Marine Hydrodynamics

Prof. Trilochan Sahoo
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture - 22
Basic Equation and Conditions of Water Waves

Good afternoon. In the last lecture, we have talked about the basics. We have given a brief introduction to water waves. In that, we have seen that among the various waves, water waves are one of the most general types of waves. Then, we have seen that for everywhere, there is a dispersion relation. That dispersion relation gives information. They are going to a further detail; we get lot of information about the characteristics of the waves.

But, like the velocity, particularly in phase velocity or the wave celerity, we used to know about the wavelength. How the wavelength and the frequency related particularly in case of water depth? How the depth is associated with the with the wavelength and the how the changes in wavelength is affected? How the wavelength is affected due to the changes in the water depth and the frequency particularly the time period?

So, with that in this background, let us today discuss in detail about what the basic equation is and what are the boundary conditions associated with water waves? In fact of when you think of any mathematical modeling for any physical model always, we look into two things. In one of the earlier lecture, I have pointed out that better mathematical; a better model has 3 aspects. If it can be verified experimentally and also, the results can be observed. If three things, the modeling information, modeling results the experimental results and the observational data analysis. If all the three things consider, then we say that is one of the best model.

In fact, in many situations, in where in environment or in marine are associated with marine technology, often we go for physical model testing and physical model testing. We do basically what here it is done miniature model is developed. The same thing is celebrated in a laboratory skill. Then, people compare this with the numerical data or observational information. All these things if it considered, then we always feel that as if it is one of the best models. So, today we will end as we know that in many situations, it is very difficult to have physical model testing particularly for last class of problems associated with water waves. Physical model testing is not possible because of the infrastructure.

Sometimes test class of the resource is because of the unavailability of the kind of a background for the people. But, many situations like major work on oscillography, all goes on that particular modeling and observation are data analysis. Comparison between the 2 only in case of motion of ships several laboratories that would testing certain physical model testing and then though numerical simulation and also mathematical modeling.

So, in this coming lecture, we are emphasizing on the mathematical modeling basically related with, related to water wave problems. When we think of a model, always we have to have some assumptions. I realize of the whole situation. In case of water waves, as I mention we always assume in the fluid is incompressible, the motion is irrotational. The fluid is also assumed in many situations in viscid.

So, because of all these assumptions, we deal with a potential flow problem particularly when the fluid is incompressible and the flow is irrotational. Then, come across the velocity potential flow satisfies the Laplace equation. That means here in the fluid domain, particularly in the water domain, we deal with here the continuity equation.

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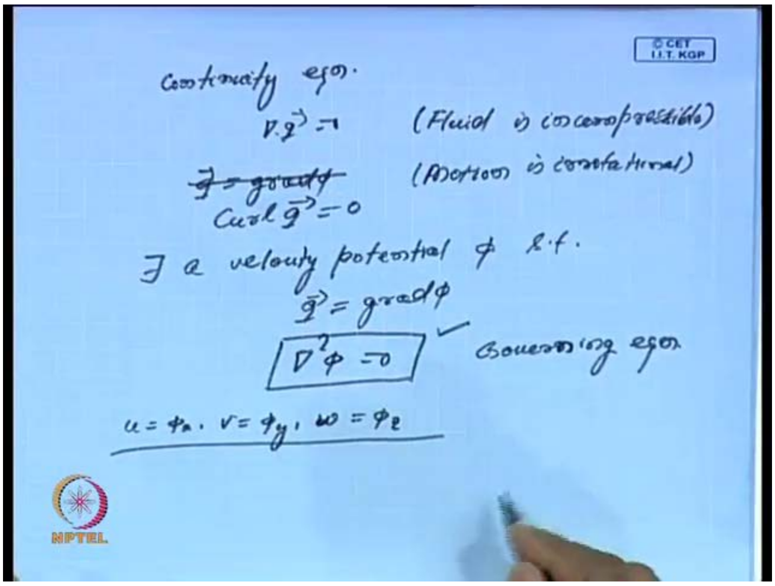

Your divergent of q is 0 because assume we have assumed fluid is incompressible. The fluid is incompressible. If it is incompressible fluid, then we have divergent q is 0. Then, additional assumption is based motion is irrotational. Basically, the fluid motion is irrotational. When we have irrotational motion, then we have q bar is equal to grad phi. Also, we have rather we say curl up q. In this case, we have curl up q that is 0. That gives me. So, the resist velocity potential phi such that q bar is equal to grad phi.

If you substitute q bar is equal to grad phi in the dimension, the equation of the equation of continuity gives us del square phi is equal to 0. So, we deal with a Laplace equation as this becomes the governing equation. this becomes the governing equation. That means in the fluid dimension, particularly for a last class of water wave problem, we assume the flow is irrotational. Particularly many problems in coastal engineering, last class of problems in offshore engineering are handle based on assumption that we deal with Laplace equation, we assume the flow is irrotational.

However, not all problems are water waves or all problems are marine hydrodynamics. We can take inter count. We can consider the flow as irrotational and fluid is in viscid dynamics. For a last class approval in this also, then and here as I have already 2 q where is grade phi that means b y u is equal to the components velocity u is equal to pi x and that my v is equal to phi y that the component velocity and w is equal to phi z in 3 dimensional. If it is 2 dimensional, we always say u is equal to phi x and u is equal to phi y in one of the next shape line. Now, apart from this, so in case of water waves, I say that I always deal with two things as I have mentioned in my one of the pervious classes.

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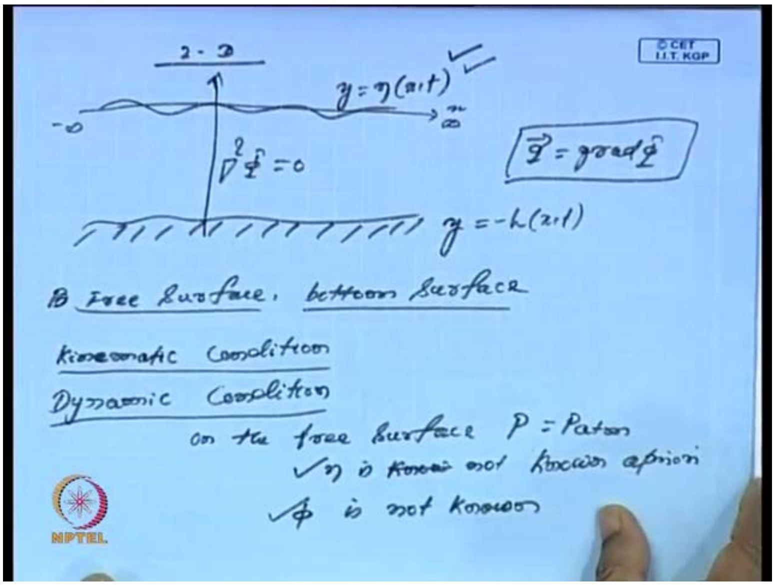

I always have a free surface that is y is equal to if I call y is equal to eta x t, I assigned that it is a 2 dimensional problem, 2 dimensional domain. I assign that in x y plane. See if I assign the flow is 2 dimensional, then I have the surface is a function of x and y. So, y is equal eta is taken always. Even that is the free surface.

Then, I will have a bottom. I can have a bottom. I will assume that, then this bottom, i also can, if i just say my downward direction is the negative direction y is equal to minus h x t, it can be the bottom. Then, in the fluid domain, we have said that del square phi is equal to 0, where q is equal to gradient of i. Now, what will happen? The 2 surfaces, assume that this is the fluid is extended from minus infinity till infinity. This is my x axis.

Let me say somewhere I have the y axis and this y axis. Downward direction is the negative direction. I assign the surface as well as the bottom surface, free surface as well as the bottom surface. They are, both are represented by y is equal to eta extend. Here, it is represented y minus h x. Now, these are the 2 boundaries, free surface boundary on the bottom surface. So, these are the 2 boundaries. So, on these boundaries, what will happen because on the free surface, there are 2 condition come. In one, I call the kinematic condition. The other one, I call the dynamic condition. There are 2 conditions; on the free surface because as I said that on the free surface, eta is not known.

On the other hand, the pressure on the free surface is pressured on the free surface. The pressure p is equal to p atmosphere. On the other hand, eta is not known a priori. So, it is not known in advance. So, here we have phi is not known. We only said that we satisfy the velocity potential phi satisfies. So, what we do here? We look into two things, one is the determination of phi eta and the other is the dimension of phi.

So, then here the dynamic condition, you have free surface p is a p atmosphere. Now, if I since I assume that the fluid is incompressible and irrotational on the fluid, in which we have already, we have taken care of that. First of all, an incompressible irrotational flow will have the equation of motion is nothing but the Euler's equation of motion. That equation reduces to the Bernoulli's equation.

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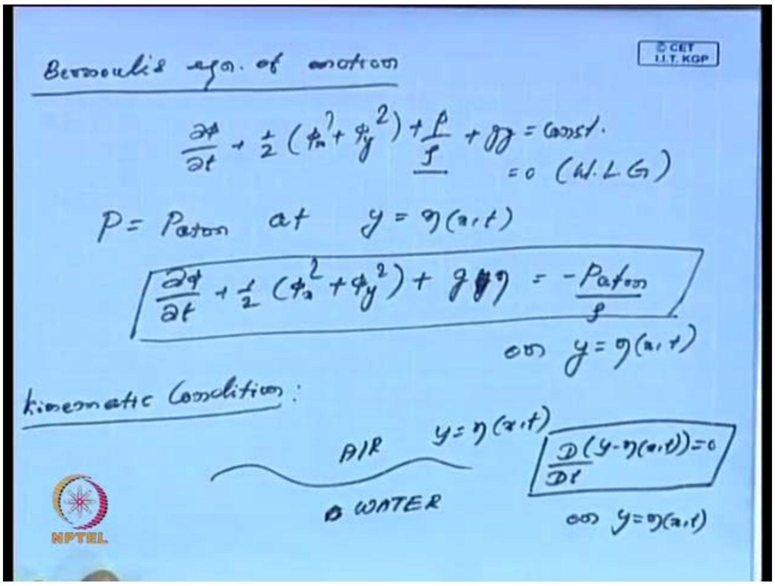

For the determination of pressure, Bernoulli's equation of motion, the Bernoulli's equation of motion says that this is a del phi by del t plus half phi x square plus phi y square plus p by rho plus g y is equal to constant. This becomes constant and this constant can be taken as 0 without loss because we are dealing with a problem where this is the dynamic. The free surface is very dynamic. This is 0. Then, on the free surface, I say that p is equal to p atmosphere. y is equal to eta.

If I say p is p atmosphere as y is equal to eta that gives me del phi by del t. That gives me. Here, we have the p is the fluid pressure. p is the fluid pressure where the density of the fluid. So, under the assumption that p is p atmosphere. So, we will have del phi by del t plus half phi x square plus phi y square plus g y. In fact, I can always say because on y is equal to eta, so I can always say that g eta. That is equal to minus p atmosphere by rho. This is on y is equal to eta. This condition on the free surface is the dynamic free surface condition. This is phi.

On the other hand, when it comes to the kinematic condition, the kinematic condition says this is one of the very important conditions for almost all wave problems. This kinematic condition says when there is if one have a fixed the boundary surface, it says that this is this is side. This side we have the water on the free surface. This side we have the air. This surface is given by y is equal to eta x t only surface.

We all know that because if y zeta is the air surface and this the water surface. For all time when space very good d by d t y minus eta x t 0 that means there is no gap. This is there is no gap between the air water. That means this is on y is equal to eta x t on the free surface. Total derivative is 0. That is there is no air and water surface. If I simplify this equation; it is because d by d t is nothing but the material derivative.

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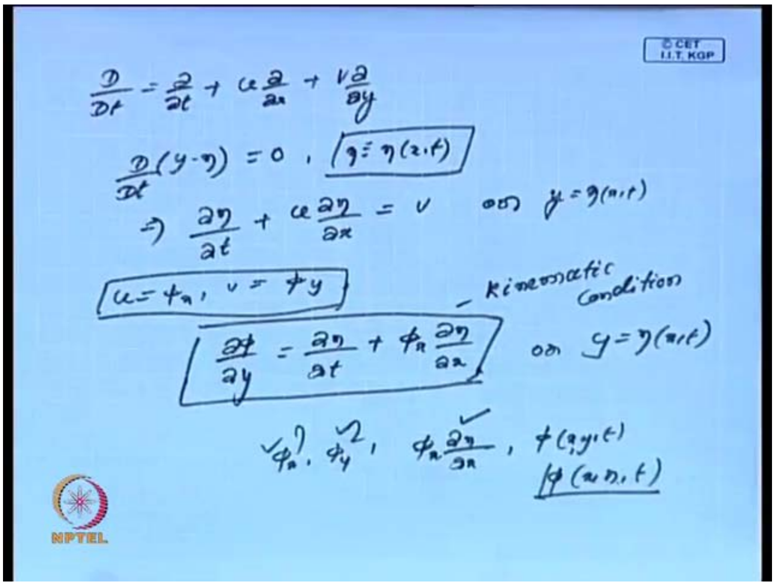

So, we all know that D by D t is equal to del by del t plus u del by del x plus v del by del y because I am dealing with 2 dimensional problems. So, when I say this is the total derivatives, so when I said D by D t of y minus eta is equal to 0, it gives me.

This will give me del eta by del t del eta by del t plus u del eta by del x plus equal to v on the surface v. When this will be delta by del t, this is because y is a eta is a function of x and t and eta is equal to eta x t. Then, we know that u is equal to phi x v is equal to phi y because we know this, because we are dealing with the potential two problem.

So, then this condition, this is on y is equal to eta x t. Then, this condition will give me del phi by del y is equal to del eta by del t plus phi x del eta by del x. This is satisfied on y is equal to eta x t because this condition I call this is my kinematic condition on this condition is satisfied on the free surface. So, there are two conditions, which are satisfied on free surface. One is the dynamic condition, another is the kinematic condition.

Now, if you look at the two conditions because of the dynamic conditions, we have terms like phi x square phi y square. Here in the kinematic condition, we have a term like phi x del eta by del x here phi x phi is a function of x comma y comma t on the free surface phi. This becomes x comma eta comma t on the free surface. So, that means phi is again depending on eta. So, because of this these terms, which are there in the kinematic and dynamitic conditions, makes the boundary conditions, the two conditions all in here.

So, here what is the complexity of the problem? Although we deal with Laplace equation or the governing equation, the two boundary conditions satisfy on the same surface, y is equal to eta. They are non-linear and that makes the problem more complex. That is why the problems become difficult to solve in general. That is one of the biggest problems associated with whatever problem. We are in the two dimensional. We assume the flow is two dimensional.

We have Laplace equation, but the two boundary conditions are on the free surface. That makes the problem more complicated because of the non-linear, presence of nonlinearity on the free surface boundary conditions mainly the both the kinematic as well as the dynamic problems. So, in a seminar manner, what will happen at bottom boundary?

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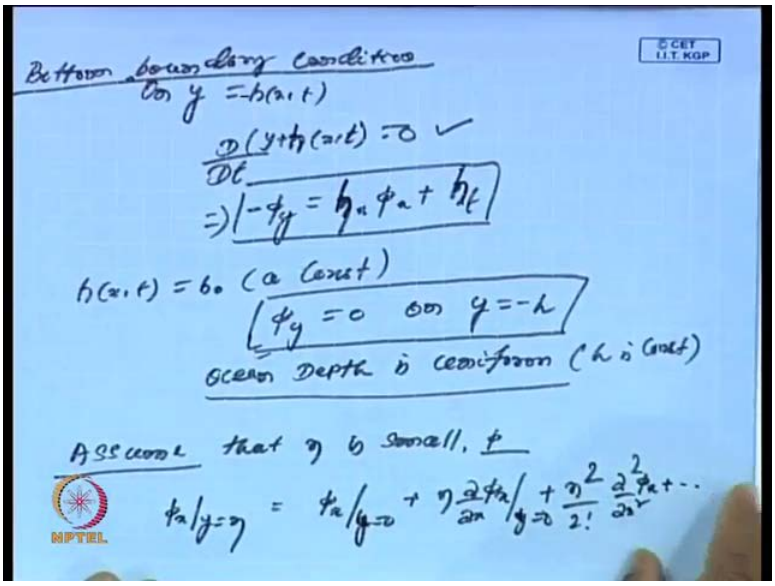

On the bottom boundary, we have we have phi is equal to on y is equal to that is free surface conditions. So, bottom boundary condition, the bottom is on y is equal to h, which is a function of x t. If we say D by D t y minus eta x t 0 that gives us phi y is equal to eta x phi x plus eta t s t that is y plus h because h negative minus phi y x. This is phi y h x plus h t from the kinematic condition. My floating y is eta is equal to minus x. We can get this condition that is on a.

As we have seen that this is on a fixed boundary, this will be 0 and the boundary is fixed. On a very boundary, this is the condition satisfied and this condition from what we have earlier seen this. Again, we see that if I say that h becomes constant, it is not a constant. That means the bottom surface is uniform where depth is uniform throughout the region. Then, I will have h x term will be 0; h t term will be 0. Then, I will have phi y is equal to 0 on phi is equal to minus h. This is the simplest boundary surface bottom boundary condition we can have.

If assuming that here, we have assumed that the depth is uniform; ocean depth is uniform uniform, so h is constant. Now, this is the bottom boundary condition. Now, coming back to the surface boundary condition because the problem is highly non-linear, we cannot handle the problem. It make in the simplest case that means associated with the two dimensional Laplace equation. Two because of the nonlinearity presence of a nonlinearity, we can handle this problem in general.

So, what we do? We try to again consider idealization particularly by assuming, we assume assume that eta is small. If I assume; that means we are dealing with small amplitude with here you have to eta is small. If you assume it as small, then the quantity like phi. All the components associated with phi will be small. So, in the process, what we can do that we can easily have it alters is expansion of the term. That means phi x y is equal to eta because eta is a small. If we assume it expansion; that will give you phi x at x is equal to 0 plus eta del by del x. This is phi x. This is also y is equal to 0 plus that y is equal to 0 plus n square by wave factorial 2 del square by del x square phi x plus higher order term. Similarly, we have phi y.

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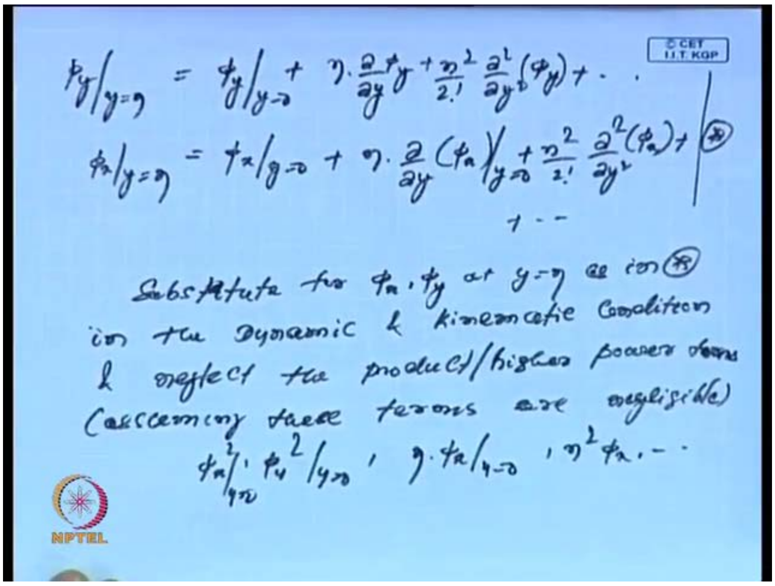

Similarly, if you put phi y naught y is equal to eta, then we can get phi y naught y is equal to 0 plus eta del by del y, del by del y del y half phi y plus eta square by factorial 2, l, 2 del square by del y square phi y plus in fact, there is a little problem in the previous phi x the phi x at y is equal to eta. Actually, it should be this, just a by the mistake.

This becomes phi x at y is equal to 0 plus eta into del by del y phi x at y is equal to 0. I am taking y is equal to eta. So, plus eta square by factorial 2 into del square by del y square phi x plus high powers instead of what I did in the last page. What we have done is we have taken del by del x. It should be del by del y. Here, we have taken del square by del x square, which should be del square by del y square.

So, substitution for these values for phi y and phi x in both kinematic and dynamitic condition takes place. In the dynamic condition, what will I get? What I will say? I will assume substitute I call this as a star substitute for phi x phi y at y is equal to eta as in star in the dynamic and the kinematic condition, I want to process this. This is because we substitute for this in the dynamic and kinematic condition and neglect the product terms, product or higher power terms assuming negligible, assuming they are negligible, assuming these terms are negligible.

So, what are the terms? This is because I will be neglecting phi x square phi y square at y is equal to 0. Then, I will be neglecting eta; eta into phi x not y is equal to 0 eta square into phi x and so on. So, basically the linear terms will return and higher powers will neglect.

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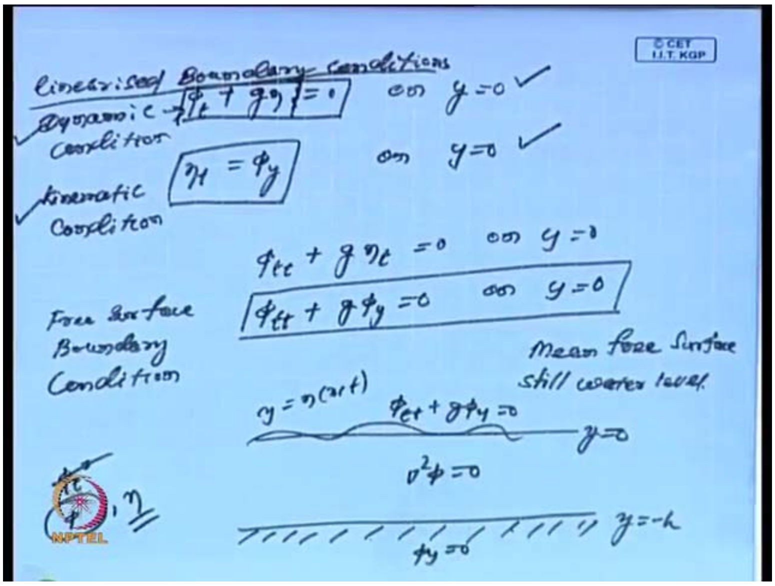

If we do that, the kinematic condition will get from the kinematic condition, that would get phi t plus g eta as 0 on y is equal to 0. This becomes the dynamic condition. The kinematic condition is inverse eta t is phi y on y is equal to 0. So, this becomes the linear zed boundary conditions, linear zed boundary conditions. These 2 are the linear zed boundary conditions on the surface. This surface is called the main free surface. This one is called the dynamic condition and this is called the kinematic condition. This is the dynamic condition and this is the kinematic condition.

So, these 2 are very important. Now, if I just take this 2 conditions phi t plus g eta as a 0, this is kinematic condition that eta t is phi y on y is equal to 0. Then, from what will get that what will happen to from the first dynamic condition? If I take time derivative, it will give me phi t, t plus g eta t equal to 0 on y is equal to 0. Then, I substitute for eta t from the kinematic conditions as phi y.

So, that will give me phi t, t plus g phi y is equal to 0 on y is equal to 0. So, this becomes the free surface boundary condition for whatever problems. This condition is a combination of the dynamic condition as well as the kinematic condition. Now, I will look at the whole thing. Then, what is y is equal to 0? y is equal to 0 is the mean x i. If I have mentioned, I call it mean free surface or my still water level, this is my mean free surface of the still water level.

Again, as we have seen that, so what will happen at the bottom? Also, if I just consider this as this is my, then this is my y is equal to 0 my still water level. I think of water of uniform depth. This is y is equal to minus h. Then, what will happen? Here, we have Laplace equation delta phi 0 and at the bottom, we have phi y is 0. This is on y is minus h. Here, I have phi t, t plus g phi y is 0. This is on y is equal to 0. So, this becomes, the governing equation is Laplace equation. We have 2 conditions; one is the mean free surface. We have phi t, t plus phi y 0. At the bottom boundary, we have y is equal to minus h. Here, we have phi is equal to 0.

Now, I just go back to a little about the partial differential equation. Here, we are dealing with a Laplace equation. We have 2 boundaries, one is the free surface boundary, and the other is the bottom boundary. On the free surface boundary, I have a Neumann type condition. Abbes type condition is a combination of phi, phi y and phi t, t. t is the independent of this is on time variable not the space variable. So, on the other hand at the bottom, we have a Neumann type condition.

So, what will it do? How will go to the solution of this problem? Before going to the solution of this problem, here also we have seen that I on the free surface that these problem is completely problem in a phi. Once, I obtain phi that means phi is the velocity potential. If I can solve it, solve it for phi. Once I know phi, then I can know eta. I need to know eta. To know eta, I can always go back, apply one of these kinematic or the dynamic condition. Once I know phi, I can find out what is eta from one of these conditions by going on the reverse way. So, this is the way we obtain phi and eta. Now, coming back to the second part of the problem, the second part is that suppose I know eta, can I know phi? The question comes suppose I say that I know eta.

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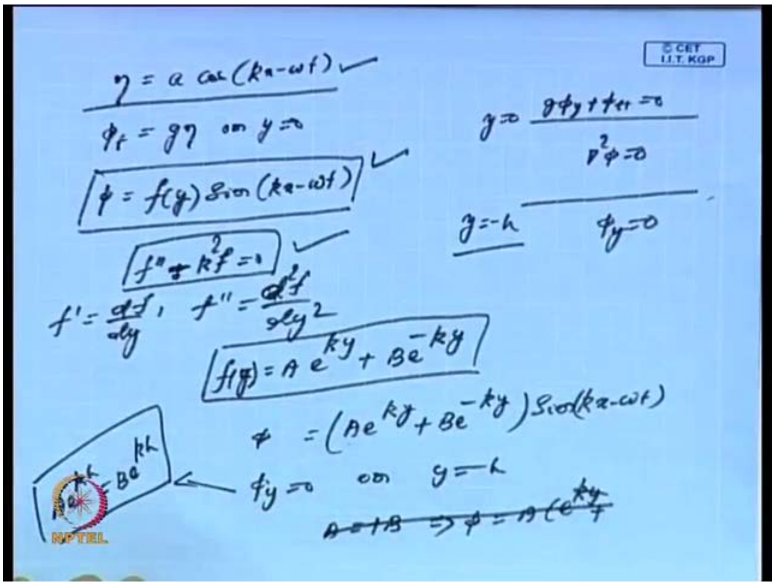

Today, we will just look at it because suppose, I take a simple problem. Suppose, I have been given eta is equal to a cos k x minus omega t. If eta is given, I have already told this is my in free surface. This is my bottom surface. Here, phi y is 0. Here, we have del square phi 0 on the mean free surface. I have phi y g phi y plus phi t, t is equal to 0.

So, now if I have this one, then suppose I have been given eta is a cos k x minus omega t because I know phi t is g eta, I have no idea what about my phi. I get from y is equal eta. My question is whether if eta is known, I assume that this is a surface. This is my free surface. y is equal to eta where eta is the surface solution. Then, can I know what is phi? But, I know phi is t is eta on y is equal is to 0.

So, that means the time derivative of the velocity potential gives me eta. So, because of that, I always can write. I am dealing with all my conditions are homogeneous conditions because of this I will take use of; I will make use of this beautiful concept. I have my homogeneous equation, equation in homogeneous boundary conditions are homogeneous. So, I always can write; because of this, I always can write my phi of this from f of y into sin k x minus omega t.

This is because if I take this form, phi is a function of x y. If I take this form, then y is equal to 0. This phi t will be g eta that will leave to, this is sin term. So, eta if I take phi t phi, it will give a cosine term and f y will give me some constant. The health constant will be similar to what g eta is.

So, because of this, I have assumed that phi is of this one. If I look at this, I satisfy the Laplace equation, if it has to satisfy Laplace equation, then f has to satisfy f double dash plus or minus, minus k square. f is equal to 0. Here, dash is the derivative with prime f prime is d by d y of a prime f double dash is del d square f by d y square. It is the derivative. So, by f satisfying this equation, if f satisfies this, then what will happen to f? f will be of this form A e to the power k y solution will be general solution. It will be e to the minus k y.

Further, this is being my f y. If my f y is this, then this f, this is my phi and because it satisfies the Laplace equation. That will give me f satisfy f double dash minus k square phi is 0 k square f is 0. Again, the solution of this gives me this.

So, the general form of my phi will be A e to the power k y plus B e to the power minus k y into sin k x minus omega t. Now, I have 2 more, 2 more constants, A, B are unknowns. Now, if I put phi y is 0 on y is equal to minus h, this is y is minus h phi y 0. y is equal to minus h that gives me. This will give me A is equal to minus B. Phi y 0 that will give me A is equal to B because this will give me A is equal to B. That gives me my phi as A times A to the power of k y plus phi y 0. That will give me A e to the power k h minus k h is equal to B e to the power k h this condition.

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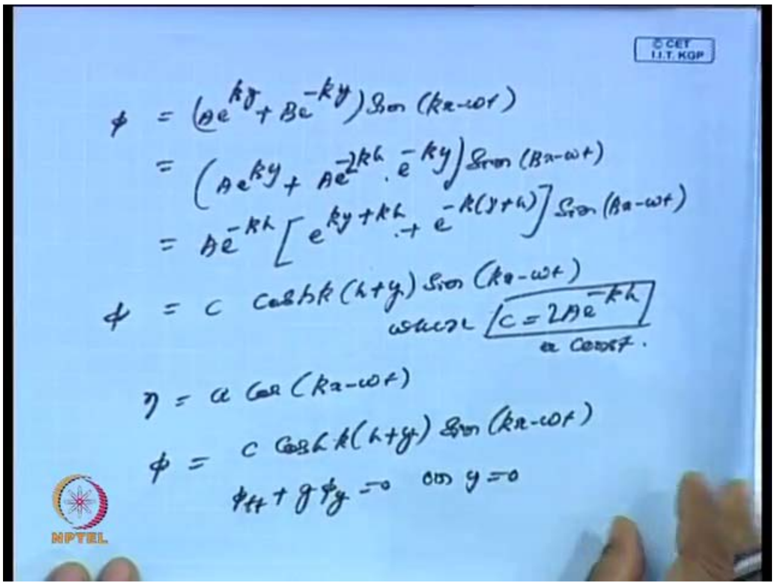

What will happen if I further simplify because my phi is given by A e to the power k y plus B e to the minus k y into sin k x minus omega t? Now, I substitute for B in terms of A, so that will give me A e to the power k y plus B will be A e to the minus k h divided by minus 2 k h into e to the minus k y into sin into sin k x minus omega t. That gives me, if I take e to the power A e to the power minus k h common, then it will give me e to the power k y plus k h plus e to the power minus k into y plus h into sin k x minus omega t.

That gives me something like c. I can call this as a constant times c cos hyperbolic k into h plus y into sin k x minus omega t. So, phi becomes where c is equal to where c is equal to A e to the minus k h 2 times this and again a constant, a constant. So, what all it is all constant? So, when I have seen that if my eta is equal to A cos k x minus omega t, then I get A phi, which is very equal to c cos hyperbolic k into h plus y into the sin k x minus omega t.

Now, again because I only assume that I have a wave of this form, I do not know what about the frequency or what is the wave number on this? Then, again here, I am finding that this is giving me in terms of another constant c. So, is there any relation between a and c? Is there anything relation between k and omega? So, to know this, if I really apply because I have to utilize the bottom condition, the Laplace equation has given me. Now, what will the free surface condition? My free surface condition is phi t, t plus g phi y 0. This is on y is equal to 0. If I substitute for phi in this that is easily we can see.

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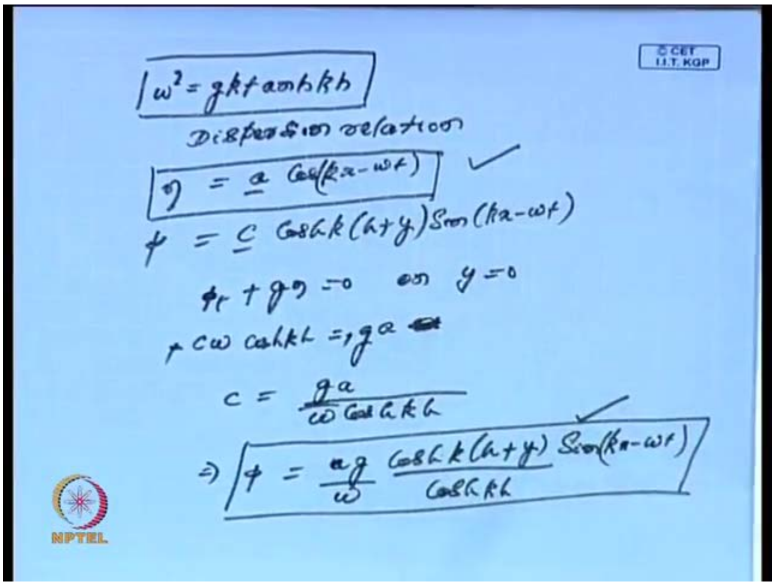

We can easily derive that phi t, t. We can easily derive that omega square is g k, hyperbolic k h. Let us substitute phi in the surface condition phi t, t plus e phi y 0. That gives us this is the dispersion relation, which I was talking about in my last class in my introduction to water waves.

So, this is the way the dispersion relation has come because just we substitute for the form of phi in the surface boundary condition. That will give us the dispersion relation. Now, the dispersion relation what I am going to do? This is because I need to because I have got the dispersion relation. I have a eta is equal to a cos k x minus omega t.

I have a phi, which is sometimes c times cos hyperbolic k into h plus y into sin k x minus omega t. I still have a relation. How a and c are related? Suppose, I say, I know that my phi t plus g eta is equal to 0 on y is equal to 0. This is coming from my dynamic boundary condition. Then, if I substitute for phi t is eta and y is equal to 0 that will give me, it will give me c omega c into this will give me sin minus omega minus c omega cos hyperbolic k h plus equal to g times minus g times a. This will give me this.

This is because cos k x minus omega t will get canceled. So, that will give me my c is equal to g by omega cos hyperbolic k h. So, that in place may phi will be a g. That will give me a g by omega cos hyperbolic k into h plus y divided by cos hyperbolic k h into sin k x minus omega t. This is my complete form of the velocity potential for the phi for the eta. So, if eta is given by a cos k x minus omega t, I have the velocity potential phi is given by a g by omega cos hyperbolic k into h. This is very important relation as per as eta and phi is concerned.

Now, by this time, we have understood that if I have a boundary condition, we have a governing equation which is the Laplace equation. We have the boundary conditions, 2 boundary conditions. Here, we are solving the Laplace equation. We still say that there is a wave because why we say that there is wave? This is because we say that the form, surface form is surface is eta a cos k x minus omega t, this surface.

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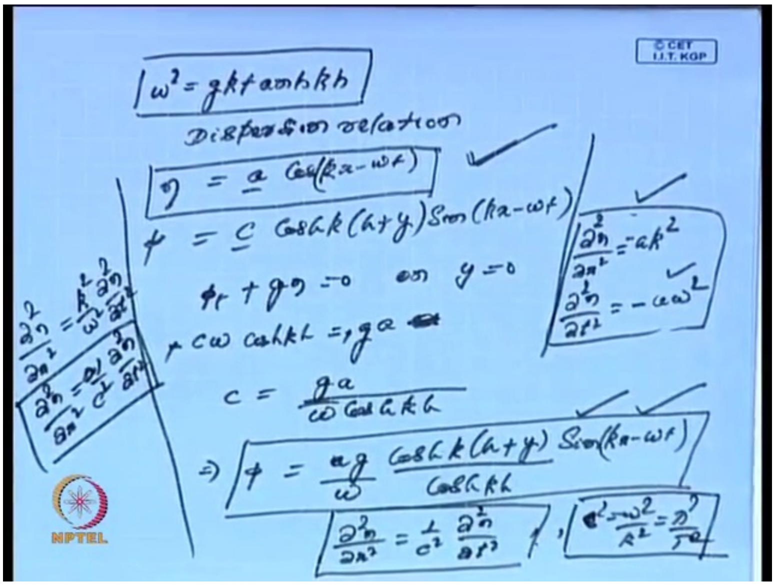

They are square eta by del x square because we have seen that that will give me and del square eta by del x square. It will give me a k square minus a k square. That will be again del square eta by del t square. It gives me minus a omega square. So, if I say from these 2, if I say del square eta by del x square is equal to 1 by k square by omega square, because this is a common, this also a common, so k square by omega square or del square eta by del t square. This is nothing but omega by 1 by c square del square eta by del t square. So, it is del square eta by del x square. So, this is the equation.

So, that means we have seen that we have, I will just write it here. That means what I am getting del square eta by del x square is equal to 1 by c square del square eta by del t square where this is what I am getting from these 2 equations. Here, c square is omega square by k square that is nothing but lambda square by t square that is my c square. So, that means if I take eta is this and my phi is this, my eta satisfies the wave equation.

So, that means I have a wave, but I am not solving wave equation. This is the question comes, but I am solving I am solving Laplace equation. Then, from where the wave is coming? The wave is coming because I have a free surface condition, where the gravity term is there.

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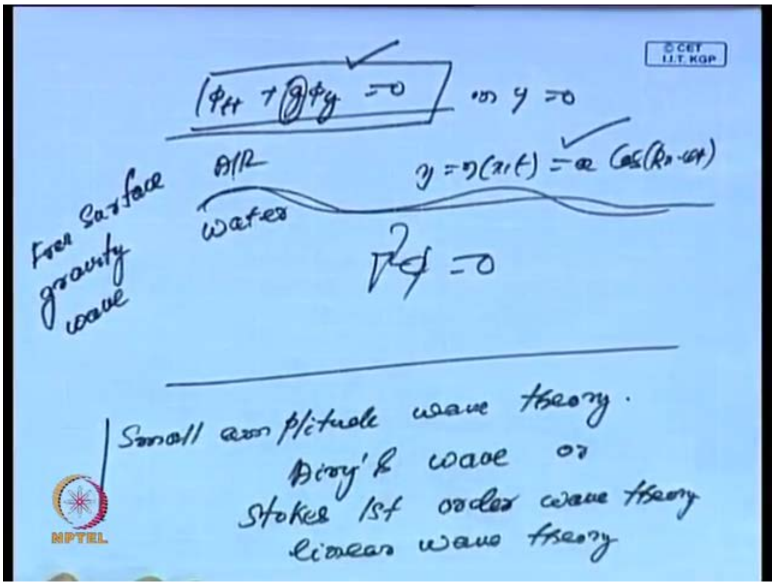

This is because in the free surface condition, we have seen that we have a term like phi t, t plus g phi by h 0. This is on y is equal to 0. In fact, because of this term, the free surface boundary condition, which has a term, the gravitational constant; it is making the free surface condition. In the process, on the free surface, we find this wave y is equal to eta x t, which is a cos k x minus omega t.

So, if you are solving Laplace equation in the fluid domain, still have a wave that is propagate on a surface, it is because on the free surface, we have the gravity is dominating. One of the terms, which are dominating and here, here, we have a surface. There is air. There is water. So, because of there, when there is disturbance, that disturbance is trying to pull, restore gravitational force. It is acting like a restoring force there. Then, in the process, we see a wave that is propagating on this free surface.

It is because of this, we call this the free surface gravity wave. Since, we have assumed the amplitude is small in this case; we call this as the small amplitude wave theory, small amplitude wave theory. Often, I call this airy wave or strokes finite stroke first order wave theory. This is a linear theory. This is a linear wave theory. Although we have assumed this is this is this is basically we have linear zed the things. So, it is also known as the linear wave theory.

So, this is the simplest wave theory in the region of the water wave theory. This theory was initially developed by airy, and then by strokes independently. In the process, they are often called as airy wave, airy wave theory or the first stroke first order theory. This wave theory gives us very important information because as I have already told that once we know the velocity potential phi, so eta if we know the velocity potential phi, then we are able to know. We can easily know what is my phi x and once we know phi, we can easily know what is phi x basically?

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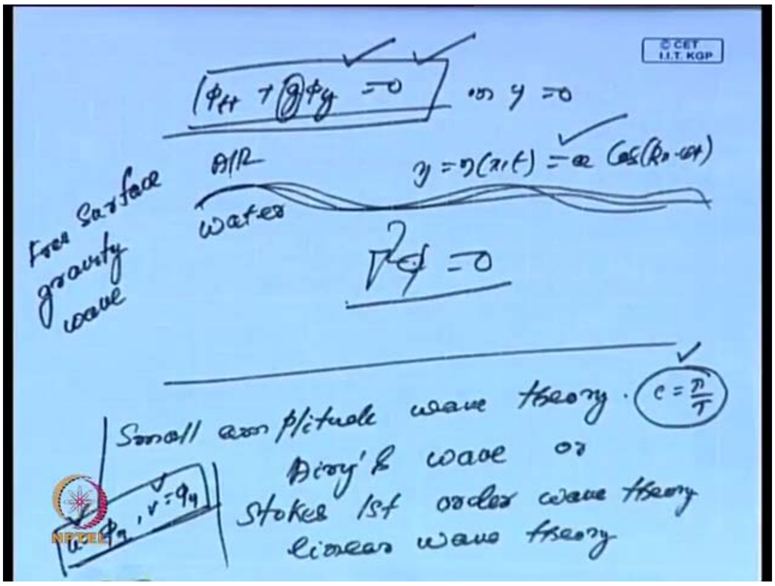

That is the component of velocity u component of x component of velocity. We can know phi by y component of velocity. Once we know the velocity component, this velocity component, this is the particle velocity, water particle velocity. On the other hand, I have talked about c that is lambda by t that is the velocity of the wave. So, here it is the velocity at which the wave is propagating here as u and v that determined eta minus that gives us the rate at which the fluid particles are moving in the x and y directions.

So, this is the difference between the two things. So, here we have two things; one is the rate at which the wave is propagating. The other is the rate at which the particles are moving. So, there are two things, which are different and this has to be remembered for all times. Then, another thing is that here, we have a fluid domain. In the fluid domain, we have Laplace equation, but because of the presence of the free surface, we have a wave that propagates on a free surface.

So, obviously, many times this question comes, that you when we are solving Laplace equation, but where is the wave? The wave is there because of the presence of the free surface where the free surface condition is responsible and because of that, this condition becomes responsible for this generation of the waves. With this, I will stop here. In the next class, we will illustrate through several examples, how for variation types of wave, how can get the corresponding velocity potential. We will able to determine other constants, other physical phenomena associated with these. With these, I will stop today.

Thank you all.