Marine Hydrodynamic

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Lecture - 19
Vortex Motion

Welcome you to the series of lectures on marine hydrodynamics. Today, we have the nineteenth lecture and today will talk about vortex motion. Although, we have, I have already introduced to you about TCT factor, which represents which gives also about the flow in the angular directions, when you have also talked about the vortex and also circulation. First I will summarize in brief about what we know and then go to the details, a little more details about the vortex version. These will start with the basic definition vorticity.

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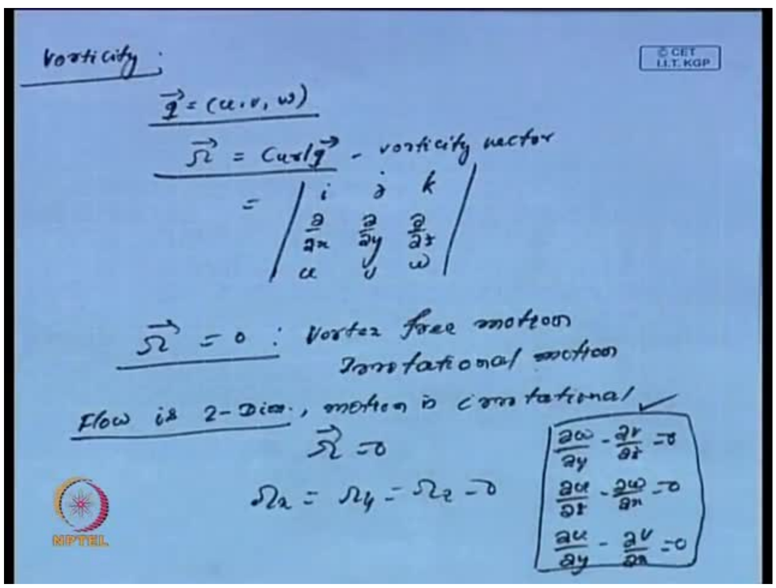

So, what I understand by vorticity? If we have listen to the q bar, these components are u v w these are the (( )) of a fluid particle the components of the velocity fluid particle then, what will happen to omega bar? This is nothing but a curl of q bar. Curl of q bar will be represent the angular velocity of the angular, curl of q bar will give the velocity in a rotational direction basically the angular. It is a measure of the angular velocity, and which is also can be written as i j k. Then del by del x del by del y and del by del z u v w, so what will happen? This is the curl of q bar, it has three components; one is the i component, one is the j components and one is the k component.

Then we will see that if which is the vorticity at the, which is the vorticity vector, vorticity vector, and then we have if omega bar is 0, we all know that we call this as vortex free motion or irrotational motion. So, in if I consider my flow is a two dimensional is two dimensional, then and flow is two dimensional and motion is rotational. This is at all we talked about, then we have omega bar is equal to 0, which gives you omega x is equal to omega y is equal to omega z is 0. That gives me del w by del y minus del v by del z is 0 and del u by del z minus del w by del z del x is 0. We have other things by, del u by del y minus del v by del x is 0. So, this is what happen when we have a vortex in motion. We have vortex three motion, where when the flow in 0 it is not behalf of this. Then we know that, once we know the vortex vorticity factor, then we can always talks about that vortex lines.

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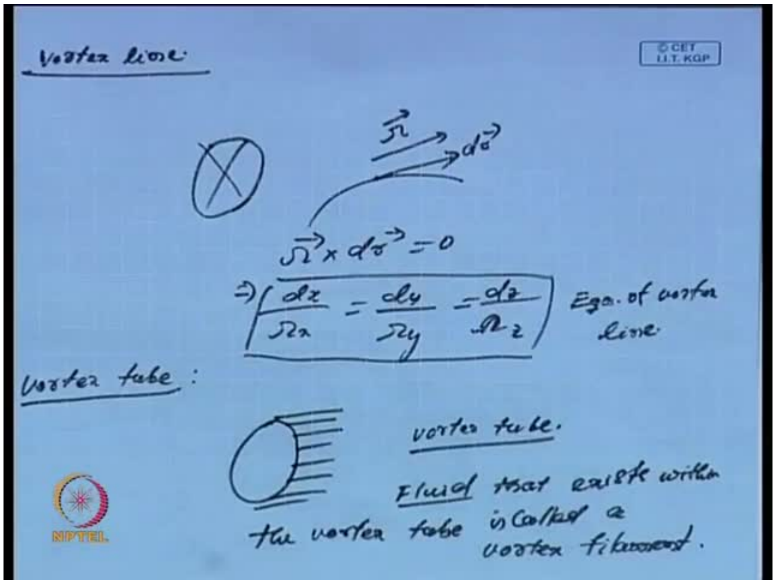

This is a vortex kind of review because we have already talked about some of this things vortex line. A vortex line is a curl think of a curl, need not be close curve or other. Just look at a curve is a curve. a curl in the fluid it is such that, the tangent any direction it is that direction and in vortcity director. So, that means this is if omega bar the vorticity director and the other the tangent at any point will draw. Then the both are parallel, that gives us. So, equation of the vortex line gives us, so omega bar cross d r bar is 0 and they have be 0. That will gives us d x by omega x d y by omega y it is d z by omega z. This will be gives an equation of the vortex line, then what will talk about a vortex tube?

Suppose, I consider any closed for, if we draw consider any closed counter in the fluid and we draw the vortex lines at each point, they are the vortex lines at each point. Then, from this closed curve, then a space bounded by these lines whole space bounded by these lines, this called a vortex tube and this is called a vortex tube. However, and the fluid, that is fluid that it just inside the vortex tube within a vortex tube is called a vortex filament. This is called a vortex filament is the fluid that remain is inside the tube, that is called a vortex filament. So, I can always say that ah vortex tube is nothing but it is the boundary of the vortex filament.

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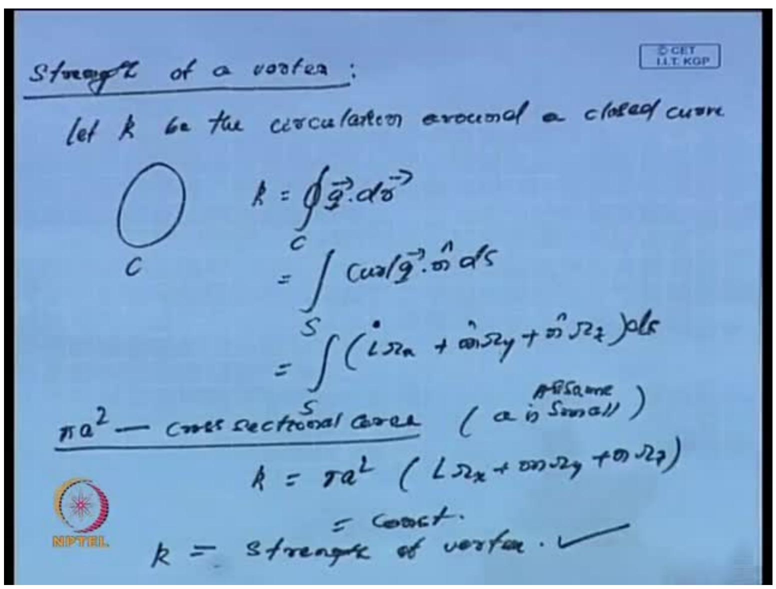

Now, if you look at the strength of a vortex tube strength of a vortex, what is the strength of a vortex? Suppose k be the circulation will get k be the circulation around a closed curve from space the circulation and a closed curve which includes a vortex tube. Then if c is the boundary of this closed curve and what will happen to k? k by definition integral over c q bar dot d r bar, that is what the definition of circulation, when a q bar dot d r bar. If I apply integral that is nothing but over the surface curl of q bar dot m hat into d s, if I put it in the component wise where it will be integral over s l omega a as plus m omega y plus n hat.

So, d z d s, now if we assume that by a square is a cross section of the area of the vortex tube, it as the vortex tube, then what will happen to k? k will be pi a square into l omega x, now already this is scalar, it is is, okay? Now, l omega x is plus m omega y plus n omega z and this is what, so this has to be circulation around a closed curve. So, this will be a constant. This is a function because then what I say, then I call k the strength of the vortex. I call this strength of the vortex, this is assuming that these one we assume a is small, this is the assumption. Otherwise, should not should then we call it as just strength of the vortex.

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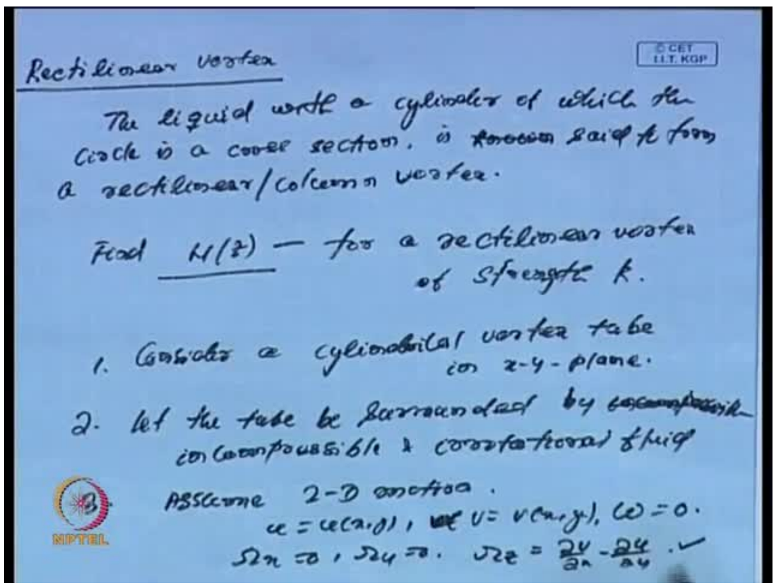

Once we know the… Now, let me save another term what is called rectilinear vortex because these terms will be used afterwards. The liquid, which is the liquid which is within a cylinder, of which in circle is cross section, circle is a cross section the liquid within a cylinder, which circle is a cross section is known as a… It is rather is said to form a rectilinear or column vortex. So, now what we will do, how to find the, have to find the w f for a rectilinear w z? That is the basically the complex velocity potential? This to a rectilinear vortex of a strength find w z for a rect, rectilinear vortex of strength k. In facts things will clarify many things, which will earlier we have introduced many of things. But we have never things are certain, there it will one’s this things are discussed.

Now, let us considering the cylindrical at a test tube in a x y plane. Consider a cylindrical vortex tube and this is first two. Let the tube be surrounded by in compressible rotational flow, by in compressible in and a rotational fluid. Let the motion between dimensional three assume two dimension of which u is equal to u x y v is equal to v x y. This is the component of velocity and w is equal to 0, then because of this then we have only thing, that means w a is 0, omega y will be 0, whereas from omega is z is equal to del v by del x. Tell us w by dell y this is the other component of the vorticity vector. We have only this term only exist on that two terms it will be 0. Now, now what will happen to u?

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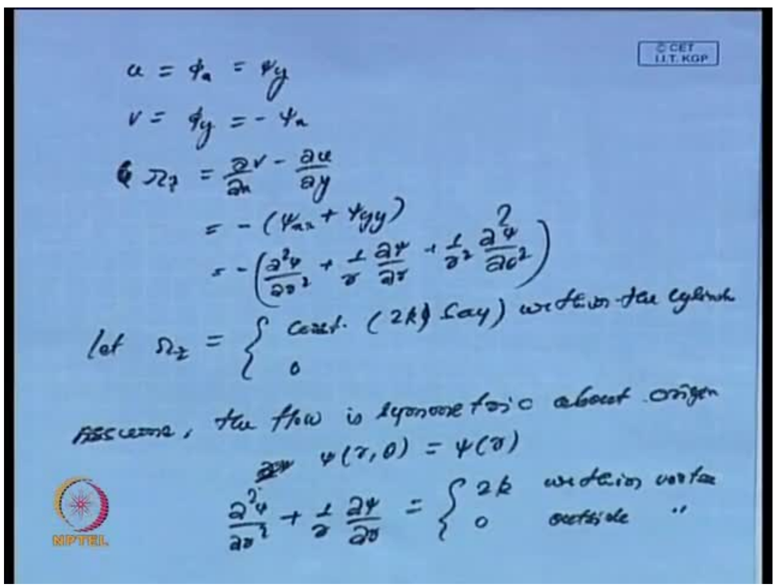

We have u is equal to nothing but phi x that is equal to psi y, the psi is the u the psi is the value psi is the stream function and again we have v is equal to phi y, which is nothing but minus psi x outside the tube. Then we have psi is a whether omega z is equal to del by del x. This is equal to del by del x del p by del x minus del u by del y and that gives us which is equal to minus psi x. So, this is minus psi x x, then plus psi y y because u put psi by, so this will be minus of this. If I put it in the Cartesian, this is in Cartesian coordinate, if I put in the this will be middle del square y by del r square plus 1 by r del psi by del r 1 cross by r square del square y by del theta square equation over there.

Now, let me talk about because this is if I say, let omega z is equal to a constant, that constant let me say 2 k. Let us say 2 k within the cylinder and this is because omega z. If it is an outside, the cylinder and we have a flow is rotational. So, since outside the cylinder fully rotational solution will be 0, whether this it will be 0. If I assume that the flow is symmetrical or is in… Now, if it is symmetrical about origin once, what will happen to del psi by del psi by del theta? So, that means psi becomes function of psi r theta becomes psi of r because theta component will not contribute to, will not contribute.

So, in the process what will happen to this omega? So, that means in a process from these equation that omega z will be, if I look at this then it will give me del square psi by del r square plus 1 by r del psi by del r because. This part is omega z this is 2, k that will be within the cylinder, within the vortex because what the cylinder is at a vortex tube. This is 0 outside vertex, outside the vortex because vortex is of radius the cylinder is of radius. Certain radius r is equal to where if I say… So, then what will happen this if further simplify that, means within the vortex we have…

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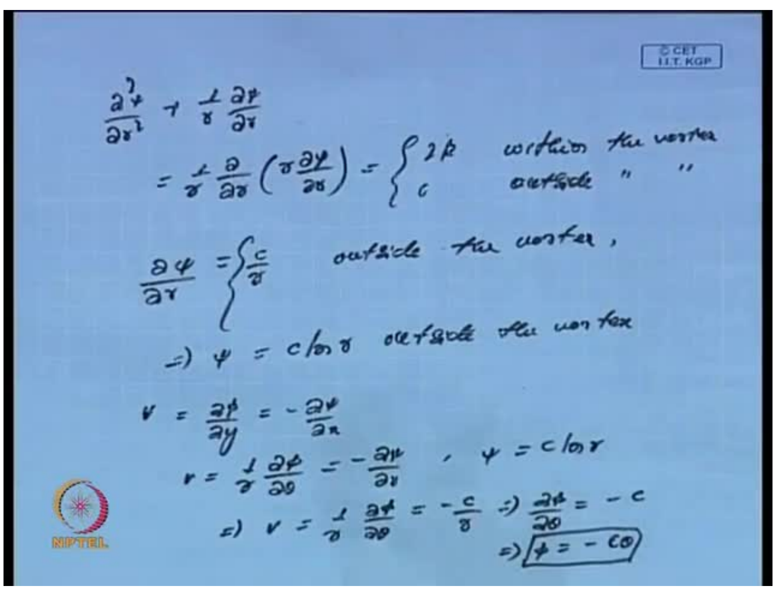

Then I can, so del psi by del square psi by del square psi by del r square plus 1 by r del psi by del r can be written as 1 by r times del by del r r del psi by del r and this will be 2 k within the vortex written in a vortex to the 0 outside the vortex. Then, because there is constant r minus sign, that I choose as I, if it is get it will just, I can take that minus and again as a plus psi, so then what will happen? If I simplify this in that within the vortex what will happen? We can easily get psi is equal to some constant c by r outside the vortex, because if I look at a outside the vortex, what will happen?

This will give me, it will give me c by r outside the vortex. This is a constant now sorry, in this will give me that del psi by del r sorry, it will be del by del r will give me c by r outside the vortex, which gives me that psi is equal to c log r outside the vortex. Because this is obviously if you take this, this should be 1 by del by del r r del psi by del r is 0. So, if you integrate with r 1 then will get some constant r del psi by del r will be a constant, so that means del psi del r will be a constant type becomes r. That is c by r, that is what c is equal to log r outside the vortex. Then what will happen to the component of the velocity?

v is nothing but del phi by del y and this is nothing but del that minus del psi by del x or we can be written also in the r theta co ordinate 1 by r del phi y del theta, which is nothing but minus del psi by del r. I have been given psi is equal to c log r, so that gives me my psi rather v will give me 1 by r del phi by del theta and that is minus c psi c log r. This will give minus del psi by del r minus c by r and that gives me, when del phi by del theta is minus c, which implies where phi is equal to minus c theta. So, this is another result, then we can look at phi is equal to I have got outside the vortex r psi is equal to log r. So, we can put it phi plus i psi, where w is nothing but phi plus i psi, that is phi is minus c theta plus i times psi is a c log r.

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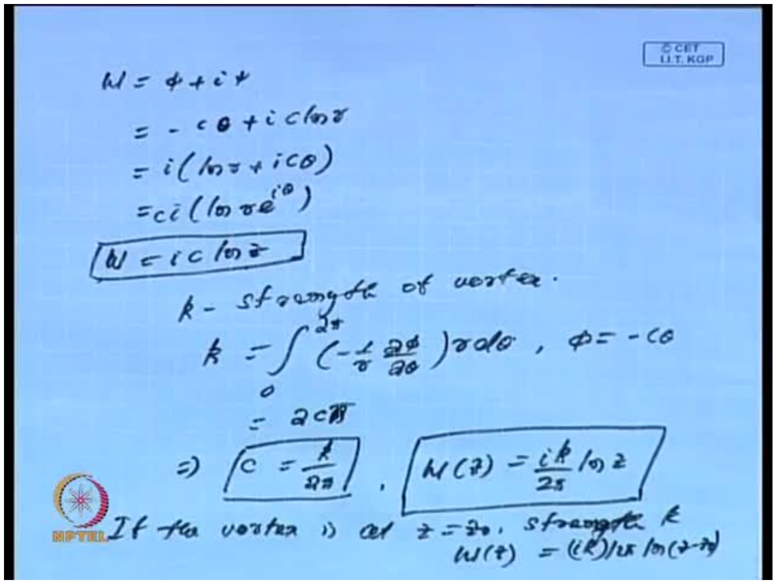

If I write it, I can once more write it here. If I take i times log r plus i c theta, so that will give me i times log r e to the power i theta i times o c is there c i times and that will give me i times c log z. So, my w becomes this is the complex potential that becomes i c log z. Now, if I look at whether definition of the strength of the vortex, what will be my k? If k is the circulation or k is the strength of the vortex, then k will be 0 to 2 pi. Thus q dot d r and here I am looking at a surface and it will give me 0 to 2 pi. That is minus 1 by r del phi by del theta r into d theta and that will give me 2 c theta 2 c pi because a del pi by del theta, whether 1 by phi is minus c theta because my phi is minus c theta and when psi is minus c theta, minus minus will be plus or r will get cancelled.

Del phi by del theta will gives c, so integral of 0 to 2 pi 2 c pi. So, which implies my c will be k by 2 pi, when c is known as k by 2 pi where k is the strength of the vortex. Then my w is in is equal to i k by 2 pi log of z. In fact in many times when we started with this complex potential and proof that, while prove that w into z, if we started with this. We have said that, which is represents vortex of a having strength k. It was we have derived by starting from the basic definition of a vorticity vector. However, basic the phenomena of rectilinear vortex of strength k. So, this is what we have obtained. So, what will happen if I change the, this what at z is equal to if the vortex, if I say the, if the vortex at z is equal to z naught. It is of the same strength k, then my w z will be i k by 2 pi into log z minus z naught. What will happen if I have multiple vortex?

Then, the summation of the vortex will also, suppose I said that will an infinite number large number of vortices.

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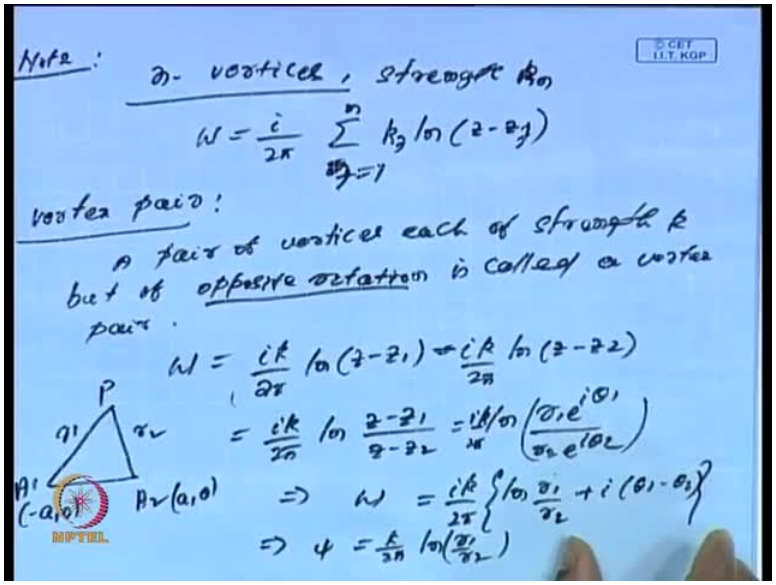

So, if I have n such vortices, if I have n such vortices, then what will happen to my w and each of the strength k? It can be easily seen that w is equal to i by 2 pi or I will say just strength k n. It can be easily seen that it will be i by 2 pi sigma or else say z is equal to 1 to n and the strength is k n, so k z into log z minus z j, where the each vortex each of the vortex is at on the point z is equal to it is a. Now, with this, now let us look at a vortex pair, what happen in case of a vortex pair? A pair a vortices, it is nothing but a pair of vortices each of strength of k, each of strength k but of an opposite rotation is called as vortex pair.

So, if I take w is that means in this case w is equal to i k by 2 pi log z minus z 1 minus i k by 2 pi because I say that there of opposite rotation. So, the minus becomes log z minus z 2 and this can be written as i k by 2 pi log z minus z 1 by z minus z 2. So, if I just say that I have p is a hyponent and this vortex is at minus a to 0, this distance is c r 1 and this is this is r 2. This is a 2, this point this is a 1 and let me call this a 0. Both are of a same strength k, then this point is a 2, okay. This will be, this can be written as i k by 2 pi log in this is called r 1 e to power i theta 1. This is called r 2 into e to the power i theta 2, then this will give me i k by 2 pi log 1 1 by r 2 plus i times theta 1 minus theta 2. This is will give me, which are if this is w, then what will be mu psi, which implies by psi will be k by 2 pi this part log r 1 by r 2. So, this is my psi and once I say psi is equal to constant that will give me the stream lines.

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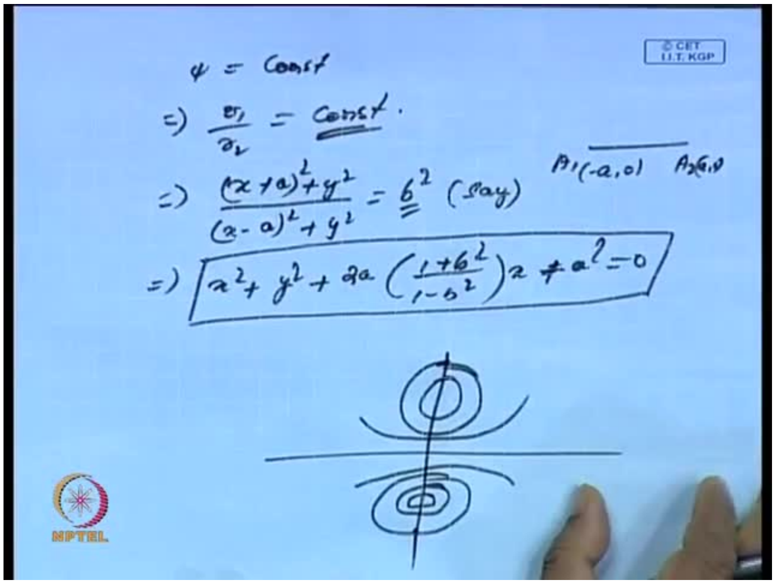

If I say psi is equal to constant, that will give me r 1 by r 2 is equal to constant and what is r 1 by r 2? That is nothing but because the vorticity are position at this is equal to a 1 is minus a 2 a and a 2 is a comma 0. So, these are the two points, so this will give me x minus x plus a square plus y square by x minus a square plus y square is equal to a constant. That constant let me call it as b square, say this b square is this constant is n nothing but b square i have to give. So, if I simplify this, then I can get it an x square plus y square plus 2 a 1 plus b square by a 1 minus b square into x plus a square is equal to 0. For this for is b I can get a and how to look like, if this is the axis, along this axis then I will have…

So, this will write this and this is the way to look like, okay? These are the stream lines, in fact these are the circle. This stream lines will look like this and if the in a same way, I will a talk about a vortex of let like a. We have seen in case of a source we have seen a w at. So, there we used to say that two pair of sources that kept at a distances apart form that multiply the strength of the source into the distance. That should give us a constant; in the same spirit will define a vortex doublet.

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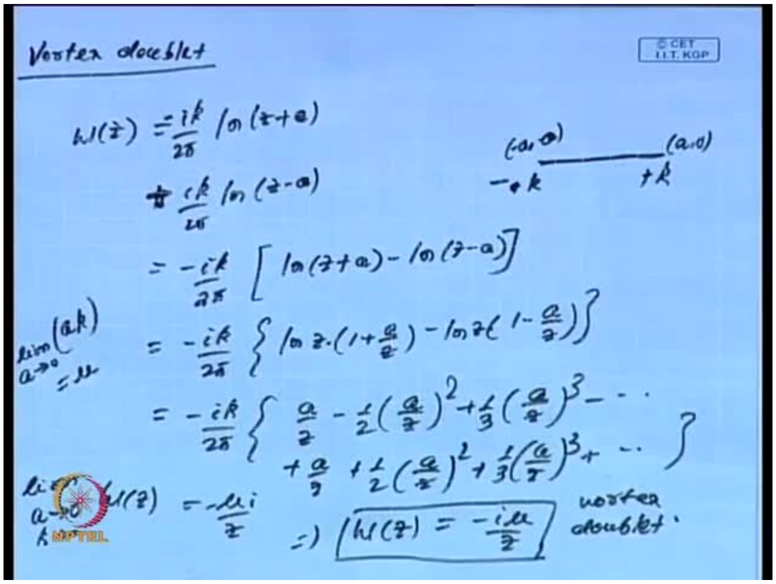

So, this is a vortex doublet or like a source we have a pair of vortex. So, for this is a vector consider a parallel vortex 1 is of strength k another is strength minus k is strength of a minus k. This is plus k and this is located at 0 minus a minus a 0. This suppose the point a 0, then what will happen to the w z? This will be i k by 2 pi log z plus a and this is has been minus because it is a of opposite strength minus i k by 2 pi. This log z minus a, if I look at this these will give me I will take minus i k by 2 pi, if I take this, then z plus a log z plus a, sorry this is minus k. This is plus k, so this is minus, this is plus, okay?

When this is minus, this is plus, so i k by 2 pi minus i k by 2 pi into log z plus a minus log z minus a. Then this can be simplified further that I called as minus i k by 2 pi into i can always write log z into 1 plus a by z minus log z 1 minus a by z. This gives me minus i k by 2 pi and this will be given because log z. Last term will be log z, there is minus log z, that will 0 and then we have log of 1 plus a by z. That will give you a by z minus half a by z square plus 1 by 3 a by z cube minus and plus a by z plus half of a by z square plus 1 by 3. This gives me, this gives me some of the terms will get cancel and again. If I put limit a tends to 0.

s is the distance becomes very close and k tends to large k is such that a k limit a tends to 0 a k. Because these value has to be k will be this strength will be such that a tends to 0 limit at a tends to 0 give a finite value k is large. So, if I do that then my w z in the limiting sense into w z and that will give me because when a tends to 0. A tends to 0, this term will be contribute to 0 and then in the process these term will because k into a by pi. So, that will give you, so mu by z and minus sign mu times pi by z and this is called the, so that means which implies w z is equal to minus i mu by z this is the complex potential. Potential due to a vortex doublet, this is the complex potential due to a vortex doublet. Now, in a same way we can called about what will happen if I have too more where is these examples will talk about rows of vortices.

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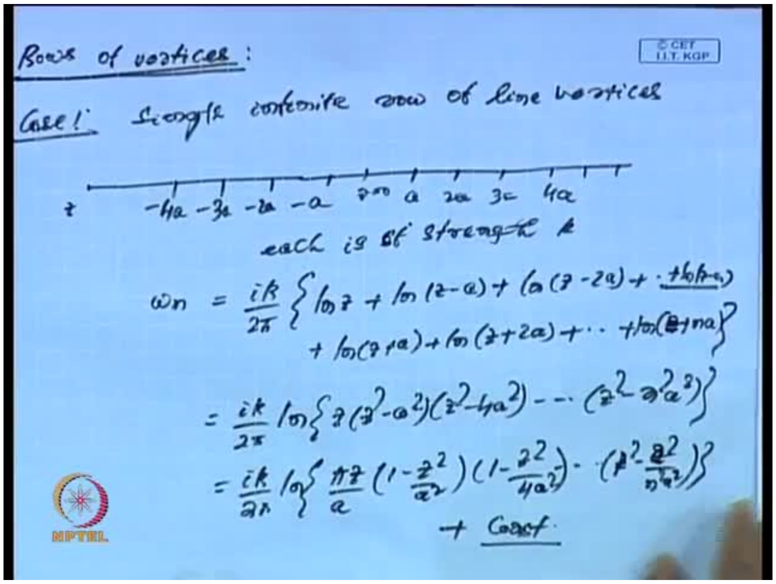

This is a very interesting often in when a body is a more slowly in a liquid, whether body moves in a liquid slowly. Then these rows of vortices plays in preservic, the receiving if control. Particularly what happen, when a body moves a rows of vertices of form in the bake of the body and in fact this is very common in many problems of marine hydrodynamics. This formation of vortices at the wake of the body and so that in this context, we will talk about the rows of vortices, but different types of a different shape of the structure. There different type of vortices, which have form on the in the wake of the body. Then, let us see case one, I will talk of a single vortices single infinite row of line vortices, single infinite row of line vortices. Suppose, I have consider a case of, we consider a single row of vortices and the vortices are formed at z is equal to let me say this is the line.

Then we say this is the z is equal to 0 z is equal to a minus a 2 a 3 a 4 a like this, this side and we have minus 2 a minus 3 a minus 4 a. So, these are the points at which infinite row of a vortices, the form and each vortices each of the vortex is of each of the strength k. If each is of the strength k, then what will happen to the sum? Let us take w n that means in the first two n plus one vortices. So, that will give me i k by each of the strength 2 a strength case i k by 2 pi log z plus log z minus a plus log z minus 2 a this side. It will go up to log z minus k a and similarly on the right side will have log z minus z plus a plus log z plus 2 a and it will go up to log z plus n a.

This is minus n a, this is minus n a, this is plus n a and if I look at this, if I look at this what it will give me? That will give me i k by 2 pi log z into z square minus a square into z square minus four square minus n square a square. These can be written as i k by 2 pi log of pi z by a. I am just multiplying a factor by a. We can always re write this 1 minus z square by a square, because I am taking a square common a square 4 a square. I will take all these common into 1 minus z square by 4 a square into z square minus a square, sorry 1 minus z square by n square a square plus some constant will become plus, constant again write because this constant will come because of this when…

Suppose, I say a term pi by a then have a term along pi by a here always written by a square 4 a square all these things, I can put it as a constant k, as a constant. So, because these are constant so the class of an we can always adjust these constant and see that these constant are assuming. In the class of general, I can assume these constant are 0 and in that case if I have limit n tends to infinity this w n that will be call w whether that case I will have i k by 2 pi. We have infinite series product series log pi z by a 1 minus z square by a square z square by 4 a square into z square n square a square dot dot dot.

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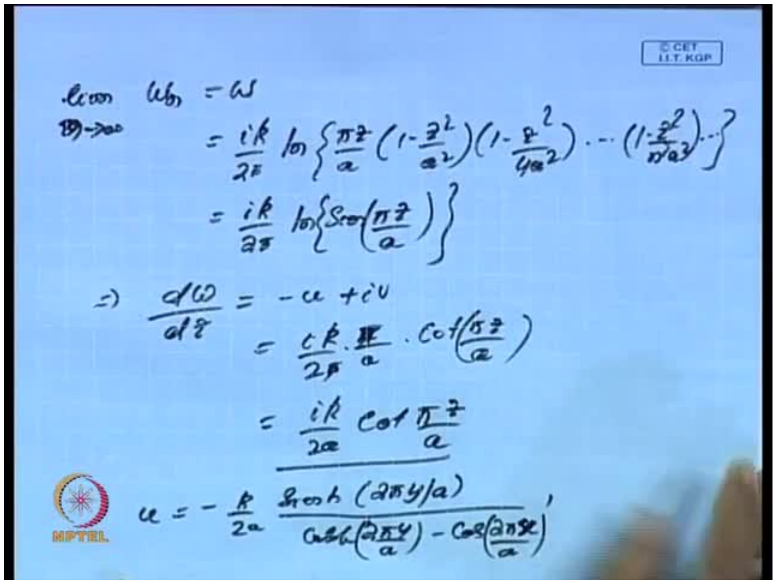

This reason is nothing but it is an interesting series basically i k by 2 pi. This is basically the series of sine pi z by a pi z by a for a infinite. This infinite series is nothing but sine pi z by a. It is a product formula of infinite series and this gives me… So, this is my complex potential but the infinite row of vortices. So, this will give me d w by d z, which is nothing but minus u plus i v. Then the right side will give me i k by 2 pi, this is into a by pi by a into this will give me cot by z by a. That is pi by that cancels, so it will be i k by 2 a cot pi z by a. If I simplify what the, which… So, if I separate in terms of cosine and sine and seperate it by u will get minus k by 2 a sine hyperbolic 2 pi y by a divided by cos hyperbolic 2 pi y by a minus cos 2 pi y by a 2 pi x by a and my similarly my g will be given.

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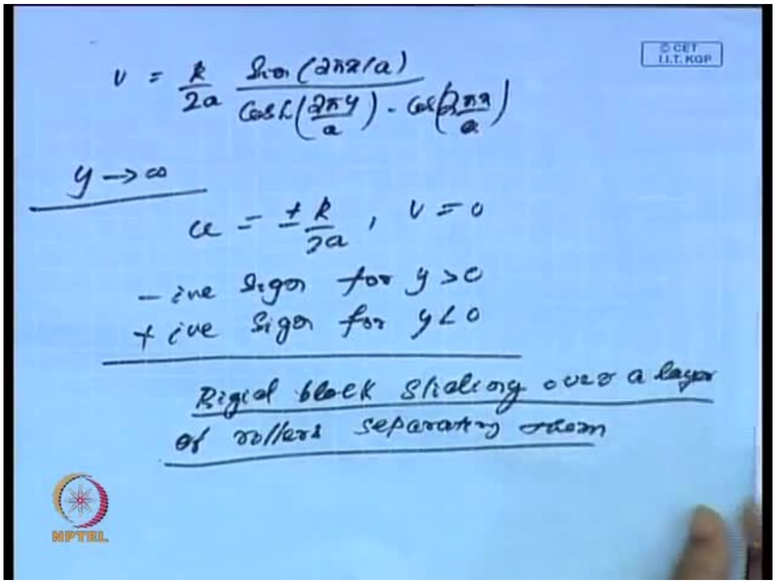

I will get the v will be k by 2 a into sine 2 pi x by a divided by cos hyperbolic 2 pi y by a minus cos 2 pi x by a. Again, if I say such y is infinity by u will be plus minus k by 2 a, where my v will be 0. It can easily seen that the negative sign will occur is in negative sign in the u for y is a greater than 0 and positive sign for y less than 0. So, what will happen that when y tends to infinity there is a uniform. So, y tends to infinity that means away from the vortex away from the infinity of vortex. We have a fluid, which is a plain at a uniform speed along the x axis. That is at a speed of k by minus k b y k by 2 a and minus k by 2 a.

So, what physically this represents at the motion is like a. It is like a rigid block sliding over a layer of rollers separates them. This is, this is motion will look like a rigid block sliding over a layer of a rollers separately them. Now, these understand, now due to this is the case of an infinite row of vortices. Now, if I will go back to a where column that is, I will think of an another vortex. That is what is famous vortex that is called Karmen Von, Karmen vortex sheet.

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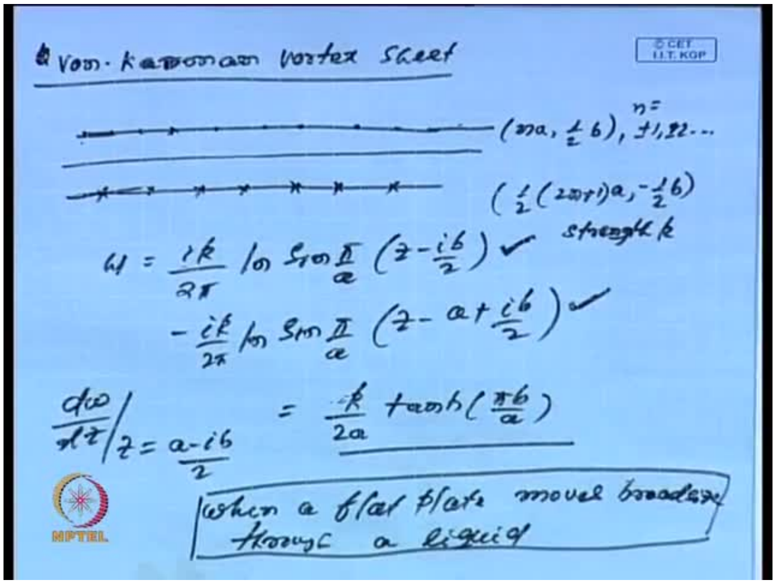

In the case of a Von Karmen vortex sheet, we have a two parallel rows of vortices, infinite rows of vortices. There apply start the first one, it is placed at the points n a half of a b. So, that means if this is the line 0, which is x axis it is on this side, this is half of b and this is, so this along this line the other one is half of 2 n plus 1 into a minus half b. So, these are the various location point wise these vortices. So, basically they do not lie on the same points, but so the distance of apart from these other a signal is here, here, here, here like this. So, then if this is the case, then we can see if you look at the corresponding w I am not spending much time is posted on the same way if you look at the corresponding w will get i k by 2 pi log sine pi by a into z minus i b by 2.

Then minus i k by 2 pi log sine pi by a into z minus a minus i b by 2. If you combine this what will happen? If I calculate it to d w by d z at z is equal to a minus i b 2, then what I will get? I will get my u. I get k by 2 a tan hyperbolic pi b by a. In fact I will here also what I will do this case also, I will proceed the same way like all the vortices. It is as the sum of all the vortices at various points and again the sum of all the vortices. In the case of a here, so here we have each of the vortices is the strength here is it each of a vortices is of strength k. If I add this two that vortices are the points in a half b will add this and both in this can be positive and negative, where is plus minus 1 to plus minus 2 like this. Then and I sum up them then, I get at this and apply the product formula for the sign series because I have the term.

Then I will a get this two sign functions and then if I simplify this, then I will get this is k by. Again, if I calculate it what happen? At z is equal to a minus b by 2 z minus i b by 2, so then I will a get this. Similarly, if I try with here other points also, then I can find at any point. I can take up in the vortices. This, then also I will get in the speed as this same as the speed will remain the same. Similarly, if I consider the, a part 0 of vortices and I can calculate the d w by d z at any point, I will also find that the speed will remain the same. That means the speed will be at by due to the… So, it will be another wherever, I consider the speed remains same. That means the vortex configuration here another with time for all time t.

Then once this is and this is a vortex configuration, which often call this is of an realized this kind of in a flat plate moves, broad size. If through c a liquid, this is kind of liquid kind of vortex, which is released when a flat but moves. If I look at and this is a, the vortex when a common vortex is and in facts this vortex theory like while will talking the motion of a ship in a water or any other body in a water. Whereas a at the wake, if I in the formation of the vortex and this vortex, this vortex it is very one can observe them.

Sometimes also we see there are flat emphasis given the, on the vortex shading particularly. It is basically measure emphasis given when a body is a moving in a fluid, is basically on the there are two aspect the importance is given flow separation and vortex shading. So, that this details will call in a specialized or and even go the details here in the desktops, this course in the desktop, this lecture, but with this background on a vortex motion, I will stop here today.

Thank you.