Marine Hydrodynamic
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Lecture -30
Gravity Wave Transformation and Energy Relation

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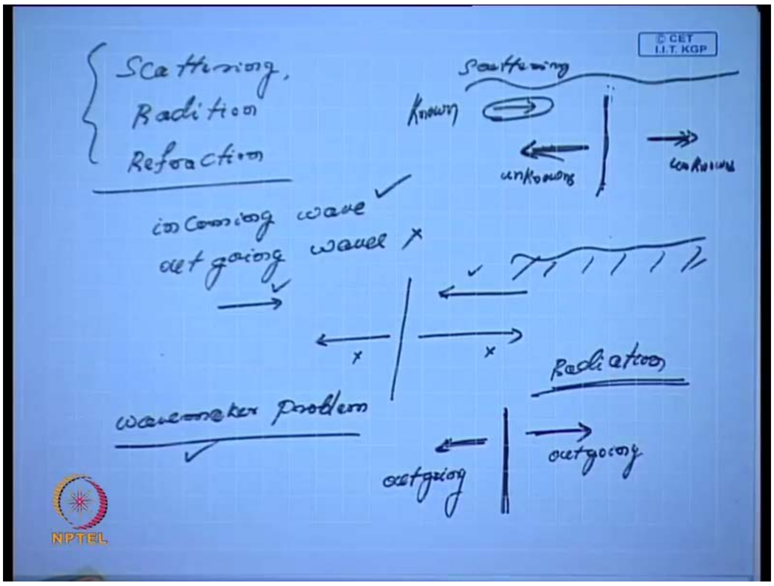

Welcome you to this lecture and marine hydrodynamics. Today will talk about wave transformation and energy relations. So, there are various transformation when it comes to waves, so here will imprecise to two things; one is the scattering and radiation, and the other one is called radiation. So, what I mean I will also if time permits only I may talk too little about to the fraction if not today, may be in the next class. So, what I mean by scattering? We what that let me talk about three waves, what I mean by in coming wave and outgoing waves. In case of a incoming waves, suppose we have a observe bar here, in case of incoming wave the wave approaches towards the observer or towards or towards the struggle bar whatever it is, it can be approaching form either direction.

On the other hand when it comes to outgoing waves, the wave that goes most away from the observe bar or the straggle, so these two are the incoming waves and these two are the outgoing waves. So, now rest on this will depend what is a scattering and then what is a radiation? In case of radiation in case of radiation what you do that this obstacle it only radiates I energy. Basically radiates the wave that means only from here initially there was it is to like a car motor. What is this obstacle? It generates the wave and it oscillates or vibrates. So, when the process because of this it generates the wave and only g g radiated in both the direction, then we call this as the in call this as the outgoing there only outgoing waves exist. Here would have only outgoing waves, so that is happened in case of wave problem radiation.

So, that one process we see it here the obstacle or the structure, which are oscillates and its sometimes we call it a wave knocker problem means the wave maker problem are problem of radiation so in this what happen this generates the wave and the energy wave because of its oscillation is an is generate a wave from this wave goes away from there obstacle. So, in this case and often we call this either the radiation problem or the wave method problem like, on the other hand in case of when it comes to the scattering problem here what happened here we have a obstacle and incoming wave will be approaching to this obstacle.

So, suppose this is the bottom domain this is the free surface and because of this obstacle what will happen because there is a gap between here and here. So, it part of the energy will be reflected back and because we have gaps. So, a part of the energy will be going out in this, so there is a transformation of energy to the other side and here there is a reflection of energy. So, here there is a incoming wave in this class we know assume that the nature of the incoming wave is known and the knowing the nature of the wave obstacle we need to very quitted determine the nature of the two outgoing waves.

So, this is known nature of the incoming wave is known. And here we are interested the these are all knows only what happens here we only assume that the nature of the structure is known. So, we need to know what are the nature of the outgoing waves? So in that case we call this as a scattering.

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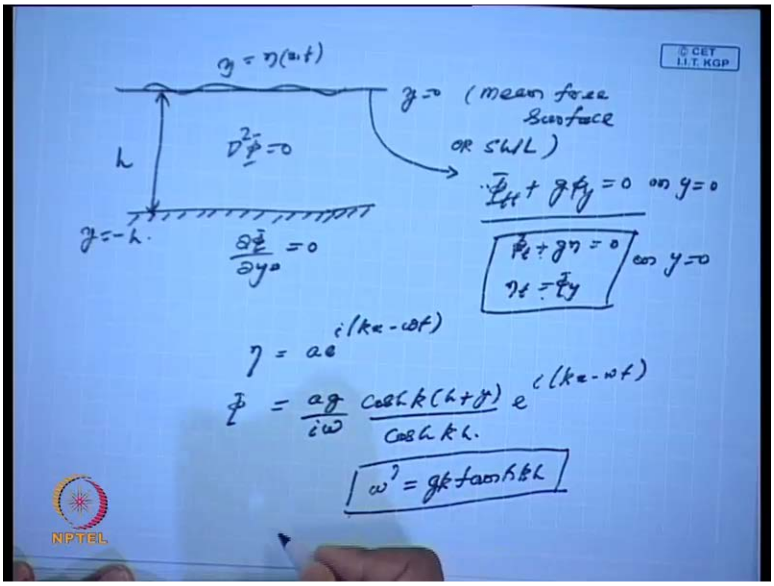

So, with this physical understanding of a scattering problem and (( )) problem. Let us see how will look in to this problems in practice in the case of what waves, so here we all are going today the basic equations. Suppose I have the wave because of, let me think of finite water depth and this is my main T surface y is equal to 0 and this is y is equal to eta x T I. Consider a 2 dimensional problem flow domain, so my web profile is a 1 dimensional in nature y b comes function of eta eta is a function of x and T and this is my surface valuation and this is my surface valuation and this is mean free surface.

We all known the fluid domain del square pi 0 and on a main free surface that is on y is equal to 0 which I called it as the main free surface. Sometimes we call it also s W L still water label are still water label on the still water label we have pi T t plus g pi by is equal to 0 pi T t plus e pi by is equal 0. Here we have this is water depth is finite so this distance is uniform depth this I call it as s here del by del y will be 0. This is 0 and this is the line is equal to minus s this assume the depth is uniform and again this condition is satisfied this is non main free surface pi is equal to 0.

And we all know that pi T plus zeta e 0 and the surface and also eta T is pi (( )). And these 2 condition are satisfied are y is equal to 0 from which we obtained this T surface boundary condition this is the clean rise dynamic condition this is the clean rise kinematic condition. And combining this we have already got the free surface boundary condition which is this and this is satisfied on this surface y is equal to 0. We have seen if you start the wave eta is equal to A e to the (( )) I times k x minus omega T, we have seen that the corresponding pi or we can see that corresponding velocity potential and will call this capital pi.

So, the corresponding velocity potential will be a g by I omega into cos (( )) into s plus y by cos psi potable k is into the per I times k x minus omega T. So, here this omega satisfied that is person lesson omega square is g k 10 high potable (( )) this is the distortion lesson now. If we look at the disport ion lesson what happen here this disport ion what about the behavior of the if I assume that the I know the period of the wave assume we all we have offered several problems on this problem this from the dispersion relation.

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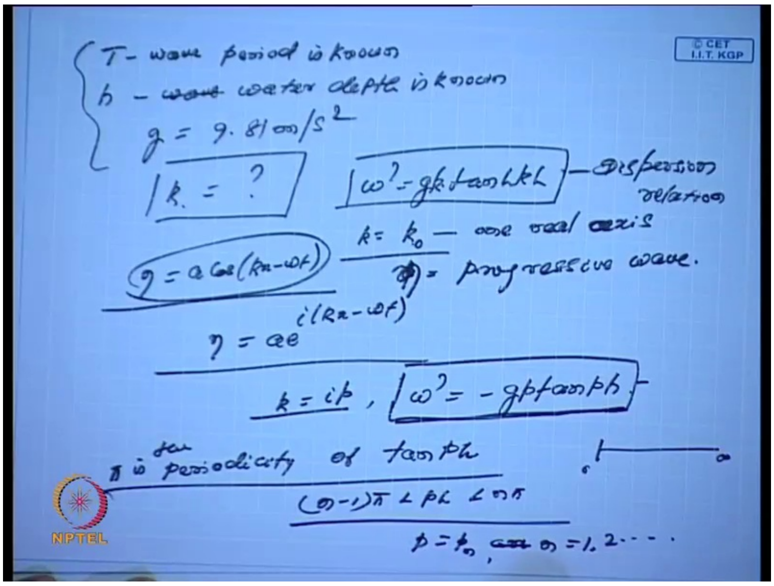

If I assume that T is the period T is known that is y period is known. And I know what is water depth h the water depth sorry water depth is a known and I have g is equal to 9.81 meter per second square, then what will be my k a that I need to mirror? So, we have already seen that omega square is equal to g k time hyperbolic case we have worked out also several problem that this as k 0 a 1 areal root in the projective develops is so 1 areal root, now what will happen? So, that is gives us the corresponding wave number it k 0 is the 1 real root of this the this number k is equal to if I call it k is equal to If I call it k is equal to 0 1.

And only would, so the corresponding pi that I call per eta I call the progress able solution corresponding it is the progress able if a part, so for k is equal to k not for real so that means this for every depth for if once for a every time b the T. What are the, we have k naught and eta if so that eta is equal to a cause k x we have k x minus omega 2. We have a wave profile in that exist. Also wish have seen that the corresponding this are even if in the general 1 what I have consider just now eta is equal to a to the (( )) k x minus omega T. So, this exist for a particular a real k exist that is k not indented.

What happen to if there any other root of this relation? This dispersion relation again if will, I will see that two analyze the roots of this dispersion relation, let me call this as the dispersion relation. If we look at this dispersions suppose, I put k is equal to I p what happen or a k is equal to I p than omega square will be minus g p 10 p h. Then now I have coming that from hyperbole function to tan function tens in function. And for we know that the tangent function is tan function is a function, which is a period city of tangent function periodicity of tan p h. p h pi is the periodicity of tan p h see that is the ph that means you can always n minus into is less than p h is less than in pi.

So, each interval I always is interval of pi I can get a different p h, so that will give me if I look in the positive axis. So, from 0 to infinity than I will have (( )) interval of pi I will have a root. So, that will give me so in this process I can easily get it has infinitely many roots, so I call this p is equal to p n are n is equal to 1 2 3 of to infinity there are infinitely many roots.

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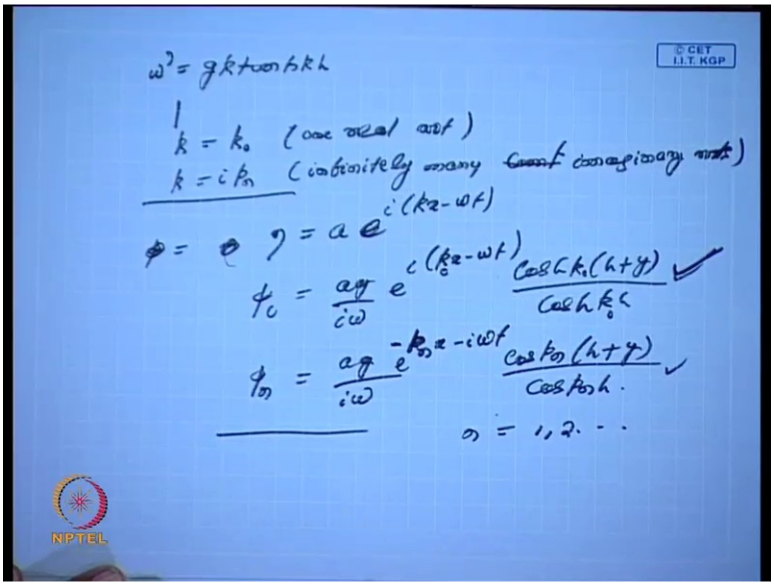

So, in the process what I get that means my dispersion relation omega square is equal to g k tan hyperbolic case if this person lesson has k is equal to k not 1 real root and the k is equal to I p n infinitely many compress are what, sorry rather I will say imagine a root. Because it has no real part. So, in the process what will happen to the corresponding, if I look at because I am dealing with a my original equation is a lab plus equation are associated with a 2 boundary condition. So, than each suppose I have seen that my pi is equal to I have seen pi is equal to or if I say my eta is equal to a, it is per I times a x minus omega T.

Then I have seen that my corresponding pi in case of the real solution in k is a real that is k not than it will be (( )) by I omega it the power I times k x minus omega T and cause hyperbolic k into s plus y because hyperbolic k h that is my pi naught. I call this as k naught corresponding solution. And if I look at k is equal to I p n than pi n is will be at the form a g by I omega T to the power when take to the x as 0 than I can have I this i k n. So, that I will give minus k n and x minus p n x minus pi omega T and than this will give me cause p n cause p n h. So, then what will happen to this, so this are for n is equal to 1 to (( )). So, I can also say, so this are also one of the solutions this is one solution. So, all this last few classes on waves we have emprises only on this solution associated pi, but now see if you look into this roots infinite many roots than this also is a solution So, it has infinitely many solution is a look at the original problem than that I will just (( )).

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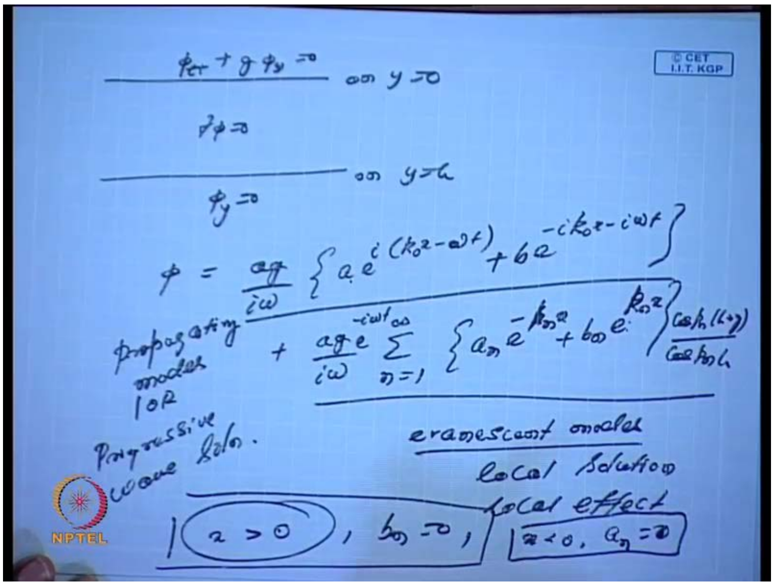

If I look at the original problem that means del square pi 0 my pi T t plus g pi by 0 on y is equal to 0 and this is pi by 0 on y is equal to h. My general solution will be of this form pi is equal to ag by I omega, I can call it a e to the per 5 times k not x minus omega T plus b times. I cannot takes minus I omega T plus a g by I omega I can call it sigma, n is equal to 1 to infinity an into the minus k n x b n a to the for k n x well other this is p n x p n x. And this is and look us p n s plus y (( )) p n h and into into the for minus I omega T.

Now, this is the general for now. Question comes, what happen because I was dealing with I this term as come. And again I was here I was taken minus p n y again plus p n x, so this I will let us explain this terms what happen actually this part contribute to the profile the take a more solution. This parts I have the local effect, which will call the evergreen modes even says centre modes and this is the profile modes, but the progressive wave solution or call the progressive wave solution.

On the other at this 7 7 modes often call as local solution or local abate, now when we say this is the other local effect what happen? Suppose, I am dealing with that my only x I am dealing with that my only x I am dealing with the x the abdomen of fluid is only positive x axis if I am dealing with x is 0. Then what will happen to this, because if x is 0 then v to the or p n x p n are positive p n, suppose it is p n x will tan to infinity. So, that means I am looking for a solution, which will not behave on boundary solution of looking only for abounded solution.

So, in that case my b n will be 0, but on the other hand if I said my I am looking at if I say I am looking at solution for x less than 0 than automatically x is less than 0. So, this part will tan to 0 where has x is less than 0 means there should be positive, so n should be unbounded x less than 0, so in that case if x is less than 0 I can call it is a n is 0. So, that will give me another set so that is a one thing than coming back to this two terms there are physically I have just be find that for a wave this are the two refer to the progression wave solution and as I have told that in case of wave.

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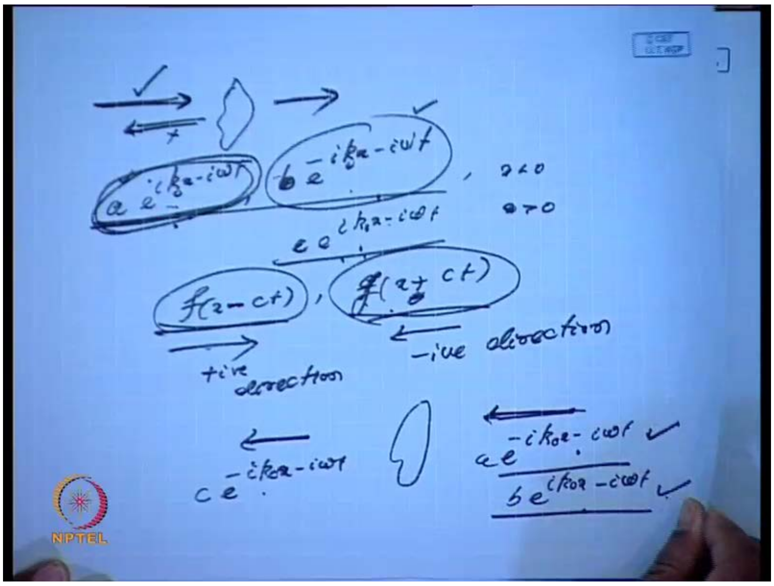

We have it may happen that wave by wave, which is a incoming wave and because we may have a structure or have some obstruction and a part of the wave exergual reflected. So, here we have a wave the nature of the incoming wave will be i k x minus I omega T and nature of the outgoing waves some a nature of the outgoing will be some b into the or minus I k call it k not x minus I omega T.

So, if we have problem of a scattering problem, so we may have incident wave and also reflected wave if we are dealing with a wave. So, in that case this will contribute to the incoming wave this contributes to the outgoing wave as well as the x less than 0 is concern with the f x greater than 0 than on this side the wave will be outgoing, but it is always moving in this direction.

So, in this case, so for so there are the toot waves this will will correspond to the incoming wave and this (( )) will corresponding to the outgoing waves for x less than 0. On the other hand if we are dealing with a scattering problem on the other hand for x greater than 0, I may have a constant c. And then my wave will be it is where i k x minus I omega T and that will be the typo clove which is the outgoing wave, here always look at it is of the form f of x minus c t and this is of the form f of x g of x plus c t. So, this wave which is always moves in the positive derivation and this wave which moves in the it gives basically this sine (( )) minus gives the direction in which direction the wave is propagating.

So, whether is a incoming wave are outgoing wave, but this plus minus sine gives us the direction, which the wave in propagating. So, that is why here we have this one is a. Although, it is a minus sign, but still it is a incoming wave where has this is the outgoing wave both are minus sign. On the other hand here we have plus for sine, this is a minus sign similar to this what this is a outgoing wave. So, this basically determine this sine sine of this particularly x minus ct or x plus city c t type. So, that determine whether the wave is progressing in the positive direction and other it is propagating always in the negative direction.

So, there are two things here whether the wave is incoming are outgoing; one as the other thing is whether the wave is propagating in the positive are negative direction that is a another things. So, when you when one deals with a physical problem one has to take this 2 aspects in the aconite which (( )). Suppose I say that I have a wave which is approaching form this side than what should be my progressive, so I will be have a to the minus i k naught x minus I omega T. If wave is if I have obstructer like this and my wave is a approaching from the right side. So, this will be my incoming wave and on the other hand when wave was approaching from this side this was becoming the incoming wave.

And here the outgoing wave will be of this type some a here some be it there I cannot accept minus I by omega T. This will be my if the structure is there, this is the incoming wave, this is the outgoing wave, this is that is the, this is the incident wave, this is the reflecting, whereas this wave, the wave will be propagating this wave. Because approaching from this side and when it approaches form this side then this will be some of the it is nature will be a to the c into the minus I cannot x minus I omega T. So, this will be the nature of the because this is this waves this two waves they propagating in the negativity direction of x axis. And where has this wave this propagating the in the positive direction of the x axis, so anything this is propagating in the positive direction will have a minus sign of the nature will be like this anything is propagate in the negative direction the nature will be like this.

So, because of that here it is these wave is a wave propagating towards the positive direction, this is a wave which is propagating towards negative direction this is again towards positive direction this is again towards negative direction this is towards positive direction this is towards negative direction. So, respective of the fact whether it is a incoming wave for a outgoing wave the direction of propagation determine the positive or negative sigh in the wave pattern. So, this two things are very clear for us now, this understanding will a look at the full solution and determine what kind of wave it will what kind of solution it will be.

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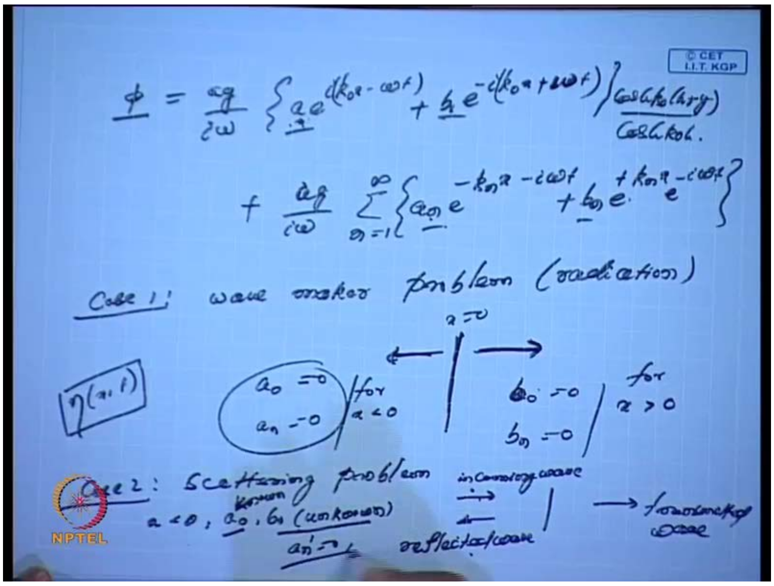

Now, if I go back again I will just try it again that the general form of the velocity potential that means pi is I have seen that this is a g by I omega a e a to the Pera I cannot x minus omega T plus. So, I call this as a 0 and b 0 minus I cannot x minus I omega T, so if I take minus I coma and I will have this. So, this is I call it omega T this into, so this is the general form, so if I am looking at a case one if I look at a wave maker problem basically I look at a radiation problem (( )).

So, if it is a problem of radiation than what will happen I will have only outgoing waves, so in that case if I say the wave is radiating energy will both the direction. So, for x greater than 0 my b 1 it is a positivity direction. So, b 1 will be a 0 will be 0 on the positive side because this is going sorry this is going in the this way, so this term will be so b 0 will 0.

And here this side will be b n will be 0 that is if this is line is x is 0 and I have a wave maker problem. So, the energy is only radiated, so your b 0 will be 0 because wave is propagating only in this deduction. So, this term will contribute because of this on this term will not contribute, whereas because wave is one of the wave will exist (( )). On the other hand here b n will be, because this term will be on the right side this term will not contribute. So, b n will be 0 on the other hand on this side for x this is for x greater than 0, but what will happen the if the wave is a generated in both the direction. In this case what will happen? This here than my a knot will be because wave is propagating the negative direction. So, a knot will be 0 and where has on from this seven side modes I will have b and a 0.

So, for wave maker problem from the general solution, this will be this is only will be and this is for x less than 0. On the other hand if I look into scattering problem case two scattering problem, in case of a scattering problem what will happen? I will have a structure, I will have one incoming wave, I will have one outgoing wave. Here I will have out going wave, so this wave is the incoming wave this is the reflected wave. This is the transmitter wave incoming wave, this is the reflected wave and this is the transmitter wave.

Here I before preceding for the right again point out high is the benefit potential, but it is the velocity potential, but finally what we are interested we are interested in knowing eta x t as I act whole earlier that many situations because whole problem has been recast wave problem in pi interims of the velocity potential. So, we are solving lab plus equation subject to the free surface boundary condition and the bottom boundary condition everything in terms of pi. So, once we obtained pi we can get eta because eta represents gives the surface profile surface valuation or the part on the waves pi dragnets by T the coefficients associated with the basically the coefficient a 0 p 0 a n and b n.

All this terms they give us and understanding about what exactly happen to unplaced waves, so in many situation because all this patrons k 0 will you know because once we solve it omega scared that is dispersion will know k 0. But what will happened in a wave propagation problem, we need to know this constants a 0 b 0 a n v n. If I looking at a radiation problem on the right side may be we have b 0. Because of the nature of the physical problem b 0 v n a 0 and here is a 0 n 0, but we need we need other conditions to finally, find what are the rest of the announce? Similarly, what happen in case of a scattering problem we have incoming wave we have reflect today we have a transmitted wave so if I login here we look at this problem in case of scattering problem on the left side for x less than 0 I will have both the terms will be so a 0 will be b 0 will be.

So, they will be, so the basically a 0 will be known b 0 unknown, whereas because this I call the incoming wave I assume the nature of the incoming wave is a known. And b 0 is the nature of the outgoing wave I am teach today associated with the outgoing wave that is the reflect that is unknown. And again on the left side x is less than 0 and out of this two because it is a less than 0 this term, so my an should be 0 all n are 0 on the other hand if I look at the in the scattering problem.

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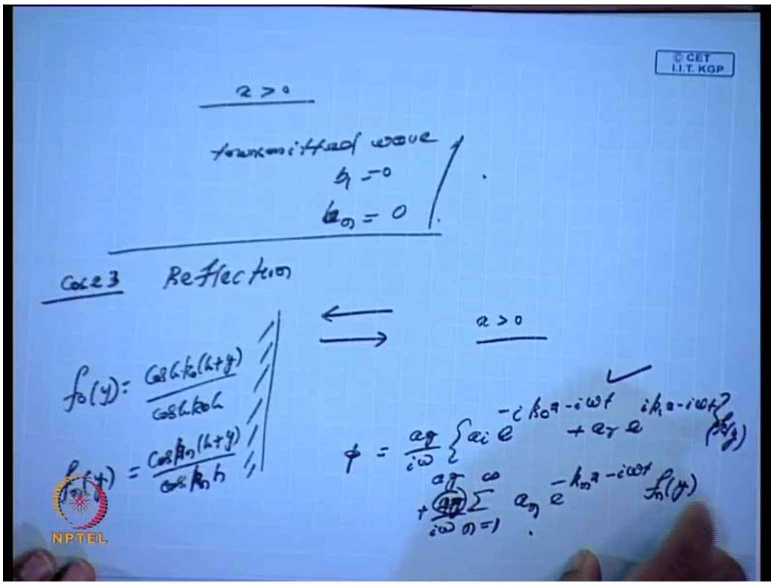

If you look at the right side for x greater than 0 in the case of the scattering problem than I will have because I have only one outgoing wave that is the transmitted wave because wave is propagating form the positive side. And then in that case my b one will be 0 the nature of b 1 will be 0. On the other hand in case of diver gents modes I will have x less greater than 0, so I have a n is b n is will be 0.

So, this is what will happen and off course in the left side constants associated constant in not be the same as the constant on the right side. They will be different than in be same I will come to a exact problem next. Now, I will go to the hard case I will come to then I will come, then I will come to 1 by 1 progress incase three suppose i case of reflection I suppose I have a (( )) I have one incoming wave is coming than that will be outgoing waves.

So, in this case of, suppose I am taking that this is I am in the profile is a x is a greater than 0. So, what will be (( )) now pi will be a g by I omega into a i, I call this as a i a to the Pera i k x minus i k x minus I omega T plus a r i k not x minus I omega T. Then we have plus sigma 1 to infinity. I call it a n and this a n a to the minus k n x minus I omega T there is a pi 0 I call it f 0 A then I call it f I f 0 y and here I call it f n y an what is my f n f 0 by is plus separable k 0 s plus y by plus high (( )) k 0 is f n y plus k n is y by cause I use the notice on p p n h. This is my general form for the replacing because I have one incident way this is the reflect today and this are the (( )) modes this is a g by omega.

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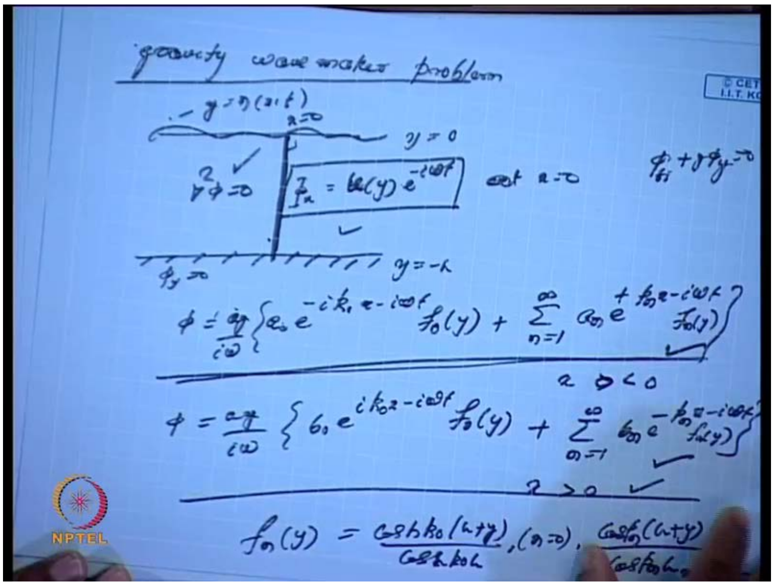

This is a g now if this are the patrons so suppose so suppose I say now I will come back to a another case of a problem of a wave maker. Basically what I say laities a make problem in case of gravity a maker problem like we have seen in a wave tank. And tank which generates wave in any hydrodynamics laboratory. One can find your wave tank particularly which generates there j (( )) to at one end of the tank and that we apple escalate and it generates wave. So, that basically almost exist worldwide any hydrodynamics laboratory that for module testing of various physical models a wave tank available.

So, at one end of the wave tank they will be wave maker and which will be generate the waves. So suppose I say let me consider this problem like this is my free surface h by g is equal eta x T and is this is my y is minus h y is equal 0. Suppose, there is a y maker here of this position is x is equal to 0 and this wave I have the wave maker exist here and it oscillates it makes a on the batik it makes oscillation pi x is u y what is a minus pi omega t.

And I mean the frequency of oscillation is the same or the frequency of the waves this is geologier. Suppose, the wave maker oscillates the speed of p i x is the valuental velocity in the horizontal direction on the horizontal direction it makes on oscillation of this nature and when I say pi x is of this is basically on and x is equal to 0. So, at the neon position about x is equal to 0 this the (( )) small oscillation small lumbtage oscillation with frequencies same as the wave frequency then what will happen to the wave? Then I have in the through hole fluid is a region, I have del square pi is 0 when this add or on this add if I say this is, so then what will happen for as usual my pi will be I am this ferm because I am dealing with only a wave maker problem this wave maker is oscillating.

So, my pi will be a 0 in to there minus nature of the I cannot x minus I omega T, then f 0 by plus sigma n is equal to 1 to infinity I will say z g by I omega this 1. This is a quantum and will be throughout f 0 y plus sigma n is equal to 1 to infinity and this will be an in to a minus p n x when the psi omega T this is minus of the should be plus this is for x x less than 0 and if I look at what happen on the right side if the wave.

The fluid is on this side this will be the form of the pi and if the fluid occupies on the other side than pi will be of this ferm L g by I omega this will be some b not u to the Pera I cannot x and just I omega T f naught y to the sigma some b n plus is for x. So, this will be the nature of pi for this is for x less than 0 and this will be the pi by x greater than 0 now the main problem remains and this pi will satisfy the free surface condition that is the free surface condition not y is equal to 0 pi T T plus g g pi by 0 all this pi are on this pi both of them satisfied the lab plus equation in the fluid domain.

And also they both satisfy pi by is 0 on y is equal to minus h here my f 0 on the fence y here. There is a f n y f n y and to I have already told my f n y is caused hyperbolic k 0 s plus y y by cause hyperbolic k 0 h that is in a 0. And this is because p n s plus by y because p n h for all in greater than one two positive. So, this is main general, now the question comes I have one more boundary condition that is pi x is e y a to the minus i omega T, if I substituted pi x here. Suppose, I look at the positive side, let me take anyone side positive side, so what will happen to pi x.

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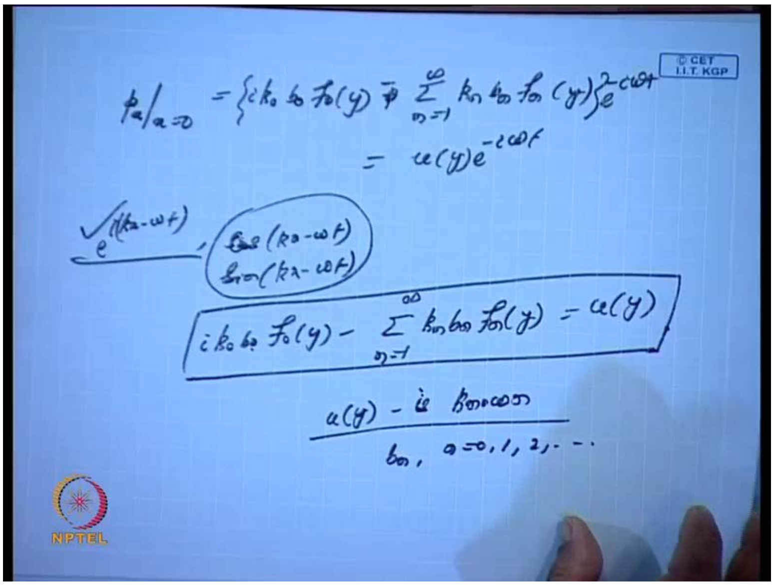

So pi x at x is equal to 0 gives me I cannot will looking at the positive direction I cannot that is b naught into f not pi plus because x is equal to 0 I am calculating plus sigma n is equal to 1 to infinity minus it will be minus k n. Then b n f n y and this is nothing but pi x x is equal to 0 into into the minus I omega T that is equal to o u y it to the minus I omega T.

I deal with a comp less exponentials, but the in a reality the solution will be either the real are imagine a part of the comp les potential of this comp less exponential function will give me the real wave part as we say that e to the Pera I a x minus omega T. The real are imagine the wave parts of the real part is sine k x cause k x minus omega T or sine k x minus omega T for easy of mathematical simplicity. You always look in to the complicity comp less form of the wave and in a reality when will go for real live programs will always when the l i conscious that real part of this.

So, if this will be this than what will happen my b y the if I simplify worthier I get I cannot b not f naught y minus sigma n is equal to y to infinity k n b n f n y this is going to e y. Now, I only have one question, what what happen here? My u y i know because I know the way the way (( )) oscillating I assume u y is non, I looking at wave radiation problems. I know that what is the how the wave maker the structure is oscillating attach x is equal to 0, so if u is known then what will happen I have, so many onwards b not v n so I have b n n is equal to 0 1 2 this are the unknowns I need to obtain them so to obtain this unknowns.

So, what I will do I only look at the because this is kind of whether you called it a in function x function. if you look at mathematically are by it is a kind of basic simple method of separational variable and this leads to and the a standard (( )) problem. So, f n y are the y again functions if I call k 0 and p n are the liagan values, then I can say that corresponding a fence will give me the liagan functions, and since it is a term level type of wonderfully problem in f n.

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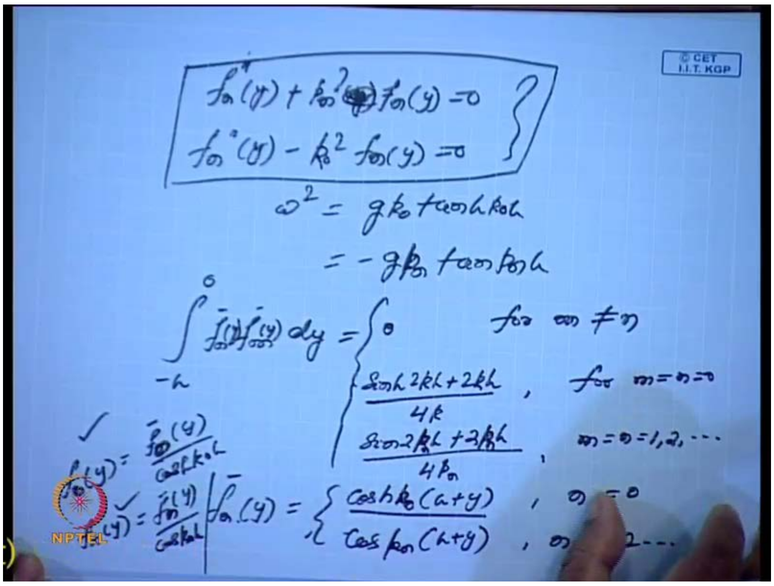

Because f n y will satisfy the differential equation fn double dash y minus plus p n square by sorry p n square f n y e 0 and again fn double dash y minus p 0 square f n y 0 and both satisfy the common thing of that p n satisfy. Omega square is a g and this is k naught g k not tan highperbely case and again it satisfy minus g p n h, so this is the the nature of f n and that satisfy. This equation to this are all standard (( )) program. It has to n point to point to find because it satisfy the free surface condition and also the bottom condition. We have simplified just under ordinary differential equation comes from and basically has comes to on lab plus equation the y component of the lab plus equation.

So, the because of this there also what is arithmetic criteria, so they are suppose two satisfy (( )) multi criteria and in this case we can see that this f n f m minus s 2 0 d y the brute is that f n y f m y d y this is 0 for z means not equal to n. And further this is equal to sine hyperbolic 2 case plus cage by 4 k. This is for m is equal to m is equal to 0 and this is sine in 2 k h plus 2 k h by 4 k p p n h p n h by 4 p n this is a m is equal to this is for 1 2 s. And here my f m because hyperbolic k 0 is plus by this is cause hyperbole cause p n into s plus y n is equal to 0 1 to (( )). So, here I am not taking the division by cause hyperbolic k naught, why because hyperbolic k not h r because that is a only only a term which should be multiplied.

Here if I divided by cause of k n h than another term will be here for separable square k n h and here cause separable cause square k n h only pi I are here this is the general thing if I take f n y is this then fm fn will be this. And this is what a rather call it f n bar f n bar and call it f n bar (( )) f m y will be fn bar pi by f 0 by y by cause hyperbolic k 0 h. My f n y will be my 0 f n bar by (( )) cause k n h. So, this will be my f 0 f n. So, since once this are authorial automatically f 0 f n, so that gives us the beauty of this that and this will be utilize.

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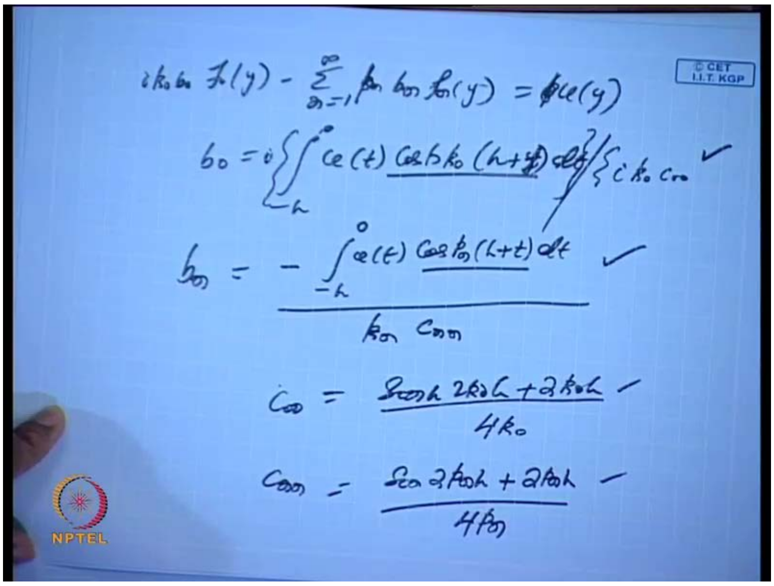

Because my original equation, if I always i k 0 b 0 f 0 y minus sigma n is equal to 1 to infinity k n p n b n f n y is equal to my u by and if have plate the (( )) then I will get my b 0 as minus s 2 0 u T cos hyperbolic k 0 s plus by s plus T d t. And b 0 will be I b 0 will be divided by divided by commas i k 0 and to I call it this c 0 0 and again b n will be minus minus s 2 0 u T cause p n into s plus T d t divided by k n c n n n a d. This c n n whether the c 0 is a 0 is I call it is sine hyperbolic to k 0 H sine hyperbolic 2 k 0 h plus 2 k 0 h.

So, an I prove will to k 0 h by plus to k 0 h by 4 k 0 and c n n and only one term. If I have here, if I will have cos hyperbolic term a division term here, if are here if i by division by cos p n h tan automatically will have a square term will be cos hyperbolic square p n h will come. Otherwise, it is remains the same this the general for, so this is my b 0 this is my b n this is my c 0 on this. So, I know all the unknown constant b n and what was my original equation.

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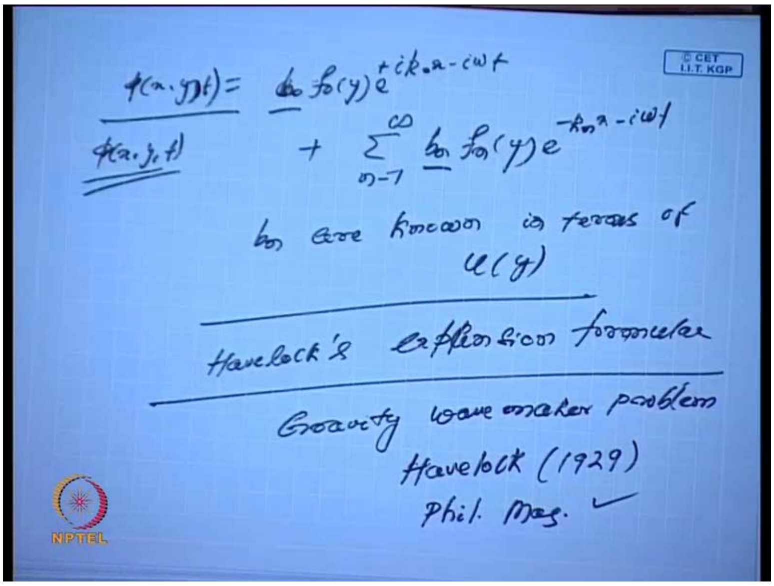

My original expression for pi my original expression for pi per its greater than equal to 0 was pi x y pi x by was this form a 0 rather on the diet side I have taken b 0 f 0 by into their minus i k 0 x plus i k 0 x minus I omega T plus sigma b n f n y minus k n x minus I omega T. This b n b 0. Now everything is it known rather it is a call a function of x y n T because this is T by pi pi is a function of x y come in T now all be ends are known in terms of u by. Because I given b 0 b n n in terms for you u T same, as because here the variable of integration T because so b ends are known, so it is very simple. In fact this is known us have locks expansion from luck have lucks expansion formula, this is very classical result are sometimes we call it gravity have locks gravity.

We make a problem, it is call the gravity wave make problem. In fact this problem was fast developed by have locks in 1929. And it was probably (( )) and this problem has a wide application today and they are in last line teers last more than T s this problem has have to simplify large varieties of problem in gravity wave associate today the gravity of OA structure (( )) problems. And almost this concept, which used in the development of the wave maker classical wave maker which is used a worldwide in a tank in hydrodynamics.

About with this today I will stop and tomorrow are in the next class rather will talk about how the energical lesson is coming into picture and how the scattering problems can be handled in simple cases over complex situation will be handled rest 2 case, but accolades the general philosophy with the radiation and scattering problem will give discussing between.

Thank you today.