Marine Hydrodynamics
Prof Trilochan Sahoo
Department of Ocean Engineering and Naval Architecture
Indian Institution of Technology, Kharagpur

Lecture - 31
Gravity Wave Transformation & Energy Relation (Contd.)

Welcome to this series of lectures in NPTEL on Marine Hydrodynamics, in the last class we have talk about wave gravity wave micro problem. Basically, we talk about the Laplace expansion formula and today we will just see a similar one application on Laplace expansional formula, that is what the reflection wave reflection gravity wave reflection by a wall basically we will consider here a vertical wall.

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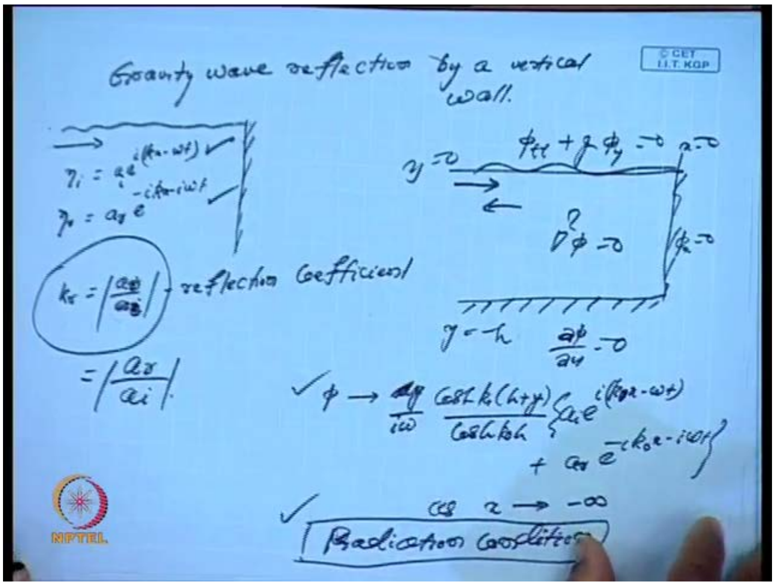

Suppose, I say that, I have a wall here suppose, I say I have a wall here, I have a vertical wall and I say that, I have a wave which is a propagating and it is hits the wall. If I say this is my incident wave, there is a incident wave I call this eta i is a e to the power a i e to the power i times k x minus omega t. Suppose, this wave when it hit the wall, there will be a you all know there will be reflected wave, on that reflector wave will be call a r e to the power i times minus i times i k x minus i omega t.

Then here, what we are looking at as an application of the expansion formula developed in the last class. I will see how what will be happen to k r that is, a i by a r that is a what, I mean the reflection coefficient when a wave basically, so here what we want to study that, when a wave and I call this as a wave reflection by vertical wall rather gravity wave reflection by a 1 by a vertical wall. So here, I am interested in knowing what is k r, a i sorry a r by a i basically, this is the amplitude of the reflected wave to the amplitude it is the ratio of the amplitude of the reflected wave to the amplitude of the incident wave, this is i.

So, this is basically a r by a i so now, what we will do here, if I look at the I will take the problem in this way. So, we have a wave, we have wall here so here, I have what is happened here, I have delta square phi 0 in the upload region and I have as usual phi t t plus g phi y e 0 on the free surface that is, on the main free surface phi is equal to 0 and here, I have a vertical wall. On the vertical wall, I have normal velocity 0 so, phi x is 0 and this suppose, the wall is look at delta x equal to 0.

And then, if I just set after my water is at depth finite depth suppose, y is equal to minus h (( )) bottom surface then, I will have dell phi by dell phi 0 here, on the bottom. So, this becomes my mathematical problems so, I have to know, what is phi and I have been given that, eta is the incident wave and eta resident reflected wave. So, the process that will give me that, what happen at the power fill this inti this information will provide me, what will happen to phi at the power fill that relative behave like from this relation of eta I, it will be a g by i omega e g by i omega cos hyperbolic k n naught into h plus y divided by cos hyperbolic k n naught h and that will be…

So, the a r let I will take a i e to the power i k n naught x minus omega t plus a r a to the minus i k naught x minus i omega t. So, this will be the behavior as x tends to minus infinity because, this side is x is 0 so, this wave this coming from minus infinity initially, like this wave which coming from incident wave is coming from minus infinity and it get reflected. So, this is dam this is this a i is associated the other a i here, if I look at these, a i is nothing but, the wave that is, amplitude of the wave that is associated with the incident wave and this is the amplitude a r is amplitude of wave that is a associated with the reflected wave.

And since a eta i is this and eta r is this like this so, my phi is of this form and already I have seen that, this is near boundary will have a problem of this type whereas, infinity and this also sometimes call as the infinity condition or the radiation condition. The condition at infinity is called sometimes condition at infinity or the radiation condition often, we call it as a radiation condition so, this condition is called the radiation condition.

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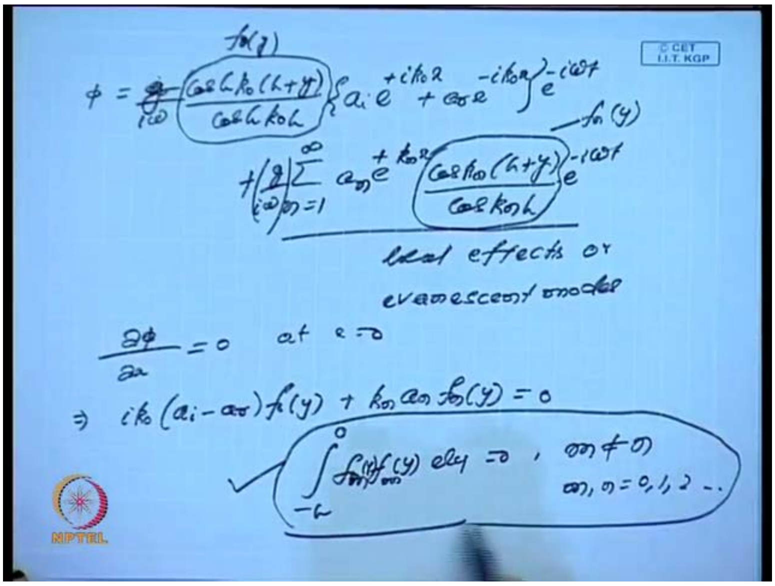

Now, if these if I know like in the pervious class, what we did if I just like the pervious class, if I have the full expansionary potential. What it will give me may phi will be, it will be i g, whether g by a m, g by i omega cos hyperbolic k cos hyperbolic k naught into h plus y divided by cos hyperbolic k naught h into, this is not a, this is g by i omega into this is a i into the minus or plus i k naught x plus a r e to the power minus i k naught x into minus i omega t plus.

Because, as I have seen in the expansion formula last time, I have seen this is n is equal 1 to infinity because, there we have local effect, which we call the a one side mouse and this also satisfy and that form will give us, I call this as a n e to the power minus k n x cos k n into h plus y divided by cos k n h and that is what, into e to the power minus i omega t. We can see that, if I look at the radiation condition then, as x tends to infinity it should be plus sign and this is a as x tends to infinity, x tends to infinity what will happen, this term will contribute to 0 we can see e to the power of x is negative, x tends to minus infinity.

So, this term will tend to 0, so all this terms will contribute to 0. And in the process, this part remains the wave part so, these are call the, this part of the solution what I said the local effect or the evanescent modes, this is all the evanescent mode and these are progressive insulation on this. So, basically this represents the wave part that is, propagated in the x direction and this represent the local effect.

So, in these sense, now if I say that, I have another condition which you said that, my delta phi because, that is a no near the wall, the wall is a vertical wall and there it is a delta phi by delta x is 0 at x is equal to 0. If I have these then, what it gives me, that gives me so, like in pervious case if I take, this will give me i k naught, which implies i k naught into a i minus a r. And I call this as, these term or there is a i g by omega g by i omega here also.

And then, I just call it i k naught into a r minus a r, I call this term as my f 0 like I have done the pervious in the previous case, this I call this x 0 y and then, I call these term my f n y. So, if I say then, what will happen i k naught a 0 minus a r into f 0 y then, plus here I will get k n and a n f n y, that will give me 0. So, if this is the case then I know, I have already seen that in the last class, we already talk that minus h to 0, f 0 f m f n f m y into f n y d y equal to 0, m is not equal to n, whether m n 0, 1, 2.

This I have seen it, last time I have shown it while deriving the (( )) formula and once use, you will utilize these terms out and substitute here. So, what will do, will integrate with a multiply with f 0 and then, integrate over minus x to 0 like this. And that will give us, if I apply this basically so I apply this relation, this relation if I apply what it will give me.

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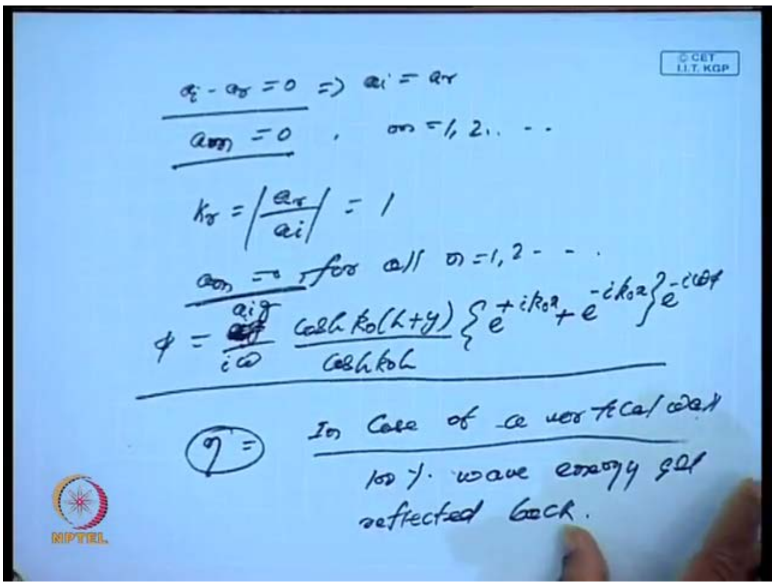

That will give me a i minus a r is 0 and a n is 0 it has first time you, first you do the f 0 multiplication, first multiply with f 0 the expression and then, you will see that the first term f 0 f 0 is non 0 where, f 0 f n is 0. So, that will give you the this term and then again, we multiply with some m f m y and then, you do the integration for all minus h to 0 for m is equal to 1 to infinity. Then, you will see that, a m is 0 in all basis because, the right is 0 and when c 0 so, that means a i minus a r is 0 that will gives me r k r, and which implies a i equal to a r, which implies k r is a r by a i modulus and let this be 1.

On the other hand, what happen when all a n is 0 0, for all n equal to 1 to these things so, my phi becomes i g by omega r g by i omega into cos hyperbolic k naught into h plus y by cos hyperbolic k naught h into I can call this a i, I will take a i comma the other call it a i sorry a i g into integral plus i k naught x plus into a minus i k naught x into e to the minus i omega t and that is what. So, here I am getting, what I am getting here, that in the process because, the m n segments all n’s are 0 m’s are 0.

So, the m n segments have n contribute so, local effect becomes there is no contribution from the local effect, only the progressive wave solution remains. And did I look at this phi then, what will happen to my corresponding eta and in this case also, I saw that k r is 1 that means, all the waves that is incident on the wall get reflected. So, that is why, in case of a vertical wall in case of a vertical wall, if a progressive vertical wall when a progressive is incident, there 100 percent reflection takes place.

If the wall is vertical that means, there is no dispersion of energy, there is no energy loss or gain of energy and all the energy, that is all the wave energy that hit the wall get reflected back that is. And impractice, it is always it has been found that, 97 what, near a vertical wall 97 percent 97 percent of a wave energy get reflected. And the other 3 percent may be converted to sound or another energy, what ever essence in the context of this analysis in the (( )), you also finding that 100 percent energy get reflected, 100 percent wave energy get reflected back. Now here, I have a question comes, what happens because, when a here 100 percent but, if you look at the corresponding eta.

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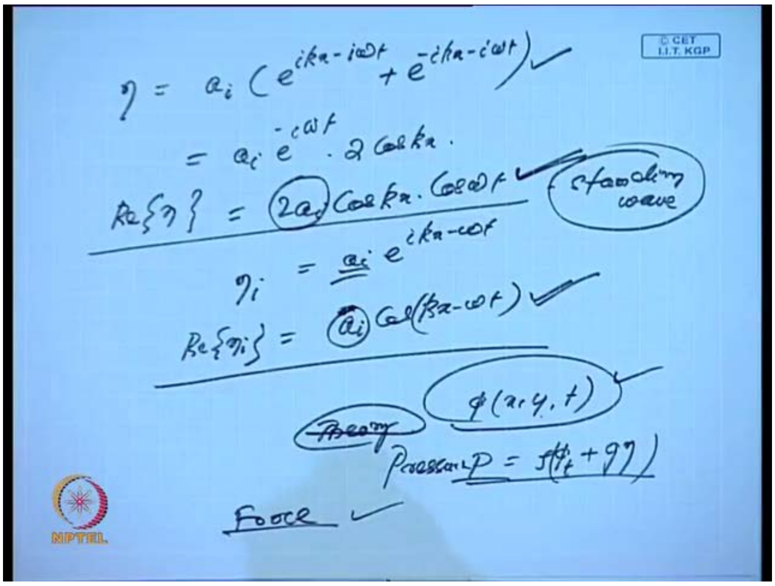

Because, we have a eta which is a a i is a r so, I call it a i into e to the power i k x minus omega t plus e to the minus i k x minus i omega t. So, it will give me a i e to the power i omega t and it is i k x plus it have minus i k x that will give you 2 cos k x. And if you look at the real part of eta, that gives me 2 a i, this is cos cos k x into cos omega t, that is what eta amplitude of this superstition of waves. That means, when the initially the wave that was hitting the wall is eta i is a i e to the power i k x minus omega t for the real part of this.

If you look at the real part of eta i that is, some a i because, I am looking at the real part into cos k x minus omega t. So and then, the resultant wave of this nature because, these icons assume that, this is a real number so, here also here also a i, as if I consider a i is as a real then, this can be will say real number so, no problem. So that means, if I have a wave which was a published wave, when it hit a wall, the resentence wave is this on a vertical wall and this is same as this and that is shows that, the resentence wave is a standing wave.

So, standing wave and I have talk about standing wave must be for in the early stages but here, we have seen also that, the a standing wave is generated particularly in case of vertical wall. No energy get, no local effect is becomes negligible and there is no loss of energy or no energy is converted to energy, does not get converted to others forms of energy. So, all the waves and then here, the amplitude of this wave is twice the amplitude of the incident wave.

So, this is what, we learnt from this example and in fact, this expansion from only have (( )), this is one of the simplest example to show, how Laplace expansion formula can be apply to where, propagation problem near a vertical wall near a vertical wall. And the same concept is used to apply to vertical structures of basically flexible for our structures and also to see walls or the quarters and any in fact, in ocean engineering also for calculating the wave load on a structure and many other applications. So, that is what we do here and often this approach is call the theory of reflection.

Now, this approach I call this this is in fact, this is approach is a straight forward approach basically, to obtain the total velocity potential phi associated with the problem flow problem. And once we know the total velocity potential phi then, you can easily know what is at the pressure, that is on a wall and once we know because, pressure we know p equal to phi t plus g eta, phi t plus g eta, that is a called the pressure. And once we know the pressure at any point so, you can again calculate the force, this I call the pressure as a P will be these.

And then, you can calculate the force on the wall by integrating the pressure about the surface area and that will give us the force. So, wave load on a wall can be easily calculated so in fact, these concepts is used in a various cases, sometimes this is this concept of problem wave scattering or wave diffraction is used to calculate the wave load on many structures. Particularly large of source structures where, the wave amplitude or wave length is much smaller than the size of this structure.

So, this is a this method approval this is although a simple example but, using this expansion formula problem, we can also calculate the wave load on a structure basically the wave force, when I say wave load, it is basically wave force. In a cell we often, what is the scatter potential, what is the reflector potential and what is the local effect and after knowing the local effects and all these individual terms in the expansion formula, in that case, they bending on the nature of the boundary conditions.

We always get the phi and once we know phi, we get find the pressure and then, find the force. Often this is called the, for wave load calculation this is called the theory of diffraction, method of diffraction so, this is I will come to that, may in the next class or later. So, with these I will just talk about because, though we have given two examples, one is the expansion formally and then, this wave is placed by a wall to understand what happing exactly when a wave, how to calculate the total potential.

Now we so, knowing this reflection and the scattering problem or the the reflection and the wave method problem, now I will go to look into the scattering problem. So, what happen in case of a scattering problem thus these two, last lecture this lecture it talks about, how the wave profanation takes place and what are two process one is the reflection wave talk today and now, I will talk about the scattering.

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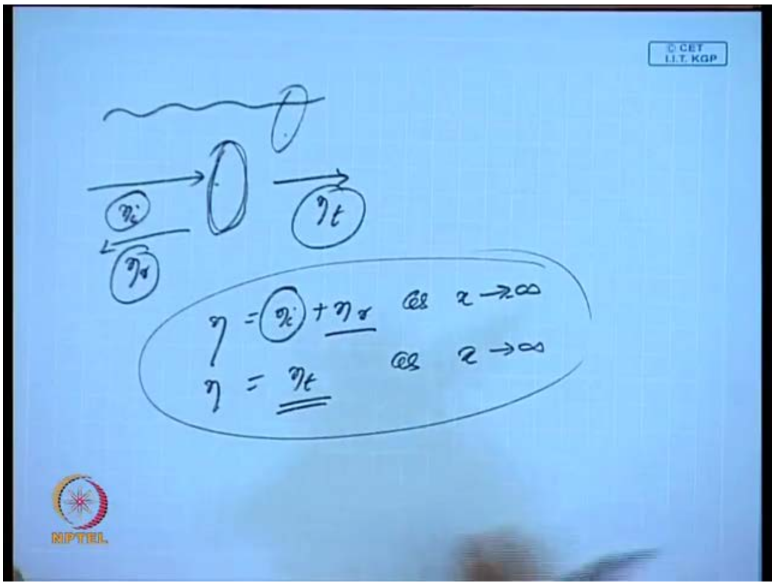

In case of a scattering, what happen as a better mention these and suppose, I have any structure, whether it is a submerge one, whether it is floating one, in any structure. What happen here, I have a wave which is incident on the structure, incident potential incident wave and there is a wave, which is a reflected wave and then, we have a wave which is the transmitter wave. So, three things here, eta i is the incident wave, eta r is the reflect wave and finally, eta t is transmitter wave.

Once we have this and here what happen, my eta becomes eta i plus eta r on the left side, as extend to infinity minus infinity and eta is equal to eta t as extend to plus infinity. So, when plus, this is the radiation behavior and then, you will have the local effects associate local effect. So, that is why and this is the process in this suppose, we know the nature of the structure if we know the nature of the structure and assume, the structure is known.

Then, how much wave energy get reflected and how much wave energy get transmitted when a non wave of amplitude, non wave eta i hit this structure, that is what we do in case of a scattering problem. Now, today I will discuss very simple example of a wave transformation basically how the, here I basically I concentrate on the scattering, this scattering of waves due to change in the bottom geometry and that will give us… And then, it related a relate the same with the energy relation so, what will do here suppose, to do so, to do this, I will just look at one of the aspect of the problem is.

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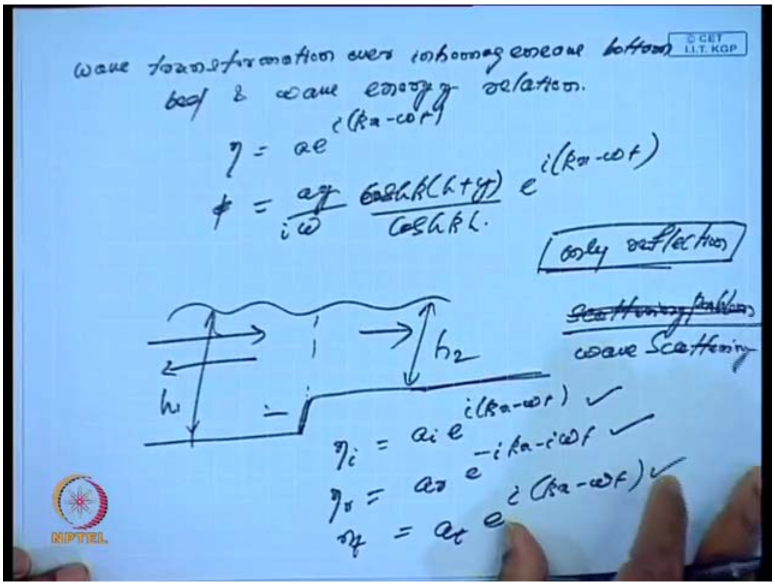

That is, I will consider let me say that, as If I will talk wave transformation over inhomogeneous bottom bed over inhomogeneous bottom bed and wave energy relation, how they are related. So, today that, we all know that, if eta is a e to the per i times k x minus omega t, a corresponding phi is a g by omega i omega rather, into cos hyperbolic k h plus y by cos hyperbolic k h into e to the power i times k x minus omega t, this is all you know.

Now suppose, I have a suppose I have wave and this wave propagate over a depth of a (( )) say that, from depth of water h 1 to a depth of water h 2 then, what will happen. When this wave and this is the point of line of demarcation because, this is where the depth abrupt changes in the water depth. And let me say that, I do not I consider only wave reflection, only reflection takes place. This is only I only consider the wave reflection, only reflection of wave take place due to the wall.

So, in that case, what will happen so, basically I have two things, wave will be reflected, one wave will be hit coming because, I have a change in the topography so, a part of the wave energy will be reflected, another part of the transmitted to the other side. So, this will be also this problem will also can recalled as scattering problem, is refer to as a, rather we will this as a wave scattering, this is a wave scattering problem. So, in this case, what will happen so, I will have my eta i and eta i will be a i e to the power i times k x minus omega t.

And I will have also eta r, that will be a r e to the power minus i k x minus i omega t and I will have my eta t, I will have my eta t that is, a t e to the power i times k x minus omega t. So, if these are the things then, what will happen here, if these are the incident wave, this is the reflected wave and this is the transmitter wave then, what happens in these case then, what will be the corresponding phi.

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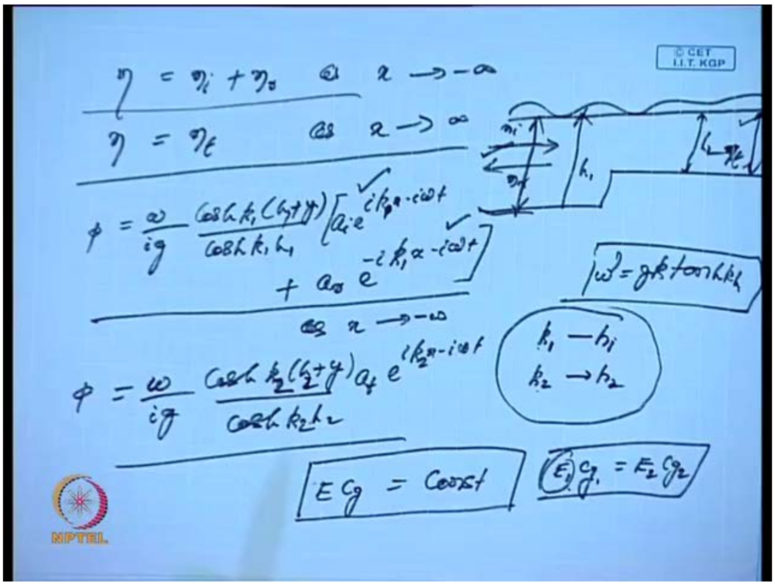

So, and in this case my eta on the left side will be eta will be eta i plus eta r has extend to minus infinity. Because, this is the wave which will propagate to the far filled far end on the x side, on the negative side and eta will be that, I call it as eta will be eta t as extends to plus infinity. So, if I just say that, the wave is the wave is propagating from here and the wave that will propagate to the (( )) is, again this is eta i, this eta r and this is the eta t, that is what wave will have.

So, here I am only considering these things now, if this the case then, what will be the corresponding phi at phi per fill, that will be again omega by i g, this will be cos hyperbolic k. And this left side, I call it k 1 because, there is change in depth I call it k 1 into h plus y by cos hyperbolic k 1 h into a i e to the power i k x i k 1 x minus i omega t plus a r e to the power minus i k 1 x minus i omega t. And we clarify here, one thing is that, why I call it k 1 because, in all these things where, here there is a depth changes, these water depth initial water depth is a h, I call it h 1.

And in the initially, this was a numeral product, there is approximate change in a water depth and then, the new water depth is h 2. That we all know that, omega square is g k tan hyperbolic k h, they will satisfy in both the regions, if we need for the same wave, the frequency of the wave remains the same. When the h changes h changes from h 1 to h 2, k will change from k 1 to k 2. So, k 1 refers to the wave number associated with the water depth h 1, k 2 refers to the wave number associate with the water depth h 2.

So, because of this evident sine wave, frequency remains the same the when the wave propagate from the deep water from depth h 1 to depth h 2, the wave number changes and this already, I have talked about in some of my earlier lectures lecture on waves. So, they sent so, this is the wave, which you propagate to the the potential associate till the (( )) this one that is, as extends to minus infinity.

Similarly, what will happen to the potential which will be, this is basically the condition at infinity, what we say sometimes omega by i g into cos hyperbolic k 1 into h plus y and this will k 2 because, the right side is I have told that, k 2 this is h 1 this is h 2 cos hyperbolic k 1 h 1 k 2 h 2 and this will be again a t e to the power i k 2 x minus i omega t. So, this symbol is very because, k 1 is associate to h 1, k 2 is associated h 2 so, this will be the potential, proper behavior on the left end this the proper behavior at the right end.

So, now, if we apply the energy relation to this, what is the energy flux flux, from the laws of conservation energy plus we have, I have already told you E C g is constant, the total energy flux is constant, wave energy flux that passes through anywhere. So that means, the wave that will be, if you calculate the energy per unit area or unit length here, is same as the wave if I calculate at this end, energy energy flux will be same. So, that may I can call it E 1 C g 1 is same as E 2 c g 2.

And what is E 1, C g 1 is the group velocity of the wave that propagate from this end, C g 2 is the group velocity of the wave that will propagate here. And E 1 is the energy that is associated here but, on the left hand, this E 1 is a combination of two waves, one is the incident wave potential, one is the reflected wave potential and both are in the opposite direction.

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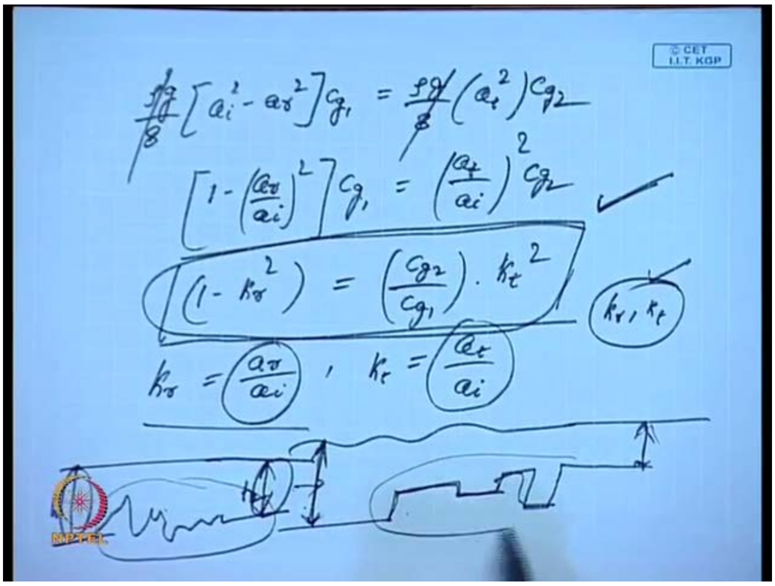

So, because of that, what will happen the same equation what it relate to E times E rho g by 8 into a i square minus a r square. Because this energy, initially the incident wave energy is a propagating in the positive direction, the reflected wave energy is propagating in the negative direction. So, because of change in direction, the speed of propagation change and then, the process this negative sign comes to the energy calculation and into rho g e into C g.

I call it C g 1 is same as rho g by 8 into a t square into C g 2 and if I put in the terms of, rho g by 8 rho g by 8 cancel because, these are same constant. And if that is the same constant then, will have I call it 1 minus a r by a i square into C g 1 is same as a t by a i square c g 2 and that case may 1 minus k r square is equal to C g 2 by C g 1 into k t square. And what is k r, k r is a r by a i and this is on k t is a t by a i, this is the amplitude of the reflected wave to the amplitude of the incident wave, this amplitude of the transmitter wave to the amplitude of this.

And this is what, we call this that the reflection coefficient and this is the general relation on the wave when a propagate from per fill. Now, here I have one more thing I want to highlight here that, when the wave propagate suppose, I have a wave, this is one of the very interesting suppose, I have a wave which propagate from these, there are various changes in the water depth and my final water depth is this. Or the it can happen that, I have a initially the water depth were some h 1 and this wave that there is bottom changes (( )) changes, what are the end the water changes, it changes to a depth of a h 2.

Because, I calculate the energy flux at this end, I calculate the energy flux this end because, that any point of time, that is a wave which reflected and transmitter. And finally, the energy calculated of this end and here, the energy calculated of the same amount of wave energy, if there is a only wave reflection is taking place, nothing is happing. Then, the amount of energy that will be going through this because, there is no loss of energy we are taking into the account.

So, same amount of energy, the energy flux because, same any cross section, we think of the total energy will be there, remain the same. So, the energy that is, if calculate here is same as the energy that is calculated here and in the process, all these differences it will not matter to the, only it will be the relation for this k r k t will remain the same and all depends. This relation this will same but, what will happen from case to case, the result for k r and k t will change, these all are will vary which will depend on the variation in the water depth.

On the other hand, the k r square plus 1 minus k r square is a C g 1 C g 2 by C g 1 and k t square, this is the relation will remain the same. Of course, C g 1 and C g 2 will depend again on the type of wave that is propagating thus, here the depth is h 1, here depth is h 2 and that will depend on that. And k r get will depend on, how the wave, what kind of what kind of disturbance this is there in the local region. These are powerful region in the local region, whatever kind of changes.

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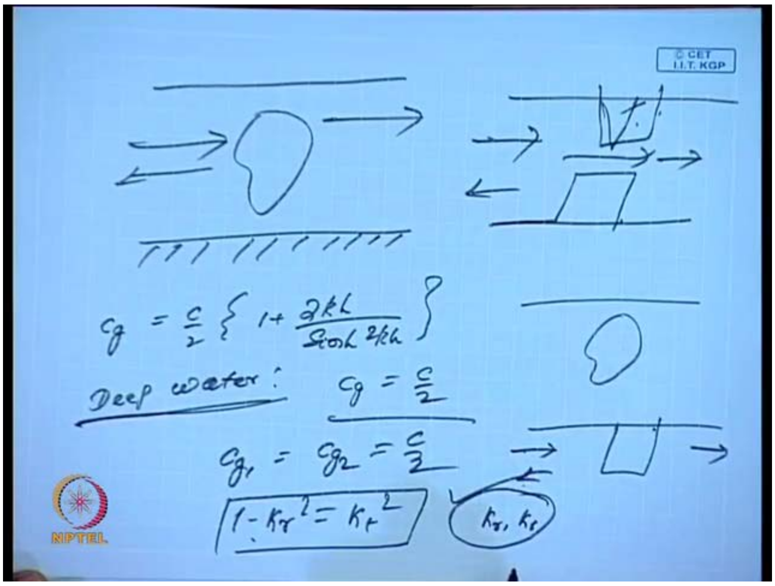

And not only it is for this result will hold good for changes in the bottom topography, it will also hold good suppose, I have any arbitrary structure, which is their whether the process of wave reflection. And the process of wave reflection, if I have I am delivering just a scattering problem where, wave is reflected and wave is transmitted or in have a bottom like a wave lock or you may have a trends, some such scene anything is there now, have a floating body, anything if it is related to the scattering of waves in all the cases, this relation will remain that k r k t relation, what we have obtained it will be satisfy.

Because, these are all the local effects, may be the body is floating here, the wave will pass through this and this will transmit to the change to this. So, in all the cases, the energy relation will be satisfied and it is of this nature. Now, I will just go from case to case that to say that, how the energy relation such as now, what will happen if I say C g, C g is nothing but, C by 2 into 1 plus 2 k h we know this, I have already find this relation.

If C g is this so, what will happen in case of a deep water, in case of a deep water we have C g is C by 2. So, whatever disturbance is there in the local region suppose, I have a structure like this or may I have a break water like this or I may have whatever I have, any disturbances here, if C g is C by 2 then, C g 1 is same as C g 2. Because, both of deep water case, wave proper getting from deep water region into deep water region then, C g 1 will remain as, C g 2 remain as C by 2.

And at the C g C by 2 then, we have 1 minus k r square is equal to k t square, this relation is again further simplified. So, that in case of deep water, when a body some most somewhere, this is the wave which is incident wave. That is a reflected wave per filled behavior on the left hand and there is a per fill behavior that is, a incident wave is there, transmitter wave is there, local effect will be there here. So, the local effect will not, it may be noted that, the local effect will not what the a 1 side mode, they are not contributing to the generation to the energy relation.

Only the per fill behavior, which is contributing to the energy relation because, the a r k r and k t, these are the relation, the values of k r and k t is deep is depending only on the amplitude of the incident wave and the amplitude of the reflected wave. Because of these, the however, although in simple cases like wave reflection by a wall, this quantity is all these a 1 side contribution comes to 0. But, it is not always true, that a 1 side modes they will not contribute, they will contribute to the problem.

However, the energy relation only will depend on the behavior of k r k t and in fact, if there are situation, which arise, this is one of the very important relation in fact, it comes in most of the wave scattering problem. Because, if it is a it may happen that, you have because, this is like in case of deep water you are telling that, k r square k t square is 1.

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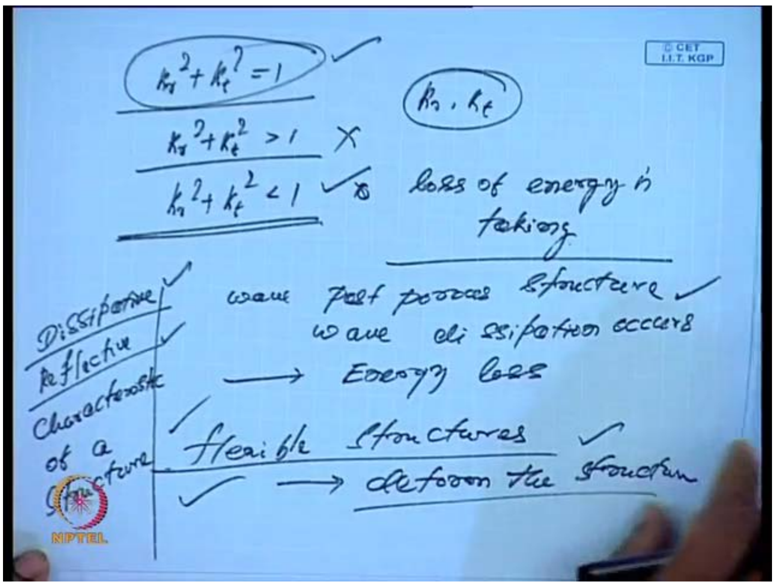

If I say in case of a it is 1, can there be k r square plus k t square, can it be greater than 1, if it is greater than 1 that means, the energy added to the system. But, there is no mechanism, by which energy we added to the system so, this which is ruled out. On the other hand, if k r square plus k t square is less than 1 then, what happen, as if there is loss of energy, energy loss is taking place. In fact, this energy loss is a factor sometimes, of evaluation of computation, we got up when a wave past porous structure, that can be a phenomena of wave energy loss and that is due to energy dissipation.

Wave past porous structure, wave dissipation type less and in the process, this is a energy loss, often it is very important in many situation, people are interested where, a particular structure how much energy is loss is taking place. In fact, sometime we use flexible structures and you will use flexible structures, floating structures then often we say that, the wave deformation, the wave energy loss takes place due to because, the energy some amount of wave energy converted to deform the structure and in the process, deform the structure in which wave energy loss takes place.

Wave energy it is not a loss in this case, it is a energy transform from the wave energy is converted to deforming the structure and in the process, refill that there is a deduction in the energy propagation both in the transmission and under reflection. In such cases, this situation can happen that, k r square plus k t square is less than 1 otherwise because, there these two phenomena here energy loss is take place, here is deformation take place, by the wave axon mechanical energy is generated, which deform the structure.

So, in that process where, energy conversation take place from one form to other and in the process, where is in all these process and these gives us a quantitative story analysis of the reflective or talks about the reflective characteristic structure or dissipative. Dissipative or reflective characteristic of a structure so, calculating this reflection and transmission coefficient, why these quantities, this k r and k t are important and apart from k r and k t, this relation is very important.

Because, it gives us lots of information without going to the detail computation, if we know k r k t, we get all these characteristic of the structure. And it is because of this region, in various branches of wave propagation problem, this first check first people calculate the such a first calculate the reflection and the transmission coefficient and they check, whether these relations are satisfied. If this is satisfied, there is no energy all the energy is there interact, only to transform from one form to another rather reflect wave and transmitter wave, how much wave energy reflected, how much wave energy transmitted.

On the other hand, if where when you think of a wave past porous structure or there is a energy dissipation, which takes place when the wave past poroces structure or in in case of the flexible structure, the structure deform. So, there is a so, in that case, in these two cases, this can may happen, this has to happen because, there is a loss of energy or energy conversation take place. So, through this simple example, we are without going much into detail in many situation, this scarcity are computed.

And first check we do two purpose, one is to check the computer whether the computer result are correct or not, the second thing to find out how much energy loss is taking place by introducing a specific structure or certain structure, this is what here we are looking for. Now, in this, this is a case of a deep water case, what will happen in case of shallow water, in case of shallow water we have if I just a remind you the the energy relation, the general energy relation.

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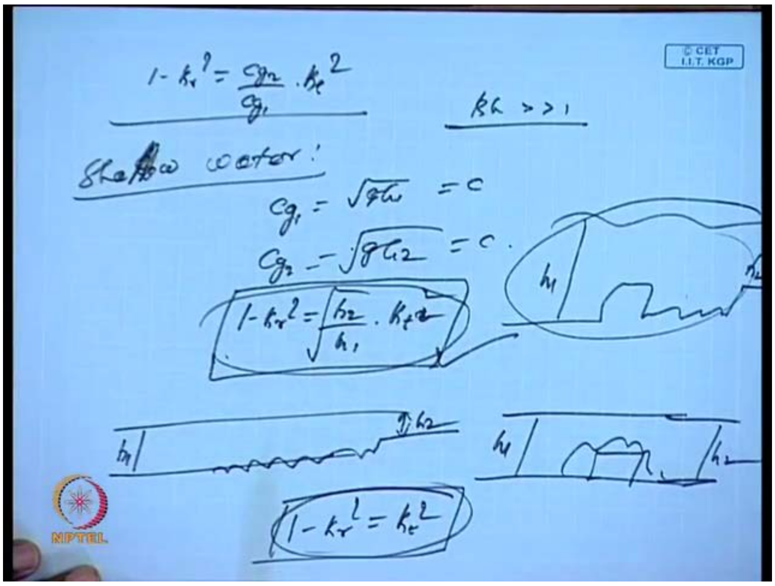

That is, 1 minus k r square is C g 2 by C g 1 into k t square and again in case of a look at shallow water, in case of shallow water this is l, in case of shallow water, C g 1 is root g h 1 and C g 2 is g h 2 sorry C g is the C, rather C g is a C. This is equal to C, this is also C then, what will happen, here also it will happen 1 minus k r square is h 2 by h 1 root bar into k t square so, this becomes a larger lesson. And if h 1 is h 2 so, the per filled may have disturbances here but, if I had per filled, if this this is my h 1, this is my h 2 so, local disturbances will not affect when it comes to energy relation.

And we put it here, initially made with h 1 and then, we have disturbances and then, it goes to h 2 so, if it is from h 1 to h 2 then, this relation would be referring so, what will happen in this case, h 2 is smaller. So, this quantity will be a quantity, which is less than 1 and so, in the process, this relation will be satisfy and that will give and when h 1 is h 2. Suppose that means, I have whatever I have have that is, in the beginning, may be I have a structure here or maybe I have a this one, which is like this where, only if I consider only the wave reflection.

But, if we again goes to the water depth h 1 to water depth h 2 then, the energy relations again remains 1 minus k r square is k t square. Here, in case of deep water, it were just becomes water depth is when we said deep water, the depth is a after certain depth for k h greater than 1, we call it deep water waves. So, even if there is a variation in depth of k h is most greater than 1, that will not effect this relation but here, in case shallow water, this will be not be effected only when this h 1 is h 2, the same relation hold only when h 1 is h 2.

If an h 1 is not h 2 then, this is the relation this will hold whole, that is what in case of shallow water. And in case if we are looking from that wave propagating from shallow water, from deep water to shallow water.

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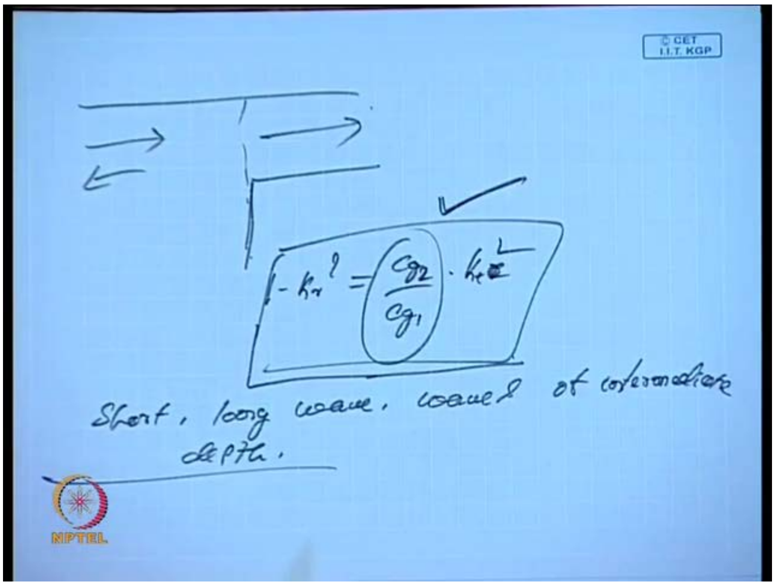

Suppose the wave is propagating from deep water to shallow water, this is a abrupt change in the water. And then, you will have again the same relation will hold 1 minus k r square, this is C g 1 , C g 2 by C g 1 into k t square and here, we need to know, what is C g 1 and what is C g 2. C g 1, the group velocity in this region on this side and C g 2 is the group velocity on the other side k t square so, this is what, I was interested to tell you the wave transformation and the energy relation.

Now, this in fact, this both in case of in fact, in case of short wave or long wave, any wave short or long wave of intermediate depth, any wave we think of, always the energy real relation has to satisfy and that is of this. So, this is one of the very very important result because in fact, we we always try to validate our computational results because, there is a one check that, if this relation is satisfied then, we say that, we are almost correctable to our computation.

And this is one of the best check or main save, when you it comes to wave propagation problem and this is what the and this is call the energy revelation. And that is was interested to tell you this now, another thing early go to. Now, suppose I have a, I want to calculate one of the interesting problem is that, when there is a approximate wave transformation due to approximate change in configuration.

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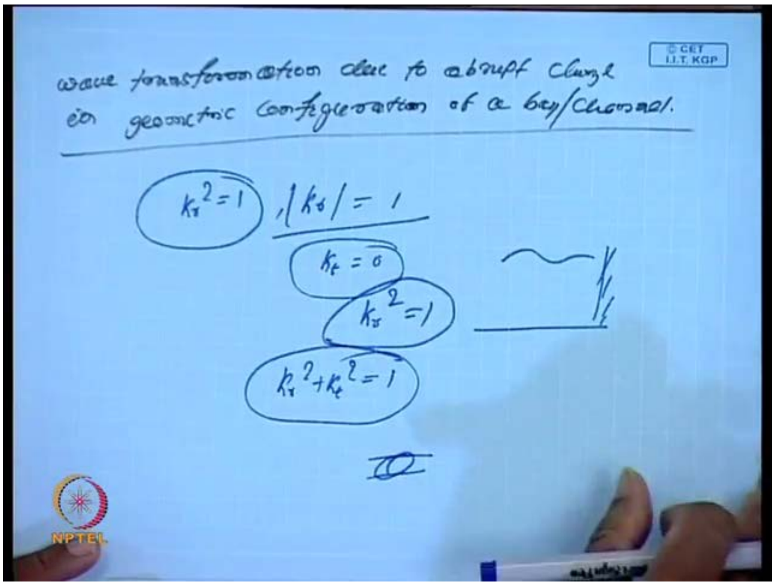

In geometric configuration of a bar of a channel so in fact, this is a another interesting problem in the. What we have got if you are going to this, that we just see one more thing but, I am interested here but, I want to tell here is that, here the interesting thing is that, in case of the bar. In fact, we have seen that, when k r square in a case of a wall near wall we have seen that, k r square is 1 or k r is 1 into the k r we have seen that, it is 1. And in fact, when k r is 1 because, they want to represent when k r is 1 because, here there is no wave when k t is 0, there is no wave which is transmitted.

So, we have say that, we can easily say in case of whether it is a finite depth or infinite depth, if we are dealing with a vertical wall, wave reflection by a vertical wall, whether it is a finite depth or infinite depth. So, k r square is always 1 and this is the energy relation in that case because, there is a no k t here, which is contributing because, our general relation is k r square plus k t square is equal to 1 so, this is these are so on and so forth. And this also check that, this also satisfy the energy relation now, what will happen now, suppose I look at the bay.

If I look at the bay, it will have or a channel then, I may I have two things, one is the bottom where, can be variation in depth sorry there is a variation in depth and the breadth.

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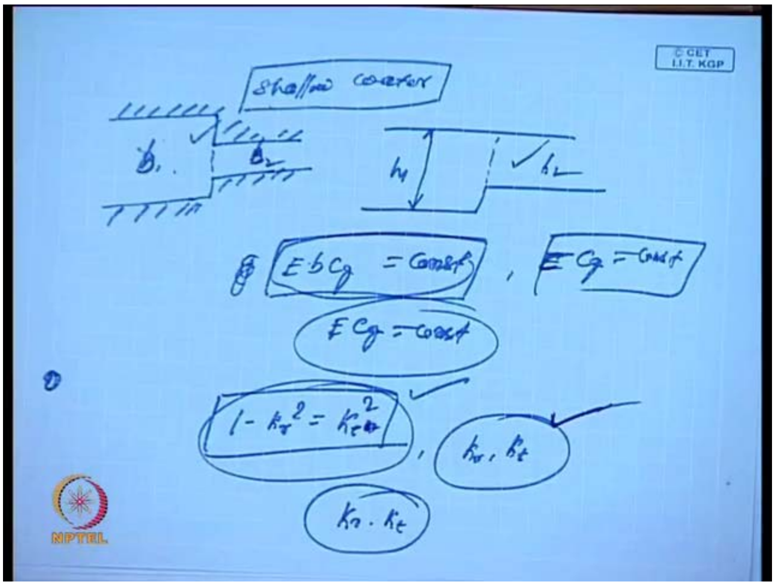

Suppose, I am considering that, I will take a plan here suppose, I have a channel of width b initially, it with b 1 and it changes with b 2. And in that case suppose, this is a channel of width b 1 and then, it changes to width b 2 and the depth of the water is initially it was h 1 and it changes to water depth h 2. And are you say that, this is same location, at the same location the changes takes place, both depth and breadth changes from initially, here breadth changes from b 1 to b 2 and depth changes.

So, in that case, how the energy relation, in that case the energy relation will change to instead of, I will use this as the small b because, that is of energy smaller. So, it will be E b C g is equal to constant and there is approximate change in depth, earlier I use to say that, E C g when I have one direction, there is a only that is considering one direction of wave is here, I am considering both breadth, E c g is the constant.

But, if there is no change in the, if the width does not change even b remains the same throughout, only the depth changes then, I will have, this also will be E c g is the constant. So, with where, considering a one dimensional channel or even in a two dimensional channel, we consider where, the breadth plays important role then, if the width remains the same throughout and same E C g constant. But, if width changes from b 1 to b t, b 1 to b 2 then, E b C g remains the constant so, this very very important.

Now, one of the another interesting point, we have we know that we have to find 1 minus k r square is k t square, this relation it has to satisfied. But, at least one of the case will walk out, out of that k r and k t, we need to obtain k r and k t and one of the wave of obtaining k r and k t is by direct application of the expansion formulae. Direct application of the expansion formulae that is, Havelock expansion formulae, the other way because, always it is not possible to obtain this k r k t in a direct manner, it depends on the geometrical structure of the a structure, by which the waves are scattered.

So, if it is symmetric or very simple then, we can easily obtain it otherwise, it may be difficult to obtain the k r k t directly. And in a such situation, we may have to take the recourse of numerical methods or various numerical methods to obtain these things and this is beyond the the scope of these series of lectures or this course, that will talk in detail later. Because, in this course, my main objective is to introduce to some of the basic things about waves and so, I will not go to that, those details been that will come in a different course at all.

However, I only try to illustrate this that, how care can be taken to obtain in some of these simple case and basically, I will concentrate tomorrow in my next lecture that, how suppose I concentration two case, water depth change in water depth and change in width, how if affect. How in this case particularly, I will consider in shallow water, in the depth of water is shallow, in this case even if we can utilize this energy relation, it will provide us one of the equation, what it determine k r k t are going to solution. Because, and I will tomorrow will in the next class we will see, how this is a helping us, this relation also helps us in obtaining the k r and k t, the reflection and transmission.

Because, there should some other relations, which will help me, I should have a two sides of relation, which will give me that to obtain this k r and k t and the detail I will talk in my next class. And there you will understand that, how this is obtain particularly, there are two things happens here, we have to always two ways to do this. Sometimes we consider the conservation of mass along with this conservation of energy equation or sometimes we consider the continued of this at the location where, there is a approximate changes in the geometry of the structure.

By using one of this along with this, we can always find out how the energy, how the reflection and transmission coefficient changes particularly, you can find what is k r and k t. And tomorrow, I will talk about that and then, I will talk about the instead of, tomorrow I will rather say that, in the next class and then, I will talk about how to generally, I briefly discuss about the wave load on structures. And once, I discuss about wave load and structures finally, just give in a discuss in a brief about wave load on structure.

And then, we conclude this part of the wave mechanics part of this marine hydrodynamics course perhaps, I not go to the non linear theory and the final attempt theory, which will be take must more time than it is excepted. So finally, tomorrow I will talk about this wave load on the structure and little about this, finish this part of this lecture and then, conclude this wave mechanics part. And after that, I will go to the details about, a brief introduction about viscous fluid and laminar boundary lathe theory, that will come in the next few lectures, with this today will stop here.

Thank you.