Marine Hydrodynamics

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Lecture - 14
Aerofoil Theory

Welcome you to the today's lecture on marine hydrodynamics, today we will emphasize on some of the application, as one of the application of the confirming mapping that is the the Joukowski transformation is hydrofoil section. So, in the last two classes, we know that by using Joukowski transformation, we can map each point on a circle to each point on a ellipse and also by using the corresponding inversion formula, we can map each point outside a circle to each point outside an ellipse.

Further, we have seen how flow past circular cylinder using the same transformation the flow past circular cylinder knowing the results of the flow past circular cylinder we can also know the results associated with the flow pastan elliptic cylinder. Further, we have seen that from the elliptic cylinder, flow pastan elliptic cylinder, we can easily get the flow past plate, so, this is… So, this is the beauty of the Joukowski transformation. Today we will look into more complex situation, where we can still apply the joukowski transformation to aerofoil sections.

And in this because the the cambered aerofoil, so called aerofoil section is more complex in nature. So, but it is quite interesting to see that by through suitable transformation, again we are able to of the suitable arrangement of the Joukowski transformation from a circle, we can map s to n aerofoil section. We will consider two cases, today we will emphasize on a symmetric aerofoil section, and next we will discuss about the cambered aerofoil. So, this let us see what exactly, I mean a aerofoil.

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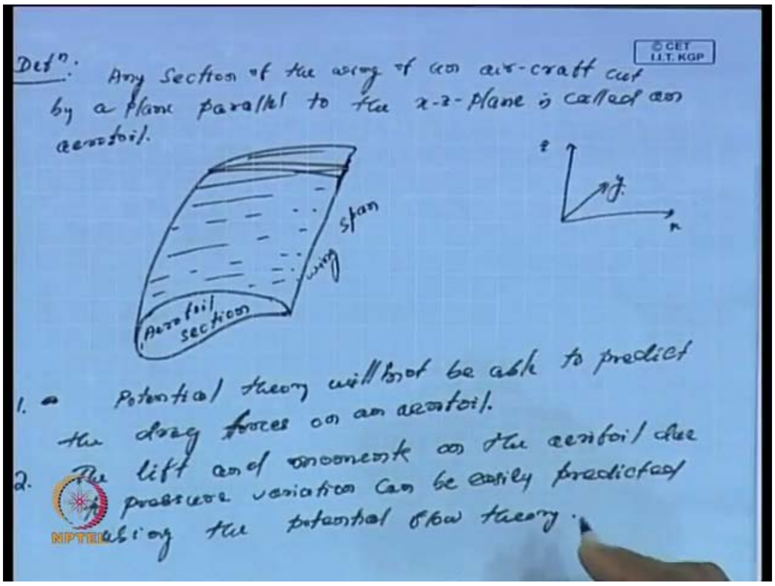

So, while think of the definition, we will concentrate what is an aerofoil section, it is a section, any section of the wing of the wing of the aircraft any section of the wing of an of an aircraft cut by plane parallel to the x z plane is called an aerofoil section. So, this is a section, I just say, if what is how you describe the x z plane, this is my x z plane and this is the y direction.

So, what I want to say that so, this is a wing span and this is the aerofoil section. So, basically this is what another, now what we are looking here, here the this aerofoil section, if we know that it gives us an idea about how, we can design the wings. And one of the advantage is that some of the points, I will mention here, that potential theory basically, in potential theory, we assume that the fluid is irrotational in nature.

This potential theory will not able to predict potential theory will not predict the drag forces potential theory sorry, I will not be able to predict the the drag forces on an aerofoil. And this is however, this is because of the region that, we have seen that this is well defined by the Dalembert's paradox, because that is it will not give you exact the potential theory where, a body some must in a fluid. So, it will be gives a contradictory result. So, this potential theory will not be able to predict the drag forces on the aerofoil.

So, to overcome this idea until develop the boundary layer theory and the details will talk later about the boundary layer theory, but. So, that the we can always calculate the drag forces there, we have to take into account viscosity wise on the other hand. The lift and the moments the lift and the moments on the aero foil due to the pressure variation can be easily computed, can be easily predicted, using the viscous in viscid flow, potential flow sounds under in the potential flow theory.

Here, I would like to highlights few more things, let me just say what are the section is when aero will be because, this is the aerofoil let me say what are the various section of this.

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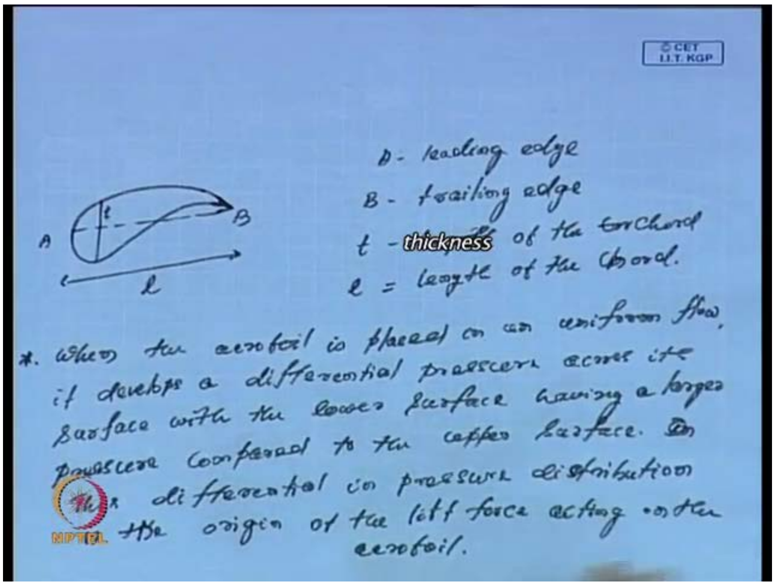

So, if I just consider this, if this is a combined aerofoil, if this is A, this is the B, this thickness that is called t. So, it call A as the leading edge, B as the trailing edge, these are some of the common terminology and t is the length of the chord and then we have l length of the chord. Now this is a some of the common terminology of this section aerofoil section.

Now, when a when the aerofoil is placed in an uniform flow, basically some more points when the aerofoil is placed in an uniform flow, it develops a differential pressure develop a differential pressure across its surfaces with the lower surface having a larger pressure having a larger pressure compared to the upper surface. In fact these In fact these differential in pressure this differential in pressure distribution is the perhaps the is the origin of the lift force acting on this aerofoil.

So, we have less more pressure here, less pressure here. So, automatically it will be act as a lift force, because on the above side, there is less pressure on the lower side, there is more pressure. So, it will try to lift the surface that is what so, this background about it will background about the aerofoil.

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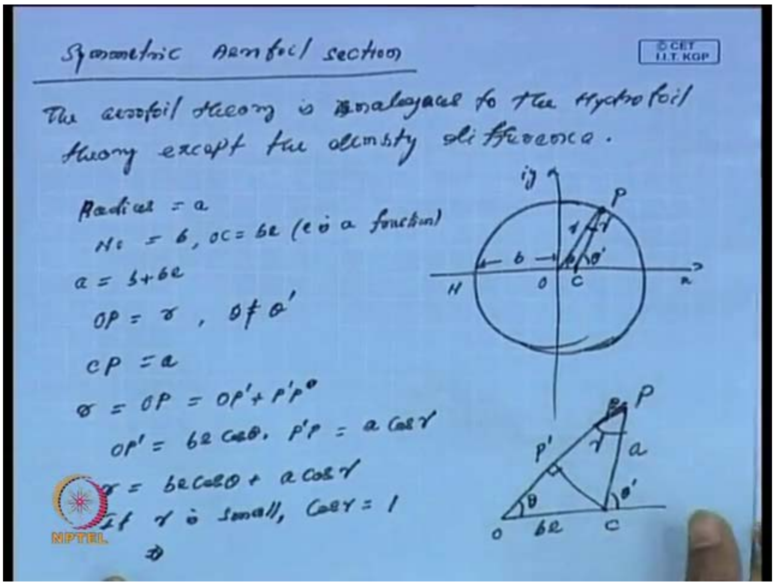

Today, we will discuss about the classical aerofoil section has truly concentrate on the symmetric aerofoil section see how we can construct aerofoil section using symmetric aerofoil section. Before going to this, because one may say that why we are talking about aerofoil theory here, because in this course of hydrodynamics, because there is analog, in fact of the aerofoil theory is very much analogous to the hydrofoil theory expect a density difference.

That is one of the reason, we try it over understand the aerofoil theory aerofoil section well then, we can easily apply the same concept to the hydrofoil, that is why the aerofoil theory will discuss here.

Now, with this, now let us see how we can apply the transformation, how can symmetric aerofoil section, how can we get by using a suitable transformation from a circle. Now, let us do one thing here, let us consider here, consider large, consider a circle and in this case, because we have seen from a circle, we can get an ellipse. So, what we will do here, I will consider as a this is the axis, x axis and this is the z plane, this is my i y and then suppose what, I will do let P bearing point on this circle. And O is the origin and let let me say c that is the center of the circle.

So, the there is a what there is a horizontal shift from the origin to the right side, now if I join this point and join this point. Now what is this distance O C, let me say this is P, this is N.

Then what will happen because, we have o p, that is the radius of the circle, let the radius of the circle a. So, if see the it is the radius of the circle is a then center is at C say radius is a then, we have let me call this distance N O as b and O C as b e is fractional shift e is a fractional, we call it a fractional shift b e.

Now, N o a b then, we have I can always say a is equal to b plus b e, we have a is equal to b plus b e. So, which clear that circle will pass through this point n, now if P is the point on the boundary of the circle then, we have let me say this angle is theta this angle is theta. So, on this is r.

So, we have O P is r, now what we will do let me recast the whole thing and let me put a and we take the whole thing here, we have C here, just a this is P, this is C, this is the point P sorry. This is the point P and then let me interact perpendicular from this to this and then I call this as P prime then, we have O C is b, let me say this angle as gamma.

So, this angle is gamma now this angle is theta then, we have let me say expand this. So, let this angle be theta prime that means, this is angle theta prime, we have theta is not equal to theta prime as then further, we have and let me say that since a is the radius of the circle and this O P C P is a. So, we have C P is a when C P is a then, we have r is so, we have r is O P, which is nothing, but O P prime plus P prime P double prime, it is this plus this is in the total r. So, then we have what is your P prime, if I look at that O P prime. So, this is ah this is a or we have O P prime this angle is gamma.

No sorry, if I say O P prime will be let me say this is b e, this angle is theta, this is a perpendicular. So, this is b means with an O P prime, we have then I call it as b e cos theta then further P prime P, P prime P P prime P is a cos r. So, in the process, we have r is equal to O P prime plus P prime P that is b e cos theta plus a cos gamma then now since the shift is very small. So, this angle gamma can be considered as very small and if gamma is small then cos gamma is equal to 1 and which implies which implies once this is small.

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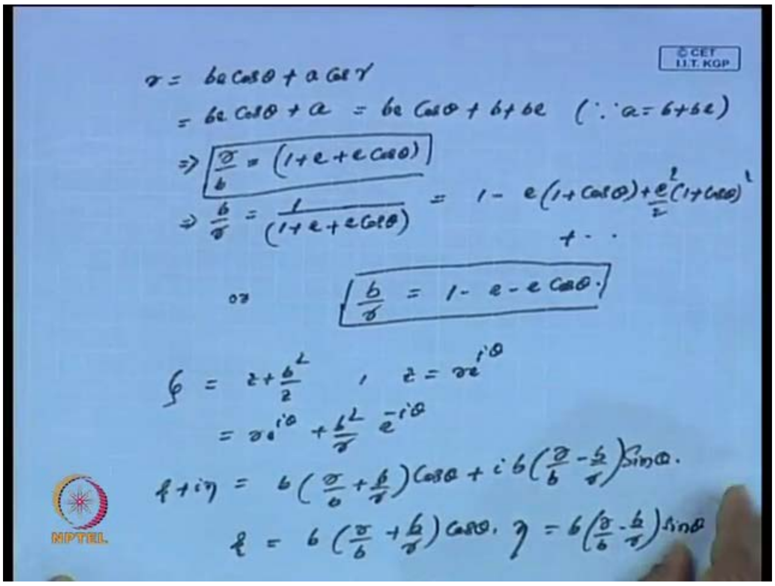

And from this we can say r is equal to r is equal to b e cos theta plus e cos gamma a cos gamma, which equal to b e cos theta plus a because cos gamma is small. Then what will happen to and again a is equal, which I can write it as b e cos theta, we have already seen our a is equal to b plus b a is equal to b plus b. So, which implies my r by b r by b is 1 plus e plus e cos theta, which implies b by r is equal to 1 by 1 plus e plus e cos theta 1, which is equal to 1 minus, if you expand it in a by number expense. if I do 1 minus e plus 1 minus e into 1 plus cos theta plus e square by 2 cos theta square plus air power.

So, which I can always write, because e is a small is a fraction so, e square will be negligibly small. So, this will give me 1 minus e minus e cos theta. So, here the ratio r by b is 1 plus e cos theta, that is 1 relation and another relation is also r b by r and again b by r is 1 minus e. Now, if this understands about r by b and b by r, I will proceed a little further.

Now, we will go back to the Joukowski transformation that means, in the zeta plane, if zeta is equal to z plus b square by 8. So, writing z is equal to r e to the power i theta, we have seen that, we can always write it, z is r e to the power i theta plus b square by r and minus i theta, which implies to because, this is b into r by b plus b by r into cos theta plus i times b into r by b minus b by r sin theta and which implies.

So, zeta is nothing, but psi plus i eta and that gives us i is b times r by b plus b by r cos theta and theta is b times r by b minus b by r into sin theta. Now, now what we will do, we will substitute for r by b and b by r.

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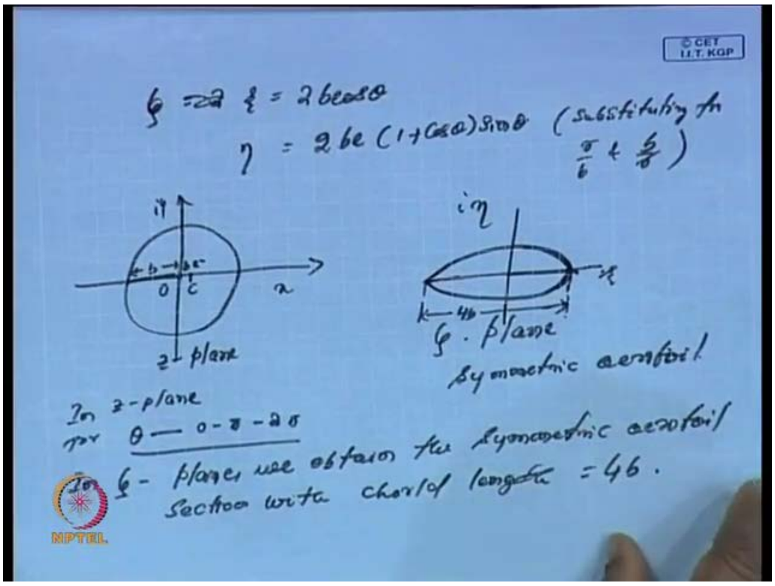

And that will give us that will give us zeta is equal to rather, I will say because, we have already phi is equal to 2 b cos theta and eta is equal to 2 b e into 1 plus cos theta into sin theta, just substituting for r by b substituting for r by b and b by r.

So, now what is happening here, we have in the zeta plane, we have initially, we have a circle and here, the center of the circle was the origin is something different than the center of the circle, that is about horizontal shift. And if this distance was this is the distance, which was b and this distance as b e and this what, we have done from this it is in the z plane. Now in the zeta plane what, we are getting, this is our i y axis this is x axis.

Now, in these plane, this is zeta plane, this is eta i eta and if I look at this xi is 2 b cos theta and then, we can easily see that this will give us. So, this is water plate in the xi eta plane, this looks and I say this as a symmetric aerofoil when the symmetric aerofoil what is total length. The length will be and again here, if is a theta varies from 0 to pi 2 2 pi then, we get in the in z plane theta varies from then, we in zeta plane, we obtain the symmetrical aerofoil section.

So, it can be seen that this is 2 b and then because, it will vary from when phi is equal to theta is equal to 0. So, this will be 2 b and if theta is equal to pi, it will be minus P P. So, the total length of this. So, with chord length equal to 4 b. So, this distance is 4 b, the total is 4 b, now what will happen to the maximum thickness, if I would know what is the maximum thickness, because let me do one thing.

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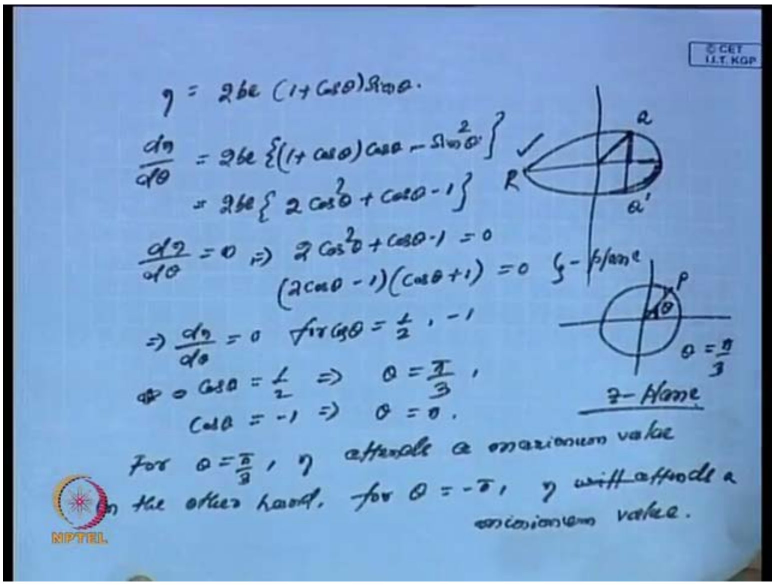

Again I put it sorry, let P b e that b e any point here, Q type of here, let extend it here, Q Q prime. Now, what will happen my eta is given by that the symmetric my eta is given by 2 b e into 1 plus cos theta into sin theta and then what will happen d eta by d theta. If, I have d eta by d theta then that will give us 2 b e into 1 plus cos theta into cos theta and minus sin square theta and that is nothing but, 2 b e that is 1 plus cos square theta minus sin square theta, so that I can always write 2 cos square theta 2 cos square theta plus cos theta plus 1.

And if d eta by d theta 0, that will give in the point at, which eta will attend the maximum value of theta for, which eta will be maximum. So, d eta by d theta is 0 gives me implies 2 cos square theta plus cos theta minus 1 is equal to 0 and if I make it as a product of 2 terms, I can always write as this 2 cos theta minus 1 to cos theta plus 1.

And that will give me 2 cos square theta plus 2 cos theta minus cos theta cos theta. So, minus 1, so this is 0. So, which implies d eta by d theta is 0, for theta is equal to half or minus 1, now what will happen, if theta is equal to cos theta is equal to half or minus 1.

Now, cos theta is equal to half gives implies theta is equal to pi by 3 and if this is the case, we can always see that, further cos theta is equal to minus 1, cos theta is minus 1 means theta is equal to pi. Now, it can be easily seen it, it can easily seen for theta is equal to pi by 2 pi by 3 eta attends some maximum attends a maximum value, on the other hand for theta is equal to minus pi eta will be attends a minimum.

So, we have seen that, if theta is so, basically eta will be 0 theta is minus pi, because this quantity will give so, eta is 0, so this point will be here, this is the point, that point I call it aerofoil, this is r suppose and that is the trailing s. So, theta is minus pi, this will lead to this point r and if theta is equal to pi by 3. So, if I say that some angle here.

So, in the in the circular type z plane, if theta is equal to this angle theta, this is the point P for theta is equal to pi by 3, for this angle theta is equal to pi by 3 in the zeta plane. So, the z plane. So, in the z plane theta is pi by 3 corresponds to maximum eta and since, if the symmetric. So, the total thickness will be 2 eta means, theta is pi by 3 then what is the maximum value of eta this point.

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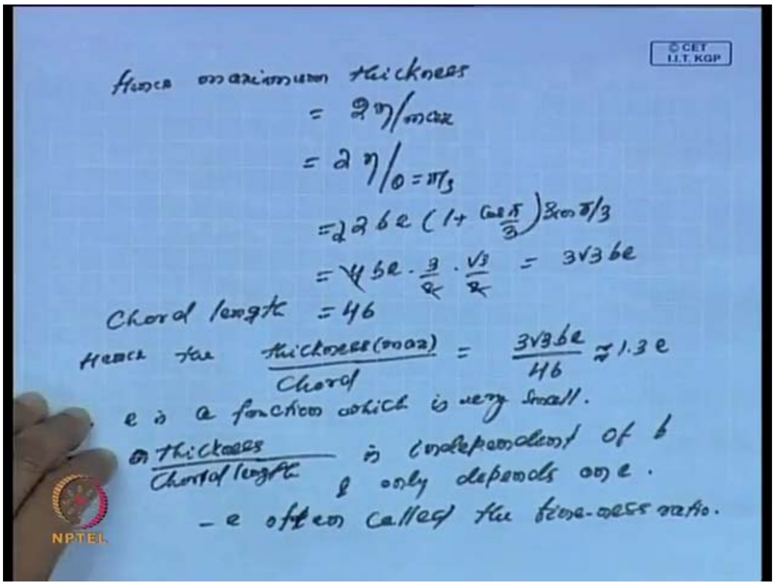

So, the maximum eta will give me, hence maximum thickness will b e 2 times eta max that means, 2 eta at theta is equal to pi by 3 and that will give us 2 times 2 b into 2 2 2 b e 1 plus cos theta 1 plus cos pi by 3 into sin pi by 3 and that will give 4 b e into this is pi by 3, 1 by 2 plus 1 is into 3 by 2 and this is pi by 3 is equal to 3 by 2. So, this is 2 2 4 cancels. So, this is 3 into root 3 into b e so, this is the maximum thickness.

Now, come here to the total chord length. So, what is the chord length, we have the chord length, that is 4 b, hence thickness to chord ratio, hence the thickness to thickness to chord maximum thickness rather maximum thickness to chord ratio and that will give us 3 into root 3 b e by b 4 b and that will give us almost 1.3 into e.

So, this is only and this e, now e is a fraction is a fraction, which is very small very small further, it may be pointed out that the maximum thickness, a thickness chord length ratio thickness to chord length ratio. This ratio is independent of is only depends on of b and only depends on e often, we call this e, e often called the fineness ratio called the fineness ratio. So, this is what, we have seen today, that by using this Joukowski transformation, we are able to transform from a circular cylinder.

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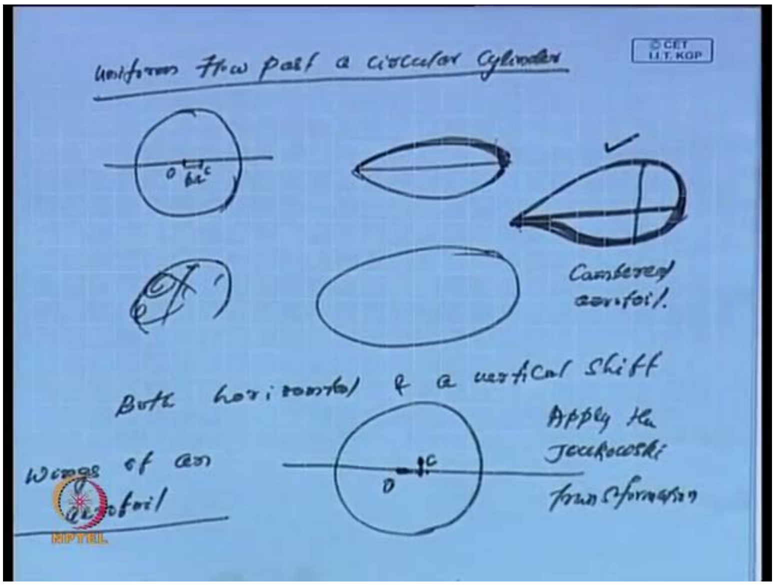

From a circle, we are able to map a circle to an symmetric aerofoil section sorry, now question comes and there, what we have done, we have done it, that we have shifted the origin, there is a horizontal shift that distance is b e, this is origin this is a center of the circle.

Now, question comes by a horizontal shift, you are getting a symmetric aerofoil section and just from a by using the Joukowski transformation. And earlier, we have seen by if we do not shift the origin then we are getting from a circle from the same circle sorry, we are able to earlier, we have seen that, we are able to get ellipse from a circle without going to the if you do not shift the origin. Now what will happen again, what will happen, if we have a vertical shift both horizontal and a vertical shift.

That means, I am looking for, if I say I have circle and this is the origin origin here and have not only vertical shift what I have horizontal shift not only horizontal shift, but have a vertical shift, if my C becomes this.

Then with this, if I apply by saying that O is the origin, C is the center of the circle where where in which case, we have a vertical shift and the horizontal shift with these understanding, if I apply the joukowski transformation, what will happen. It is obvious that from, this was symmetric, this symmetric characteristics will be last, because we are putting it a little of so, that means, my bottom side is appears to be.

So, there will be a shift in the bottom side and then horizontal part from the access from discover, upper side the distance the height will go up, but the lower side, it will be there will be. So, this kind of aerofoil section will call it as a cambered aerofoil, it will call it as cambered aerofoil. And this cambered aerofoil is the 1, because we have as I have mention that the pressure from the lower side will be higher than the pressure on the upper side and perhaps.

This is the part, we are looking that means, our we are looking for aerofoil section where, the, which is a cambered 1, not a symmetric 1 and which will be more appropriate because, we are looking for a surface where, the surface lowers differential pressure is on the upper side Pressure difference is less compared to that only lower side. And therefore, there is a uniform flow and that will give help us in the design of the aerofoil section.

And with this and here also, we will see that here, the the leading edge does not have, if the trailing is have a sharp, because the thickness of the trailing edge is becoming 0 whereas, the leading edge is not so. And it has a maximum thickness not at the end about some where in the right side of the zeta axis phi axis rather.

So, now then like, we have done, but will do, that we will compare this then, we knowing the flow past a circle, suppose I have say, I have a uniform flow, past a circle circular cylinder. So, knowing the uniform flow past a circular cylinder like, we have done it, we can, we have earlier, we related the uniform flow past a electric cylinder. So, in the same manner knowing, there is else for the uniform flow past a circular cylinder, we can, we will be able to calculate the uniform flow past, I aerofoil symmetric, aerofoil section as well as and this as well as cambered aerofoil section.

And once, we are able to know the complex velocity potential associated with the flow of a cambered aerofoil then easily, we can easily find out. How will be able to calculate the forces lift, force drag force, that is acting on the aerofoil and since I have already mention that drag force will not be the most, it will not be predicted properly by the potential flow theory.

So, we will be able to at least get a clear picture about what about the lift force and the moments that is acting on the aerofoil section. So, the details of which will be from the next few classes. However, one of the major understanding of this aerofoil section will give us a bit good understanding about, how to design the wing section.

So, wings upon a aerofoil and as I have mention, there is analogue between the aerofoil section, the hydrofoil section expect that, there is a change in the pressure sorry, change in density. Hence, we by knowing the flow past and electric cylinder, we can always calculate the flow past a hydrofoil section. And it will also help the designer to design a proper aerofoil section a hydrofoil section for more, which will which will be able to use almost all the hydrofoils.

Another point here is the hydrofoil section is that, this is this hydrofoil sections there unlike, the bluff bodies like a cylinder or ellipse. This hydrofoils sections, they are sometimes streamlined bodies. The hydrofoils are they are streamlined bodies the streamline body.

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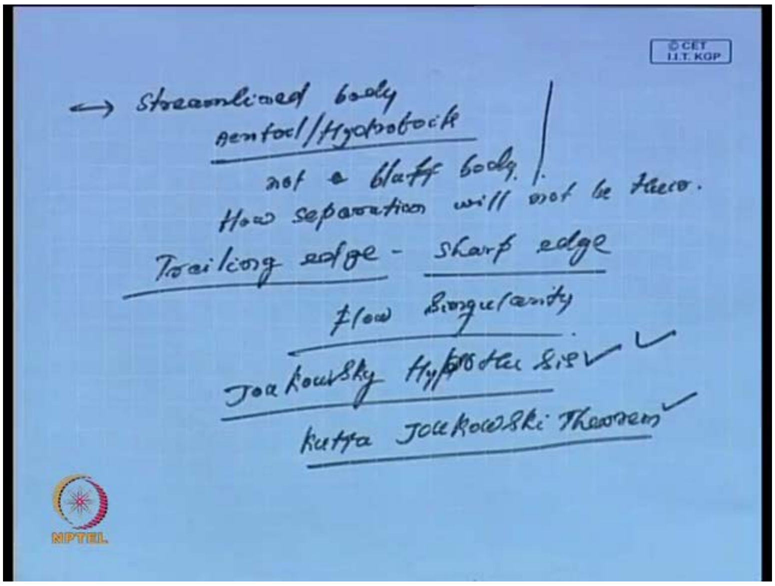

On the other hand not a bluff body, they are streamline body, but they are not like a bluff body not a bluff. So, this is another basic difference that so. In fact, it is because of this reason, because the structure is streamlined body and this hydrofoils are hydrofoils are streamlined body. So, one can easily, there will not be much of a flow separation flow separation will not be there.

But, one of the another question comes, what happen near the trailing edge, what will happen near the trailing edge, because it has a sharp edge. There is a question that there can be flow singularity that can be flow singularity and these will address in the next class once, we discuss in detail about the cambered aerofoil then will come to this to calculate. There is something called Joukowski hypothesis and then we will talk about Kutta Joukowski theorem.

So, Joukowski hypothesis will give us the idea, what will happen how the singularity in the flow will avoid near the trailing edge where, there is a flow singularity. And again when, we will go to Kutta Joukowski theorem by using this will be able to know what is the flow gap will be able to know what is the lift and the moment, lift force and moment, that is acting on the aerofoil and that, we will discuss in the next classes.

So, today we will stop here by assuming that by using the by we have already discussed that using the Joukowski transformation, we are able to get a symmetric aerofoil. However, we are looking for a cambered aerofoil and that we can do by applying the

Joukowski transformation to the on a elliptic ellipse with a suitable from a circular cylinder, we can get a cambered aerofoil by shifting the origin to the there is a little left or little right and then little in the vertical direction.

So, that we will discuss the next classes and then, we will come to the Joukowski hypothesis and Kutta Joukowski theorem, that is will give us a good understanding about the hydrofoil theory or. The first initially, the aerofoil theory and the same concept, we can use for the hydrofoil theory with this, we will stop today.

Thank you.