Marine Hydrodynamics

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Lecture - 15
Aerofoil Theory
(Contd.)

Welcome to a fifteenth lecture in the series of marine hydrodynamics. And in the last lecture, we are talking about, I have introduced about aerofoil structure, and then we talked about, how to transfer, a symmetric from a circle to a symmetric aerofoil by using the Joukowsky transformation. So, in the first the end of the lecture, I had mentioned that we can if we shift, because we had done vertical horizontal shift, so from a circle, if we are applying the Joukowsky transformation. So, we are able to transform a circle to a symmetric aerofoil. But what will happen, if we transform the circle, using the Joukowsky transformation and the circular shift, both there is a horizontal shift and there is a vertical shift, and and then we will apply the Joukowsky transformation. Then it is obvious that, there will be a shift, towards the and upward direction and that only for that will give us the cambered aerofoil, which is look like a typical aerofoil section of a, of an air will. So, let us look at this, that how, we are going to do this, we proportioning further about the theory of aerofoils, so with this, so let us first do the transformation.

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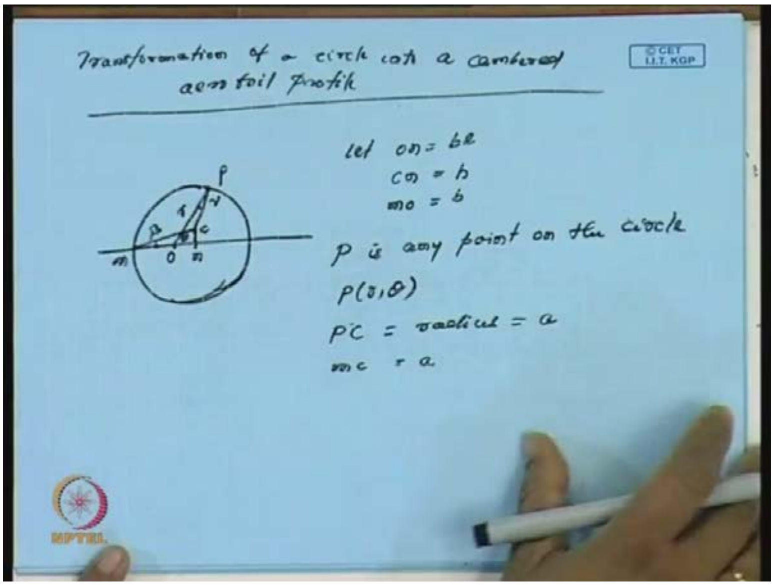

The transformation of a aerofoil transformation of a circle, in to an, of a circle into a cambered, cambered aerofoil profile. So, to do, so to do so what, we will see that suppose, we a have circle, I told you in the circle part, we are doing suppose this is the line. So, the origin then I will say that, o is the origin in the circle, and this distance and then what I am doing my say, I say that let the horizontal shift b and and this is and then there is a vertical shift, that is and c somewhere here. So, o is the original of the circle, and c the center of the circle, o is the origin and c is the center of the circle. In that case, what I will do, I will say fled the distance, as usual let o n as b e, and again o n as b e, while the shift is a distance c n, and let c n is equal to h. This is the vertical shift and o n as b e the horizontal shift, and then we have m o, we have my m o is b m is point here m o is b.

Then what will happen, now what I will do, I will take any point, then we say, suppose p is any point on the circle, point on the circle. If p is any point on the circle, then what I will do, I join this o p, then my, and then p is if I say this is angle is theta, and this distance is r. So, p as a coordinate r theta, and c is the center of the circle, and we call it join p c, so p c is the radius of the circle, and I will call this, this radius of the circle is a. Then we have and then again, if what I will do, I will joint the point m with c, so my m c again, because this is point, this is the center m c is also a. And then I have two things here, my with this understanding, let me say that this angle, this angle let me say this is beta, and then the angle it makes with this let me, call this as gamma. So, if I take this, and what will happen, now what will I will just separately plot these points, so and we call this it would separately, that will be very clear.

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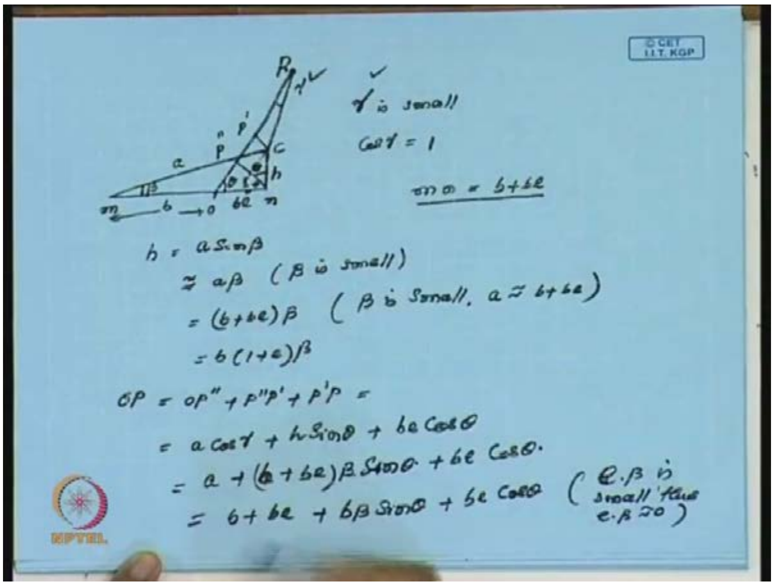

So, basically I have this point is m, then I have which somewhere, here adjust then I have n is here, and then this is there is a vertical section, that is h and then this angle is beta and this point is c. Then I have p somewhere a point here, and then I will call it p is the point here, p is o p and then this angle is, this angle is called gamma, this angle is theta. And then if I draw a perpendicular here to here, and that I call this is perpendicular angle, and I will call this p prime, and this is another perpendicularly draw from here, I call this p to the power n. So, my vertical shift is, is this angle is beta, so what will happen to h will be, this angle is beta and then this is what is m n, this is the vertical shift.

And then this will be, and then this distance is a because this is the radius of the circle, so h is equal to a, because h by a is sin beta a sin beta, h is equal to a sin beta. And again my a is equal to, so which also I can write it as, h is equal to a sin beta, again along this distance, further I will come back to this. Now, now gamma is small, gamma is small, if gamma is small then we have, then we have, gamma is small, then we have first gamma is then my 0 cos gamma is almost 1. Now what I will do, so a sin beta, since beta is small I can call it as a beta, that means beta is small which can be equivalent to a beta, and e is a is nothing but we have o n is b, e o n is b e and then we have this m o is b this is b.

So, I have m n equal to b plus b m n is b plus b e, and then it is because of that, since beta is small, since beta is small, I can also call this as b plus b e into beta, this beta is small. I can call a, because h is the small shift, so I can always say a is equivalent to b plus b e, so this is same as b into 1 plus e into beta. Now, this is h, now what will happen to o p? o p is o p double prime plus p prime double prime p prime plus p prime p, and this is nothing but this is so this will give me or we can write it as, so this will give me a, because this is p prime p, p prime p is a cos gamma. Because this is a, and this angle is this is a cos gamma plus p prime p double prime p prime p double prime its now this angle is theta. So, this is pi by 2 minus theta because this is a perpendicular line, so this is also theta, because this is pi by 2 minus theta, this angle is again theta.

If this angle is theta, then if I draw it parallel here, I draw a line here, then it can be easily see that p prime p double prime will come as h sin theta plus because, so this will be if I draw a perpendicular. So, this angle this will be parallel to this, this is again perpendicular, this is a perpendicular, this is a perpendicular. So, this line is same as, this line, so this is nothing but from this to, this, this distance is, is called a sin theta. So, similarly, p as the level this have parallel line, so this will be p prime p double prime will be sin theta, then plus o p prime o p double prime, and this will be give you this is b e this angle is theta, this is the perpendicular, so this will give b e cos theta s.

So, if I do that because cos gamma is gamma, then this becomes a plus a plus, we have h is a sin beta a into that is b plus b e, b plus b e, b plus b e into beta, b plus b e into beta into plus b e cos theta beta into sin theta plus b e cos theta, and this gives me, this gives me a is plus a is again my a is b plus b e. So, this becomes b plus b e, again beta is small, and e is small, so this is b beta, then since beta is small, this also I can write it as b plus b e plus b beta sin theta b beta sin theta plus b e cos theta, the similar b e into beta is small thus neglected. So, e beta will be 0, so this term will go, so b plus b e b plus a is b plus b e, so b beta sin theta plus b e cos theta, so this is what o p and what is o p, o p is nothing but my o p is nothing but this is a angle gamma, this is angle gamma and o p is r. So, this cartage becomes r is equal to, so if this is the case, then what we will get?

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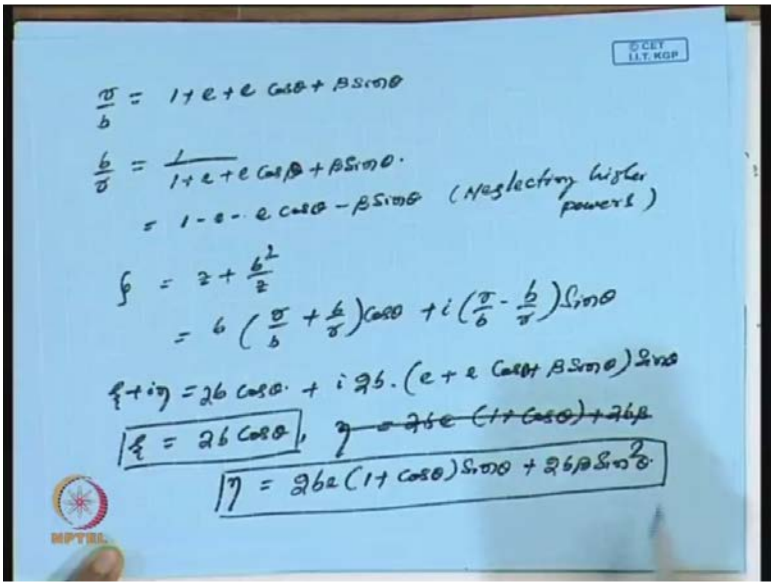

Then, we get r by b is equal to 1 plus e, 1 plus e plus e cos theta plus, plus beta sin theta. So, in the same manner, we can get, what will be happen to b by r 1 by 1 plus e, 1 plus e plus e cos theta minus b sin theta plus beta sin theta. If I do this, then I expand it in a polynomial expansion, if I do then it will give me 1 minus e minus e cos theta e cost theta minus beta sin theta. And once b by r is this because I am neglecting high powers, high power then if I go back to the Joukowsky transformation my beta is equal to z plus b square by z, and that is nothing but b into r by b I am not going to mention, we have already done this, plus b by r cos theta plus pi into r by b minus b by r into sin theta this gives me.

So, now if I substitute for, so my this will give me, if I say b into r by b plus b by r, if I add this 2 that will give me 2 b cos theta plus i times 2 b r by b minus b by r so 1 1 will get cancel. So that will may that will give us e plus e cos theta or this is e e cos theta plus beta sin theta, e cos theta plus beta sin theta, so then it will be 2 b 1 plus e e plus e cos theta plus beta sin theta, sin theta into sin theta. And thus my xi is equal to 2 b cos theta, zeta is desired for psi eta, and hence and my eta is equal to 2 b 1 plus e cos theta, 2 b e into 1 plus cos theta plus 2 b beta sin theta into cos theta cos theta into sin theta eta is equal to 2 b e 1 plus cos theta 1 plus cos theta into sin theta plus 2 b beta, this is sin theta, this is again sin theta, sin theta theta. So, my that is my eta, and this is my xi, so this is my xi and eta, if I plot this how it look like, so then what I will, what I will get?

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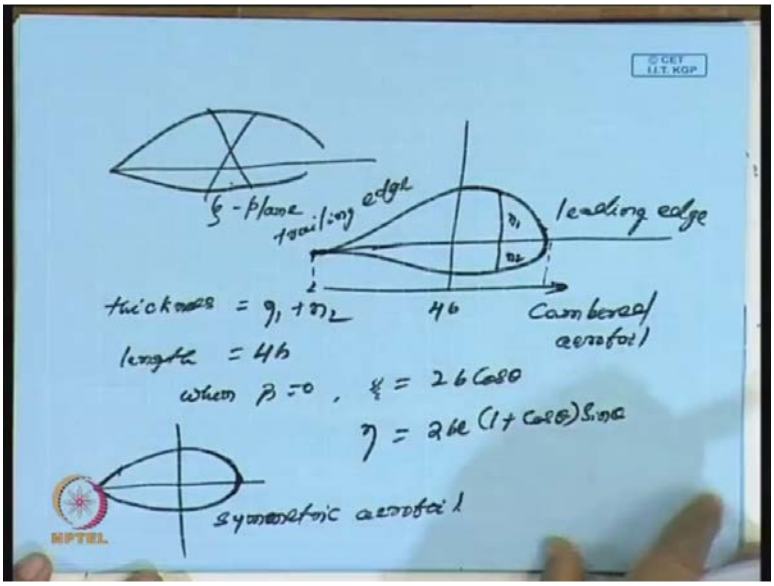

In the theta plane, this is my zeta plane, in this is the zeta plane, and for this will look like, this will not be like this rather I will put it in a proper way. It will be something like this in the previous case, we have seen that this was symmetric, along the about xi axis but here they just shift to the right, to the upward direction. So, this point, I call this as my minus b or rather I will call this, this is my if I take anywhere in the axis, and this is my eta 1, and then I say this is my eta 2. So, anywhere the thickness of the aerofoil will be eta 1 plus eta 2, and then what will be my total length length will be 4 b.

So, this distance, the total distance this is 4 b and this is just because 2 b 2 b 4 b, and further I can have, what will happen length is 4 b and the thickness will be eta 1 plus eta 2. Another thing, we can see here, that when beta is 0, if beta is 0 then my xi will be 2 b cos theta, and my eta will be 2 b e into 1 plus cos theta into sin theta, and that case my aerofoil will be asymmetric. This should be like this, because this side by just half edge, and this is the, this is what my symmetric aerofoil. Whereas, this is my cambered aerofoil, and if we will see that always, in this case the pressure distinction, will be uniform, because it is symmetric, in the both the, are sides about xi axis. On the other hand here the pressure will be, because this side the pressure will be maximum, compared to this side, because where is a this is there is a vertical shifts.

So, the pressure it will be, so for this is the region pi, and the pressure because there is a maximum pressure, we have pressure is higher in this side, so it will it is the one of the reason, why it will be acting as a lift force. Because the pressure is more here, on this side of the surface compare to the upper upper surface, so always this will provide here, additional pressure thrust, and that will act as a lift force in the aerofoil. On the other hand, if we will look at this, this one we call this that the, this is the leading edge, this is also call the trailing edge. And soon, we will see that, this edge there is a singularity, that I will come to say compare to this here, there is no singularity but at this end that, we will come to see. Now, now let us look at the, what is the thickness, maximum thickness for aerofoil can have, if you look at the thickness t.

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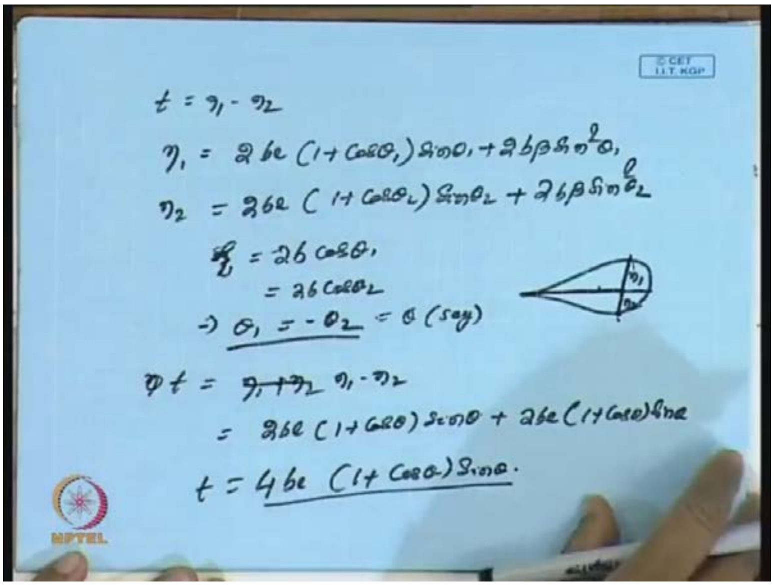

As I have mentioned t will be eta 1 minus eta, and eta 1 minus eta 2 rather, then my eta 1 will be for the same angle theta, it will be 2 b e into 1 plus cos theta, 1 plus cos theta 1 into sin theta 1 plus 2 b beta sin square theta 1 and by eta 2, will be 2 b e 1 plus cos theta 2 into sin theta 2 plus 2 b beta sin square theta 2. But since my eta 1 is equal to 2 b cos theta 1, is same as my xi 1, because if I look at this, if any point, I am considering this is my eta 1, and the same perpendicular point is eta 2. So, my xi is fixed, so xi is 2 b cos theta 1 is same as 2 b cos theta 2, which implies my theta 1, if minus theta 2 theta 1 is theta 2, that means in the, this will be minus theta 2.

So, the angle in this way, if I say this, so the theta 1 and theta 2 are negative, so in that case my t will be eta 1 minus eta 2, because this, this distance minus this eta 1 minus eta. So, the total distance, and that will be you can see, that will be give us, if r eta 1 minus eta 2, that will give you rather eta zeta 1 plus eta 2, this is eta 1 this is eta 2. So, then that will give me 2 b e into 1 plus cos theta, so this is eta 2 this is eta 1; this is eta 1 minus eta 2 where 2 b e 1 plus cos theta and this I call it as theta say, 2 b 1 plus cos theta into sin theta, because 2 b beta sin square theta, this will be minus, this both will be same. So, this will contribute plus 2 b e into 1 plus cos theta into sin theta, and that is nothing, but 4 b e cos theta into sin theta. So, now if you it t is the this, this is the max this is the thickness that is t, then what will happen to the total chord is 2 b, chord length is 2 b.

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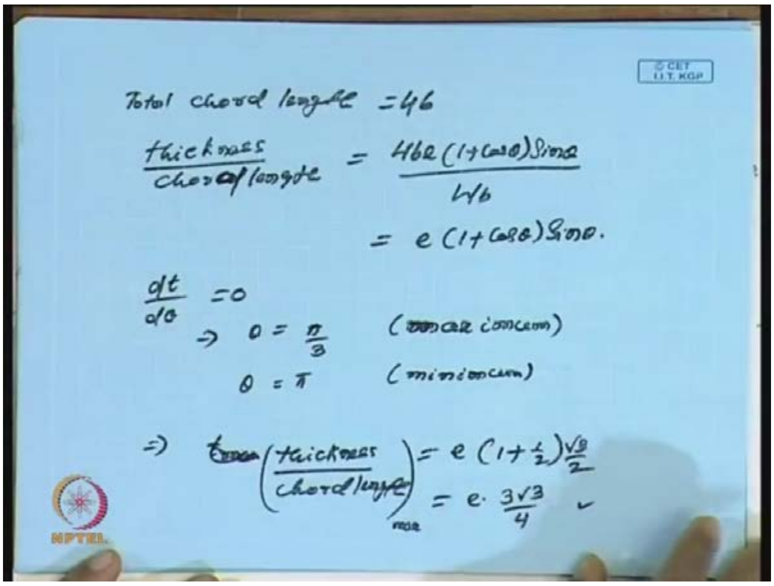

So, thickness total chord length is 4 b, so if I just say that thickness to chord length received thickness to chord length, that will be 4 b e into 1 plus cos theta into sin theta depended by 4 b, that is nothing but e into 1 plus cos theta into sin theta. So, again it can be like, if you look into the same way, what will happen to when d t by d theta, if you look at d t by d theta is 0, we can always say that theta is equal to pi by 3, and theta is equal to pi by 3, will give us the maximum thickness. Give a maximum for t and if theta is pi by (( )) pi by 3 will give a maximum and again, we will see that theta is equal to when would becomes pi theta is equal to pi. Again this is that will give minimum, this will give us minimum, and that shows that in fact, we have seen in that case, then what will the maximum, thus what will happen then in that case t maximum, will be thickness to chord length ratio maximum will be again, we will see.

So, this thickness to chord length, and that will be e into 1 plus cos, cos pi by 3 that is 1 by 2 into sin by 2 root 3 by 2, and that will be into 3 by 2 3 into root 3 by 4, and this will be valid. This is the maximum, this, the maximum of this only, on this and this is what the same result, we had got in case of a symmetric aerofoil. Now, now again, we can see, suppose so this is what, we are looking for so we have got the reason for a cambered aerofoil and we also see that in a case of the cambered aerofoil also, we get the maximum volume and theta is pi by 3 and it is again minimum and theta is pi, another thing what will happen if there is.

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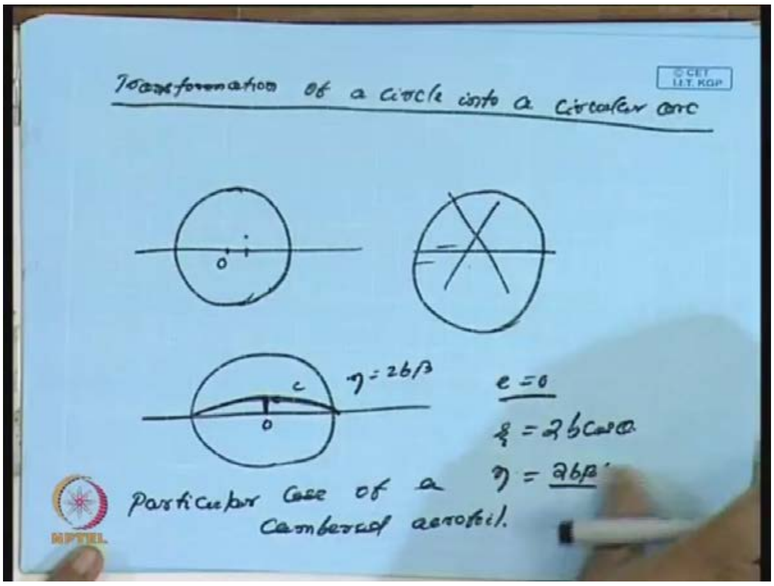

The next transformation, I will look into of a circle into a circular arc, let me say what I am looking for this, in the previous two cases, we have seen that, we have shift the center of the circle, we have origin is . We have always shifted the center to the first initially to the right, and then initially there is a vertical shift but what will happen there is no vertical no horizontal shift, there is no horizontal shift so this is the rather I will say. So, suppose there is no horizontal shift the center is shift here, only vertical shift, this is origin, then what will happen. In that case if, we will go back to the previous formula, then from there, we can easily see, that there is no vertical shift. So, e will be 0, and once e is 0, then what will happen to my eta e is 0. So, my xi will be 2 b cos theta whereas, on the other hand my eta will be 2 b beta sin by theta 2 b beta.

So, that will be 2 b beta, because theta will be the same as any point, there is nowhere have shift, so in that case what will happen, my I will get foil like this, whereas, this distance is to be 2 b beta maximum it will here, we taken b maximum 2 b beta. So, in this case here only shift is in the for vertical shift, and that vertical shift is nothing but 2 b beta, where there is nothing only horizontal side, so this is a, so from again I am just taking this result, from the cambered aerofoil as a particular case, as a particular case of a cambered.

So, what, we have understood by now, we have seen that by using the Joukowsky transformation, from m using a from a circle, we can always come back to a cambered aerofoil a symmetric aerofoil or a circular arc, arc. And now, let us see how this because, we have seen that in case of a, in case of a aerofoil, we have a trailing edge, we have a leading edge, what happen at the trailing edge, particularly when a z is equal to minus b.

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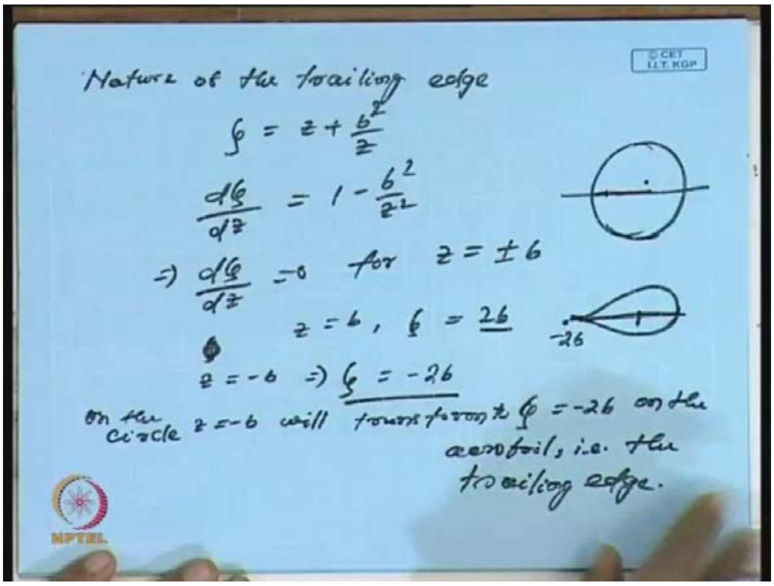

So, the nature let us look at the nature, of the trailing edge of the nature, of the trailing edge in this case, we have seen always zeta is equal to z plus b square by z. Then what will happen to d theta by d z, this should be 1 minus b square by z square. So, which implies d theta by d z will be 0 for z is equal to plus minus b, so z is equal to plus minus b. Now, what will happen to the point z is zeta is equal to, whether now what will happen when z is equal to b, if z is equal to b my zeta will be z is b means beta is 2 b z is equal to b beta is 2 b, and z is equal to b is a point, because of the circle it is the point that look at my circle the point is z is equal to b.

Because I have taken this point as center is somewhere, c is somewhere here, and the this distance total distance is 2 b b plus b e, so always the point b will be somewhere inside particularly. Once if you look at the aerofoil, because this distance total distance is 4 b, and somewhere 0 is here, so it is minus 2 b 2 2 b, so my b will be a point inside. But what will happen, if z is equal to minus b, if z is minus b which implies zeta is equal to minus 2 b, so minus 2 b as a point here, so this point minus 2 b. So, that means the trailing edge, when z is equal to b zeta is minus 2 b, and in that situation, so this will be the point z is equal to minus b will transform. When the circle will transform to zeta is equal to minus 2 b, on the aerofoil rather on the say on the circle z is equal to b, will transform to zeta is equal to minus 2 b on the aerofoil. That is the trailing edge, and now I will look at how this will behave, we have the trailing edge, now let us see, we have already seen.

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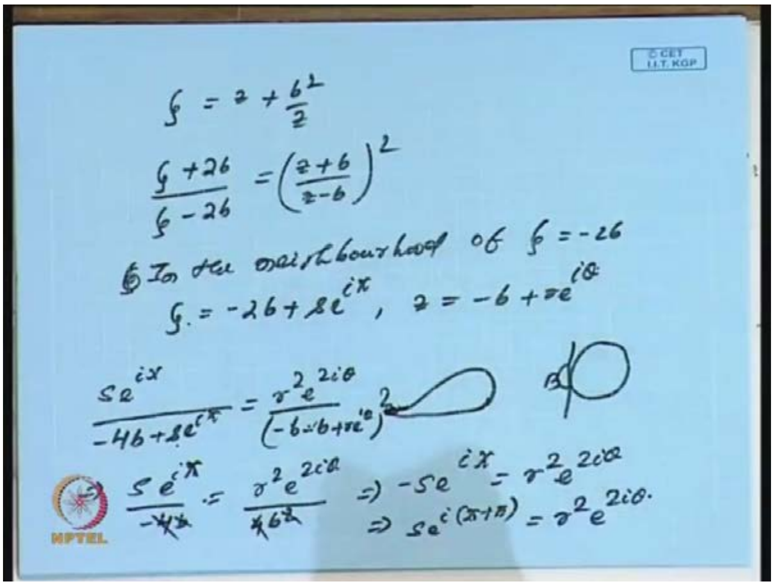

We have zeta is equal to z plus b square by z, it can be easily seen that, z plus 2 b by zeta plus 2 b by zeta minus 2 b, it will be can be easily written as, if for zeta is z plus b square by z, you will see that it will be z plus b by z minus b whole square, so the point. So, this itself, so that now what I will say, let me take any neighboring point of zeta, zeta in the neighborhood have zeta is minus 2 b, if I say zeta is minus 2 b plus s e to the power i chi and my z is minus b plus r e to the power i theta on the circle that means I am considering a circle, this is a point b.

And then I consider a aerofoil, and I consider this is the trailing edge, corresponding point here, then if I substitute for this zeta and z in this expression, then I will get s e to the power i chi by minus 4 b minus 4 b plus s e into i chi equal to this is z z is equal to minus b minus, it this will be r square e to the power twice i theta divided by that is a z is minus b minus, b into b minus b rather minus 2 b z is minus b, minus b minus b then minus minus z minus z is minus b plus plus r e to the power i theta.

Now since r and it r and s are small, so which I can write it as s e to the power i chi by minus 4, because this is a small quantity compared to this, so this can be neglected by 4 b. It will be r square e to the power 2 i theta, 2 i theta by the because this will be, this is square, so this will be by 4 b square for this square so 4 4 get cancel b b get cancel which implies s e to the power i chi is equal to minus s e to the power i chi is equal to r square e to the power 2 i theta, which also I can write it as s e to the power i times pi plus pi is equal to r square e to the power 2 i theta. So, that means if point at this edge, if there is a shift here, there is an in this, in theta then a small increases here, that means here, it would be pi, so what will, what it will show me that will show me that.

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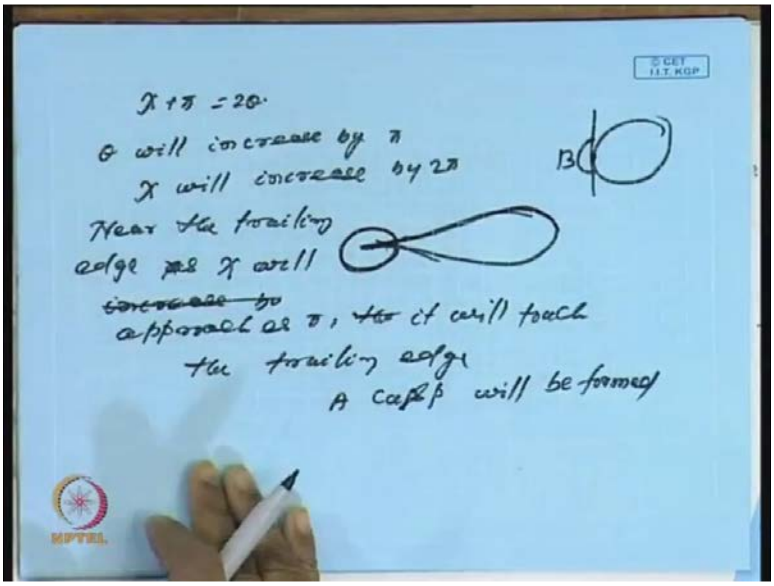

As if my chi plus pi if I look at angle chi plus pi is 2 theta, and that shows if I move around the point b, if theta increases by pi, theta will increase by pi. Then what will happen? chi will increase, theta is increases from 0 to pi, that will be 2 pi and that means, and then chi will increase by pi increase by pi. And again another thing is that but I have two edges of the, I have two edges of the airfoil, there are 2 foils. And then when again what will happen near the trailing edge, as chi will increase by pi, as the angle pi will increase to pi, increase by increase rather will tend to pi approaches pi. Then the, it will touch trailing edge, that means, this angular length only this here, and here. So, it will form a cos, cos will be form, we have 2 surface; we have 2 surface here, on their meeting at this point. And this is what, now with this understanding, I will say now I will go to the another aspects and what, we call the, because we understand that here, we have understood that in a circle at the point b. Then angle is reaching there is a theta increase, theta is increasing by pi, then this same angle here is increasing by a chi, will increase by to pi. Because this is the 2 pi, now, now with this understanding, let us go back to the Joukowsky hypothesis.

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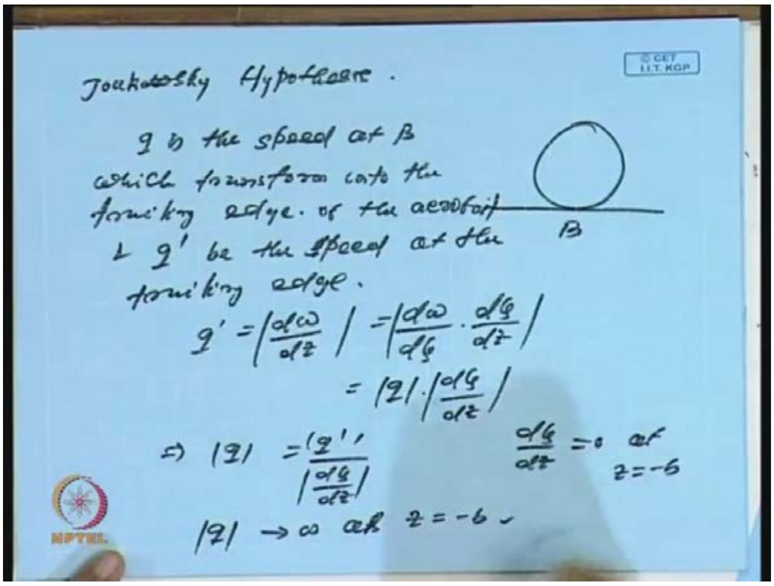

This is very simple you understand the Joukowsky hypothesis I will do one thing, let me say that q is the speed, suppose look at this at b is a point here. Let q is the speed, speed at b which transform, which transform to the trailing edge into the trailing edge of the aerofoil, and q prime. Let q prime be the corresponding speed is the speed at the trailing edge. Then what will happen to q prime, q prime is d w by d z, and that is nothing but d w by d theta into d theta by d z, on this modules and this will give us d w by d z is q bar into d zeta by d z and d zeta by d z, we have seen this.

We have seen that, so which can also be written, so which implies my q will be q prime by d zeta by d z, and this d zeta by d z, we have seen d zeta by d z is 0 at z is equal to minus b. So, that means q will tend to infinity at at z is equal to minus b, because I have d zeta by d z is 0 or z is minus b. So, this will be q will tend to zero infinity as so this is important, so that means now q is the speed at z is equal to minus b. So, that what will happen in the process, the corresponding point because, the point the z is equal to minus minus b, will corresponding to the trailing edge. So, we have a the speed will be infinity the, so but.

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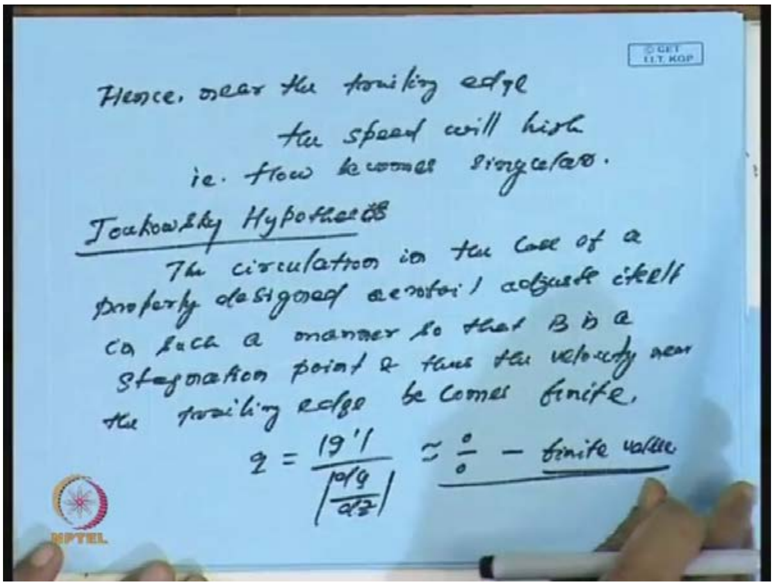

So, thus hence near the trailing edge near the trailing edge, because speed will be large, speed will be extremely high, will be singular, will be high, that is the flow is singular flow becomes singular. So, but in reality if this is of the trailing edge, we should have a flows would be regular, so to avoid this to make the flow regular but Joukowsky propose that, that then comes the Joukowsky hypothesis. The Joukowsky hypothesis says, this hypothesis, the circulation in the case of properly designed aerofoil, adjust itself in such a manner. So, that b is a stagnation point and thus the velocity near the trailing edge becomes becomes finite in fact this is obvious because, we if, we have, we have already seen that our q is q prime modulus by d zeta by d z and this will be 0. So, these has to be fab of 0 by 0, and then only it can be to a finite value, it has to be finite value, if this has to be 0 by 0, in determine form. And so for that q prime always q prime is zero, we cannot, we cannot achieve this.

So, for this, this is what Joukowsky, so this is what Joukowsky hypothesis, says that somehow, we have to introduce a circle circulation, in case of properly aerofoil in adjust itself that means, we have to introduce a circulation in the flow, so that in such a manner, so that b is a stagnation point. So, the point b what I have described that will behave like a stagnation point, and once it is a stagnation point of flows it will be 0 here, once the flow, flows it will be 0. Then this would be d zeta by d z will be 0, so that will be give 0 by 0 which in limiting case, then in the limiting case, this will give us a finite value this is Joukowsky hypothesis.

Now with this understanding now, we have understood that in case of a aerofoil, near the trailing edge, will have a singularity for singularity, and to avoid the flow singularity a the near the trailing edge the circulation has to introduce in such a manner. So, that the near the trailing edge, now flow there will be a stagnation point will be develop near the, near the point, and that will need to the velocity near the trailing edge, which will need to the velocity near the trailing edge to be finite, and that is what Joukowsky proposed in a hypothesis. So, this is what I want to talk in this lecture because, we have already seen that how, that behaviour of the trailing edge, now another major result in this case of aerofoil, is the theorem of Kutta Joukowsky.

And once, we know the Kutta Joukowsky theorem, then that will give us because, we have already been exposed to that in case of a cambered aerofoil, we have, we have a lift will be there, because in the analytical case of a symmetric aerofoil. In the case of a cambered aerofoil, the two sides of the aerofoil, where is pressure difference and that will lead to, that there will be a pressure difference, and that pressure difference gives us, because on the upper side there will be less pressure on the lower side. There will be more pressure and in the process, there will be a a lift, that will be the main reason, for the development have lift in the aerofoil, in case of a cambered aerofoil. And this lift and movement as I have yesterdays like, earlier lecture I have told that the lift can be and movement can be calculated by using this blasius theorem.

And on the other hand if, we will go back to the drag because, we have seen that in case of a circular cylinder, we have seen that d'alembert's paradox says that, there is a no drag force in case of a cylinder. Since, we have transferring a circle to aerofoil so we will not able to get a correct picture of drag force, in the case of a cylinder. In this case, in the case of aerofoil, however the lift force, will get almost, will get almost accurate relation of lift force here. And in the next lecture, I will tell you, about the Kutta Joukowsky theorem, which calculate lift and the moment, that is acting on the aerofoil. With this, we will stop here today.