied u by t. So, this becomes the full solution, in this case and it may be noted that here, I square is chosen as sorry, rather minus i routable is chosen as e to the power 3 pi by 4, 3 pi sorry, 3 pi pi by 4 and that is what we chosen in this case, that comes from the physical requirement, which is a this is the solution associated with poiseuille flow, in case of oscillatory motion. Now, let us look at what happen in case of transient motion, in case of a poiseuille flow.

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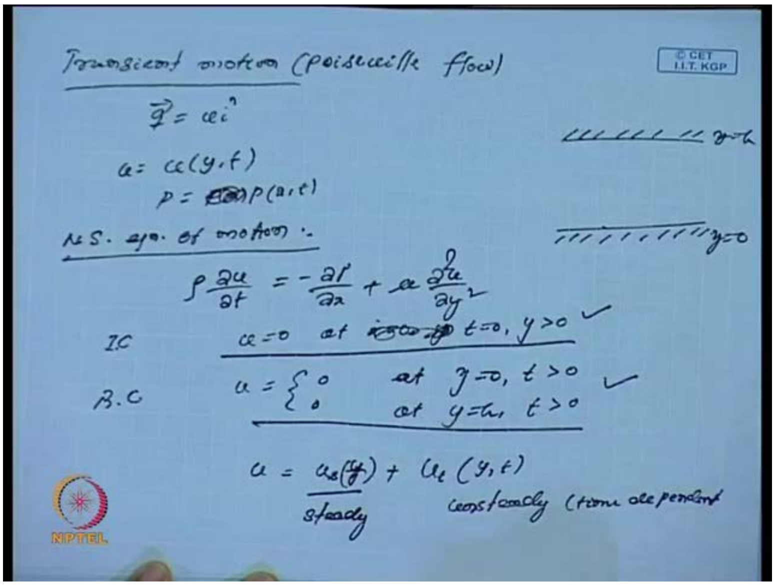

So, if I look at the transient motion, basically for the poiseuille flow, now in the same case proceed in a same manner. The 2 plates are fixed and here, this is at y is equal to h and this is at y is equal to 0 and we have already chosen q bar is equal to u i hat. So, we have u is equal to u y t and we have p is function of p x only or rather, because it is steady motion function, p is a function of x t. And then we have over naves tic equation of motion gives it is just like the previous case only, the I have the del u by del t rho minus del p by del x plus mu del square u by del y square. This becomes the equation of motion.

And here since, we are looking at the transient motion. So, what will happen the solution and the boundary conditions here are also, u is equal to 0, at x is equal to 0 sorry, u is equal to 0, at t is equal to 0 and y is greater than 0. So, initially the transient, initially at time t is equal to 0, u as 0 and u is equal to u is equal to 0, at y is equal to 0, t is greater than 0 and again is at y is equal to s, t is greater than 0. So, this becomes now. So, initial condition, this becomes the initial condition and these 2 becomes the boundary condition. So, this is the I C and this is the boundary condition, thus this gives that at the 2 walls there is no flow and this gives at time t is equal to 0, but there is no fluid does here, it is coming from the no flow condition, but the time t.

So, at t is equal to 0 y greater than 0 is this is 0 and here at y t is greater than 0, y is 0 is also 0. So, now, look into the type of solution it has. So, if you look into that then what will happen and I have. So, what I will do like earlier my total u, the result will be u s plus and this is u s is a function of y plus u t, that is a function of y and t that means, this is the steady motion and this is the unsteady motion, basically this is the time dependent motion, time dependent part. So, this is unsteady, this is steady. So, if I substitute for u equal to then what will happen to my u satisfy, because when we have unsteady motion.

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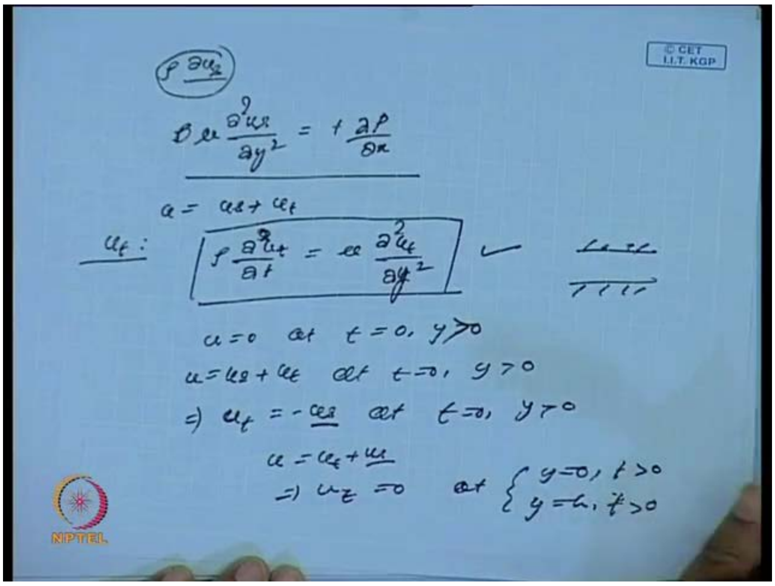

We have seen that my u s satisfies the rho del u s by del unsteady motion, in case of unsteady motion sorry, in case of our unsteady motion my u s satisfies rho u s satisfies gives del square u s by del y square equal to minus a plus del p by del x into mu. And if this satisfy u s satisfy then what will happen to because u is equal to u s plus u t, if u is equal to u s plus u t then what will happen to u t, u t will satisfy it can easily seen that u t will satisfy rho, that is del u by del t rho del u by del t equal to mu del square u t by del t square del y square.

So, this becomes my u t will satisfy, because if add this 2 thing to then, you will get it. So, u t will satisfy this equation. Now, if you look at the boundary condition and the initial condition, we have given u is equal to 0, at t is equal to 0, y is greater than 0 and and our u is equal to u s, u is equal to u s plus u t at t is equal to 0 y greater than 0. So, it is implies u is 0 means that means, my u u t becomes minus u s at t is equal to 0, y is greater than 0.

So, that means, at time t is equal to 0, the time dependent on unsteady motion is same as the the steady state solution on the other hand when that is u t is a minus u s. Further, we have because we have the if you look at the boundary condition at the wall our u is equal to u t plus u s and u t is already u s is 0, in case of a steady motion. So, that means, again u is 0, so that means, u t is also 0 at y is equal to 0, further y is equal to h and for all t greater than 0. So, this becomes when the process what will happen to my. So, in a process my u t becomes, so u t will satisfy.

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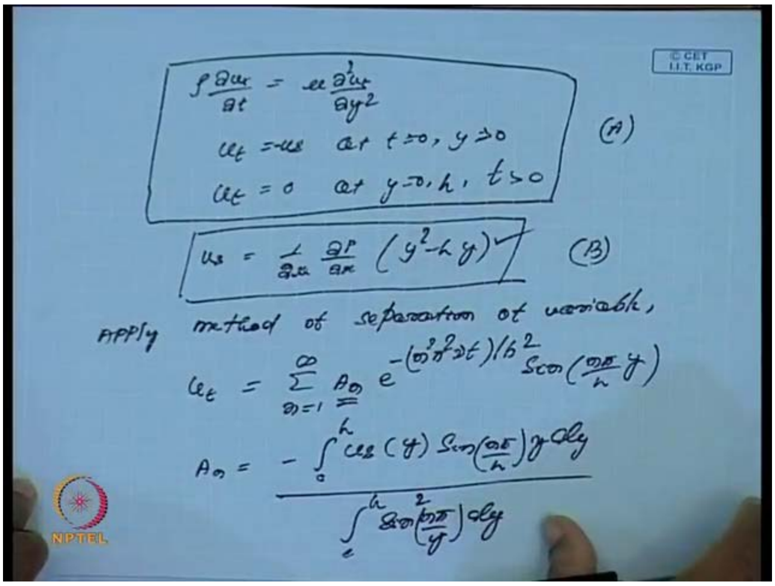

Rho del u by del t equal to mu del square u by del y square and then you just object to the boundary condition at we have u t is equal to u s minus u s at t is equal to 0, y greater than 0 and u t is equal to 0 at y is equal to 0 comma h at t is greater than 0. So, this becomes now initial boundary value problem associated to the u t on the other hand what will happen to u s, u s is the solution that is associate the unsteady motion unsteady poiseuille flow and we have just that earlier, just we have obtained, we have 2 classes, we have just obtained the and poiseuille flow solution. And that solution is nothing but 1 by 2 mu del p by del x into y square minus h y, this is the solution, if I have a this is my u s. So, if I substitute this u s, this is A and this I call. So, substitute for u s in this 1 then I know the initial condition at u t.

And if I do that then apply the method of suppression of variable to this problem, I substitute u s from B in the initial condition in A and after substituting. If, I apply the method of separation variable apply method of separation variable, then your u t will obtain that means, the solution of this here, obtain as integer of this 1 sigma n is equal to 1 to infinity a n e to the power of minus m square pi square nu nu t by h square into sin n pi by h into y.

This becomes measure gen solution where a n's are to be obtained and n substitute that obtained, because we can see that, this u t satisfy both the boundary condition at y is equal to 0, this becomes 0 at y is equal to s. And but u t is minus u s using this initial condition where u s is given by the this, we can obtained what exactly, because I also have this these functions sin n pi by h y is this functions have orthogonal functions. So, that will give me that will give me my A n and that will give me my A n as minus 0 to h u s because I have substituting here u t here, u s that is function of y into sin n pi by h y d y divided by 0 to h sin n pi by y d y sin square n pi by y and if I simplify this. If you simplify this result then substitute for a u s from here.

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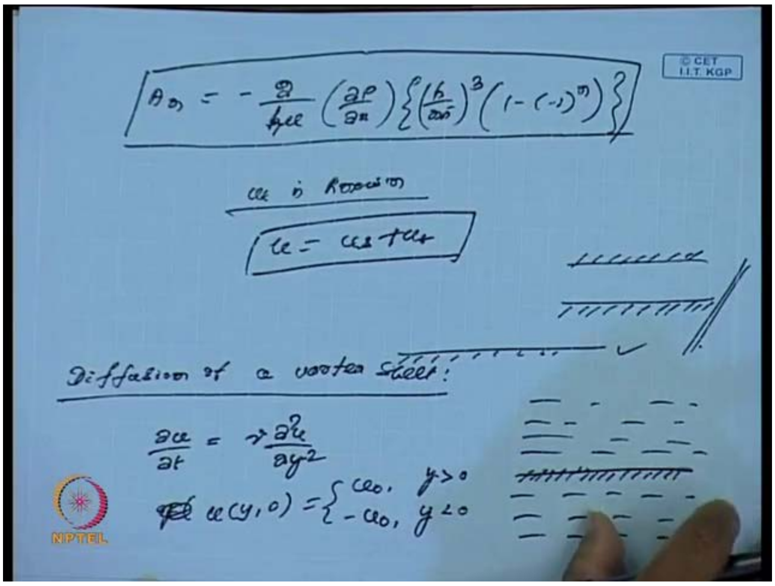

And that will give me that will give me A n as minus 2 by b mu 2 by h mu sorry, 2 by h mu into del p by del x to the power half del p by del x into is h by n pi cube 1 minus minus 1 to the power n and this is A n, these are the n's. So, once n set down. So, obtained u t is known and once u t known. So, my u is nothing but A u s plus u t. So, the full solution is obtain in this case also. And so this is the case of the unsteady solution. So, we have seen the 2 cases of unsteady solution particularly when we have the there are 2 plates or 2 flow between a channel when the motion is unsteady and also we have seen the case of a when just a plate infinite plate, which is oscillating either transient or oscillatory.

So, we have seen that unsteady case and these are all some of the work example for fully developed flow associated with unsteady motion with these only just let me go for another case, which is little more complex. But, I will be method, I will derive this, that is the what the problem is called the diffusion of a vertex sheet, here what exactly happen like, we have seen that when we have applied in the plate was oscillating that is transient motion.

And only, we are looking at the flow that was when y is greater than 0 what will happen, if the plate is oscillating and these plate is oscillating and we have fluid in both the direction, this plate is oscillating and we have fluid in both the direction. That is a transient motion of this plate and this is A. So, here what, I will say that basically my I have a I look at a there is no pressure gradient. So, I have a del u by del t del directional flow. So, del u by del t minus is nu into del square u by del y square.

And here what I propose that my I say that plate is oscillating in such a way that y u, y is 0 what time t is a equal to 0, y is 0 means at a t is equal to 0, this becomes u naught for y is greater 0 and minus u naught for y is less than 0. So, this is oscillating in the positive direction y greater than 0 means the plate is oscillating in these way and if it is y less than 0 as if the plate is moving on this way. So, the plate is moving for y is greater than 0, the ocean is the plate is moving in the positive direction where as the plate is moving on positive direction where as for y is less than 0, the plate is moving on the the plate is moving on the positive direction and then as usual the this is the initial condition apart from the initial condition.

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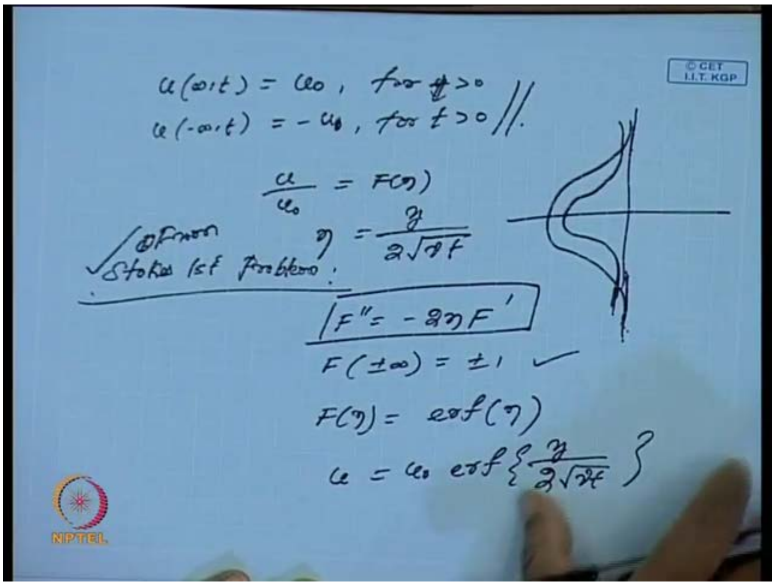

We have u at infinity t is u naught and u at minus infinity t is minus u naught. So, this is the this is for y greater than 0, this is for t greater than 0 and this is here also t greater than 0. So, with a so I have one initial condition at t is equal to 0, there also fluid motion is these under the infinity, this is the motion is like this.

So, then which as like what we have done from the transient motion for a the stokes problem, in a similar manner particularly stokes first problem then, we have substitute by u by u naught is equal to F of eta will follow the self, similarity solution approach as we have done. This is y the case of a single plate, which is oscillating then eta is this then as we have I just follow stokes approach, stokes first problem where a single plate, which is oscillating. Same approach, I will follow then, I will the I obtain as F double dash is equal to minus 2 eta F dash and here, the because in such infinity then see that F plus minus infinity will give me plus minus 1 and further I will have.

So, this will give me to this boundary condition, so that means, I have looking for the solution, I have this equation subject to this boundary condition. And this will give me, if I look at the solution of this, this will give me F of eta e r f eta and just forming the notice on a stokes first problem, I am ensuring that, I have single plate which is a making transient motion. And then will see that u is equal to u naught e r f that is eta is y by 2 root over of eta t, which is the kind of solution will have in this case.

Here, stokes first problem from either I will say from stokes first problem and if you look at the flow. In fact, a we have seen that the flow will be like this like this into let that the line between like this and 1 is for various cases, because the positive as we go ours from this plate, this become u naught, here it will be minus u naught. And again on this side u will be u naught this plate will be moving. So, this is the way the plate, you will be look like and from these, if I look at the because what will happen to, this is kind of flow where, what will happen to is the flow what we look at the vorticity.

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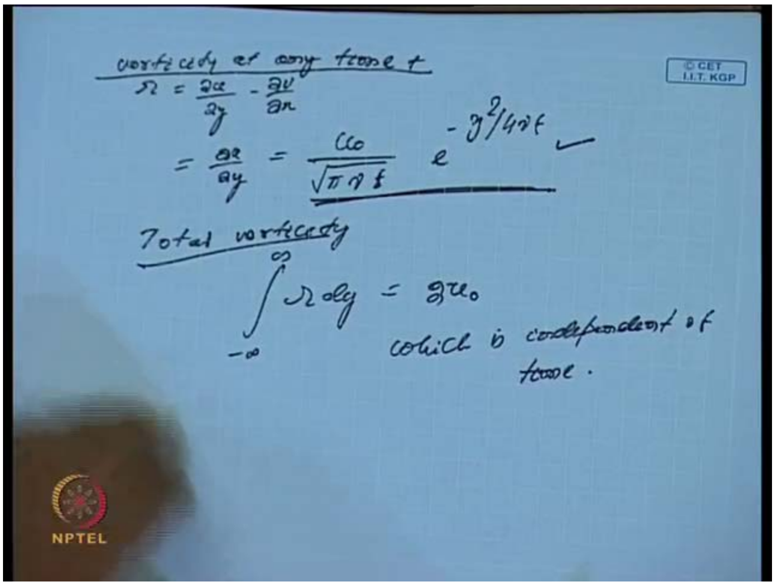

The vorticity will be del u by del y minus del v by del x and these will be 0. So, it will me del v del y, if I look at this then it will give me u naught by pi nu t into the minus y square by 4 nu t. And del v del y will be given this then again, if I look at the total vorticity. This is the vorticity at any time any time t, total vorticity that will give meinus infinity to infinity omega d y.

And it can be easily seen that, if I substitute for these values of omega here then this will give me 2 u naught, it can be checked. So, the total vorticity and which is independent of time. So, with this example shows that the flow is there is a, it is no more a vortex free motion it is a there is a vorticity, vorticity is non zero and a total vorticity at any point, the vorticity is given by this whereas, so it is a rotational motion. So, motivated by this that in case of a unidirectional flow, which can have a vorticity, now let us go to derive one of the very important result, that is called vorticity transport equation.

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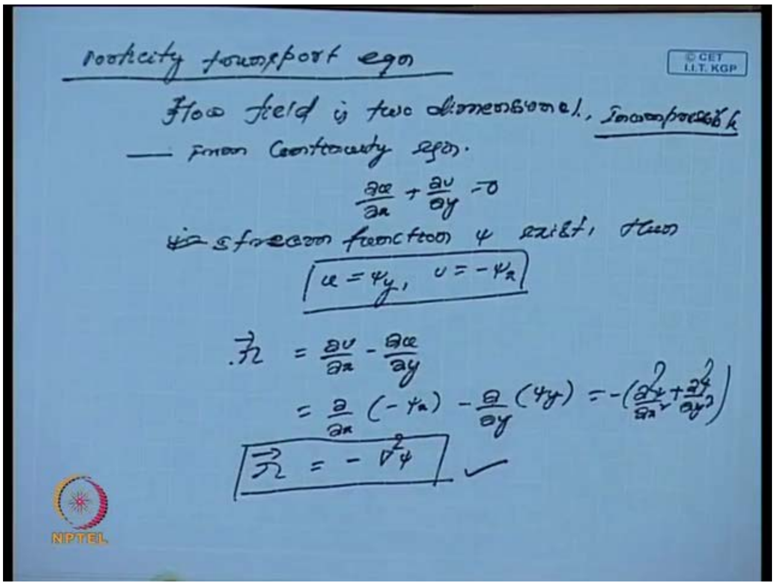

Equation for a compressible in compressible fluid. So, basically if the flow is 2 dimensional, we have seen that when the flow in compressible and discuss even, if the flow is laminar sorry, if the flow is unidirectional vorticity is non zero. And now let us look at the vorticity transport equation, basically what will happen when the flow is a in compressible and vicious then what will happen to the vorticity equation.

2 equations basically, let us consider this as the fluid is flow field is 2 dimensional, if I have the flow field 2 dimensional. So, we have form continuity equation from continuity equation, we have del u by del x plus del y del v is 0, if you assume that I have psi of a stream function that exist, because is the flow field 2 dimensional, I can have a stream function of psi. The stream function psi exist, then I will have u is equal to psi by and u is equal to minus psi x, we all know for a fluid flow.

If this exist then what will happen to omega flow field dimensional flow that will be nothing but del v by del x minus del u by del y, if I do this and my v is what, v is minus phi x. So, this will be del by del x v is minus psi x minus del by del y u is psi y and that gives me minus del square psi by this gives minus del square psi by del x square plus del square psi by del y square. Thus omega is equal to minus del square psi, this is omega bar, we call it and this is.

So, on like the case of a rotational motion in case of a in compressible fluid by where, which is the fluid is rotational is to say del square psi is 0, but here when the flow is 2 dimensional and we have fluid is in compressible and basically, I have not yet. So, for in compression fluid, the vorticity satisfy this. Now, looking at the if you look at that comes from the continuity equation, now if you look at what will happen in case of the equation of motion particularly, in this case of in compressible in visible fluid.

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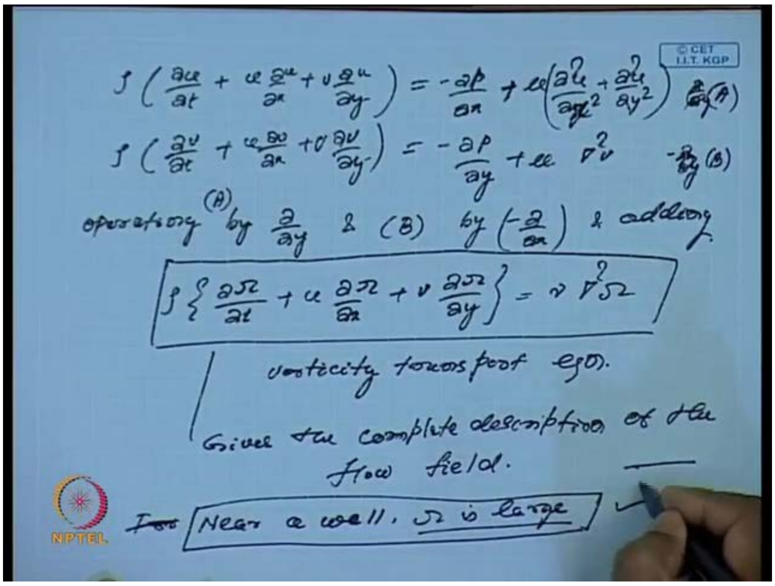

The equation of motion becomes, we have rho del u by del t plus u del by del x plus v del u del u by del x Plus del u by del y that is equal to minus del p by del x plus mu del square u by del rather, I will call it del x square plus del square u by del y square, this is a x square. Similarly, I have the y component it will be rho del v by del t plus u del v by del x plus v del v by del y and that is minus del p by del y, if I observe the body forces in pressure plus mu, I can call that z square the it can be also call it del square f.

So, what I will do, I operate the first equation del square by del y and second equation minus del by del y. If I do that that operate by del by del y, the first equation and second equation minus del by del y and r 1 and 2. Further I call it, this is equation A equation B operating by del by del y operating A by del by del y and B by minus del by del x and adding therefore, add this then what will get will get rho into del omega by del t plus u del by del x omega plus v del omega by del y and that will give me nu.

There is this 2 trans basically, you will see that the pressure transfer will be eliminated and that will be nu del square omega this becomes the equation. And In fact, this is called the vorticity transport equation. So, the 2 equation of motion reduces to single equation. So, in case of when, we have vorticity transport equation of motion becomes a single equation. So, the 3 equation, so one is the equation of continuity and 2 equation of motion introduces, the 3 equation and introduces to 2 equations and another thing is that in case of the vortex of the data transport equation.

This case a this provides the complete report full, this gives the complete description of the flow filed. So, this omega when the motion is small, even, if the motion is a small the omega is undisturbed. So, even if I say omega is a small, this equation, because u is there omega is only one term. So, this equation remaining for so and another thing here, I want to say that the when near a wall omega is large, it can be seen that, this is one of the important observation though. So, basically when you have flow around a body then will see that just near the wall, there will be the vorticity transport, the vertex formation will takes place and there will be propagate.

So, there is a formation of vortex takes place just adjacent to the wall in case of a laminar flow and and these vorticity, these vortex that respond the vorticity, that is produce near a wall that starts propagating in the flow direction. And that is similar to that as if the energy transfer from heated surface. So, what happened again, I will repeat this near a wall the vorticity, that will be produced particularly when omega is large, near a wall omega is becoming the vorticity becomes large and that vorticity. It it get transportant along with a fluid and this the energy associated with these into a similar to as if a heated surface is a that like the transfer of energy in a heated surface, that is the way vorticity transfer takes place.

And with this understanding, I will because will come to this vorticity transport equation little in detail about when, you will talk about boundary lag theory, because the formation of a vortex formation takes place and even if as you give away when see that in case of boundary lag theory how vertex shading flow. Vertex shading takes place vortex shading and flow separation takes place, that will come as a part of the when you will look in to the boundary lag theory.

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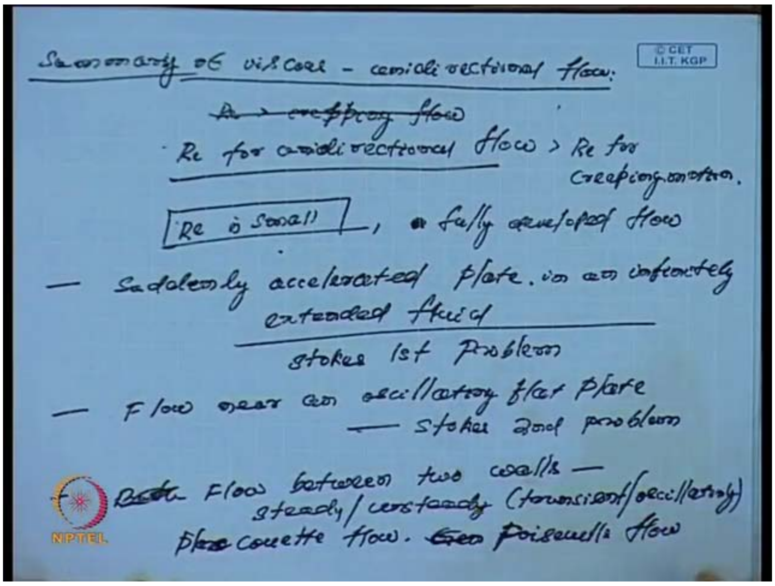

Now with these equation of vorticity equation, now I will go to that to let us me summary among the result will look into the summary of the some of the result what, we have discuss today and in the case of the flow associated with the first I will a briefly summarize, but all we have discussed. So, will call this summary of viscous unidirectional flow, in fact, when I started the unidirectional flow, I say the Reynolds number is greater than creeping flow motion.

The Reynolds number associated backup rather, I will say Reynolds number for R e for in unidirectional flow is greater than R e for creeping motion. However, basically when motion is slow motion. So, even if it is R e is greater in this case, but still the R e is a small, because here again, we are also not considering the convective wave of set of term, R e is small, there is a convective, we are not including here and again we are considering, the flow is unidirectional in this case what if the fully developed flow.

We concentrate 2 types of problem, the first type, we call when 1 wall is first, we talked about there are 2 cases. Suddenly accelerated plate flow due to suddenly accelerated plate and then we also talked about and particularly. So, suddenly accelerated plate in an infinitely extended flow fluid and this problem is is I put as stokes first problem.

Basically, we have only one plate, which is oscillating at the time t is equal to 0, suddenly there is no speed and suddenly, it start to oscillate. And then we grouped into in case of flow near a oscillating flat plate that means, in this case flow was transient that is infinitely extended plate at the flow was a transient plate. And in this case again the plate was there, but the flow was oscillated in nature, because the flow was oscillated and this problem is we call stokes second problem.

And then we talked about the third category of problem, we talked about third category of problem, we talked about today that is a what I call this diffusion became, today is I talked about to, but what vorticity diffusion equation, I talked about today. So, then I have another problem, we talked about flow between the wall, flow between a wall 2 walls, basically flow in a channel.

And here, we also talked about when the walls are placed and here, we also talked about when the wall surfaced both the steady and un steady summation talked about in case of unsteady problem, we talked about, which flow, which are transient or oscillating. And in case of a and there, we there are flow, which we have consider circle stokes that is coquette flow generalize coquette flow plain coquette flow then we talked about generalized again, we talk at a hagen poiseuille flow. Then we talk about, we have talked about to generalized these 2 cases, we have talked about the steady and unsteady motion.

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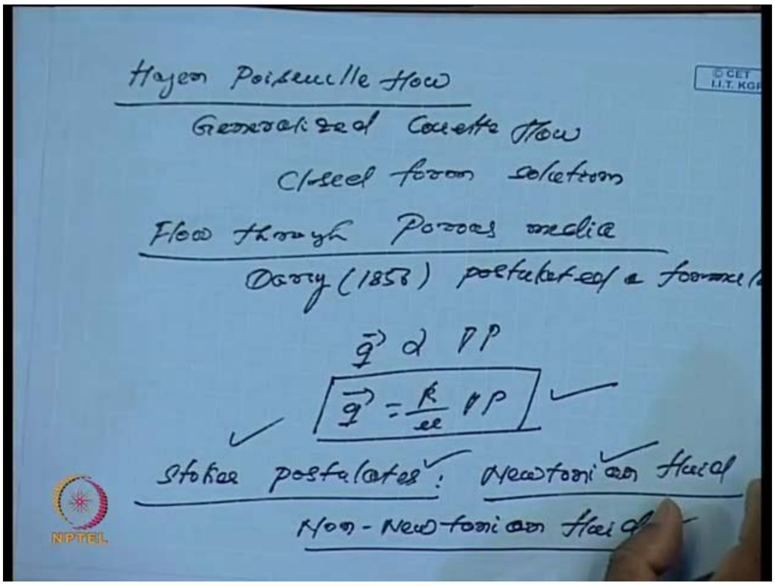

Then we have talked about hagen poiseuille flow, we have talked about hagen poiseuille flow and after hagen poiseuille flow, we have again talked about generalized coquette flow. So, all these cases, we have a we have been able to get closed form solution and closed form solution where, there of you problems and impact, we assume the flow is unidirectional or a fully developed flow.

Now, whether further flows like, we if we look at flow through porous media, flow through porous media basically, which is based on darcy's law. Basically, the darcy's postulated this formula by darcy's in a 1856 get the postulated and a formula for flow through porous media, that formulas as q where is equal to there is grade of P that means, q bar equal to k by mu that P where k is first to the permeability of the medium and mu is the viscosity dynamic viscosity of the fluid.

So, in these basically when you look at the flow in the rocks, even if a will see that when flow passes through some structure, they will follow this characteristic. It is the first 1, it is a motion is slow. So, this is a was case of a then another type flow, because all these thing our flow is based on stokes postulates as as a you hema navel stokes equation based on a stokes postulate.

And these are called as newtonian motion, newtonian fluid, it is not necessary that all flow will follow Newtonian, newtons flow are stokes postulates. There are flows, which will not follow stokes postulate and will call this kind of fluid as non Newtonian, there are be as if non newtonian flow, but we are not going to such details, but I say that the fluid, which is in the navel stokes equation is based on the stokes postulates and these fluid are newtonian fluid and ah there are other fluid. The fluid, which will not from the stokes postulate will be called as non newtonian fluid, basically then we have talked about to is actually, 3 dependent methods basically, all these problems, we talked based on 3 measured methods.

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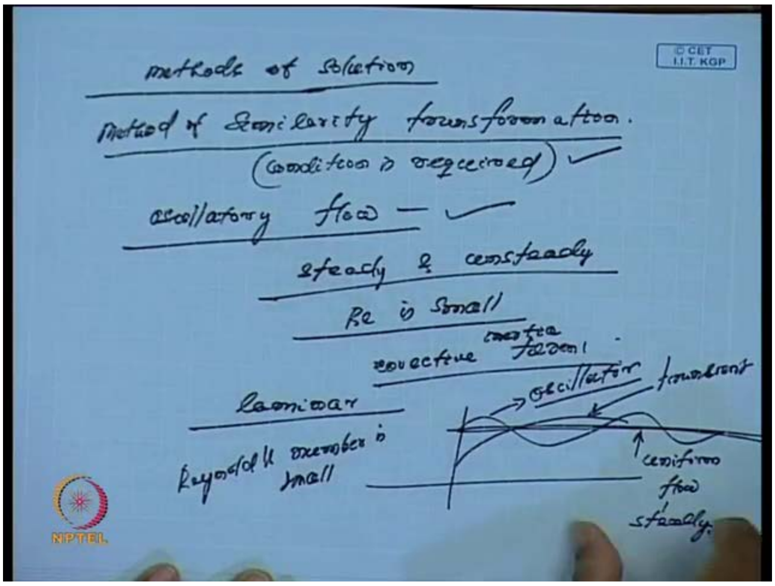

Analytic methods of solution, if you looking to we have based on 3 methods, basically one we have talked about similarity transformation method, similarity method of similarity, transformation. And this method in fact, this is one of the method, which is a reduces the dimension of the problem to large extent and of course, this is a there are certain condition to be followed for the flow on a certain condition is required when we have to follow the similarity transformation.

But, this is a 1 of the advantageous that of this method is that it reduces the dimension of the problem by 1. And basically, this is more suitable for large class of problem for laminar flow can be handle by this and another in case of a oscillatory methods for oscillatory flow. The problem illustrate to boundary value problem only and having a non homogeneous, it is becomes a non homogeneous boundary fully problem and it becomes very easy to solve, but we have seen in case of simple problems.

And then another aspect of these the 2 kinds of flow what, we have discussed that is a steady and unsteady. In all the cases, R e is small or concentration goes on unidirectional flow and also R e small, we have taken the convective terms, we have neglected. Convective be in a set term then and this called as laminar as I have already mentioned for a laminar flow, the flow follows certain direction and follows the regular path. The flow can that can be a transient, it can be oscillatory, it can follow a steady path.

This is this is oscillatory, this can be transient motion and can follow a uniform speed uniform flow. So, that the bases laminar flows and this oscillatory flow, it follows a periodic pattern and this is a uniform flow or a steady flow, basically this motion in study, that all becomes in case of laminar flow and when Reynolds number is large small.

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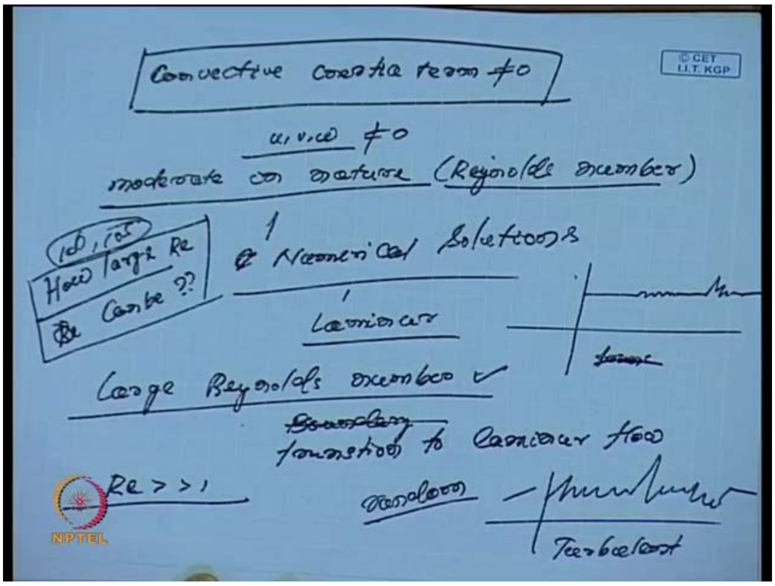

That is a large class of problem particularly, if you look at, if I say that convective term, convective inertia term is not convective inertia term is not equal to 0 that has a large class of problem where, the convective inertia term is not equal to 0. In such situation the flow will exist the components of u v w will exist and in fact, large class of problem can be handle on these and in most of the cases, it is the Reynolds number is moderate in nature for a moderate in nature.

R e Reynolds number and large number of and here, the closed form solution may not be a possible and most of the time will go for numerical solutions and even if although, we go for numerical solution and in this case the flow can still in laminar, but again. So, this is here the Reynolds number is moderate, on the other hand again sorry, for a larger, Reynolds number, this will Reynolds number becomes large then what will happen there is a bounded layer theory.

So, before going to that when there Reynolds number becomes large then there is transient total laminar comes like suppose, we have a flow, which was initially going on the steady flow whenever Reynolds number will be large. And suddenly, there will be disturbances will be formed and this is called the transition intermediate as a Reynolds number goes on increasing, this is transition tool laminar flow laminar flow of course.

And beyond that when the Reynolds number becomes extremely large then the flow pattern, they will not be again a basically for a various largely Reynolds number the flow will be very it is random. And particular does not follow any particular path and then the flow is called this as a turbulent, basically again here. In fact, there are situations when these Reynolds number is large that varies from case to case the question comes how large it can be, how large the Reynolds number can be, R e can be.

And these all depends or the nature of the physical problem one is dealing with. So, and these Reynolds number, because it can go up to 10 to power 6, 10 to power 5 that order and it can be more than that. And even if there are situations like this since particularly when you look at a flow, on a flat plate laminar flow again, we have in between this a in this large number, we can have a situation where, we have boundary layer formation of bounded layer that will take place.

And in the next class will talk about how the boundary layer formation takes place what is boundary layer thickness and how the equation of motion particularly, the laminar boundary layer, the laminar flow what we have discussed the equation of motion for viscous fluid, how it will change then there is formation of the boundary layer. And then beyond boundary layer when the Reynolds number goes increases, further then will see that turbulent layer flow becomes turbulent. So, let us stop here, the next class will emphasize on boundary layer theory.

Thank you.