**Marine Hydrodynamics**
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**Lecture - 23**
# Basic Particle Kinematics in Wave Motion

Welcome you to this lecture series in marine hydrodynamics, today. In the last class we have talked about the basic equation as well as the associated boundary conditions related to wave motion and we have seen that that the free surface which represent by a y is equal to eta. Then we have the velocity potential phi that there is a relation between them and there are two such relation which relates them one is the dynamic boundary condition the other is the kinematic boundary condition. I have seen that the governing equation is Laplace equation and it is the free surface which is responsible for the generation of waves at the water surface particularly and we have considered the case of uniform depth of water.

We have seen that once we know the velocity potential phi by solving the governing equation Laplace equation along with the free surface boundary condition on the bottom boundary condition, then we can easily find out what is the surface elevation that is eta. So, once phi is known the velocity potential is known we can always find what exactly eta is. So, with this background today let us see how the, because we have seen there are two things associated. There are two terms here one is the phase velocity that is the rate at which the wave propagate and the other one is particle velocity that is q bar which has component u and v.

So, today let us see how the particle motion takes place because here we have a transfer of energy, but there is also water particle which also moves and there also in motion. So, how the motion of the water particle, what kind of path they follow and how the waves are generated which path the wave energy propagation takes place, to understand these let us have a look at again the wave motion problem.

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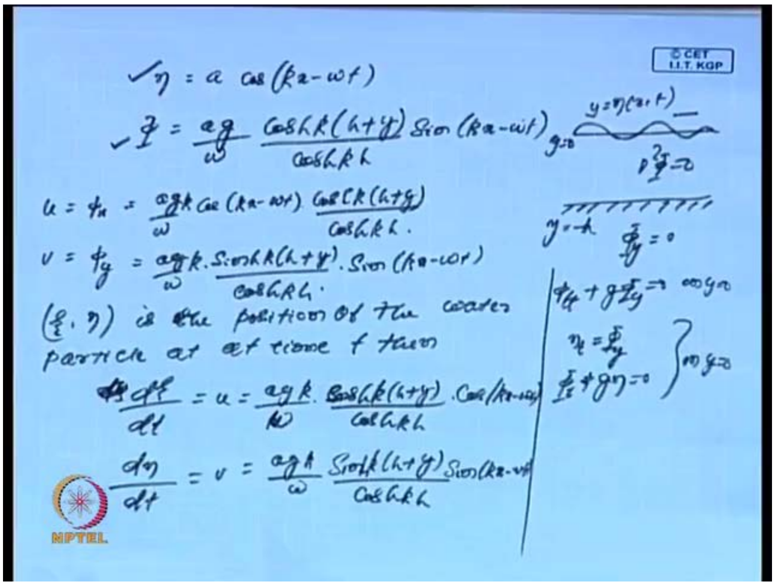

So, this let us suppose we have seen that when we have eta is equal to a cos k x minus omega t. In the last class, I have found out phi corresponding phi will be a g by omega cos hyperbolic k into h plus y by cos hyperbolic k h into sin k x minus omega t. Here, I always relate it that this is my free surface mean free surface this is my bottom belt and bottom belt is this is by y is equal to minus h and this is y is equal to 0. Then my free surface, I always say that this is y is equal to eta x t is my free surface, I have del square phi is 0 here phi y is 0.

We have only free surface here on the free surface we have phi t plus g phi by a 0 that is eta clear y is equal to 0. We have seen that relation eta t is a phi y and pi t plus g eta is equal to 0 these two conditions are satisfied on y is equal to 0. So, basically then we have seen that if we have a even eta then we can find what is the corresponding phi, once we know phi here I always mean p square phi capital phi.

So, what I want to see that suppose once I know phi what will be my u, u is a phi x and the u is phi x, once phi x is left then what will happen this will give me a g by omega then k cos k x minus omega t this is into we have this tau cos hyperbolic k into h plus y k h. Similarly, have v velocity component provided that will be phi y and this will be again a g by omega into k sorry phi y into k this is sin hyperbolic k into h plus y by cos hyperbolic k h into sin k x 1 as omega t.

Then we all know if psi and eta is the any point in the fluid domain is the position of a particular position of the water particular time t, at time t. Then we have u is nothing but d is d psi by d t at the same as u is equal to a g by k a g by omega into k this is cos hyperbolic into is much by cos hyperbolic h into cos k x minus omega t. Similarly, we have d eta by d t that is by v and that will be n a g k by omega this is sin hyperbolic into h plus y divided by cos hyperbolic h into sin k x minus omega t. Now, what I will do if I integrate this.

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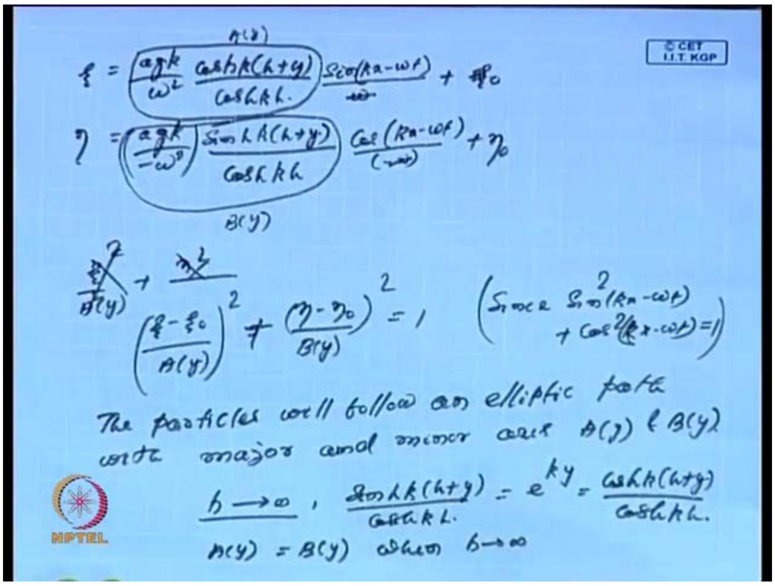


So, which implies this will give me my psi will be I have to integrate with respect to t and that will a g k by omega into cos hyperbolic into h plus y by x cos hyperbolic h, this is by omega by omega this is cos mention this is sin k x minus omega t divided by omega this is called by omega. Then we have eta is equal to this is minus when will come minus will come here, so it will my omega. Similarly, eta will be a g k by omega this is sin hyperbolic k into h plus y by cos hyperbolic k h into this sin means this becomes cos k x minus omega t then divided by minus omega.

Once then what will happen then what will happen, if I consider this as this term by omega square I taken this I call it this once I call it this term I write minus omega square I will take it. So, then if these term I call it as A y and these term I these term I call this as my B y then my psi is psi by A y square psi square by a square y eta square B y. There will be constant and that I call it as psi naught plus it will be a constant term associated with eta naught these are integration functions which depend on the original time t 0.

So, then in that process what I will have then psi minus psi naught divided by A y square equal to plus eta minus eta naught divided by B y square and this is equal to 1. We have used the relation z since sin square k x minus omega t plus cos square k x minus omega t equal to 1. If you use this from here we will get and now what will happen that means the particle these are the position of the particle these are the position of the particle psi naught and eta naught are the fixed point at time t is equal to if it is a these are fixed position of the particle at time t is equal to t naught.

Then at any time t the particle we will follow and this A y B y since, A y is B y in general different then we can always another particle will the water particles will follow an elliptic path. Elliptic path with major and minor axis A y and B y in fact both A and Y they are all dependent on the water depth position dependent. Then again we can see that we can easily see that when y is tends to infinity momentum x tends to infinity then what will happen then we have sin hyperbolic k into h plus y by cos hyperbolic h will be e to the power k y.

Similarly, then we have this is a much cos hyperbolic into h plus y by cos hyperbolic h and then since what will happen because of this, this can be easily obtained. So, as a result we have u y becomes b y when x tends to infinity and in the process when A y becomes B y.

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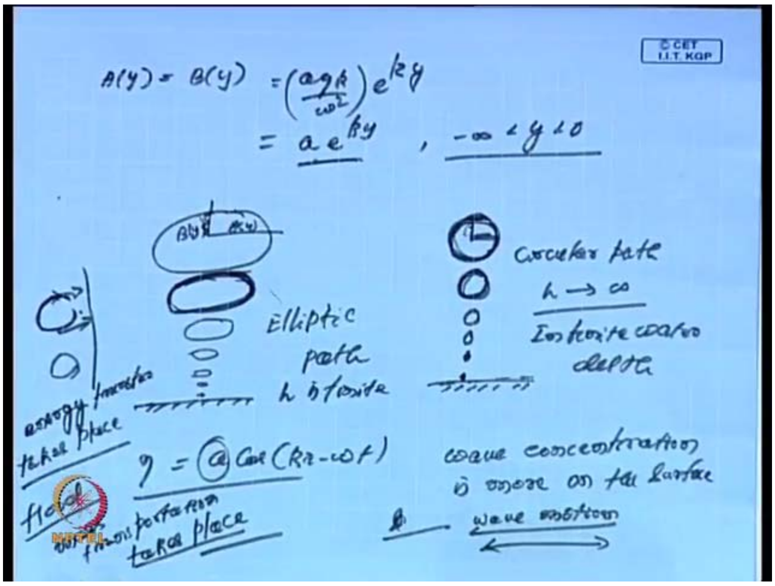


So, that means my A y is equal to B y is equal to what will happen a g k by omega square a g k by omega square into width e to the power k y and we also know that a g k by omega square omega square is g k in case of deep water. So, omega square is g case there its becomes a e to the power k y so minus infinity is less than y is less than 0. So, A y becomes this and that means the particle motion will be, so I can call this as since in this case when A y is becomes B y that means initially the particles were following circle a elliptic path.

It was depending on this was initially the particle motion was there following elliptic path for finite water depth and it was decreasing as the water depth was going on these was the A y part this was my B y part and this was a decreasing as we as going down. On the other hand, when this is elliptic path for phi h is finite, the other hand when h is becoming infinite then they follow a circular path.

So, here it become a circular path and here h is tending to infinity the depth is infinite, infinite water depth and here the radius of the circle is always a is the amplitude because we have started with the wave we have started the wave eta is equal to a cos k x minus omega t. So, this a is the amplitude of the wave and the once it is this radius of this is decaying exponentially because y is in the negative direction, it is y is always negative. So, it is decreasing exponentially it is decreasing as that as we go down in the water so at the particle motion becomes 0 at seabird this is the seabird.

So, if both the cases the particle motion is 0 and there becomes there is no fluid motion, that is possible at the bottom due to the generous along the waves at the surface. And here it is therefore the circular path whereas, in other case in this case the following elliptic path and here this is the radius that is always depend on the position of the particle. So, in the process what happen here the surface concentration wave, so in free surface gravity wave concentration is more on the free surface is more on the surface. Particle excitation also is more on this surface because this x eta is more on the surface, which goes down as you go down it comes two x dimension it comes to there is no excitation on the particle.

So, there is no wave motion and this is another aspect is that they follow all elliptic closed elliptic path and here they follow circular path here because there is only one motion although I will not go into the details. But I make it point is there question comes whether can they follow open path, will suppose they whether they always complete they complete the path or they still they can before they complete the path, if they whether happens to that. So, here actually what is happening here the water particle there is if I will not go to the details, but I just mention here that here in this case only energy transfer energy transfer takes place only energy transfer takes place takes place.

So, the each particle excide the nearby particle and that is the way the energy get transferred. On the other hand, if the situation in which particularly they will not follow they will not complete a closed path. That means in that case before it completes a closed path the particles moves to another point and that is what happened in case of a fraud wave. In case of a fraud where the mass transportation takes place because in this case the particle before it comes to the same point these particle same particle instead being a exited it has already shifted to another point.

Then the circle complete circle closed circle is not found and in that case and that is the only possible and there is a transportation of mass. So, in energy transfer particularly in case of waves in the ocean in general where there we have the situation of wave propagation only there is no transfer of mass. On the other hand when there is a transport transfer of mass the one point to another, in such a situation the particle will change the before the complete cycle is formed the particles changes its position and in the process a h particle changes its main positions.

So, in the process no closed path is formed and that is the basic difference between basically the wave motion in a sea wave motion in a sea without mass transportation and with most motion with mass transfer, so that is one of the very important this is one of the very important point to note. So, with these now I will look in to another aspect that what happen, we have seen that a stationary we have.

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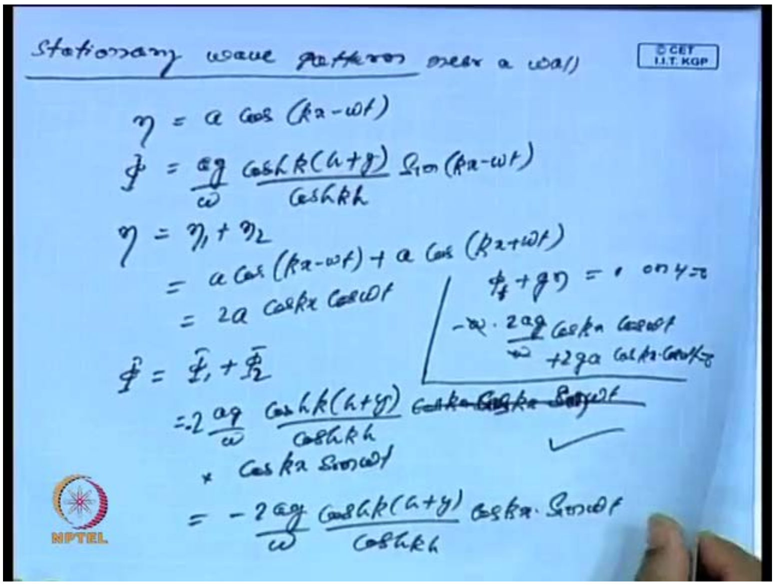


Let us relate the stationary wave pattern in case of a when a wave propagates near a wall, stationary wave pattern near a wall this is another interesting point to be noted. Now, we have seen that if I just say eta is equal to a cos k x minus omega t these are very simple examples, but the other relations are very important eta is equal to a cos k x minus omega t. Then we all know that the corresponding velocity potential will be a g by omega cos hyperbolic k in to h plus y by cos hyperbolic k h into sin k x minus omega t.

So, then what will happen if I just say that I have another wave where eta is a combination of two such waves eta 1 plus eta 2, where eta 1 is a cos k x minus omega t plus eta 2 is a cos k x plus omega 2, omega t. And the resultant here, it will be 2 a cos k x cos omega t and the corresponding velocity potential phi will be phi 1 plus phi 2 phi 1 is the velocity potential associated with eta 1 and phi 2 is velocity potential associated with theta 2. So, if I do this then what will happen here then I will have a g by omega 2 a g by omega cos hyperbolic k into h plus y by k h this is sin k x minus omega t. The other case it will come across cos k x minus omega t the other one will come as sin k x k x eta t by t plus eta.

So, here cos k x cos omega t and this will give us sin will give me cos k x sin k x sin omega t sin k x sin k x cos omega t. I hope this is clear because if we have let us have a look at it, we have phi t plus g eta is equal to 0, I will say phi t plus g eta is 0 and y is equal to 0 if this is 0 that is my phi t is on y is equal to 0 this is y is equal to 0. These term will be 1 phi t will give me minus its minus omega into sin omega t phi t plus g eta.

Now, this will be again, sorry this will be it will not be sin it will be again this is cos phi t plus g eta, so this will remain as cos k x this will be sin, sorry this will be into cos k x sin omega t and may have a look at this minus omega minus omega this will be minus sin. If I take minus omega phi t is minus omega this is plus omega, I will minus sin minus omega.

Then we have 2 a g by omega and this is 1 cos k x sin omega this is becomes cos omega t and then omega omega cancel to a g by omega plus g times 2 a cos k x cos omega t is 0, so this is the right answer. So, that I will let us put it in a proper way minus 2 a g by omega into cos hyperbolic k into h plus y divided by cos hyperbolic k h into cos k x sin omega t and here what is happening. Let us look at this one what will happen to because this is a standing wave, the corresponding wave is a standing wave.

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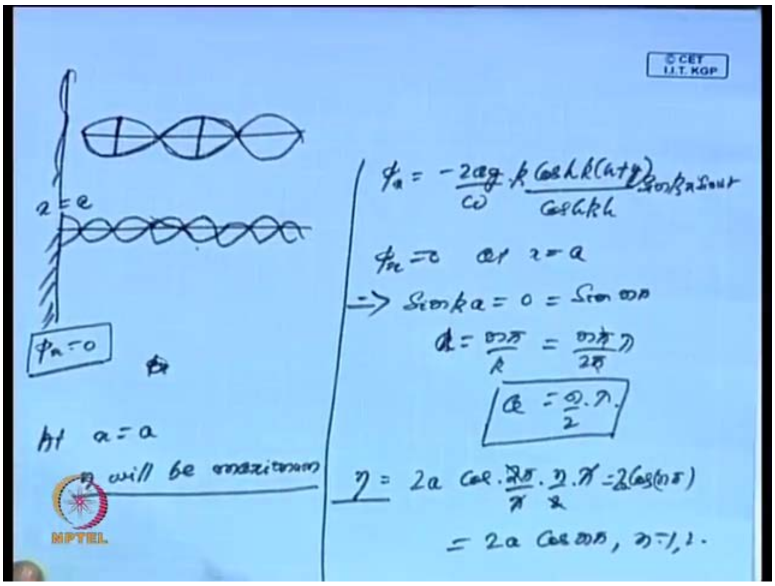

We have as we know when the two waves collide to progress we have collide they will form the standing wave like this. The question comes, if I have a near a wall if I have a vertical wall then near a wall I have phi x will be 0 and if phi x is 0 phi x is 0. Suppose, this is the position of the wall phi x some of the wall is located at the position x is equal to a, so at x is equal to phi x is 0 that will give me that because my phi is my phi x will be in this case I have phi x my phi x will be minus 2 a g by omega. Then I have a k this is cos hyperbolic k into h plus y by cos hyperbolic k h into sin k x into sin omega t sin omega t.

Then this phi x is 0 at x is equal to a, a phi x is 0 at x is equal to a implies sin k a is 0 sin k a is 0 and when sin k is 0 this will nothing but sin n pi that means k is equal to or a is equal to n pi by k and k is nothing but 2 pi by lambda n pi. That, means k is equal to or a is equal to n pi by 2 n pi by k and k is nothing but 2 pi by lambda n pi by k is 2 pi by lambda and that is nothing but if pi get cancel n by 2 into lambda. So, this is my a and what happened when a is n by 2 into lambda that is then what will happen to my eta there eta a t. So, 2 a cos k is k is 2 pi by lambda into n by 2 into lambda this is this, this becomes lambda this becomes cos 2 a cos n pi. So, basically this is become 2 a cos n pi and for all n is equal to 1 to 1, so the maximum.

So, what will happen to eta, so at a we have seen at x is equal to a, so eta will be maximum because for all n it can be maximum of value of eta. So, if eta is maximum in a standing a maximum of eta amplitude eta becomes amplitude will be maximum here. Maximum amplitude will occur only at the nodal points and the point x is equal to a has to be anti node point, so like this. So, what we are concluding from here that obviously it can also be seen that here the horizontal velocity is 0. Here, the wall as per the fluid particle is concerned and if the formulation of standing waves we have seen that eta attempts the maximum eta will be the maximum the amplitude becomes the maximum.

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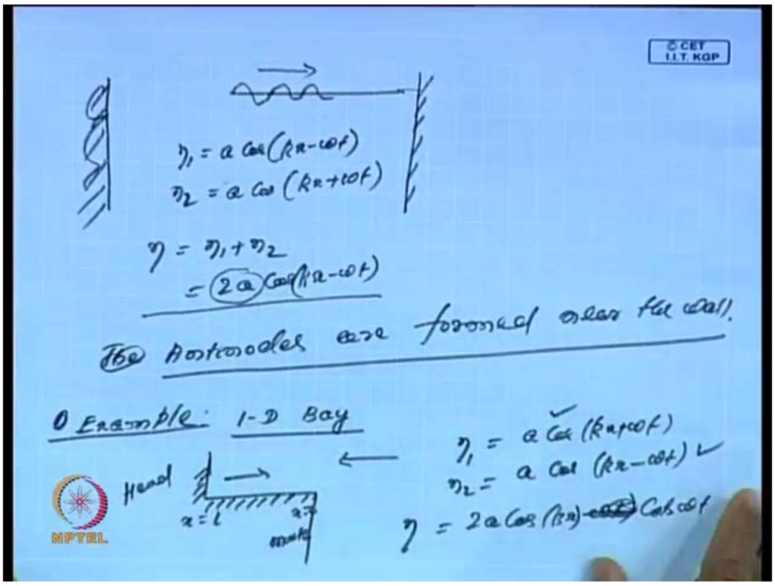


So, what it says that if I have a suppose, I have a wave which is propagating if I say suppose I have and I will put it this way even if I say, I have a wave which is a suppose the initial wave which was propagating this way. I say, I have a 1 in any direction you have suppose I say eta 1 is equal to a cos k x minus omega t, this was the wave which was approaching in the positive direction and eta 2. It hits a wall particularly vertical wall and that will be once, it hits the wall the wave will be back a cos k x plus omega t because it is a vertical wall whole wave will be back reflected back and then the resultant wave will be eta will be eta 1 plus eta 2 which is a nothing but 2 a cos k x minus omega t.

So, what is happening here, so that means the formation when a progressive wave is propagating hit at a wall then the wave will be return particularly, it will be reflected back because the amplitude some both waves as same only direction is opposite. So, it will firma standing a whose amplitude is twice that of the individual waves at the same time what happen near the wall because there is a standing wave formation has taken place. So, what is happening near the wall in this standing wave formation near the wall the antinodes are formed the antinodes are formed are formed near the wall near the wall and that is one of the very interesting observation which has been clearly derived.

Now, what happens suppose with this understanding of formation of antinodes, I will just look at the oscillation of wave. Let us look at illustrate this way example, how it helps us in understanding problems of ocean engineering. I will take typical example consider a one dimensional bay when I think of a 1 dimensional bay, I just look at suppose I have a suppose, this is x is equal to l this point is x is equal to 0 this is a 1 dimensional bay here this is the mouth and this is the head of the bay this is the bay mouth.

Then, now if I just say that a wave initially it was propagating from here, so my eta will be a cos k x minus omega t because it is propagating from this side it will be and this eta 2 will be a cos k x plus minus omega t. Once, it will it will hit the wall bay head then it will back this is my second wave this is my first wave. So, the resultant will be 2 a cos k x minus omega t and then what will happen what will happen to my, sorry this is cos k x into cos cos omega t. So, what will happen to my eta 0 if I look at what happen to here the bay head?

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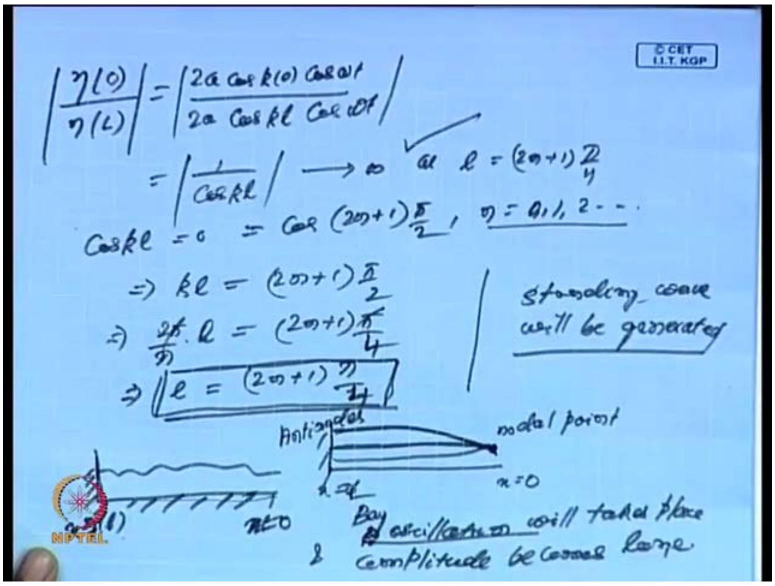


My eta 0 by eta l if eta 0 by eta l eta 0 is amplitude of the wave they have the mouth whereas, eta l is the amplitude of the wave near the head. So, eta 0 by eta l modulus will be because this is 2 a cos k 0 cos omega t becomes 2 a cos k l cos omega t and that gives me if I look at the modulus it will give me 1 by cos k l and that 1 by cos k l. What will happen to this, now what will happen cos k l will be 0, if cos k l is 0 that means this will be cos 2 n plus 1 into pi by 2.

So, if this is the this will be 0 at these points and then that will give me k l is equal to 2 n plus 1 into pi by 2 and case 2 pi by lambda into l is, so which implies this n can be 1 0 1

2 then the integral value with their natural number. So, that pi get cancel, so which implies by l is equal to 2 n plus 1 into lambda phi this is lambda by 2, so what it says one is there is a or there is a 2 here. So, this will be 4, sorry there is two here it will come as 4 so 2 n plus 1 into lambda by 4.

So, that means when l is equal to this is a very important relation then l is equal 2 n plus 1 into lambda by 4, my eta 0 by eta wall only tend to, so this will tend to what this will tend to infinity as l becomes 2 n plus 1 into lambda by 4 and then what does it mean. So, this becomes very large that means and my this was my bay this is my eta l this was my eta 0 this is my this is my x is equal to 0 point this is x is equal to l. So, these may stands if there is a wave which was propagating and then what will happen in this x is equal to 0 to x is equal to in fact x is equal to l, x is equal to 0.

So, the distance is lambda by 4 and here it is the bay head they are this vertical wall, so there are will be standing wave should be found there, so that means these distance is one-fourth of lambda or when n is equal to 0 this is one-fourth of lambda because there is so near the bay head. There is a standing wave should be generated, standing wave will be generated and will be generated near the wall and here this distance of total wave and then again here there this distance is lambda by 4. So, at this point what the eta 0 by eta will has 2, if it has to be infinity then these has to be nodal point and as usual antinodes will be found antinodes will be found here.

So, that means when a wave propagate is case of a 1 dimensional bay, so when a wave propagate from the deep sea to the bay area. So, there is a chance that when l is equal to 2 l plus 1 into lambda by 4 if the bay is vertical one and this distance is l then there will be a condition of the water particle resonance. It will resonant oscillation will takes place and that resonance condition of the bay oscillation will take place and amplitude in the sense has to be loss becomes loss once the amplitude.

That means the bay will bay regional’s will take place in that situation we always say bay oscillation has taken place and amplitude becomes loss so this is one of the very important point. Now, with this I will I will highlight few more things, so the second thing I want to tell here is that what happen to the higher modes of oscillation.

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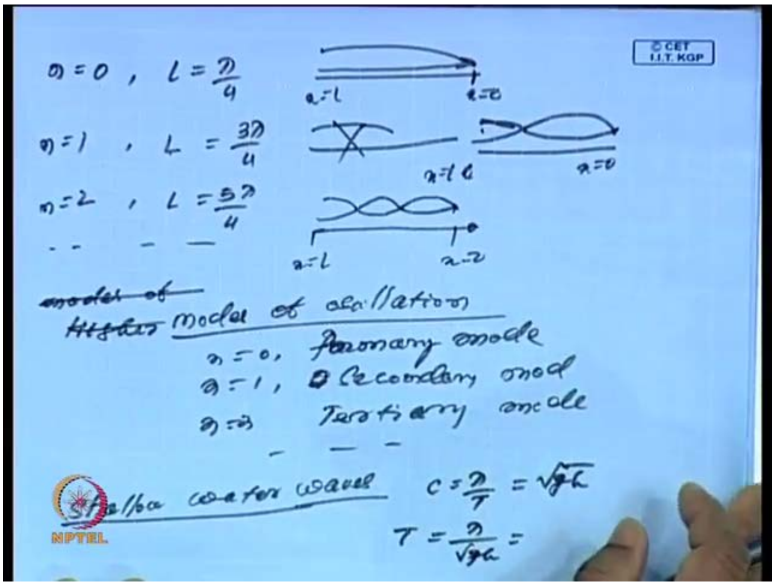


We have seen when n is equal to 0, we have l is equal to lambda by 4 and then we have seen that the total only one-fourth of a wave which has taken place, this point is x is 0 this is x equal to l. Now, when n is equal to 1 then what will happen to l, l is equal to 3 lambda by 4 in that case suppose I say, sorry l this is 0 x is equal to l this is x is equal to 0 and then because these point resonance this will be a nodal point. This, will be a anti nodal point, so what will happen this distance is 3 lambda a 4, so this much will be happened. If I take n is equal to 2 then l is equal to 5 lambda by 4 and in that case, if I look at it this is x is equal to l and x is equal to 0 then, so what is happening the lambda is going on decreasing with we have a increase in values of n.

So, in fact these are called if I go on increasing n then will see that smaller and smaller waves will be form and that means the wave length lambda of the higher modes of oscillation will go and decreasing. Initially, it is one-fourth l is lambda by fourth then l is becomes l is width. So, as we go on increasing n goes on increasing the length of the wave particle goes on decreasing the individual modes.

So, these are calls modes of higher modes of oscillation or basically n is rather call it modes of oscillation and in fact in these case, when n is equal to 0 we call it fundamental a h primary mode of oscillation primary mode of oscillation. Then n is equal to 1 called secondary mode secondary mode n is equal to 3 this is called tertiary mode and so and so on.

So, this is higher modes of oscillation now I will just take another typical example in this case. Suppose, I think of a shallow water waves these are consider case of a shallow water waves, if we consider the case of a shallow water waves then what happens I have already seen in case of shallow water we have already seen c is equal to lambda by t and for shallow water c is a root g h. So, what will happen to lambda, so then if c is lambda wave then T will be lambda by root g h and lambda becomes or lambda is because t is lambda by root g h. Let me just do a little homework further.

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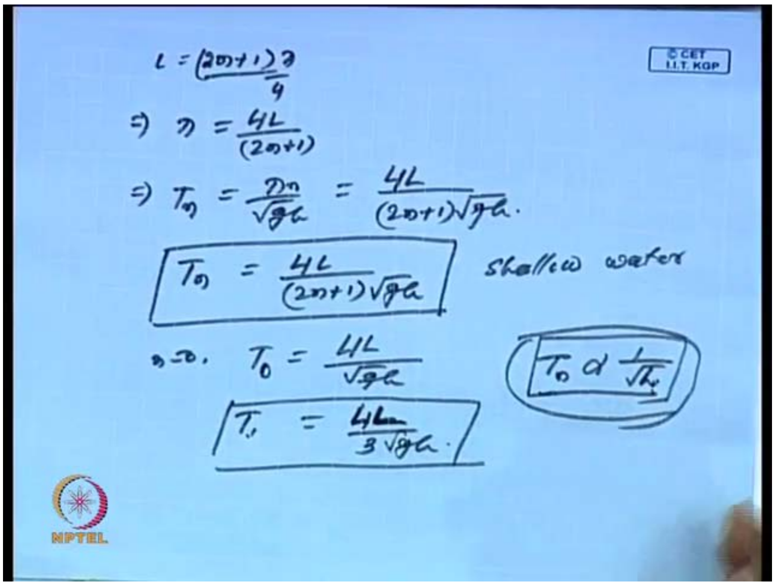


So, T is lambda by g h and we have seen l is equal to 2 n plus 1 into lambda by 4, so that means my lambda is 4 l by 2 n plus 1, so when is a y T is equal to lambda by root g h and this becomes 4 l by 2 n plus 1 into root g h. So, this is I call this for a each n, I have a T n, so I call this as a T n because for each values of T corresponding to n is the fact to this us and this also I will call it as lambda n.

So, T n that means my period of oscillation T n becomes 4 l by 2 n plus 1 into root g h that is in case of a bay. So, this is the period of oscillation in case of a bay and in case of shallow water that means when n is equal to 0 my fundamental mode of period of oscillation, that is called the primary mode as period associate with the primary mode is 4 l by root g h. Then that is t 0 my t 1 will be 4 l by 3 into root g h and another point here to observe not only that the period of oscillation t n the head is as 1 by root h.

So, when the water depth is increasing the depth is increasing h is increasing, T n is decreasing, higher the water depth lower the period of oscillation. So, basically that is another observation here is what we have made and again the period of oscillation this one is anyway fixed. So, this will be T 1 will depend on the water depth and that will because for a particular bay the length is fixed.

So, once the bay oscillation when a standing to 0 that means when the water depth is reducing the period of oscillation is going on increasing. So, in fact this is a situation which is often observe in case of a bay, even if in the shallow water under the assumption of shallow water region here we have seen that how it is happening.

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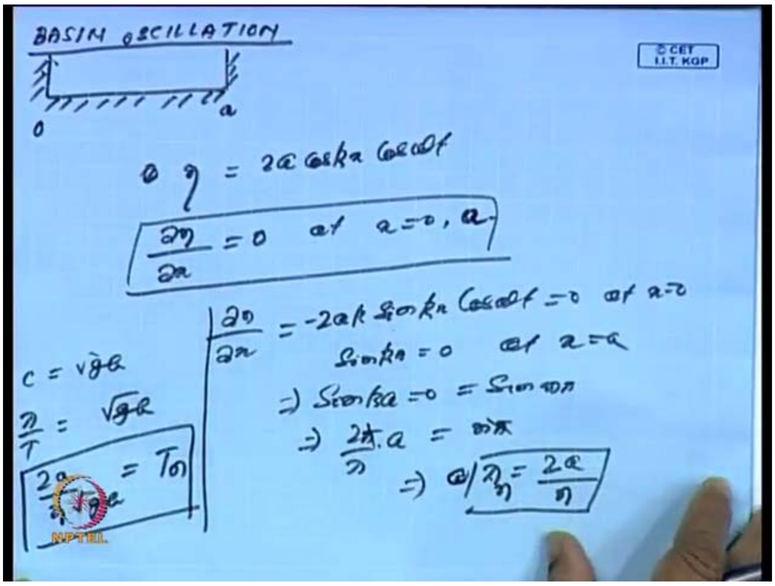


Now, with this background we will go to see that what happen in case of a basin, in case of a basin again I will consider the same case. Let us take a basin which is of the basin is of length l suppose this say this is 0 this is the length of the basin is a and we have if I again call it my eta as standing waves will be formed. So, if I say eta is equal to 2 a cos k x cos omega t then what will happen to my because at the two ends del eta by del x will be maximum because all set can be seen that del eta by del x will be 0 at x is equal to 0. The x is equal to 0 and a because here the amplitude will be maximum and amplitude will be maximum then del eta by del x will be 0 at the two ends because antinodes will be found in the both sides. We have a vertical wall, so antinodes will be formed.

So, in this case, this is basically I am talking of a basin oscillation basin oscillation, so if this is the case that is that will what will happen here. So, once del eta by del x 0 at x is equal to a that will give me this if I look at this substitute for it del eta by del x is equal to 2 a k sin k x and cos omega t and 1 sin k this is 0 at x is equal to 0 this is automatically satisfied. On the other hand, if sin k x is 0 at x is equal to a which implies sin k a is 0 on that is given sin n pi and which implies k is 2 pi by lambda into a is equal to n pi, pi pi get cancel.

So, implies my a rather I will put it lambda, lambda will be 2 a by n 2 a by n will be lambda 2 a by n. So, that means I call this type of this as corresponding lambda is lambda n then if I look at this and what will be my c is root g h c is lambda by T and this is 2 a by root g h and once this is root g h lambda is 2 a by n. If I put lambda is lambda n has 2 a by n into root g h this becomes T n.

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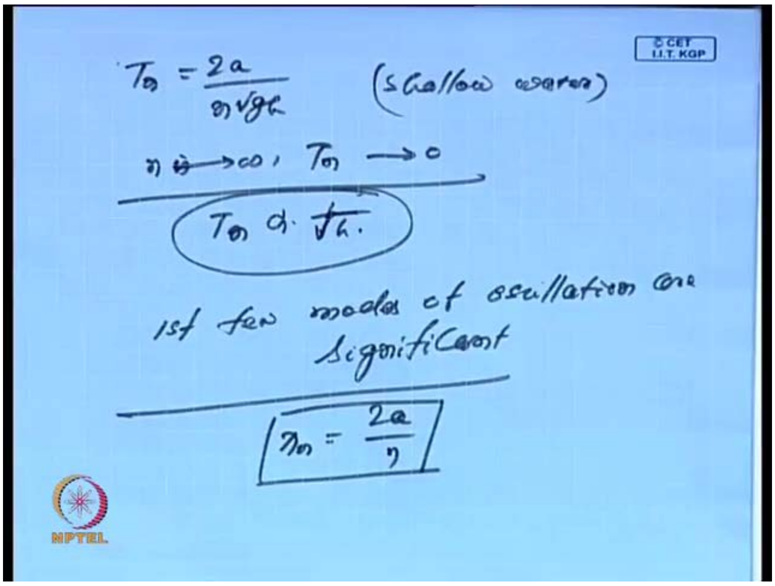


So, in this case what we are observing what we are observing that again in this case I have seen that T n is 2 a by n into root g h. So, when n is increasing n is increasing n is tending to infinity t n is tending to 0 and further here also we have seen t n also where is as 1 by root h. So, again smaller the water depth and here I am considering the case of shallow water also as an example, so here also we have seen that the higher the modes of oscillation smaller the period, the period will decrease in the mode oscillation case.

So, in both the cases the first few more of oscillations will be very important. We have just modes of oscillation as significant without and when this situation happen that means when I have lambda n is becomes 2 a by n happens I have a situation of harbor oscillation closed bas in oscillation or I say harbor oscillation. In fact this harbor oscillation and basin bay oscillation or a basin oscillation because have a results I call it a rectangular harbor and it can be told as a bay in it can be called as a lake.

So, this lake oscillation or a basin oscillation takes place under this condition and in both the cases we have seen both in case of bay or a basin oscillation. We have seen that first few modes of oscillation plays a dominant role and higher as the higher modes of oscillation does not contribute much to the oscillation of because impact will be very less wave length will be smaller and smaller.

So, in process one always look into the these higher modes of oscillation particularly the that higher mode of oscillation rather the first few modes of oscillation and that is that is why when you think of a tank or any basin, we always look into the first fundamental mode of oscillation, a secondary and tertiary mode of oscillation. That becomes very important aspect in fact in all wave resonance problems and it plays a very significant role in the harbor oscillation.

In fact it has been observed that this during a storm or during a high wave situation the damage in a harbor or boat is not much rather it is very high when boat the after the storm subsidized because when the fundamental frequency of oscillation on harbor matches with the first frequency of oscillation. Then harbor oscillation takes place and during such period the damage becomes more because of the bay oscillation and the harbor oscillation which takes place and this is that is way what is in all design, one has to take in to account this distance.

So, if one knows a particular nature of a wave and a particular bay or a basin. So, one always has to any new design or even if design of various structures one has to take this into account, so that there harbor oscillation is avoided or the bay oscillation is avoided if possible in while design or developing any structure near a bay or a basin, with this today I will stop.

Thank you.