Marine Hydrodynamics

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Lecture - 16
Aerofoil Theory (Contd.)

Welcome you to this series of lectures in marine hydro dynamics. In the series, now last two lectures were talked about to how using confirm and mapping particularly Juco say transcription, we can transform the circular cylinder, we can map a circular cylinder to a aero pile. Initially we talked about symmetric aero pile, then we have seen how we can relate it with Cabadoado file, then in the last lecture we have talked about the glucose k hypothesis. Today will talk about Guta Geotherce theorem, which plays a very significant role in the design of the airing sections particular with the aerofirm.

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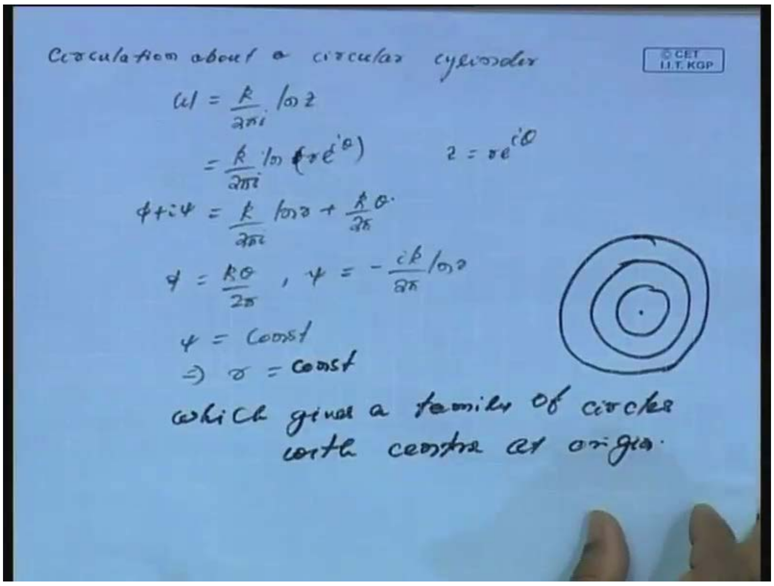

So, with these if you are going to the Guta Geotherce theorem, let us talk about the circular, the circular cylinder read meaned circular cylinder circular cylinder. In fact I have talked about these consider a complex potential w is k by 2 pi I long back. If we consider this and what will happen we should write this k by 2 pi I.

Then if you put z equal to r to the I theta along with z. However, we will call it along r e to the I theta, then that will give you k b y 2 pi I for r plus k y 2 pi k by 2 pi into theta, which if it called pi plus I psi. Then what my pi is equal to k theta by 2 pi and psi is equal to minus I by I k by 2 pi and log r.

If you say psi equal to constant psi is equal to constant, which means implies r is equal to r is equal to constant that gives which gives a family a family of circles with center at origin. Now, now this needs a simple (( )) circle with center at origin. So, this is the product in fact I have talked about this in some of the other lecture. Now, let as look at this from the point of view of circulation.

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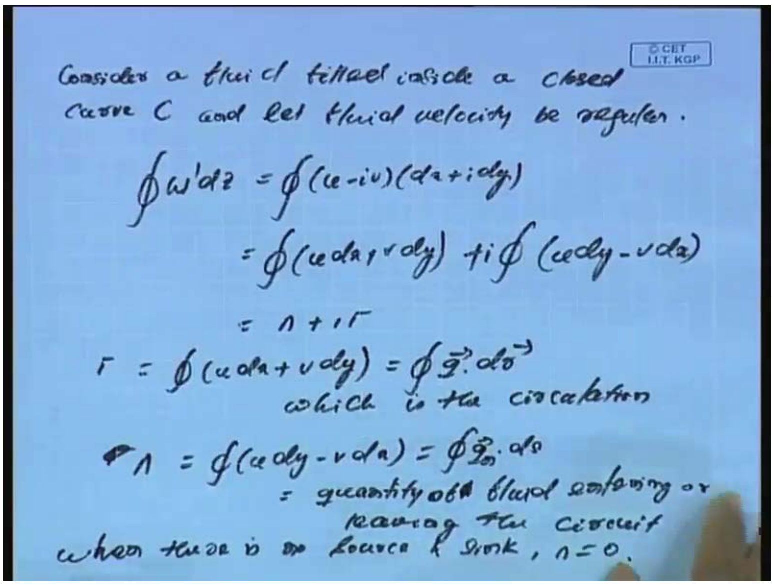

Now, what will happen if I let me consider a fluid consider a fluid fill inside a circular closed curve basically called towards c. Let a fluid electricity inside is a ruler the fluid velocity be regular be regular, then what will happen should not happen along the clothes w data d z as we know w data is is u minus I v and d t is d x plus I d y, which gives us u d x plus v d y plus I times u d y minus v d x. These all give us if you call it has I gamma now we have gamma u d x plus v d y is nothing, but q word d r bar.

This is, which is the circulation, which is the circulation. On the other hand if we look at gamma this is sorry, this will be u d y minus v d x. This is what this is equal to integral of q n bar data d s and this is the same as quantity of fluid of the amount of fluid entering and leaving fluid entering not leaving leaving the control circuit. Hence, when there is no source on sink when since the glow is regular when there is no source on sink sink. We will have because here the flow is regular, so we have n is equal to 0.

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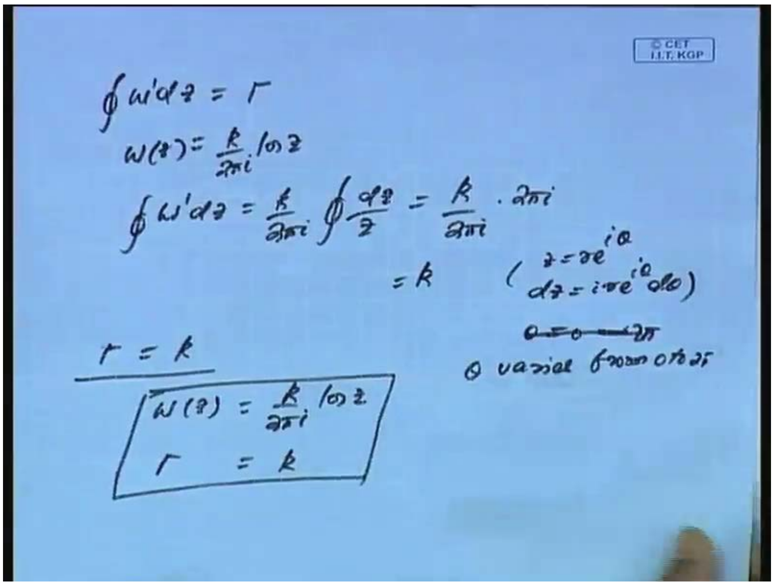

Thus these gives us, hence we have integral when there is no source in sink integral over a closed square w d z is nothing but gamma, which is a circulation. Now, we will go back to my previous problem were I have taken w z equal to k by 2 pi i log z, so in that case integral W dash d z will be k by 2 pi I this is d z by z and that will give us, if we look at a circular of radius r and this will give us k by 2 pi r into 2 pi i. Because if you put z equal to r e to the pariadat theorem, this you can easily get is equal to k. This can be obtained by putting z is equal to re to the i theta, so you have d z is i r e to the i theta d theta and the control equilibrium theta will have 0 to 2 pi theta is equal to 0 to pi 2 pi theta varies from 0 to 2 pi.

So, what we have seen from this is nothing, but gamma. So, we get circulars in gamma is equal to k, so if I say that if circulars no, so if I have w z if by w x I give it by 1 by 2 pi i log z. Then is see by circulars in gamma is equal to k. This is what I am looking for by 2 pi. So, this is an important result and we will use this now this understanding I will go back, what I will do?

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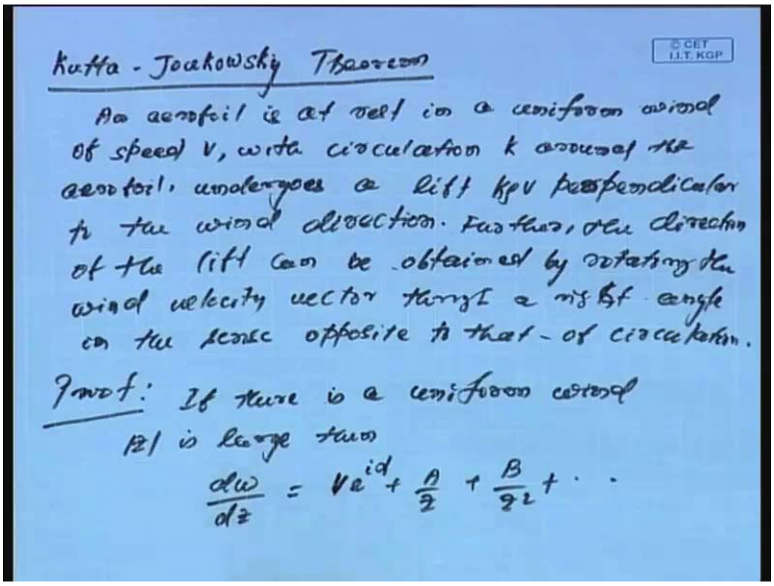

I will go the circle that is what I call the Kutta Joakowsky theorem. Kutta Joakowsky theorem, in fact this is a very important theorem in the story of a hero dynamics or in the hydrodynamics because both of them have an anallo where the anallo has to each other. So, the theory says an aerofoil is at rest in a uniform wind an aerofoil is at rest in a uniform wind of speed v speed v with circular from k around the circulars in k around the aerofoil. Then with circulars from k around the aerofoil will undergoes a lift and with a lift k row v perpendicular to the wind.

In direction and that is just not the lift factor further the direction of the lift can be obtained by changing, by rotating the wind velocity vector by rotating the wind velocity vector through a right c f right angled through a right angle in the sense. Opposite to that occurs.

So, this is a very important result, how to prove this? So, basically when we have a aerofoil which is attastedly in from the speed v and we have a circulars in k around the aerofoil, then the aerofoil will undergoes a lift k row v, which is perpendicular to the wind direction. Again the direction of the lift can be obtained by rotating the wind velocity of the through a right angled in the sense opposite. So, if that is the case, so then that is how to go for it suppose I have a wind from wind I will just give u a proof of this because this is a very important result I will give u a proof.

Suppose, if there is a wind from wind if there is a uniform wind, so from large distance at large distance also the wind speed will remain the same. So, and if I say mod z is larger enough is large, then we can write complex velocity depends on we can write d w by d z is something like because I am asking that if there is a uniform z is speed with which makes an angle alpha. So, I can call it d w by d z is a speed and suppose it makes an angle alpha the the direction the direction makes an angle alpha with the x axis in the downer direction.

Then v to the angle alpha let me say that a by z plus b by z square, so if take this representation of the complex velocity positional at the velocity. So, this will give us a wind speed of which moves with a speed v and with an angle alpha which subtends an angle alpha with the x axis at large distance.

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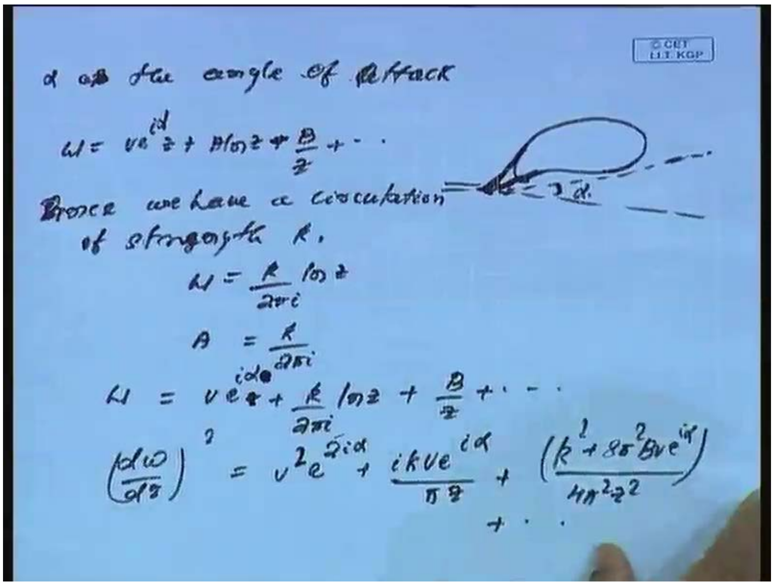

So, if d w by d z is this then from these I can always get and I can always call alpha is the angle of attack because it so basically, so this angle is alpha this angle is alpha. Then what will happen then what will happen. W here w will be v e to the power I alpha into z plus a log z plus here minus b by z plus so on. Then since we have said that there is a circulation, since we have a circulation of strength k, we look at the corresponding w as a result. Just now we have derived this can come k by 2 pi I log z. That means my a will be k by 2 pi I. So, if k is a a by 2 pi I then then we have d w by d z, so if you substitute for w is equal to these, so my w will be v e to the power I alpha plus k by 2 pi I k by 2 pi I I will call this k by 2 pi I, this is log z. K by 2 pi i log z. Then plus b by z plus, so on will stopped during this what will happen to my d w by d z d w by d z square will be v square e to the power 2 I alpha because plus plus by first term will be a log z sorry d w by d z this will be minus k square by a.

This will give me rather valid to this because so my w square is v square e to the power 2 pi alpha plus k by 2 pi I this will give me v square b y z square I k v e to the power of I alpha by pi by pi z plus. In fact you can see that k square plus 8 pi square b v e to the power of I alpha divided by 4 pi square z square plus so on. Because you have d w by d z this term will give us w is into z, sorry this is into z. So, then once this is this, then if we apply what will happen to the Blasicero theorem.

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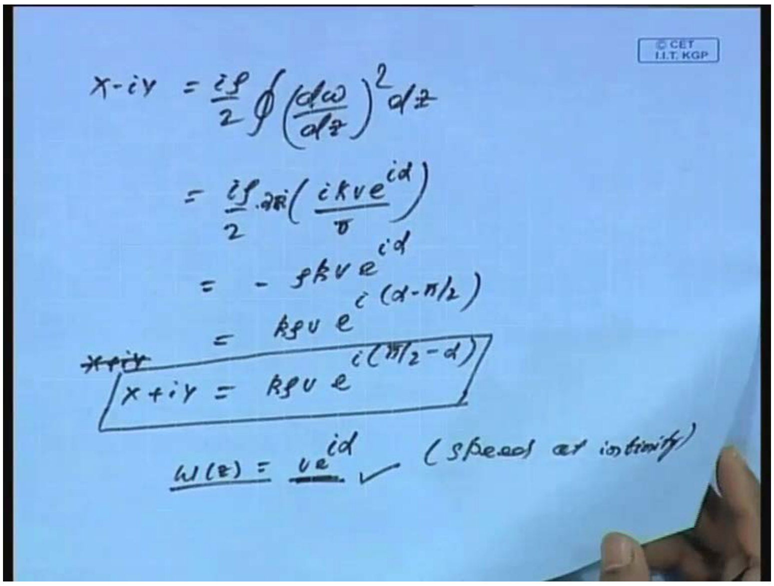

If we apply Blasicero theorem, then we will have x minus i y x minus i y is equal to I row by 2 integral over the closed circuit d w by d z square d z. If you evaluate this, in fact substitute for d w by d z and evaluate this by using Blasicero theorem will get I row by 2 into be the first term equal to be to 0 and the set third term will also herodato terms will contribute 0. So, the only term contribute which is associated 1 by z and that will give is i k v e to the power of i alpha by pi and that gives me minus row k v e to the power of i alpha i row by 2 into 2 pi I which is the sum of the recede.

So, that will give you k v because this will be i i minus row k v e to the power of i alpha pi pi will get cancelled then. These can be written as k row v e to the power i times alpha minus pi by 2. Then if I put it x plus i y from here, sorry then rest when entered k row v, e to the power I into pi by 2 minus alpha. This is what the final say because we have started with over v was we have started w z. We have seen that we have started with w z is v e to the power of I alpha that rest the speed at infinity infinity.

Now, if w z was that now look at the lift force that is the y component of the force then that gives us pi by 2 minus alpha and this makes an angle perpendicular direction and it is just perpendicular to this. So, this is what we have got now, so we says that the lift force is perpendicular to the direction of the low velocity in fact this is observed in most of the when you have a that is the this concept is used in fact in case of a popular at particular in the case of the aero philer hydrophile, this is what the theorem says.

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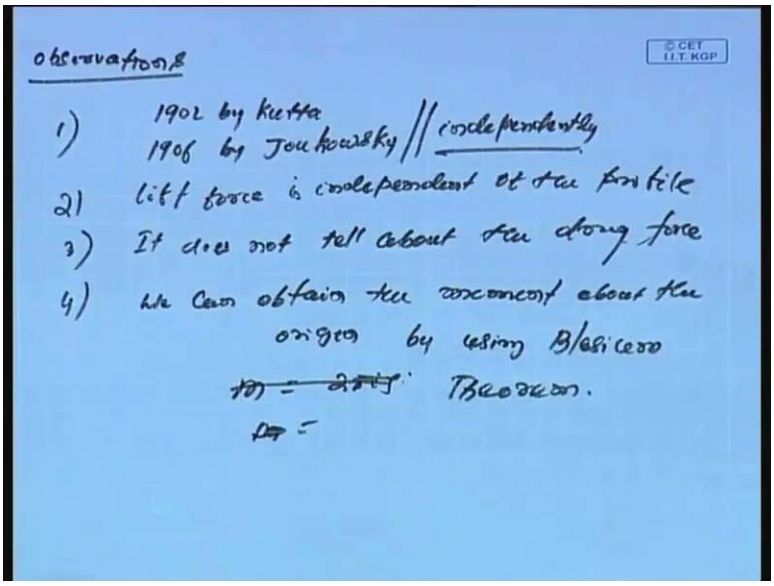

Now, with this theorem now if we will go back to, some of the observations will make, some of the observations some of the observations if we will make in the observation theorem, in fact this theorem as discovered a 1902 by Kutta and then by in 1906 by Joukoursky. So, and then it is in the put it was independently though results erect in independently.

That is its known as the again the lift itself lift force is a independent of the profile other profile, but it all depends on the speed as well as the circulation. The third point I want to say it does not tells it does not tell about about the drug force because on the aero foil along with the bounded way we are not we have consider the flow as in compressive.

Then if it is a, since a it is in the fluid is the air and then the will say that and drug will be 0. On the other hand you have in the in this case we have not taken because we have not taken the (( )) into account. So, it will be not proper to say that the difficult to find the drug here. On the other hand 4, in the same manner in fact like the (( )) we have obtained we can obtain the movement about the the movement about the origin obtain the movement about the origin. It can be easily checked that it can easily check that movement m will be 2 pi I a row v b.

This it can be obtained by not mentioning exact value when there is a function, so we can easily obtained by using again Blasicero theorem that is m is equal to using that formulae, rather I will say I can obtain the movement of the origin by using Blasicero theorem. I am not going until here, but this will now, these understandable. Let us see that how can we obtain on? Or the application as an application of the message independently we have obtained about the process that is particularly the lift force acting on the aerofoil. Now, let us look at these from the point of view as an application of the glucose k transformation equally.

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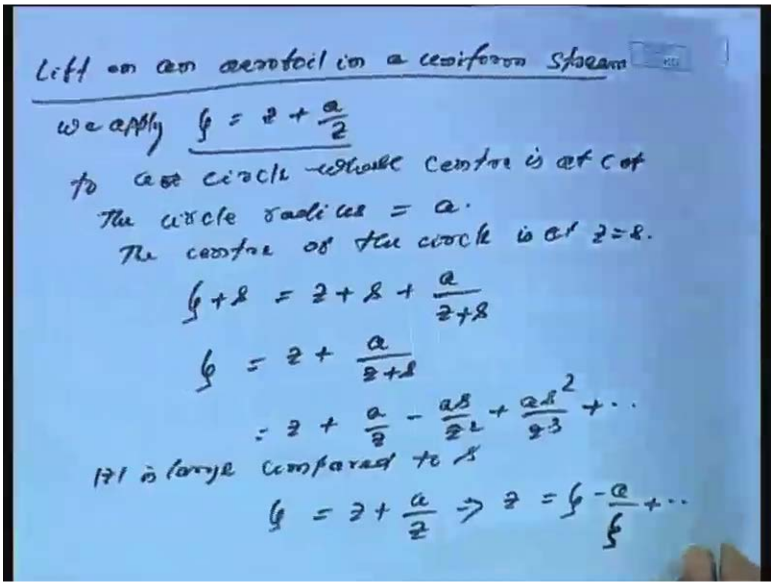

Suppose, we want to find out the chemical aerofoil in a uniform stream, so this a another result where the important result in an aerofoil to calculate the lift aerofoil in a uniform stream. So, we know that from glucose k transfer we have geta is equal to z plus a by z, suppose we shift the origin. Let us say the circle there is a circle, this we are applying we we apply that geta transition to a circle whose center is at c and of the radius with the radius the circle radius. Let us say this is a. The center let us say the center the centre of the circle is at z is equal to s where s s is very small, then what will happen? If I will put geta plus s, then I will have z plus s plus a by z plus s. So, whose guess me when geta is equal to z plus a by z plus s. If we expand it we enters z plus a by z minus a s by z square plus a s square by z cube.

So, you can expand it because it if have a Binomial theorem of that 1 by z plus s, then I will get it. Now, when z is large mod z is large because s is small, then what will happens my geta? Geta will be z is large, s is small, so this will be negligence so my geta will be again z plus a by z. Now, if I just get the reverse transformation I can easily get when z is equal to geta minus a by geta plus 5 atoms then now we have got this.

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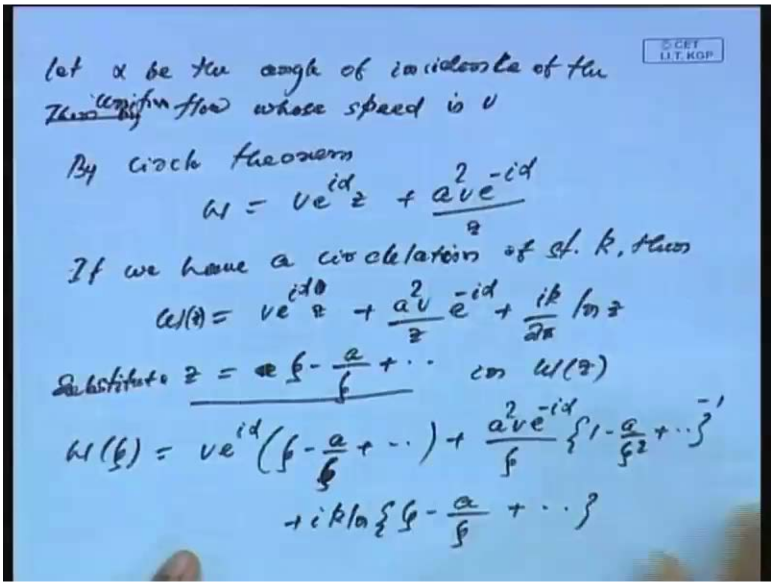

Now, let alpha be the angle of incidence angle of incidence, if alpha is the angle of incidence then what will happen to them by circle theorem (( ))? Angle of incidence of the flow whose speed is of the uniform flow whose speed is v, then we have by circle theorem. By circle theorem must my w will be v e to the power of I alpha into z plus a square into v into minus I alpha by z u a square. That will give basically u a square by z so this is my w this is the flow in from flow faster circular cylinder with this moving with the speed v. Now, if I have to this if I add if we have circulation what will I say circulation of strength k then then we have w is equal to v e to the power of I alpha into z plus a square v by z into e the minus i alpha plus i k by 2 pi into log z.

So, w becomes this now if this is for a circular cylinder. Now, you apply this to the aerofoil section, then I have to use the transformation. If I substitute for z equal to that is geta minus a by geta plus higher formats, then substitute for this in w in w z if is substitute for this. Then I will get my w z is w z or w geta as v e to the power of I alpha into geta minus a by geta plus a square v e term minus I alpha by geta. I can write it has 1 minus a by geta square minus 1 plus I k this because log z z. I will put this I will put this as I can now I can take geta minus a by geta plus.

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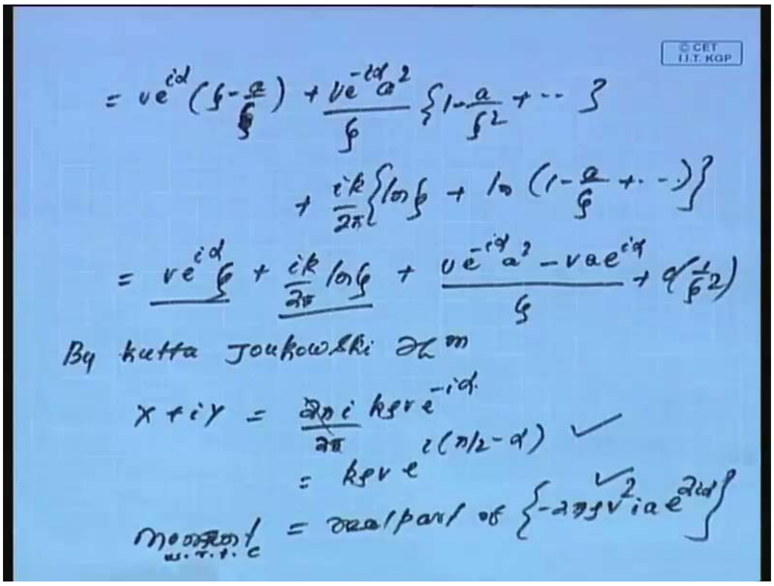

If I do a little simplification, then I will get which is equal to the v e to the power of I alpha. Geta minus a by geta plus v e to the minus I alpha a square by geta into 1 minus geta square plus 3 s plus I k by 2 pi brought I have 2 terms I can have log of geta plus log of 1 minus a by geta plus the high powers. If I do this in fact little simplification v e into the power I alpha into geta plus this term, I will take it I k by 2 pi log of geta plus v e into the minus I alpha v e into the minus i alpha a square minus v. This term v a e to the power i alpha. This is by geta plus 1 by geta square, so this is the first term the second term is this first term second term. This is the these term first term then finally, this term so this is, so this is the way it has come.

Now, it applies again by just apply Kutta Joukowski theorem. We have seen that this is like velocity term 10 this is the circulation term. The term with this have terms the same member like what we have done, then if we apply the Kutta Joukowski theorem will get x plus I y because the term that will contribute only these term and that will give us 2 pi I by 2 pi into k row v e to the power k row v e to the power of i alpha and that will give us as a. That will give a 2 pi 2 pi cancel, which will give you k row v e to the power of I times pi minus 2 by alpha, which is and again you can sign. If you will go the movement that is the in this case you can get the moment that will be equal to real part of minus 2 pi row v square into I a e to the power of I alpha. This is the moment, so this moment is always calculated for the centre moment with respect to the centre see the centre thus if l is the lift. So, the lift force this will give us the lift and then we can get the movement and the lift force if you have a flow and this here we have applied the directly the Kutta Joukowski theorem. Also, directly by the glucose k transformation, so this is the way we are able to calculate the, we are able to calculate the person movement when we have even to flow past a aerofoil.

Now, with this understanding what is this this Joukowski theorem, this Joukowski theorem tells us, in fact the when we have a aero foil that are similar structures like in the site of bats for the site of aero plane. Even the hydro piles there are certain other application in Mervin hero dimensional that is the craft particularly the red or… The other moving variant vaccines it always want to calculate it the the hydro pile of other hydro piles. Then to this Joukowski transmutation always are, sorry this Joukowski theorem always gives us the important information about the lift force as well as the movement.

That is acting on the aero pile and that plays a very significant role in the design of the aero foil, so with this background I will not go to further detail about the Joukowski transmutation detail about the design of the aero foil, because this is other design of the hydro pile section because the design is always based on the Joukowski hypothesis. Always the lift and lift force and the movements are calculated based on the Kutta Joukowski theorem. This concept is used in the design, so this is another aspect the whole aspect of the design of the aero foil or the hydro foil and various sections of the the view of an aero foil or an hydro pile.

That is all together different vapor in aero dynamics or in marine hydro dynamics particularly. Those who for the design and I will not go to those details, but rather will stop here when this back grounded aerofoil that. Since, this is analogous as I am again and repeating, since it has certain analogy with the hydro foil the same concept will be used in the design of various marine vehicles. Mainly as per as the aero foil section is concerned when you stop here, but in the process we are going. Now, we are going to show other applications, we will talk about other applications of the hydro foil particularly of the Joukowski not the Joukowski transformation.

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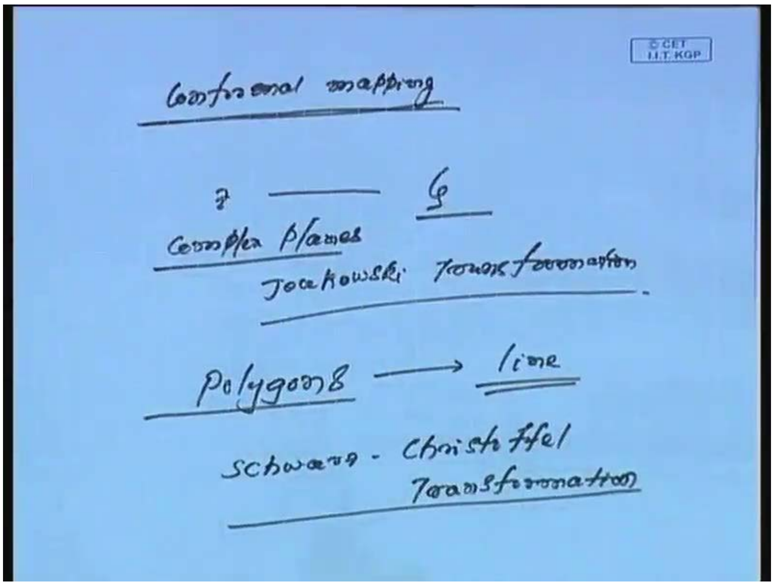

Rather we will go the other application of the conformal mapping conformal mapping like we have seen in the I just give you a review because we are spend a good amount of time and conformal mapping we have seen that from a just using a conformal mapping various conformal mapping we can always transform from 1 plan in the z plane a problem the z plane to another problem in the geta plane both are complex plains both are complex plains, but we are able to map the z plane into the geta plane.

The question comes and we have seen it before by we have seen from a source or from one of the important transformation. What we have used is the Joukowski transformation apart from some of the other transformation, which initially talked about Kutta Joukowski transformation. We started about trasforming a circle from a circle we can go to ellipse from a circle we can go to a aero foil and (( )) foil now the question comes, can we can we suppose we are never talked about polygon hexagon particularly we are never talked about polygon, so can we talk to about that can we transform a polygon to a line.

By a treated conforming mapping, because if you can do that from polygon to line, then last class problem can be handled in the proponaes because there are several kinds of polygons. So, we can look into particularly channels (( )) problems, which are very common. Also like even if circles, so if we say that a circle can be dependent by a line and, so basically we will look into various like sometimes we see that (( )) changes in the depth and width of our channel. Can we so that? So, I think we can easily do that and one of the major transformation is called Schwarz Christoffel transformation.

By using the Schwarz Christoffel transformation, you can always have a polygon line and in the process we can solve large number of problems associated with (( )) problems particularly in marine hydro dynamics. So, now let us go and tell what exactly his happening about the Schwarz Christoffel theorem. Christoffel transformation or sometimes we call it has Schwarz Christoffel theorem. I will just give you a detail about I will not go the detail about the today, but I will just give you the theorem.

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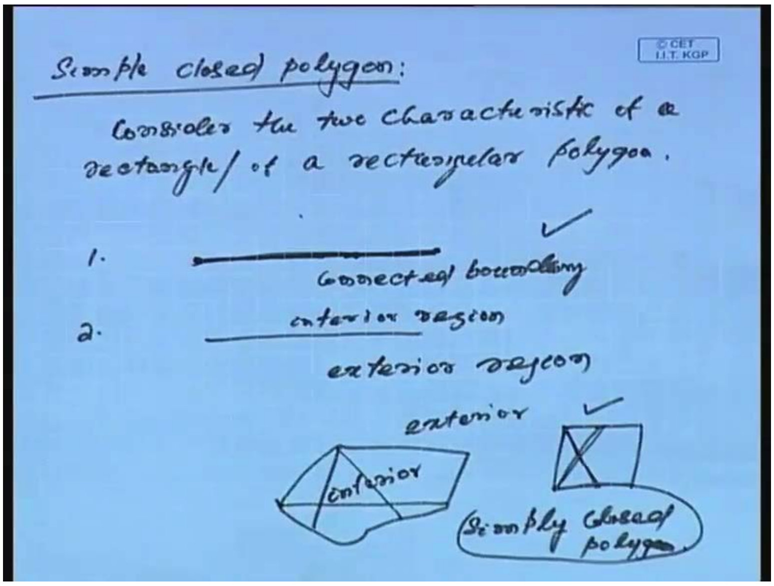

In brief, if I am going to Schwarz Christoffel if a am going to the Schwarz Christoffel theorem, then you called about what I mean by a simple closed polygon, simple closed polygon. Now, that at let us look at a rectangular action. Let us consider a rectangle consider a rectangle 2 characteristic of the rectangle (( )) of a polygon.

We can always we have a point, small point from 1 point you can always move to, suppose this is the point on the boundary and from 1 point, we can move to another point on the boundary just without on any. If this is the point here on the boundary, then we can move suppose this I have another point here. So, we can always move without leaving the surface, which I can always follow a path I can reach here. It by reaching from here to here, I never leave this path this path if I reach from one point to another by leaving the path I call this as a connected path the rather the boundary is called connected boundary.

This is 1 point, so from 1 point on the boundary if we can lift this to another point just by following a path which never slips the boundary, then I call this path as a connected path the second point. I say these boundary always divides the plane into 2 regions; one is the upper region and one is the lower region. Sometimes we call this as if we have a boundary, then it will always we call it has a interior interior region and one is the exterior region. So, let us look at any (( )), so this we can call it has a interior and we can call this at the exterior at the boundary.

The interior points are such that, so again since the path is a connected path then any 2 points always can be joined by an interior, you can always reach to any 2 points any 2 points we can join by without going outside the boundary. Because this is a connected boundary, so from any two points can reach without going to the boundary and this is by a. Then this and we need not cross the from reaching one point to another point. We need not cross the boundary this is an another characteristic particularly in case of a rectangular polygon or a polygon if you look at a rectangle.

From any point you can reach to another point just by using following without going to the outside the boundary particularly in the interior. We can always rest interior path we can choose and we can the rest. So, these are the two very important characteristic of a simple closed polygon particular to major characteristics. These characteristics without going to the exterior we can go from one point to another in the boundary and that is why any configuration (( )) lines in a plane, which satisfies this kind of characteristics that means we can connect one point to another without leaving the boundary. The second is any two points in the interior region can be connected without going outside the boundary. Then we call this as a simple connected region simply closed region closed polygon.

That in process when we have a simple closed polygon, so there are if it is a either does 2 regions either it is a exterior point or it is an interior point. So, the exterior points or the interior points, we can always go from by choosing any path and this is what we talked about simply closed polygon. This points of each class from a connected system so with these understanding, I will not go to further details today, but with these understanding about a closed polygon. In the next class we will go and talk about how, what is the first Christoffel transformation?

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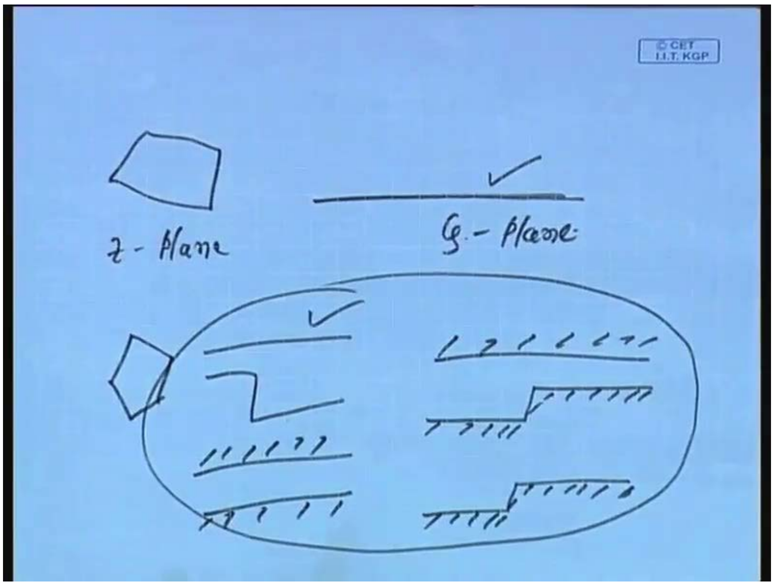

How this Christoffel transformation helps us in transforming any point on a polygon to in the z plane on the geta plane integior airline? So, this is a geta plane and it can be on a next line, so this is what on any kind of polygon, it can be it can be path like this it can be two straight lines or boundary it can be, so all these things a large class of problems can be handled by this. There are some on the simple cases so, but has quite large number of application, if all these class have problem can be transformed to a just a in the geta plane on airline.

Then it will be able to simplify large class of problems physical problems by these transformation that is what the Schwarz Christoffel transformation. In the next class, will be mention about the Schwarz Christoffel transformation find the respective how? But how this Schwarz Christoffel single transformation is able to give us the idea about transforming? All various shapes, various types of polygon bringing it to a line on a plane like a Joukowski transformation. This was Christoffel transformation is also another very important theorem, a transformation, which plays a significant role in Merin hydro thermodynamics. Fully work out this in detail in the next class.

Thank you.