Marine Hydrodynamic
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Lecture - 28
Worked Examples on Wave Motion

Welcome you to this lecture on wave motion particularly this is the part of the course of the marine hydrodynamics under the NPTEL program. In the last couple of lectures, we have talked various aspect of water wave motion. Today, we will try to work out to few examples and illustrate some of the applications for better understanding, for this wave motion problem. Now, let us start up with the very simple example.

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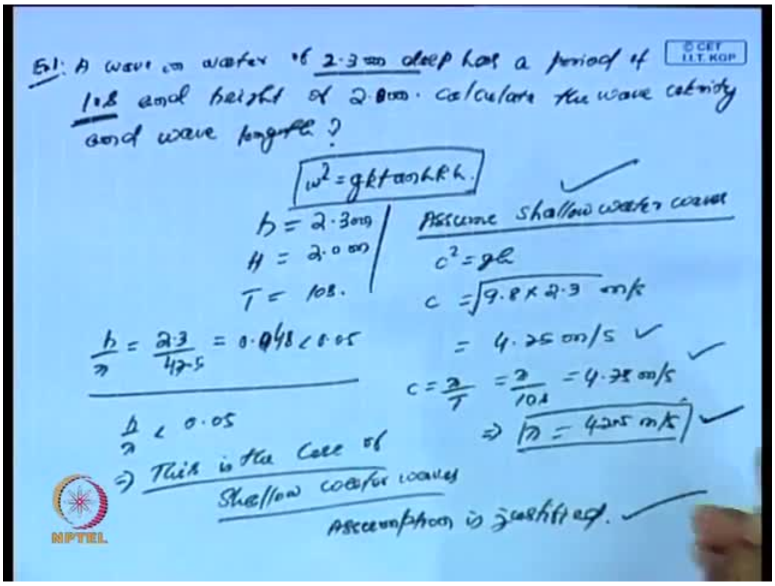

Suppose, a wave in a water of 2.3 meters as deep has let me call this example one, now we are to conduct this as a period to a period of 10 second and a height of 2.3 meter. Now, we need to calculate wave clarity and length basically and in the wave length. This is the question, so in this class what will you do? Because if you look at the omega square for the new wave for the despise wave length and omega square is g k tan hyperbolic h, so because I have a wave length to much of a deep water depth is very small and period is 10 seconds.

So and it has height of sorry, 2.2 meter. So, if it is height is 2 meter, where are the water depth of water vapor h is given as 2.3 meter and capital H to the water wave height that is 2.0 meter and we have been given t as 10 second. If I assume suppose, I assume, assume shallow water, I assume shallow water waves and this water wave I assume, it is of shallow water waves if I assume this, then what will happen? Then as per definition c square is g h as c square a h is 2.3 9.8 into 2.3 and this is c meter per second and that becomes 4.75 meter per second.

If c is this, then what will happen to? c is nothing but, c is equal to lambda by 2, lambda t is 10, t is 10 second, so this becomes 4.75 meter per second and which implies lambda is equal to 47.5 meter per second. This is my lambda, so wave solidity we have already seen wave solidity and wave length. Now, I have started with assumption this is swallow water case, what will happen in case of swallow water? Then it will be h by lambda will be if it is shallow water case, so my s by lambda will be 2.3 divided by lambda is 47.5 and this ratio will give me, it can take it 0.048 and this is less than 0.05.

So, this is satisfied and the assumption so this is satisfied. So, h by lambda is less than 0.05 which implies this is the case of shallow water case, of shallow water waves. So, I have started with an assumption shallow water waves and I have also, this assumption is a justified, assumption I can always say the assumption is justified. This is true, that means this is lambda is equal to 7.5 and this is 4.75 is true because that is the assumption. So, because of this, because of this assumption and this becomes easy and then otherwise I would have solved this equation on this equation of this omega square, but this has simplified to a large extent of problem. Now, I will go to another example.

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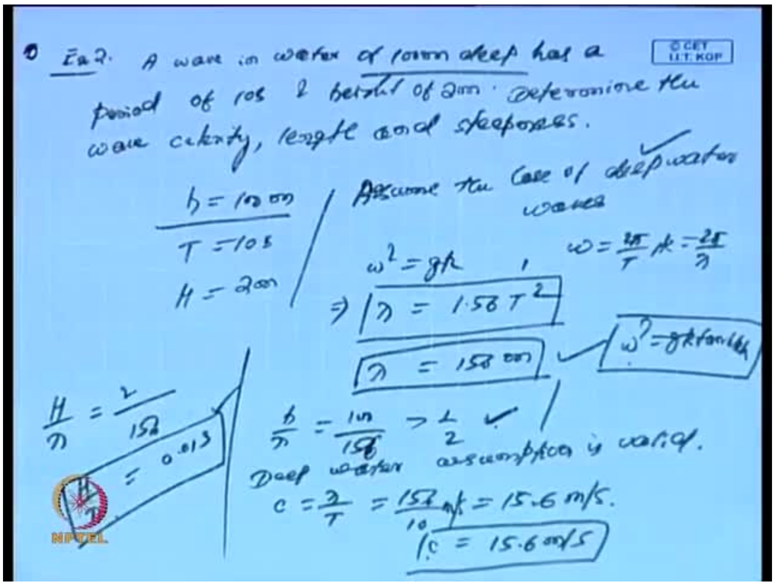

We call this as example two, suppose a wave in water of 100 meter deep has a period of 10 seconds and a height of 2 meter, then it is determined wave celerity to the another wave celerity length. Basically I mean to here the wave length and stiffness. So, to do that, what I have to do? I assume because it is a 100 meter deep water my h is around 100 meter and it has a period of 10 seconds, t is ten seconds and height of capitol H is 2 meter. Simply I assume that, assume the case of deep water, assume the wave motion is in a case of deep water waves.

If I assume that this is a deep water case, when you have omega square is g k. In the case of deep water, which can give me easily that I can easily get it from lambda 1.56 into t square this wave 2 pi by y is equal to 2 pi by t and k is 2 pi by lambda, then one usually you can get this from this equation. So, I will even t is 10 second, so if I put t is equal to 10 second then lambda is 156 meter. Again lambda is 156 meter, then I will look at h by lambda h is 100, lambda is 156 and that is equal to the half. Hence, this assumption of a deep water, the deep water assumption justified is valid.

Now, once this is now what i have been asked, again i will say what is my c c is nothing but lambda by t 156 by t 10. So, this is 15.6 meter per second, then again and wave stiffness, wave stiffness that is your h by lambda H by lambda H is a given as a 2 by lambda is 156 and that gives me 0.013, that is my h by lambda, that is wave stiffness.

This is c we have a c 15.6 per second and m lambda is 156 meter. Then usually, so lambda is this and c is this and this, the lambda be this.

So, here in the this two examples are same, the assumption that whether it is the shallow water assumption because fortunately we are lucky that our assumption becomes justified, so that usually you could do this calculation. On the other hand if it is not be satisfied, suppose this assumption will not be value assumption and if it will not be valid this will not hold good. This condition will not hold good in that case, I may have solve the originally original equation to find, what exactly the wave area is?

That means, I have to start 10 hyperbolic, so this is the, what we have discussed earlier. So, and we have to approach the numerical methods, even if in the previous case, because if we pursue it we are able to handle it directly, assumption of shallow water is also (( )). Now, is this two simple example, I will go for another example where the propagation of a wave from deeper water to shallow water. Let us see what another example of a wave from deep water to shallow water.

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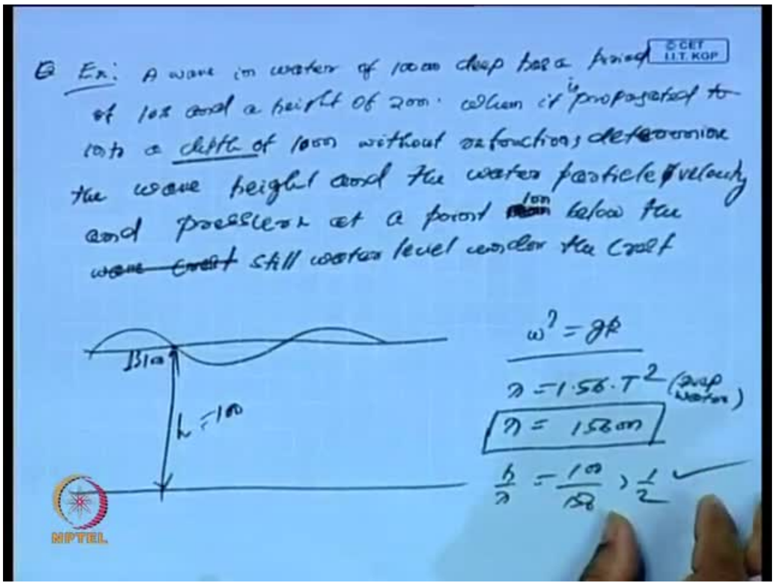

Suppose, in the last two examples we have seen that, in the last few examples we have seen that, wave propagate there is no change in the water depth. Now, suppose I have wave propagate, there is no change in water depth. Now, suppose I have a wave, suppose I say wave in water of 100 meter deep has a period of 10 seconds and a height of 2 meter. When it propagates, when it is propagated to, when it is propagated to into the depth of 10 meters, when it is propagated to a, into a depth of water into a 10 meter without any perfection or without reflection, that determine the wave height.

The water particle velocity water particle and pressure at a point 10 meter below below at a point 1 meter below, the 1 meter below the wave crest. 1 meter below the still water level, under the wave crest under the crest, that means I am looking at this is the inverter level. So, I am looking just at this point, if this distance is 1 meter below the water depth by initial water depth sequence. Then initially the depth was 100 meter, this depth was 100 meter is was initially 100 meter.

So, in this case what will I do? If you look at this omega square is g k, assuming that this is past initially, it was in deep water wave design. Then we have, we all know that a lambda is equal 1.56 into t square and t is 10 seconds, so it will be 156 meter, so lambda is 156 meter initially. This wave was propagated and if you look at this assumption because I have taken this under the assumption of by deep water. Deep water, this is under case of a deep water and we have just under previous examples we have seen that this assumption is a valid option. We can say h by lambda is 100 by 156 that is divided by half. So, this is an assumption, now from this wave has propagated to a depth of 10 meter, then what will happen?

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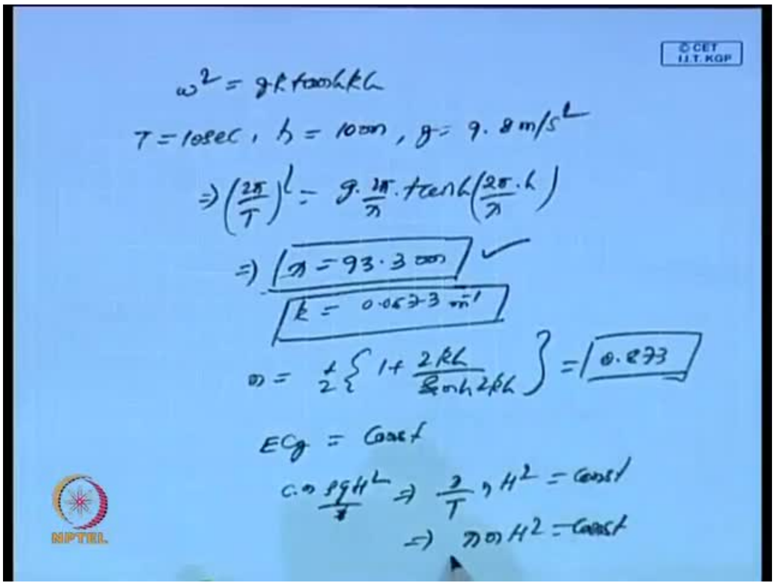

If you look at for original equation omega square is g greater hyperbolic k h, we have been given t is equal to 10 seconds and our h is 10 meter. If h is 10 meter and g is 9.81

meter per second. second square. Then you can easily see if you solve for t is equal to that is omega is 2 pi by t square is equal to g into 2 pi by lambda to 10 hyperbolic 2 pi by lambda into h. And if you substitute for t g and h then and the solution for lambda, then one can easily seen that lambda will be 93.3 meter, this will be lambda.

It can be checked, one can apply the method, but the bi section method to find this lambda from this starting this one because t is known h is known as (( )). Once this lambda is known, then we have k is equal to 0.06 73. That is square which is nothing but 2 pi by lambda and it is by minus 1 and that is is my wave length, wave number. Then my n is in general, n is equal to 1 by 2 1 plus 2 k h by sine hyperbolic, 2 k h and this gives me if I substitute from k and that will give me 0.873.

So, in fact we know that the n is equal to half, in case of shallow water, in case of deep water where n is equal to 1, in case of shallow water. So, this n itself the value shows that it is the case of intermediate water depth point and 73 because it lies between 1 and half and 1. Now, again if you go back to the energy when there are some match E C g is equal to constant. E C g is a wave velocity is the energy density associated with the wave, then we have from this C g is a c into n into e is rho g H square by 2 by 8 and that gives me c. It is nothing but lambda by t and this is n, this is H square that is constant because at the period of the wave is same.

I can always say that lambda n H square is equal to constant, so if I take this values of, that means lambda, that is lambda n H square at H is equal to 100 meter is same as lambda n H square and that is H is equal to 10 meter.

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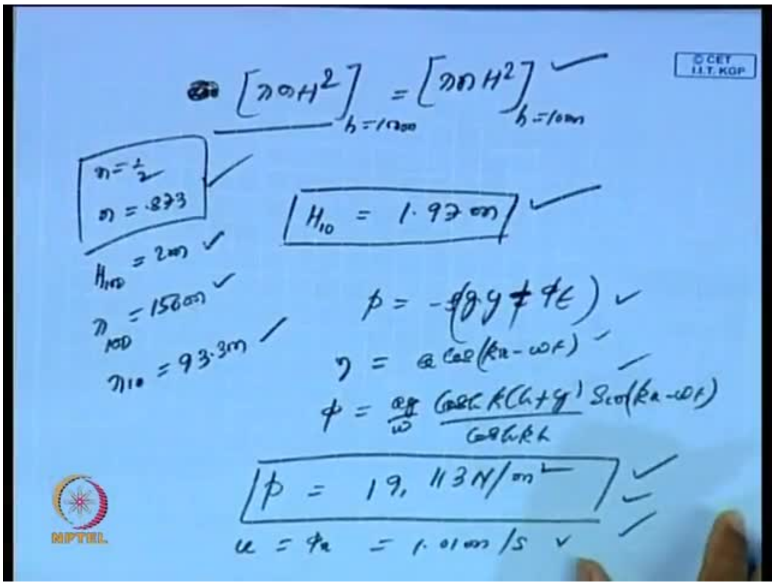

Because initially the wave lies in a 100 meter deep and then it has come back to a depth of 10 meter. So, if I substitute for and in case of an h is 100, I have seen that we have seen that this is the case of a deep water, in case of a deep water n is equal to half and in case of a when h is 10, we have already seen n is equal to 0.873 substitute of n in both sides and initial wave height was we have initial h 100. The wave height initially it was 2 meter and my lambda was 156 meter. So, this was lambda 100 and then I have seen that my lambda 10 is 93.7.

So, my lambda is 93.3 meter. So, if I substitute for this values this, this and this with all about this and then I substitute here, then I will get h 10, where I took the what when, that other depth of a 10 meter that will give me I substitute 1.97 meter. This is the wave height, so there is a small meter 2, 1.97 meter, this is what and this we have obtained by this result and this is nothing but the energy, conversation of energy force. Now, to alert new request because, I am interested in knowing the pressure.

I have p is if I apply the Bourn lie’s equation p is minus s g y plus pi 2 minus pi 2 and if this is the case, then I have eta is suppose I have eta is equal to a cos k x minus omega t. Then my pie will be a g by omega cos hyperbolic k into h plus y pi cos hyperbolic k h into sine k x minus omega t. Then we have this, if I substitute, substitute eta and pi and p and again y is equal to minus 1 below the crest. Below the crest means, so this will give me my p, will give me it can be easily seen that 19 113 Newton meter per meter square. So, this will give me the pressure, hydrodynamic pressure.

Again, if I look at calculate the (( )) velocity partical velocity because pi is the pi is u, it is nothing but pi x and the below the crest 1 meter below the water waves below the crest. This will give me 1.01 meter per second, so this can be checked by substituting for these values. Then this will be 1 can easily obtained, so we have the three examples what we have solved. First case we have started the example very simple example of a shallow water waves and then we went to case of a shallow deep water wave.

Then third example is very interesting, because here we are able to know the water, how the water depth changes to give the change, how we apply changes due to the change in the water depth and also how one is able to calculate the pressure and the components component of velocity of the water particle. This is very clear example in a similar way, one can work out in a such example, but in this understanding I will just go to work out a little more theoretical problem, particular at a look at the capillary wave motion.

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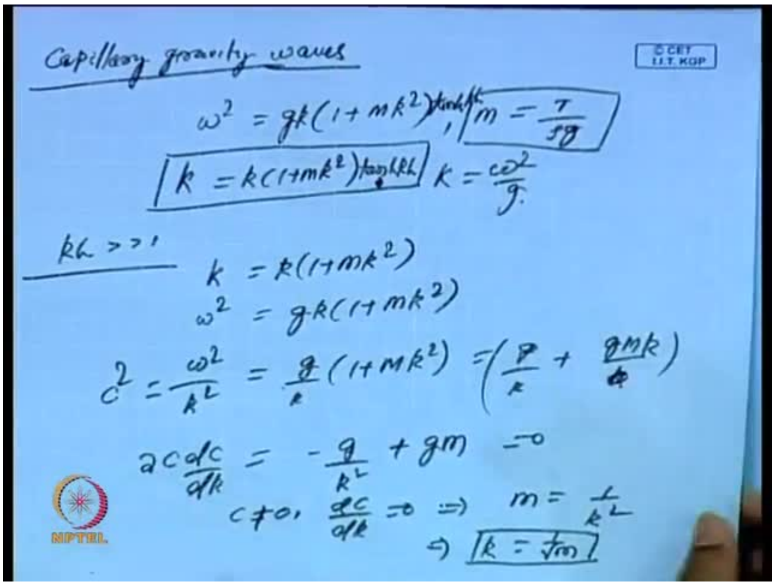

One problem associated with a capillary gravity waves is, problem is like this. So, we all know that in case of capillary gravity waves that is personal relation is omega square is g k 1 plus m k square. Whereas or sometimes we call it m is t by rho g. This is the coefficient of extension g, is the gravitational acceleration due to gravity rho is the density water. Omega is the wave frequency, whereas k is the wave number associated with the capillary gravity waves.

Sometimes we put it in this one, we call it capital K is equal to k into 1 plus m k square, whereas capital K is also omega square by g into 10 hyperbolic k in general. Now, what will happen in case of deep water k h is what larger than 1, then we get k is equal to small k into 1 plus m k square and if k this k is nothing but omega square lies g. That will give me omega square is equal to g k 1 plus m k square, then omega square by k square g by k 1 plus m k square is can be written as g by k plus g g m by k square by k g m into k. If I say this is nothing but c square, so what will happen to t c d c by d k?

So, if I take that is respect to K, then this will be minus g by k square plus g m. If I say this is 0, that means c is that equal to 0. This c by d k is 0 that will give me optimum value. Once this by d k 0, which implies that m is 1 by k square implies k is 1 by root m. So, it can be easily seen by this value, that k is 1 by root m, that c will attend the minimum it can be checked. That this will be optimum either minimum or maximum, but it can be seen that when c is equal to k is equal to 1 by root m c becomes c minimum. So, this can be checked and once this is a where some k m is t by rho g.

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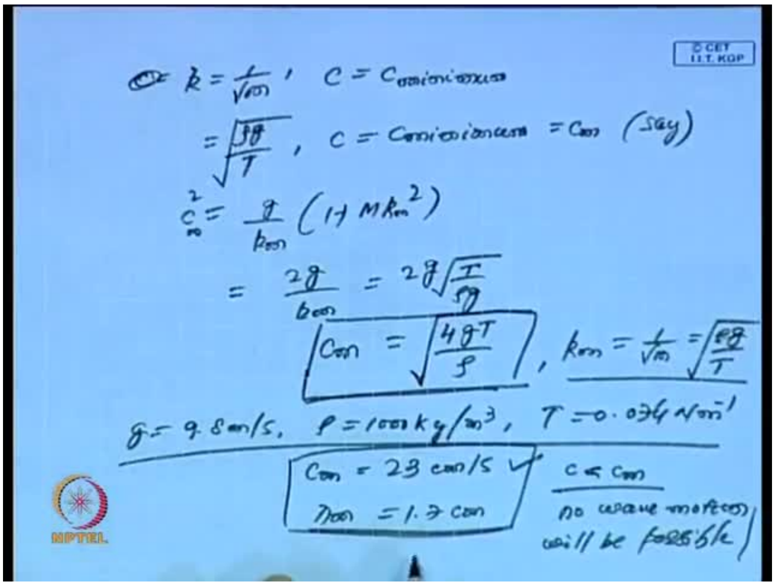

So, this will be rho g by t, so that is c is equal to c minimum. So, what is that c minimum? I call it c m say, so c is equal to because this is basically we are consider in the case of shallow. This is the case of deep water, so c m square is equal to g by k g by k m 1 plus this is g by k into 1 plus m k k m square. That will give me, so if I take g m k m square is 1 because k is 1 by root m k m is 1 by root m c is equal to c m. So, m k m square is 1, so that will be 2 g by k m and that can give us and then again k m is rho g by t 1 by sorry, k is equal to c is equal to c minimum. This is rho g by t, so t by rho g. So, that will give me 2 2 g into t by rho g

That will give me wheatear that g g t by row, if I put into this 1. So, this will give me, so c m that means by c m c minimum becomes this 4 g t by row. My k m 1 by root m rho g by t. So, for this k is equal k m c become c m, that means minimum value of speed of the of the capillary waved waves that comes up. That means and if I just take one typical case that if I take g is equal to 9.8 meter per second, then rho is 1000 k g per meter cube and t is equal to 0.074 Newton per meter. Then we can see, that c minimum equal to 23 centimeter per second lambda m, because k m is this.

So, lambda is 2 pi by k m and lambda k is 1.7 centimeter. So, for this values of this parameter values can get c m is 23 centimeter per second lambda is 1.7 centimeter. That means when wind a blows, this is minimum speed which can attend below that I will for, but in this case when the for a capillary waved waves, when the wind blows within to blow over a flat c surface, no wave will be possible below this c is equal to c m. c is less than c m, no wave motion will be possible because it is the minimum.

The speed is minimum here, so below that no wave motion will be possible. In fact it has been pointed out that for us that is the smallest possible wave, I mean all wind generated wave, this c m this lambda m wavelength lambda, the correspond same for the smallest possible way that exist in the ocean, particularly if you look at the gravity waves, the wind generated waves. So, this is one of the very important example of the wave propagation problem. So, that shows how small a wave, length can be 1.7 centimeter minus 1. Now, this is one aspect give me workout on the another case in the same length that is…

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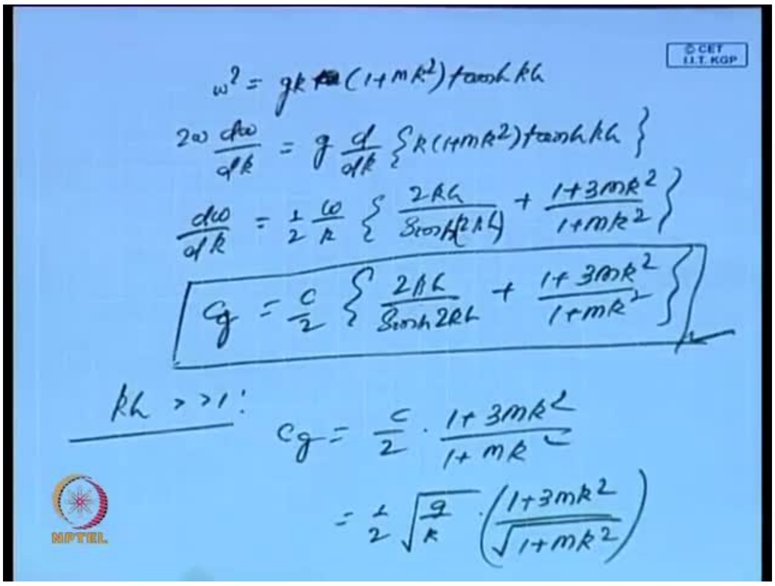

Suppose, I have been given omega square is a g k 1 plus m k square, then hyperbolic k h this is 90 percent less than. So, what will happen here two omega d omega by d k? That will give me g by this. It is my derivative, I have taken and when I take that derivative after lot of simplification I can always get d omega by d k is equal to 1 by 2 omega by k into 2 k h 2 k as sine hyperbolic 2 k h plus 1 plus 3 m k square 1 plus m k square. That same as this is nothing but c by 2 2 k h by 1 plus m k square. So, this is this is, this by d omega by d k that is c g.

This definition is c g that is the group velocity as a centered with the capillary waved waves this is one of the most general result. If I say that k is much greater than 1, then this quantity will be 0 and my c g will be c by 2 1 plus 3 m k square by 1 plus m k square. This will be into this, then this further can be simplified to write it 1 by 2 into g by k root over 1 plus 3 m k square by root over 1 plus m k square. This is the c g and it can be seen even if I assume that c g will be minimum.

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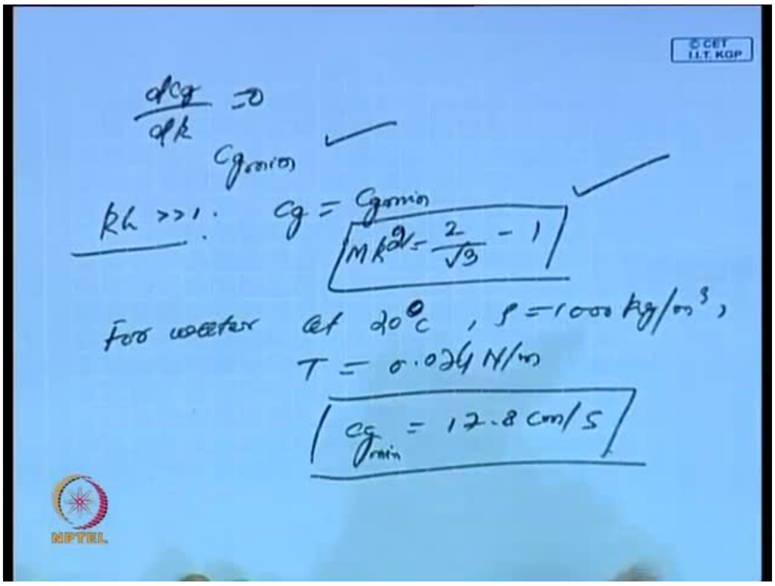

So, if I say that d cg by d k 0 that will again give me c g minimum. It can be seen that in case of deep water k h is much greater than 1 than c g. Again, c g is equal to c g minimum that means wind velocity will attend the minimum only when m k square 2 by root 3 minus 1 for this value the c g is minimum. Now, you go back to know that what exactly happen? We have seen that for water of for water at 20 degree Celsius, we can say that rho is equal to 1000 k g per meter cube.

We have t is equal to 0.074 the same example Newton per meter, then it can be when you say in that case c g will be 17.8 centimeter per second. That c g is c g minimum, so this is what happened in the case of deep water. This case, so this is one of the another example which a suggest me that, how the wave propagation, how the group velocity can be obtained? Also minimum in the group can be obtained by knowing the from the in case of capillary. Here in both the cases, we have only seen that we have only analyzed that is for same result.

Now, if this two examples I will work out one or two more example because I say this, this are all worked examples, but I have considered in my last 8 to 10 lectures, so I am going to various aspect of the problems. Now, I will we have already seen the case of a wave oscillation in a tank in a 1 Dimensional tank.

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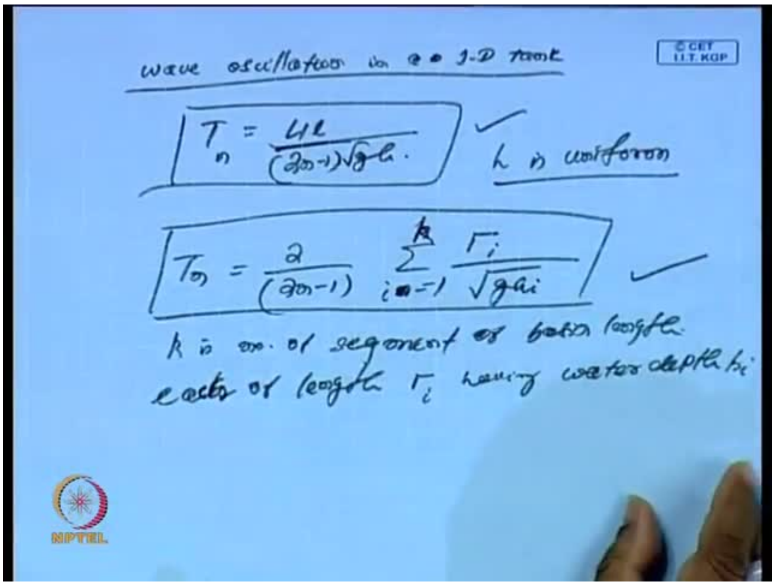

The question comes, what happened if the tank is having length and width or 1 dimensional tank. We have seen, we can say we have seen the case wave oscillation in a… We have seen that in case of a rectangular wave 1 on the t h t omega t n is 4 l by 2 n minus 1 into root g h, but if you look at the like recently are or in for another other reasons l is come across this is one water depth is vary from h is uniform. But if the water depth is varying, then there is a when h is varying over the… Then what will happen to my t n?

So, it has informed that some empirical formula that a t n is such a case, this is one of the imperial formula which is used 2 by 2 n minus 1 into sigma n is equal to sorry, rather I call it i is equal to 1 to n or i equal to 1 to k. k search anyways in the water depths or as gamma i by root over g h i. So, that gamma is the k I, rather k is the number of seconds k is the number of segments of the ration length k is the number of segments of the ration length each of length each of length gamma i having water depth h i.

So, this is the formula normally we use when to calculate the period of first lesson of the wave during the wave oscillation, when there is the bottom depth, which is having and which has case of different segments. This segments is of length gamma a and width a sorry, depth h i. Now, this is one result, which is in fact charge many times. This kind of result are used now, I will go to a, this is a 1 Dimensional result. Now, what will happen in if I just consider the case of a 2 Dimensional lesson.

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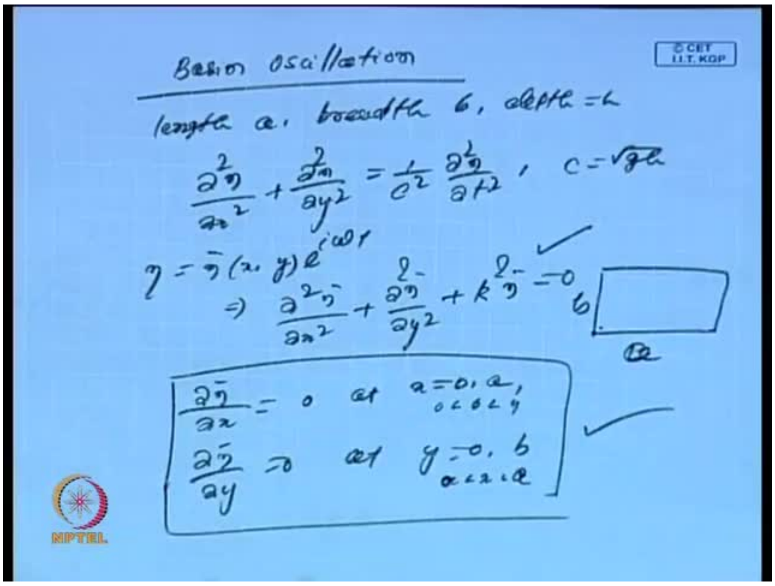

In case of a 2 Dimensional problem, suppose start with a basin wave oscillation in the basin. If in basin oscillation and the basin is up length a and breadth b, length a breadth b under depth, this is a line when depth is h. So, in that case we have, if I will look at the the equation dell x square eta by dell y square 1 by c square dell square eta by dell t square plus c equal to g h c is a root 3. Then in this case, if I assume my eta is from this form eta x y into by omega t, there also satisfy simpler relation dell square where I am separating my eta by eta bar.

So, it is dell square eta bar by dell x square plus, then this will give me plus k square eta bar equal to 0. This becomes the dependent eta bar is the special component of the surface elevation. Then we can see that curves I have basically the length is l length is l under the breadth length is a and breadth is b, b. So, what will happen? This is this is a plan here, then I have what will happen, when dell eta by dell x will be 0? Because (( )) is 0 and from that we can get dell eta by dell x is 0 at x equal to 0 and a and again dell eta by dell y and 0 at y is equal to 0.

This four conditions it has to satisfy, so that means we are solving this equation subject to this. These are the four conditions on the four sides of the wall. They will be, it will satisfy. Here this is a 0 less than b is less than y, where a is less than x is less than 0 less than x is less than a.

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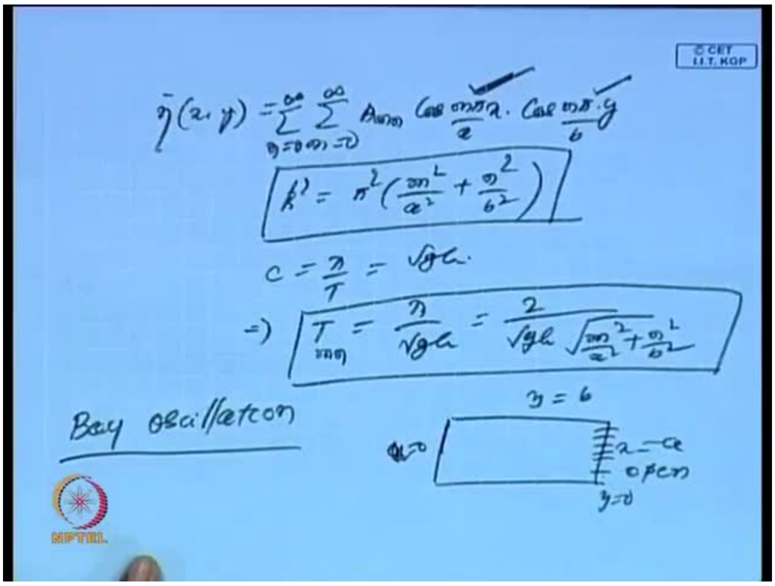

So, if I look at this, then one of the typical solution this equation will be the radiation equation will be eta x y. That means eta bar x y sigma m is equal to 0 to infinity A m n cos m pi by a into x into cos n pi by b into y eta by x y will be this. Again we can see that this has to satisfy the radius to equation. Then it can be relation that k square is equal to pi square m square by a square plus n square by b square. This is my k square square, k has to satisfy. And further we can see that by c is lambda by t, and c is nothing but root g h. So you can see that c is lambda by root g h, and by substitute k is 2 pi by lambda that will give me 2 by root g h into m square by a square plus n square by b square. So, this is what my T becomes and here t is no more a, because T is depends on to the both surface. Lesson one is the associated with the 1, is general deduction 1 is associated with the because it has along the x axis.

This is the y axis in the process my t m n depends on this. Now, if you look at this then, when m is equal to in particular you can see when m is equal to in particular, we can see when m is equal to… Now, we have t m n it depends on the the waves that oscillates in the… Here in the process this eta bar it has to because it will all depend on m and n. So, there will be two summation, because m will value from a 0 to infinity. This is also 0 to infinity, so there will be double summation. In the same way suppose, I look at bay, if I look at bay which is open on one side, then suppose this is the x is equal to 0 or this is x is equal to 0 and this is a line.

This is y equal to 0, this is y is equal to b, this is x equal to a and suppose this side is open. This is open side, then this is a bay oscillation. Again in this case, it will satisfy, satisfy the same in this equation and you can see that in this case, so will have eta is equal to sigma, sigma m n, that is a m cos m pi by a into x into cos 2 m plus 1 by into m pi by b 2 b something like this into y.

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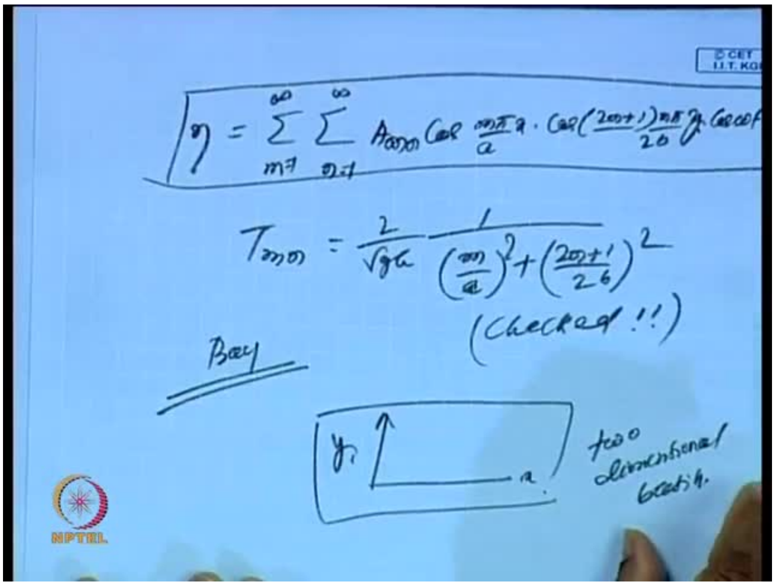

So, this will be the, into cos omega t, which will be the nature of the wave eta and again t m n. In this case t m n will be 2 by root g h, 1 by m by a square plus 2 m plus 1 by 2 into 2 be square or something like this check, this can be checked. So, this is in case of a bay and here it depends on a modes of oscillation. Both are on the surface as along the y axis this surface is 2 Dimensional. This is in case of rectangular basin, what happen in case of… If I look at the case of a circular basin circular like, in case of a circular wave, I will not spend much time on this, but I will briefly mentioned if the bay become circular, then instead of x y it will be satisfy if equation of motion dell square eta by dell r square plus and if I write my eta r theta is f or r e to the power i s theta, first to synchronization saying that certain symmetrical theta.

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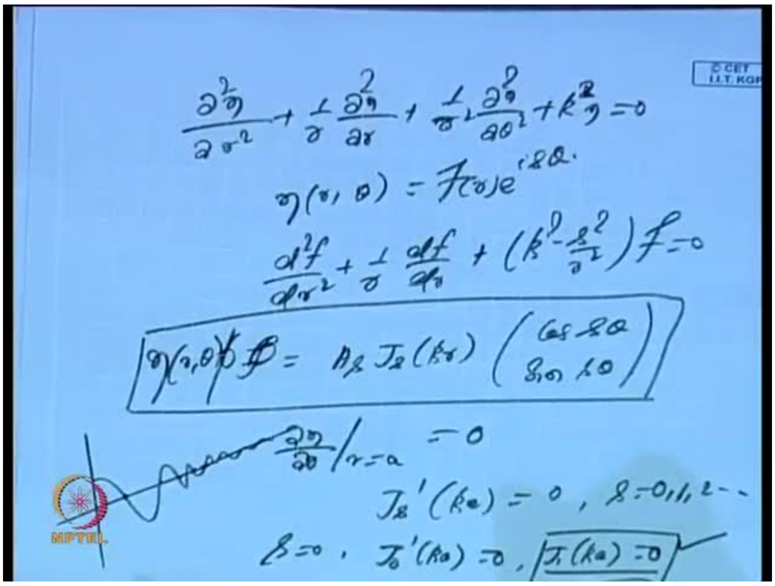

Then I will have if you satisfy in f del square f by d r square plus 1 by r d f by d r plus k minus a square by r square f is 0 and that will give me and this gives me my f will be of this form a s j s k r into cos s theta sine s theta. One of them, so this will be general form of the eta and again this will general form of eta, eta r, theta will be of this form. This can be easily checked, in fact this can be found in most of the tech broken water waves and once this is this, so what will happen? For that, if I say that a dell eta by del r r is equal to a and the boundary is 0 and that gives me it will give me is am prime 0, for s is equal to 0 1 2.

You can easily see that particularly when in case of s is equal to 0 by 0 prime k is 0, that is what j 1 k is 0. In this j 1 k is 0 as been roots it is a because j 1 k functions. The functions are like this, the function waves like this. So, it will all zero's are here, here, here, here on this axis although decays at far field, but finite amplitude. That the one when it decays with the how many roots? But if you look at, what will happen to k? Then we will see that this infinite number of root will give us infinite course of oscillation and because of that will like, once will get infinite modes of oscillation. That is what we have seen also in case of a rectangular basin or in case of a rectangular bay, whether it is 1 Dimensional or 2 Dimensional.

So, in a and her for s, you have taken the case of 0. If you take s is equal to 1 2 32 onwards, then also will get, then will also get several such values of the, because for each s have whether the corresponding j is time k is 0. Then you get if you look at the solution of those j 0 j basin function, then you get in fact many solutions that will give us in finite mode of oscillates mode of oscillation. So, whether any closed or open basin whether of any kind of a geometry, whether it is circular, whether it is a rectangular, whether it is open basin, whether it is close basin, whether it is 1 Dimensional, whether it is 2 Dimensional we have several modes of a lesson. That and in this cases, first few modes of this lesson plays a significant role.

We have earlier seen in case of a 1 Dimensional bay or basin, now we have seen same thing in case of a 2 Dimensional bay basin as well as in case of circular basin, close to circular basin. So, this is a and, in fact in case of the difference is in case of a 2 Dimensional basin you have or a bay you have two modes of oscillation; one is in a horizontal direction, one is in the do direction. Because on the one, if you take surface plan along x axis, you have a one mode of oscillation, the y axis again you have another mode of oscillation and the combination becomes more complex.

So, and again in case of circular case, we have seen that for that in case of a circular basin also. We have seen the first few dominating mode of oscillation are the first few modes of oscillation also, because at the, that is very natural from the, that is, that is clear from the root of the basin functions. This three examples today on bay always this lesson as been very good idea about the case of the 2 Dimensional. Other examples I have discussed today gives us very good idea, about various aspect of problem analysis in case of water waves.

Particularly the coupler wave to a motion problems associated, we have find out the minimum of group velocity that has given how small a wave length of a wave can be? We have seen that in these two problems, and with this today, we stop here without much discussion today. In the next class may be we will talk about some of the disturbances that propagates surface or the bottom. How this bottom and surface disturbances affect the wave motion in case of a, in case of a gravity wave motion? Also I will give talked to you briefly about little about the finite amplitude wave theory before completing the discussion of wave motion. With this today I will just stop here.

Thank you.