Marine Hydrodynamics
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Lecture - 3
Streamlines and Flow Direction

Now, today we will go to the 3rd lecture in a series, the first two lectures we have talked about first we talked about the basic Marine Hydrodynamics, it is introduction, motivation. In the second lecture, we have talked about basically the continuity equation. And while talking about the continuity equation, we come across talk about what is the vector, then irrotational motion and from that we could get that existence of potential phi which exist in case of a irrotational motion in a fluid flow problem. And today again now we will go to, what about the flow direction? In which direction how the flow, how to know the direction of the fluid flow, so in this context we will talk about stream lines and path lines.

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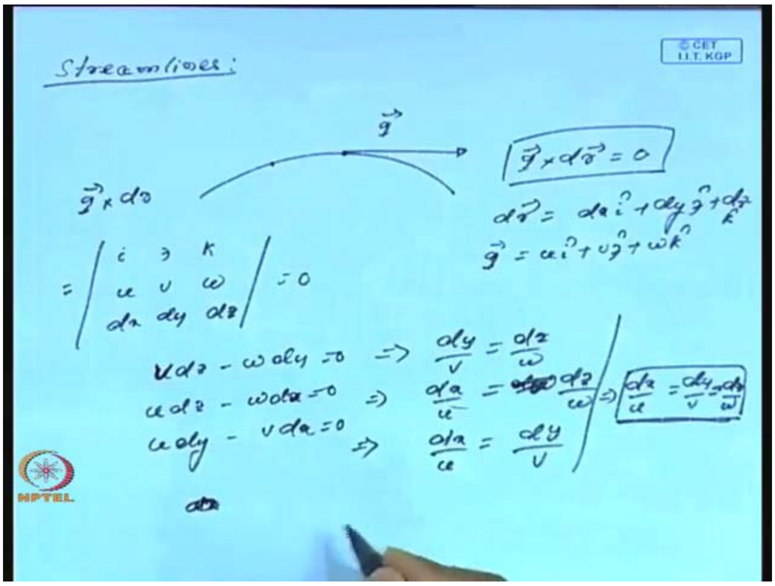

Stream lines - for a fixed instant of time, let us draw a path in the fluid a space curve at a particular point, let us draw a tangent to this. And if it q bar velocity vector, if a velocity vector q bar is same as the direction of the tangent; that means, q bar cross d r bar equal to 0 and then this line, this curve we call the stream line, because that means along a stream line, the flow it gives the direction of the flow of the fluid. And then in as a result there is no fluid usually across a stream line there will be no flow, no fluid will pass through the streamline, until or unless gives the direction of the flow.

So, if u simplify this one, then you have d r bar is equal to d x i hat d y j hat, these are the components d z k hat, then we have already q bar is equal to u i hat plus v j hat plus w k hat from q dot d r bar is 0, this will give us i j k you have u v w, you have d x d y d z. And if this is 0, because this is what you have q bar cross d r bar means and if this is 0, then and gives us from this you have u d z, rather if I go to this then v d z v d z minus w d y is 0. Then you have j then you have u d z that is u d z minus w d y w d x is 0 and then we have k k component u d y minus v d x is 0, from this we can easily get d y by v is d z by w. And from here, we will get d x by u equal to d y d w by sorry this is called d z by w and then you have from here, you will get from the third one you will get d x by u is equal to d y by v.

So, if you combine that three things, you will get d x by u d y by v is equal to d z by w. So, this is what. So, this is what we call the they are just equation of the streamline. So, you can take any two of them and solve it and, so that will give us two sets of constant, if i say that I will take either any two either this two this two I can take, I will get two sets of equations and I can take this two or I can this take one and third. So, that will give you often. So, because of that we say that, it gives two infinity set of constraints and this is the equation of the streamline. Now, let me just take a simple example to understand what exactly how it means.

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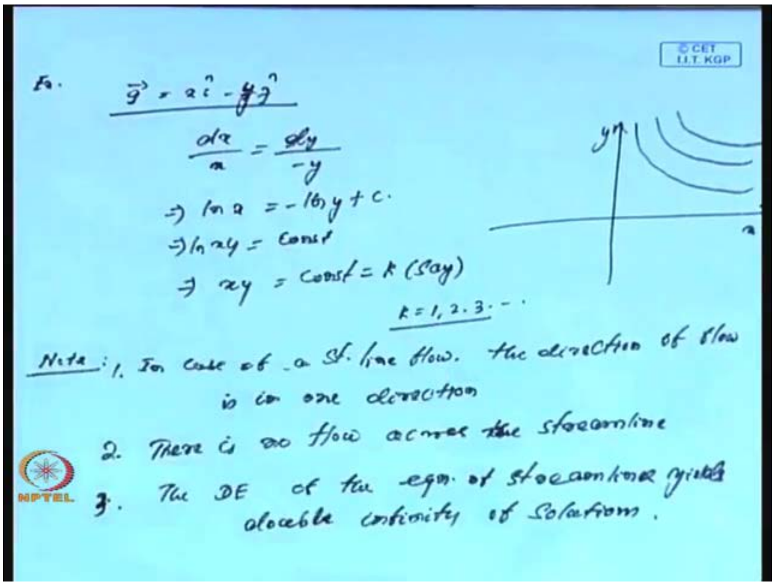

So, if I say I will take a simple example, like if I say my velocity vector q bar x i hat minus j sorry y j hat, if this is the velocity vector, then what will happen to my streamlines equation of the streamline will be d x by x will be d y y minus y, once this gives me. So, it gives me l n x minus l n y plus constant which gives me l n x y is equal to constant and that which gives me x y is equal to constant and since, if x y is constant then what will happen, if I draw these lines.

So, this is for each constant suppose, we call it k and this constant say since it is. So, what will happen then, what if I draw these lines let me say in that straight line this is x axis, this is my y axis and if I draw it, this will be my curves lines is bar there is well is a constant. And again because, similarly we can get in a now, I will go to this is one example for x positive, y positive we can get when k is a constant or k is equal to 1, 2, 3 it can have any values and also, so we can have this is one of the example.

So, now some of the points to be noted note, so in case of a streamline in case of a streamline stream line stream line flow the flow direction of flow is in one direction is in one direction, as I have already mentioned there is no flow across if there is no flow across a streamline across the streamline. And I have mentioned the the differential equation the equation of the streamline, yields double infinitive solution and in this case you have, this q bar is x i hat this is a two dimensional flow, this is the one. Now, once you know what is a streamline, what will happen like in case of a vertex line you have got a vertex stream what will happen, if we take up any closed curve.

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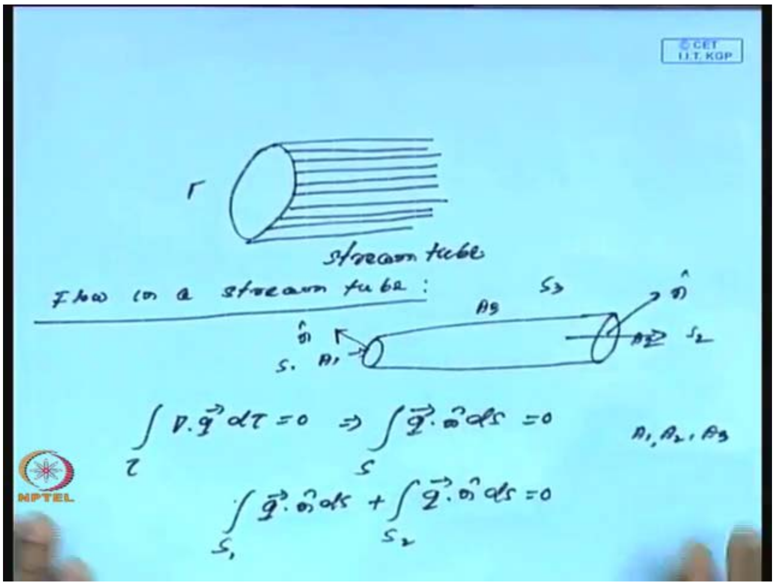

If we think of any closed curve and draw a stream line, in the inside the fluid you have a closed curve, let it a curve (( )) like in case of vertex lines, vertex tubes we have seen that we have taken any closed curve inside the fluid and we have drawn vertex lines, we got a vertex tubes. Similarly, suppose I say I had draw I have any closed curve and that each point of the curve, if I draw a line, the stream line, then this will give a tube and this tube we call this stream tube and let us look at the flow in a stream tube, if I look at the flow in a stream tube. Then let us say that lets look at the stream tube of various cross section, this is cross section A 1 this side is A 2, let this A 3 is the boundary of the tube and then these are the two cross sections. So, there are three one is A 1, A 2 and A 3.

So, let us say that fluid is flowing along this direction and it is going out in this direction and let me say that, at this point if n hat is the normal unit normal outer drawn direction and here also, we have outer drawn direction normal n hat. So, if that is the case and the vessel that the fluid is entering through the cross section A 1 and it is going out through the cross section A 2. Because, it is and there is a flow what will happen, if we will say from the continuity equation, we have for a in compressive fluid we have got this is equal to 0, we have seen in case of continuity equation.

If this is the case then which is same as, integral over the surface q bar dot n hat d s that is 0. Now, if I look at now, cross section now, this surface we have three surface we have A 1 we have A 1, A 2 and A 3. Since, we have three surface, since we have a stream line flow, this is a stream tube and there is no flow which will cross through A 3. So, because of that we will have, integral over this is called the surface s 1, this is my surface s 2, this is my surface s 3. So, there is no flow across s 3. So, what will happen my this will be same as, integral over s 1 q bar dot n hat d s plus integral over s 2 q bar dot n hat d s will be 0 because, there is no flow which is crossing the surface s 3 and if that is the case, then because, we have one is the inward direction.

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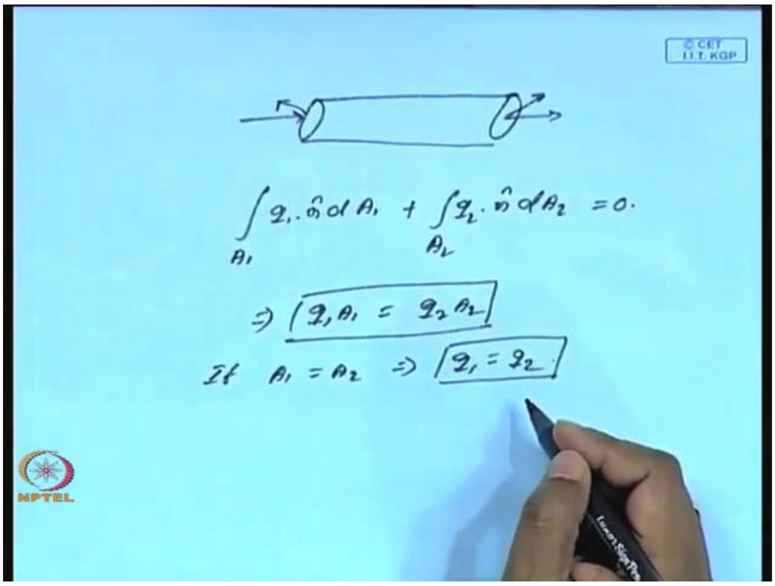

Because, this side you have the stream tube, this side your flow is entering, this is the way flow is entering and this is the flow so outer direction is upward. So, we can have this is the this is the outer direction, here it is outer normal. So, if then in that case, so we will have integral over s 1; that means, A 1 q 1 dot n hat d A 1 plus integral over A 2 q 2 dot n hat d A 2 this is 0 and which implies q 1 A 1 because, the normal's are in opposite direction. So, we will have a negative sign so, that will give you q 1 A 1 is q 2 A 2 and this is what in case of a stream tube, the amount of fluid that will enter through this and in that that same amount of fluid will go out through this.

So, now, if I say A 1 is A 2 which implies from this relation, we will have q 1 is q 2; that means, when you have a tube, stream tube of uniform cross section the speed remains the same. Now, to demonstrate this let me take an example, let us consider a stream tube a circular pipe.

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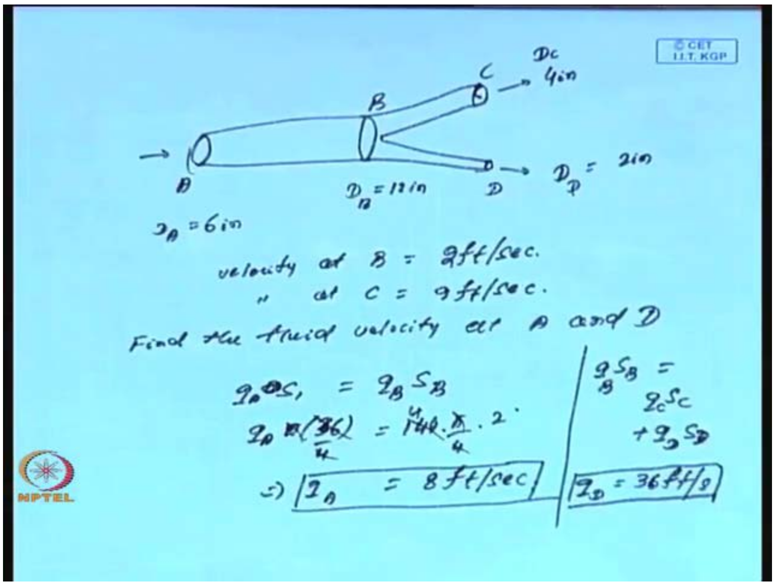

Circular pipe, so this is a tube, it has fluid will enter through this one and it will exit in this way, it will enter this way till fluid release, let this point be A, B, C and the D. If i say if I say my diameter D A is 6 inches and here, D B the diameter at the point B is equal to I will set 12 inch at C I will set this is 4 inch D C that is 4 inch and diameter at this point D D, this is I will set 2 inch, so it is clear.

Here, this is diameter is 6 inch here, the diameter is 12 inch from there, there is a bifurcation and here, diameter at point C it is a 4 inch and here it is 2 inch. Now, if I say the velocity, the fluid velocity at B is 2 feet per second, if it is at 2 feet per second and again velocity at C and we say, this is 9 feet per second, my question is, find the fluid velocity at A and D. That means, at which rate the fluid will be entering through this tube and a, which rate a fluid will be going out through the D tube, through the face D.

Now, since we have seen that incase of a stream tube, we have seen that the q 1 A 1. So, q A D A q A rather I will call this as s 1 is q B s B s 1 s 2 are the surface area and this is a cylinder here. So, my q A into s 1 is pi into D square by 4, 36 this is 36 by 4 and this is q b q B, here it is 144 into pi by 4 into q b q B at this point is 2 feet to this gives me, pi will cancel 4, 4, 4, 4 then I have q A will be if I cut this with this then it will be 4. So, 4 into 2 is equals to 8. So, q is 8 feet per second.

So, my speed at here, becomes 8 feet per second. Now, in the same way, what will I will do that I will be because, I know this speed here, I have the fluid speed if I say that at q B at this will be q B s B will be same as q C my s C plus my q D s D and if I utilize this substitute for this, then I will easily get I am not going to the detail I can get my q D as 36 feet per second, this is what I will get, it can easily check that, just a simple calculation I am leaving it to you.

So, this two example suggest that how the flow, fluid flows inside a tube of certain cross section, if the cross sectional area changes then, the velocity of fluid changes, fluid velocity changes and here that is why sometimes when have a very the flow through unusual the speed because, very high it is because, that the unusual, the area squeeze we squeeze the area and in the process, the speed of the flow speed will be high, fluid will flow water higher rate, layer the speed will be high. Now, this I will go to, what is path lines particularly, path of a particle.

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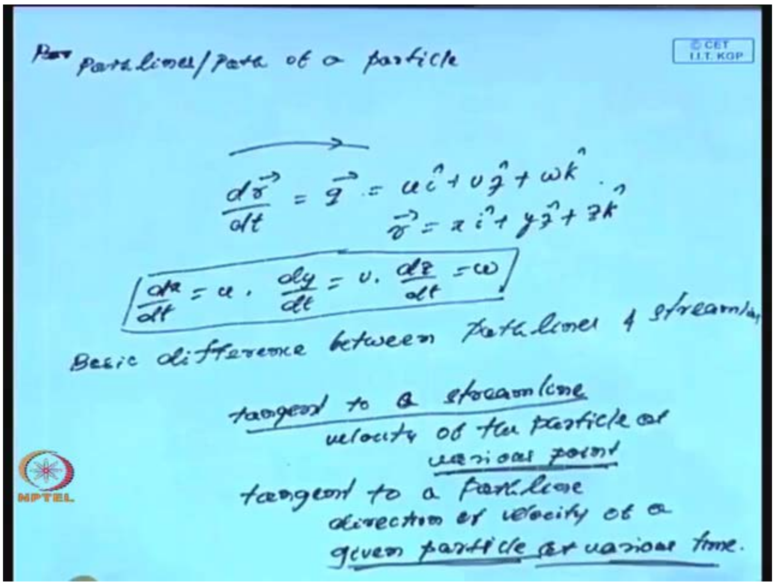

Basically, what we talk about here the path of a particle is a curve again it is a curve the path of a particular a curve which a particular fluid will describe during its motion. So, the differential equation of the path line will be d r by d t that this will represent the fluid motion. So, d r by d t will give you q bar and what is q bar q bar is u i hat plus v j hat plus w k hat. So, that is; that means, and if you has I put it in r as write it for r as x i hat plus y j hat plus z k hat, then we have d x by d t equal to u d y by d t that is equal to v d z by d t that is w, these these are the equation for the path lines.

Now, the question comes we have already seen that what how how far the path lines and stream lines are difference, basic difference between path lines and stream lines. So, the tangent to stream lines, we have already we know that the tangent if we draw a tangent to the stream line it gives us the direction of velocity, tangent to a stream line, if we draw a tangent to a stream line it gives us the fluid particle, it gives us the velocity of the fluid particle, of the particle basically the fluid particle at various point on the other hand.

The tangent to a path line, if we draw a tangent tangent to a path line gives the direction of velocity, it gives the velocity direction of velocity of a given particle that, there is at various at a various time. So, basic difference is that here, at here it gives the that when you draw a tangent to the stream line, it gives us the velocity of the particle at various points at a same instance of time on the other hand, if you draw tangent to a path line direction of the velocity of a given particle this is the difference at various times.

Here, various points we have at various times here, the given particle for a particular particle at various times whereas, in case of stream lines, it gives the same particle at various points. Basically, your time is fixed and in this case your time is varying space position is fixed. On the other hand, if I will say that when the motion is steady, if the motion is steady then stream line do not vary with the time and it coincide with the path line. Now, with this background of stream lines and path lines, I will just go to two dimensional flow.

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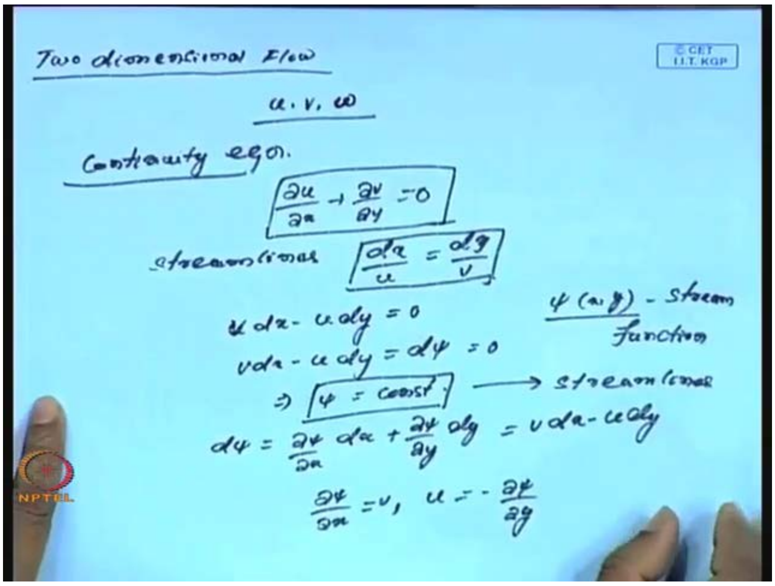

Why the need of two dimensional flow? Because, everything nature is three dimensional why should we think of a two dimensional flow. When it comes to particularly we have the three components of velocity u, v, w suppose, one of the component is very small or negligible. So, in such a situation what we will do, then we can consider if one of the component is small, then we can consider the flow as two dimensional and in fact, it has been found that in many situations when the flow is symmetric along the particular direction then it is easy to analyze consider the flow as two dimensional.

And the from experiments and observations it has been seen that while physical modeling in many situation, the two two dimensional flow is a very accurate realization of the flow situation. Even if there are axis symmetric flows, here the flow can consider as a two dimensional flow. So, because of that there are two aspect, one is either the certain symmetric characteristics of the flow, if certain symmetric character characteristic of the flow is there or when the one of the components of the velocity is negligible.

Then then we can always consider the flow as two dimensional and that gives a very good accurate realization of the flow characteristics. Suppose, if we consider one cross section, particularly what happened in the large ocean when you do with large problems related to ocean waves, are often we consider the flow as if the flow characteristics is similar in each section, each cross section, particular cross section we always consider the flow symmetric assuming the symmetric characteristic of the flow, we always consider the flow as two dimensional.

So, and the it has been seen in many cases that, this gives a very good realization, particularly for large class of marine hydrodynamics problems and it is easy because, the moment you reduce the dimension by one, the complexity of the problem reduces and that helps us to understand a get a very accurate realization of the situation, in a very simplify manner, often we try with a two dimensional problem, once we get a proper feeling about the problem, then we go to the general case that is what we do.

Now, with this I will now go to few problems related to the two dimensional flow problems, particularly we have already talked about the what will happen to the continuity equation in two dimensional, continuity equation particularly here I want to say, emphasize the often the flow is incompressible because, as I have mentioned earlier that here, we will be considering for a large section of this force as the basically marine hydrodynamics, we will concentrate on incompressible fluid and for a. So, for a incompressible fluid if that would have continuity equation will be del u by del x plus del v by del y is 0 you see the continuity equation.

And now, let us consider the differential because, and then we have the streamlines, we have come across the streamlines that is the equation of the stream lines is stream lines are d x by u is d y by v, this will give us the stream lines for a two dimensional flow. If that is the case, then what will happen let us consider the differential u d u v d x minus u d y if this, consider this as 0, if this is 0 that gives us from this we can always get this v d x minus u d y is 0.

If v d x minus u d y is a exact differential d psi then what will happen. So, that will be 0, then psi is a constant and that psi because, this psi is equal to constant again what will happen, if I relate with. So, further we can see d psi as del psi by del x because, if psi is a function of x y, if I say psi is a function of x and y then d psi is del psi by del x d x plus del psi by del y d y and this is same as v d x minus u d y see, if you compare this component wise, we will just compare the x part d x part and d y part, then we can say del psi by del x is equals to v whereas, u is minus del psi by del y.

So, in fact this psi is called the stream function, psi is equal to constant, psi is the stream function, this psi is called the stream function and psi is equal to constant as we have seen that they will give the stream lines that will give the stream lines. Another point of view here, now I will relate phi with the, the stream function with the velocity potential.

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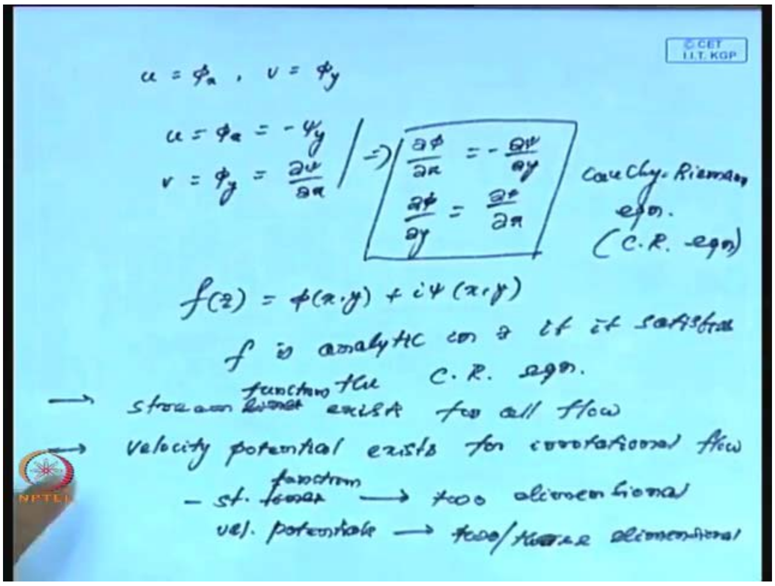

We have already seen in in case of the two dimensional flow, we have already seen u is equal to phi x v is equal to phi y. Now, from here again, if you relate with the stream function. So, we have u is equal to phi x and phi x is nothing, but we have u is nothing, but minus del psi by del y minus psi y similarly, if we say v that is a phi y and v is again del psi by del x.

So, which gives us del phi by del x is equal to minus del psi by del y and again del psi del phi by del y is del psi by del x, this is the sometimes say complex function theory, this is the Cauchy Cauchy Riemann equation, particularly if f is a function of a compression f of z that is given as phi x y plus i psi x y then f is analytic, we say f is analytic in z provided, if it satisfy the satisfy the Cauchy Riemann, the Cauchy Riemann equation. So, this is the Cauchy Riemann equation, in short we call it as Cauchy Riemann equation.

So; that means, for a two dimensional flow, we will have if the flow is potential. So, here it is very clear; that means, I have already mentioned that a stream lines exist for all flow exist for all flow on the other hand, we have velocity potential will exist only when the flow is irrotational, it will exist for irrotational flow, on the other hand for stream lines they are only for two dimensional flow, stream lines they are associated with with two dimensional flow, when the flow is two dimensional.

On the other hand, when it comes to the sorry stream lines particularly I will say that a stream function this is not stream lines, stream function, exist for all flow whereas, this is again stream function, exist for two dimensional whereas, velocity potential velocity potentials here, both two it exist for both two or three dimensional flow. So, this is a very, very this is a very clear difference between as I say the stream lines, stream function and velocity potential.

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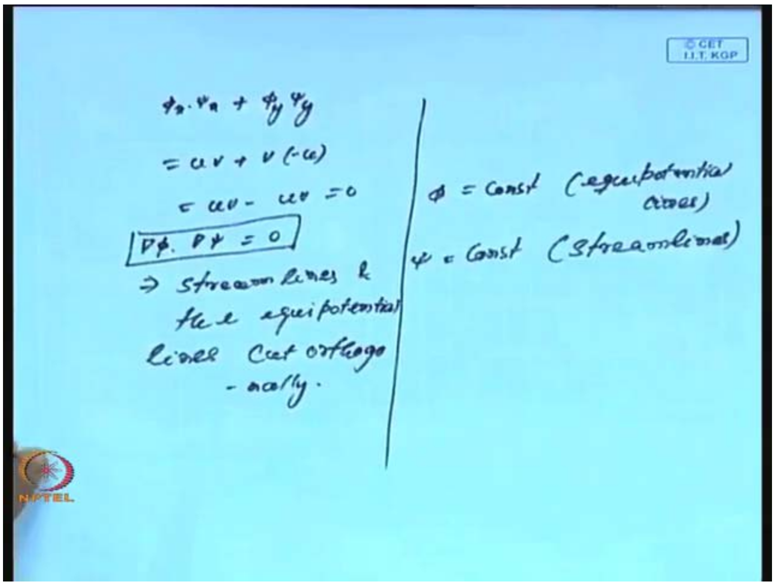

Now, another thing here that what is the another basic difference now let us just say that if we say, phi what will happen to phi x psi x plus phi y psi y this will give phi x is u we have seen we have seen phi x is u and psi x we have seen it is v whereas, phi y is v we have seen psi y is minus u. So, this is same as u v minus u v that is equal to 0.

So, which even one must say that grad phi dot grad psi is nothing, but grad phi is 0 which implies that the stream lines, when phi is equal to constant gives the stream lines psi equals constant it gives the phi is equal to constant is equal to constant, it gives the equip potentials equip potentials I have already mentioned where as the psi is equals to constant gives the stream line, it gives the stream lines.

So, this suggest me that the stream lines and the equip potential lines, cut orthogonally this is another. So, this is another very important result. Here another point I work I would like to highlight that I have already mentioned that the stream function is independent of the fact whether the flow is irrotational or not, on the other hand the velocity potential phi they exist only for irrotational, when the flow is irrotational and that is of potential kind.

So, irrotational flow is are often called as the potential flow. Now, I will come to two examples, couple of examples rather to understand that how the flow characteristics are defined in case of various examples are few examples rather I will concentrate on to understand the path lines, stream lines then flow irrotational flow stream functions and how they are related. So, to do that I will go to this one by one let us work out few examples.

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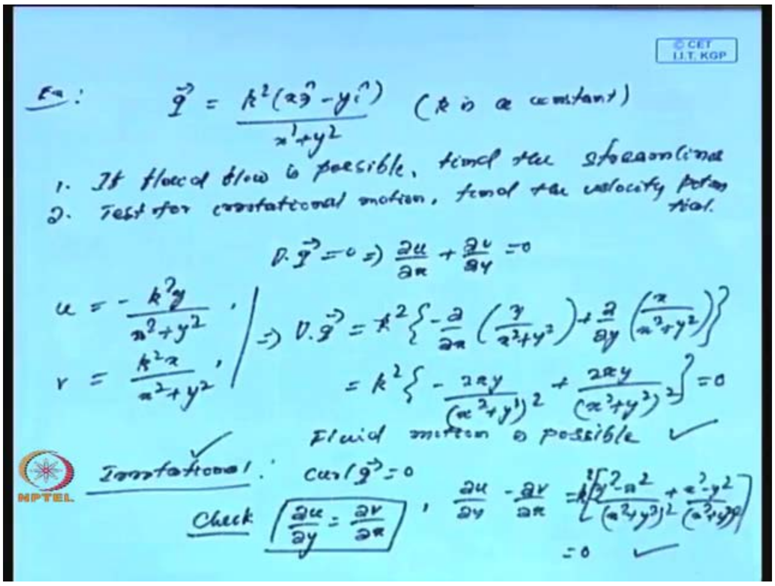

And, so here I will first four five examples let us work out. Let me say q bar equal to k square x j hat minus y i hat divided by x square plus y square, here I assume k is a constant. So, now, whether this q bar represents, whether it has a it relates with some fluid motion or not then if, so if a motion is possible, fluid flow is possible is possible find the stream lines.

And then that is the first part, second part test whether the flow is irrotational, test for irrotational motion. Again if the flow is irrotational, then find the velocity potential. So, here I have preferred this given the velocity vector, I have two questions, first question I am emphasizing whether to test whether the flow is irrotational flow is possible fluid flow is possible or not, once we say that the fluid flow is possible.

Then, we will find that what is the stream line because, that will give as a direction of the flow and then, we will check it for whether the flow is irrotational, if the flow is irrotational, then we will find the corresponding velocity potential. So, here we will find the stream lines, we find the velocity potential at the end we will check whether the cut orthogonally or not that is what we will do, then it will be clarify some of our whatever we have theoretically discussed, let us check for it.

First this is a two dimensional flow and for the two dimensional flow we have divergent of q bar is 0 that is del u by del x plus del v by del y is 0, this we have to check because, it is two dimensional and it will, so del u by del x what will happen here, here your u is minus k square y by x square plus y square and your v is k square that is x y x square plus y square and then, what will happen your which implies del dot q bar will be k square into minus del by del x y by x square plus y square plus del by del y into this is x by x square plus y square.

And this gives us k square and if you take the derivative with this is minus 2 x y by x square plus y square, square this will gives plus 2 x y divided by x square plus y square, square and that gives us 0. Hence, fluid motion is possible in compressive fluid, we have a possible fluid motion, motion is possible and once the fluid motion is possible. Now, whether we say that, what will be the stream lines now check for irrotational let us see first, whether the flow is irrotational.

If it is irrotational, we have to check it for curl of q bar is 0 curl of q bar is 0 because, for irrotational and that is same as because, the flow is two dimensional here, we have to check it as if del u by del y is equal to del v by del x this we have to check and here, we can see that, easily here del u by del y minus del v by del x this equals to y square minus x square del v by del y, this will give you y square minus x k square into by x square plus y square, square then plus x square minus y square divided by check it and that is this as 0.

Since, this is 0. So, the flow is irrotational. So, this is satisfy and we have we have already seen, this satisfies dimension q is 0 fluid motion is possible and here, we have seen fluid is irrotational. So, now, we will go for what are the stream lines and what are the velocity potential, to find the stream lines what we will do.

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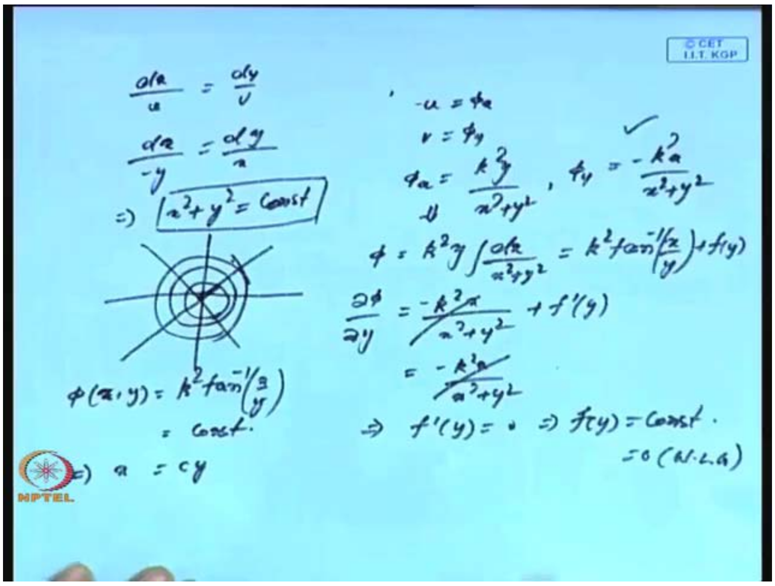

We have a stream lines will give us equation of stream line is d x by u is d y by v and that that gives us that is why two dimensional flow. So, that will give us d x by minus y d z y by x square that will give you d y by x which gives us x square plus y square equals to constant. So, this gives the stream lines. So, it is concentric circles, this all will give us concentric circles on the other hand, if we look at the velocity potential.

So, in that case, we will have u is equal to phi x and v is equal to phi y already, we know u v. So, we have phi x is equal to k square y by x square plus y square and phi y minus k square x by x square plus y square, if you integrate it then you get phi from this one, we can get phi is equal to k square y integral d x by x square plus y square and that gives us k square inverted tan inverse x by y tan inverse x by y plus f of y.

Now, what will happen to if I take del phi by del y of this, this gives me k square del phi by del y if I go far del phi by del y minus k square x by x square plus y square plus f dash y and this is same as, already I have been given phi y is this minus k square x by x square plus y square. So, which implies because, this implies f dash y is 0 because, this term will get cancelled into this term, when we have f dash y is 0 and; that means, f y is constant and this constant, we can always say it will 0 without loss of generality because, a constant, we can always take it as 0.

So, we have taken arbitrary constant we can take it as 0 that gives us which gives. So, phi we get it phi x y we have got it as k square tan inverse x by y. Now, if I say that phi x y is constant will give me the equip potential lines, if I put this as constant which gives me x is equal to c y. So, if these are the concentrated stream lines now, x is equal to c y is a centroid lines, if x equals to c y then, what will happen they will just pass through the origin any line will pass through the origin sorry it will go through this origin and you can have, number of lines and this is a line passing through the origin these are all circles.

So, if you look at at draw at any point a tangent you will see that, these two lines the stream lines and the equip potentials they will cut each other orthogonally. So, this is what it shows. The another point is that here, we have seen the flow the stream lines they are circles whereas, the flow is rotational, the stream lines are circles the following circular path the fluid follows the circular path whereas, the flow is irrotational. Now, I am showing in another example that, the flow is rotational they are circular as well as the flow is here rotational in nature, they are not irrotational, let us look at this example.

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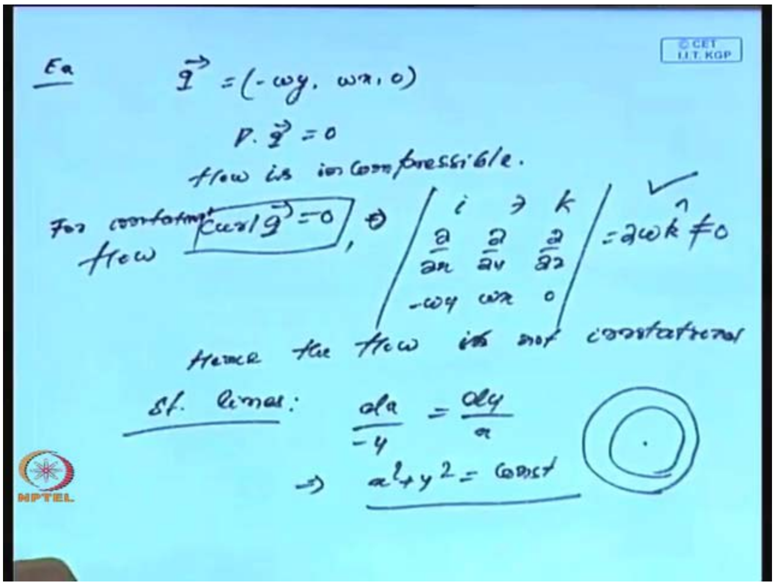

Suppose, my q bar is minus w y, w x, 0 let us say omega y then we can see, easily that del dot q bar is 0 and again. So, which shows that flow is incompressible fluid flow, flow is incompressible; that means, there is a fluid motion if it is possible and if you look at curl of q is 0 which is same as, which implies i j k if you take del by del x del by del y del by del z and then u is minus omega y omega x this is 0 and then this you calculate, it to 2 omega k hat and which is not equal to 0.

So, curl of q sorry it is not 0 for irrotational flow, this is for irrotational flow here I am finding here, curl of q is not 0 hence, this flow flow is not irrotational. Now, look at what is the stream lines, if you look at the stream lines, we have d x by minus y is d y by x and again which gives you x square plus y square is equal to constant. So, if this is the stream line for each constant you have a circle passing through origin and centre is at origin on the other hand here the, curl is not 0.

So; that means, we have a fluid flow that the fluid particle follow circular path whereas, the flow can be remain irrotational whereas, in the previous example we have shown that the flow can be irrotational, but still the flow can follow circular path, with this I will stop today.

Thank you.