Marine Hydrodynamics

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Lecture - 13
Uniform Flow Past an Elliptic Cylinder

Welcome to this series of lecture in marine hydrodynamics. In the last class, we have talked about Joukowski transformation. We started with the conforming mapping. Joukowski transformation is 1 of the very important transformations, which plays a very significant role in analyzing several problems as I menti1d. Out of various problems, we will be doing 1 today will consist flow past an elliptic cylinder. How you all use the Joukowski transformation to study the flow past an elliptic cylinder from the topic pro past of in a circular cylinder? What to do that let us concentrate on what is the elliptic coordinate in system.

Once we clear the elliptic coordinate system, then it will be the helpful. This is because we have already known that why elliptic transformation point on the circular is can be relate point on the ellipse and again from a point outside a circular cylinder outside an elliptic cylinder ellipse. We can always relate to a point outside a circle. So, this concept will utilize in this lecture today to analyze the past an elliptic cylinder. Let us see how it works.

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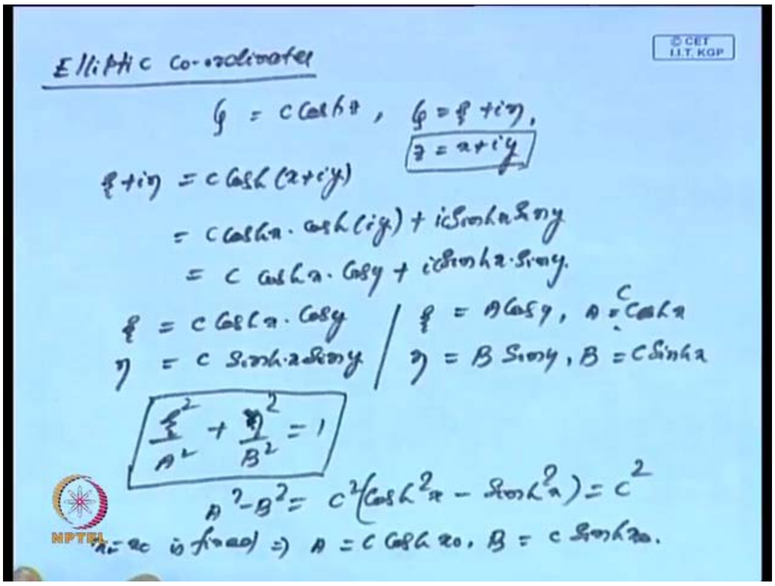

So, before going to that, let us think of another elliptic cord in a system. So, in this suppose that I say geta is equal to c cos hyperbolic z, where my z is x plus i y. So, then what will happen this if I substitute for this implies my geta is i closer i eta and that is A c into to cos hyperbolic z is x plus hyperbolic? If I simplify this that will give me c cos hyperbolic into cos hyperbolic i y plus i times sin hyperbolic is equal to sin hyperbolic x into sin y. This can be written as this.

I can put it as c cos hyperbolic x into cos y plus i sin hyperbolic x into sin y. If I write this, if I delete the real element parts, I will get my get a c cos hyperbolic x and the cos y and my eta will be c. There is A i c i c c sin hyperbolic x into sin y and coming itself. If I write, I will write i is equal to A cos, cos pie eta is equal to B sin y A B y c cos hyperbolic x. B will be this as c times and my hyperbolic x. So, from this, I can always get i square by A square plus i square by B square is equal to 1. This i square by A square is cos square by this i square by B square sin square. So, this together is 1.

So, that means these through transformation also and since I can get, that is any point on a circle. Either I will get the get a plan in points on ellipse. This coordinates system; I call this as the elliptic coordination system for a point on a circle. A square, this is eta square. It is not a z square. This is eta square. Now, what will happen when you look at here A square? A is A c hyperbolic c square cos hyperbolic square x minus sin hyperbolic square x. Then, the tan what symbol square? So, as usual, we have seen A square minus B square is c square. If A, B is the semi measure on minor axis, then we have seen because we have seen c c 0 minus c 0.

They are the focus of the ellipse. If I just say again if x naught is a fixed point, if x is equal to x naught fixed, if the point is fixed that means I can get A is equal to c cos hyperbolic x naught. B is equal to c sin hyperbolic x naught. So, they become fixed point and the constant instead of renewal, they become constant. Once A and c are, A and B are known, the c is known.

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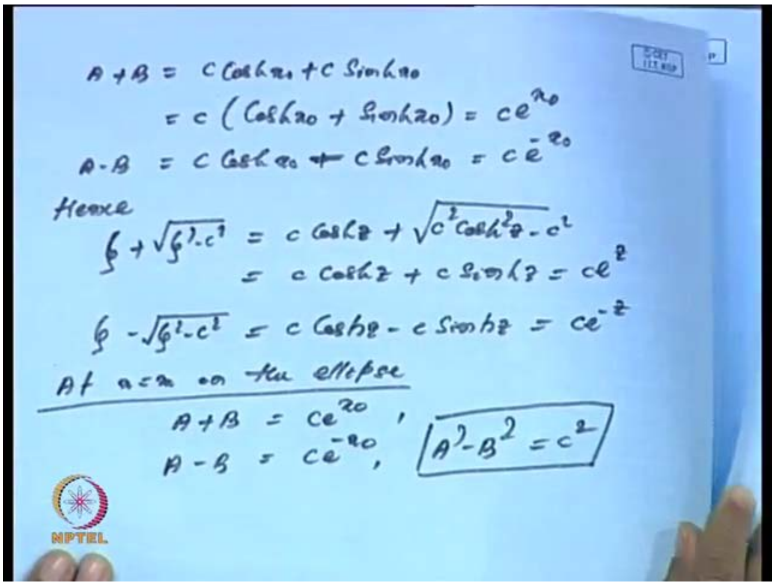

What will happen to A plus B? A plus B is c cos hyperbolic x naught plus c sin hyperbolic x naught. It gets c cos hyperbolic x naught plus sin hyperbolic x naught. That will give me that will be tamper x naught cos c 2 minus x naught by 2. Here, it will be able to c to the power of x, x naught in a similar manner. If I just look at A minus B, A minus B will be c cos hyperbolic x naught minus c sin hyperbolic x naught. Then, we can easily say that it becomes c e to the minus x naught. Hence, what will happen? It may be previous talking about elliptic coordinate. What will happen?

This i plus i square minus c square should give me because I have taken this i is equal to c cos hyperbolic z. So, this will be c cos hyperbolic z plus c square cos hyperbolic square z minus c square. That will give me c cos hyperbolic z plus this will be c sin hyperbolic z. From the previous x plus, I can always get c is to the power of z. Similarly, if I say i minus, this will again give, I can easily get c cos hyperbolic z minus c sin hyperbolic z. So, that will be m c. It is the minus z because it obvious forms here.

Hence, at x is equal to x naught, x naught on the ellipse will have what will happen to A plus B? We have already seen A plus B is c e to the power of x naught. A minus B is equal to c e to the minus x naught. Now, we have also seen this is 1 thing. Also, we have seen A square minus B square is equal to c square. These are the lessons. These are the things, which will be very helpful for our cylinder, especially when we will learn about the elliptic cylinder. With this understanding in the elliptic coordinate system, we will go to the very past and elliptic cylinder, past elliptic cylinder.

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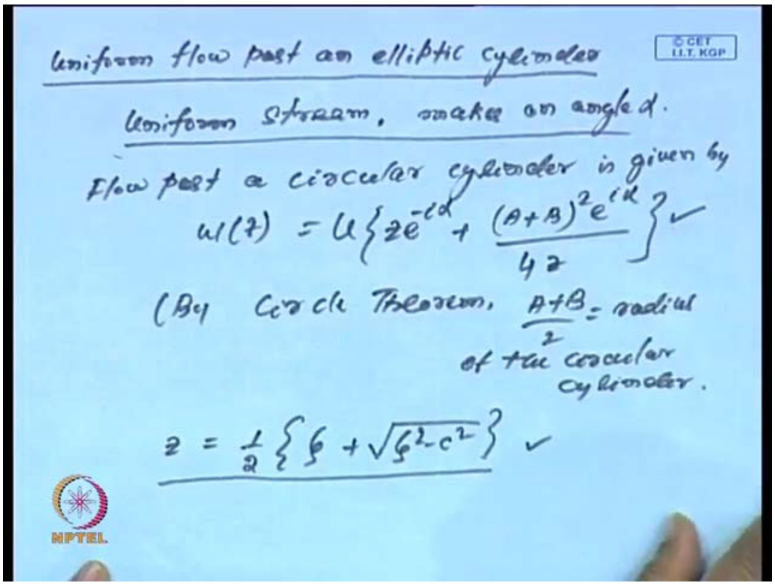

Now, we will discuss about the uniform flow past, a uniform flow past an elliptic cylinder. So, to do that, 1 of us knows that if I have a uniform flow, if I have a uniform stream, it makes an angle alpha. Uniform stream makes an angular alpha with x axis. Then, the uniform flow past a circular and the flow past a circular cylinder is given by w z is equal to u z e to minus i alpha plus A plus B. This is because A plus B in this concept of elliptic cylinder; I am considering A plus B by 2 is the radius of the cylinder by alpha by 4 z.

This becomes u z to e to i alpha plus u, u A square by z. This is by circle theorem. By circle theorem, it is A plus B by 2 to the radius of circle to radius of the circle, in this, the radius of circular cylinder. So, w z is a flow past a circular cylinder. Now, we know that the transmit ion geta, if I say z is equal to 1 by 2 geta plus geta square minus c square, then these transformation. In the last class, we have seen that this transformation transform re elect in a point outside a ellipse to in a point outside a circle. Now, if I substitute for z in these terms geta, then what will happen by substitute z in these? This z if I substitute here, then what will happen? I can get w geta w geta.

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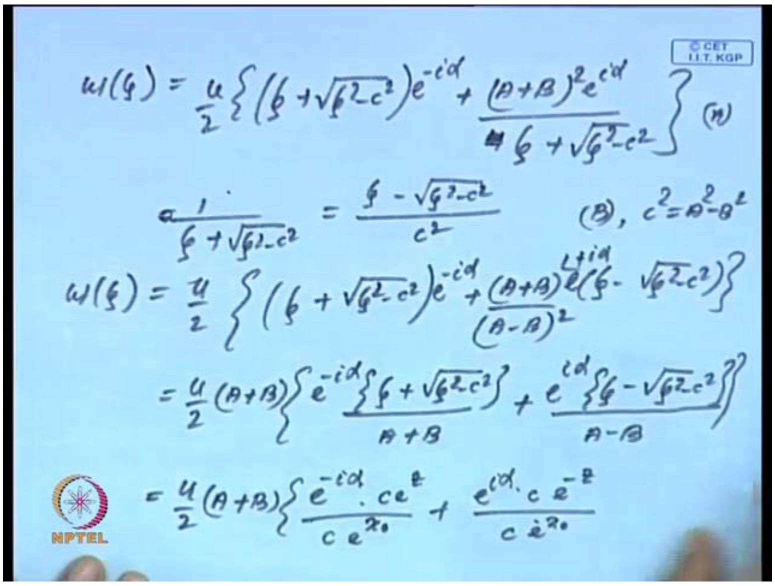

It will be giving me u that is u by 2 because there is A. It will give me geta plus geta square minus c square e to the power of minus i alpha than A plus B square. That is i alpha divided by 4. Already, we have taken this. It will be A by 2. We have taken. So, it will give us A 4. There is a 4. Here, it will get canceled that is geta plus minus c square. So, this will give a space. Now, we have seen that if I just relate 2, some of the previous results that I will write some examples geta plus geta square minus c square. Then, I can get already d1 this algebra.

This will give me before that. What I will do? What will happen to 1 by geta plus geta square minus c square? This I can always write that geta minus geta square minus c square geta square minus geta square minus c square by c square by substituting this value. If I say this is A, this is B, A substitution for B of this value in the lesson from B, A, then what will happen to my w geta?

w geta will give me u by 2. This will give me and we all know that again c square is nothing but A square minus B square. Substitute it. So, here we will get geta plus geta square and c square that e to the power of minus i alpha plus A plus B square by A minus B square. This is because this is by c square. So, this will give me geta minus geta square minus A square. So, this gives me u by 2 into, let us say A plus B combined, I will have e to the power of minus i alpha geta plus geta square minus c square divided by A plus B. It will give me A plus B minus rather plus.

There is A e to the power of minus i alpha will be there plus alpha will be there. So, that will give us A plus B. So, c square is because c square is A plus B square. So, this is A, B to the power of i alpha into geta. This will give me minus geta square minus c square divided by A minus B s because we have taken A plus B combine. So, this will give me alpha plus A alpha alpha plus A plus B whole square this plus minus.

We will see that. So, this will give me u by 2. Here, I have A plus B square A minus B square that is by alpha. This is why, I get w geta. This will give us that. We have seen A plus B c to the power of plus x naught. So, we will see that this is A plus B to e to minus i alpha c square x by c square. So, let us c e to the power of minus i alpha c e to the power of z. I can write it divided by c into e to the power x naught because A plus B, A plus B c to the power of x naught and geta into geta square minus c square c times c to the power of c z. So, this is plus. If I put e to the power of i alpha, this is again c into the minus z by c e to the power of x naught minus x naught. So, there is, I will see that total. I will make a correction.

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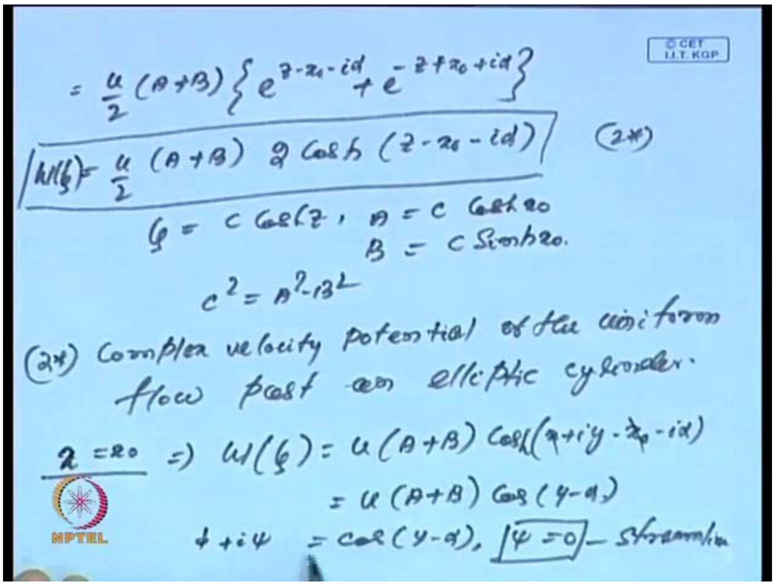

This is a little complex calculation, u by 2 into plus B. so, we have e to the power, we have already c. c that cancels will get it by z minus x naught minus i alpha plus e to the power minus z plus x naught plus i alpha. This is we can always write it as u by 2 into A plus B into this 1 e minus 2 cos hyperbolic z minus x naught z minus x naught z minus x naught minus i. So, this is the z minus. This will become minus. So, what does it present?

This is by w geta. Here, my geta is c cos hyperbolic z. A is equal to c cos hyperbolic is n naught B is equal to c sin hyperbolic x naught. Again, we have seen that c square is a square minus B square. This is what represents the complex velocity potential for the flow. Thus, we stared with what we have done. We had the circle theorem. Now, we substitute for z in terms of a geta because each points in a circle, ellipse outside the circle naught outside the ellipse relate to 1 point outside the circle.

In the process, we got w geta in this form. This form is the complex velocity potential for the flow 2 star gives the complex velocity potential at the uniform flow past an elliptic cylinder. Now, what will happen if z is equal to x naught? If I say z is equal to x naught than w geta will be u A plus B, which will be cos rather I will say if I put x is equal to x naught.

So, this will give me cos z is x plus i y x plus i y minus x naught minus i alpha. That will be u A plus B because x naught, x naught will get cancelled. So, this is called hyperbolic. This will be cos y minus alpha. So, this represents, if this I say that means w eta is phi plus i psi that is cos y minus alpha that means phi equal to 0. This will give me a streamline. That gives me a streamline psi equal to 0 that equals to streamline.

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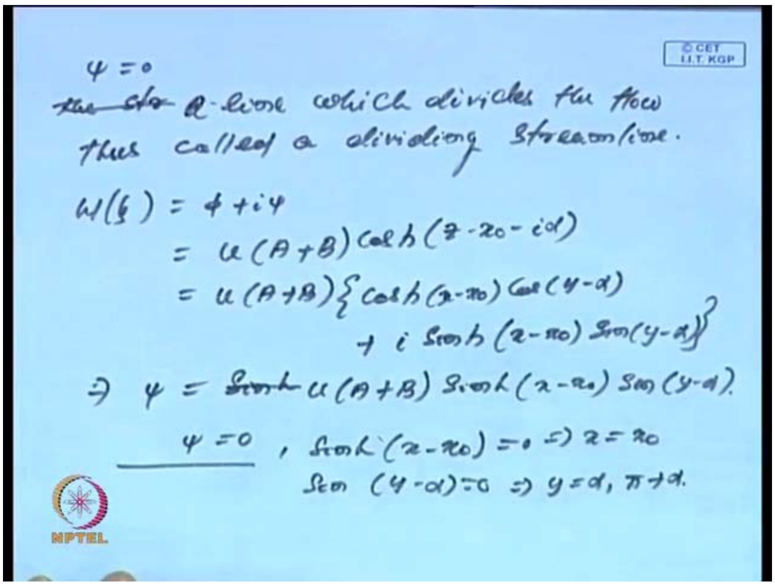

These streamline, I call this as the line for psi equal to 0. The streamline is called a dividing streamline. It is a basically a line, which divides the floor. This is because since psi is equal to 0, it is a streamline that there will naught be any flow across this line. Hence, this line is called a line, which divides the flow. Thus, it is called a dividing thus called thus called a dividing streamline. This is called a dividing stream line.

On the other hand, what will happen to w if you look at the alpha that is pie plus i sin in terms you have u into A plus B? We have already that as cos hyperbolic z minus x naught minus i alpha is can be written as u into A plus B u into cos hyperbolic x minus x naught cos hyperbolic x into x naught into cos y minus alpha plus i times sin hyperbolic x minus x naught and this y minus alpha. This implies that if I relate pie and sin, then my sin will be sin hyperbolic, rather it will come as u into A plus B into sin hyperbolic x minus x naught into sin y minus alpha.

Hence, this size constant, if size is equal to 0, if size is equal to 0, then again if I say size is equal to 0, 0 is stream line divided by 0. Then, in this case, it will give me sin hyperbolic x minus x naught is 0 and sin y minus alpha is equal. If this is 0 means x is equal to x naught, which we seen that x is dividing by stream line and again y minus alpha is 0 with this. y is equal to alpha or i plus alpha. So, that means these are the dividing stream lines.

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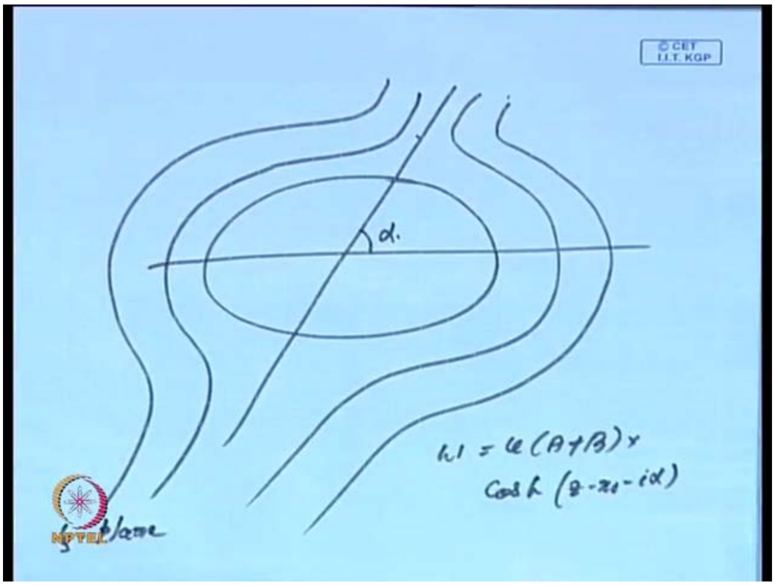

So, if I look at a flow, if I look at a flow, when we look at a cylinder, we see the, we see the x axis. This is because my flow is making an angle with alpha. Then, we should do deduction invasion when the fluid is flowing. This line shows how the flow will be. So, this is the way the fluid will be flowing. This is the w is equal to u A plus B. This angle is alpha.

These angles invest the fluids flowing and the deduction of flow A plus B into cos hyperbolic z minus x naught minus i alpha. Alpha is the angle of influence and this is in the data flame. Now, with this, what will happen if there is any stagnation point? This is because when we were discussing a flow, we need to know whether there is any stagnation.

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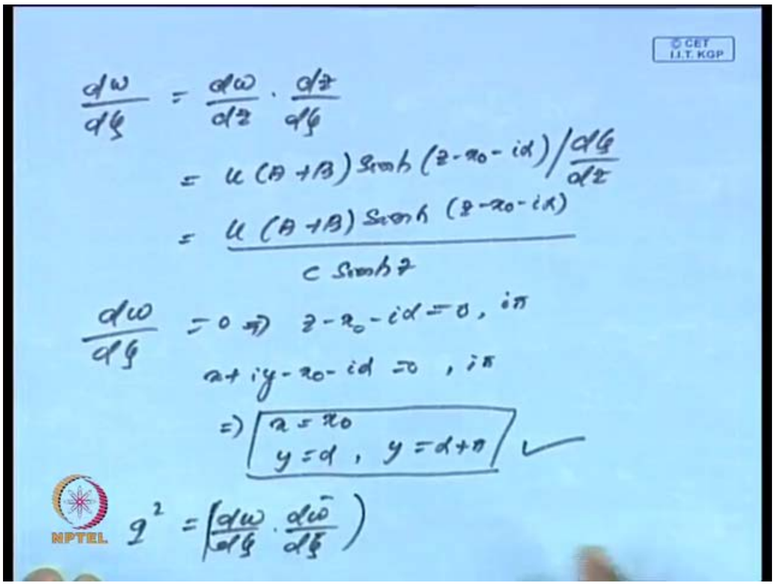

So, what will happen to d w by d g is equal to that d w by d z is equal to d z. It can be seen that this is nothing but u undo A plus B sine hyperbolic z minus x naught minus i alpha divided by d g data by d z and this again d g by d z. This has to do for this. So, this is A plus B sin hyperbolic z minus x naught minus i alpha divided by d g by d z. This again d g by d z, this has to subtract to this. So, it is u into A plus B sin hyperbolic n minus x naught minus i alpha.

We will see that this is by c sin hyperbolic z. So, d g data by d z would be c sin hyperbolic z. Hence, the stagnation point if d w by dg is 0, so if the stagnation point that gives me that this is hyperbolic. So, that will give me z minus, which implies z minus x naught minus i alpha is 0. That gives me either it can be 0 or it can be i pi. If this gives me, if this is 0 that means y z minus x is equal to z is x minus pi x plus I y minus x naught minus i alpha that is 0 or i pi.

So, it implies x can be x naught, y can be alpha. Again, y can be alpha plus pi. So, these are the points for which d w by dg is 0. That will represent the stagnation point. So, that means there will not be any flow across these points. Now, again we know, if we want to know the speed q square is equal to d w by d g, d w by d g. So, it can be easily seen that if we again substitute for d w by d g data, then I can easily see.

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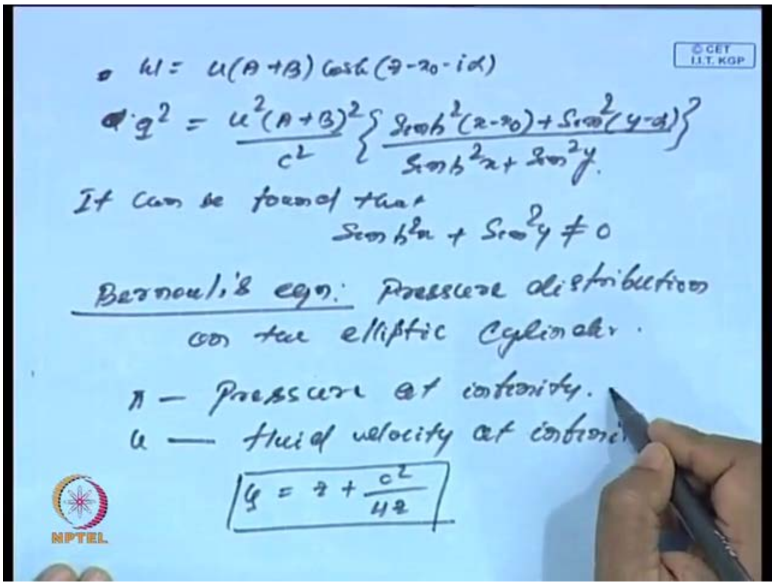

This is because we have the w u A plus B cos hyperbolic z minus x naught minus i alpha. From this, I can get easily q square and q square is u square into A plus B whole square by c square and not going to congest compact form. This is hyperbolic square x minus x naught plus sine square y minus alpha.

This is just a sum of the algebraic sums to do it sin hyperbolic square x plus sum square y. So, this will give me Mimic Square. It can be easily seen that it can be found that sin hyperbolic B square x plus sine square y is not equal to 0. This is because x is equal to 0 only at focus. There are tresses. There it should not be 0. So, if this gives the q, then how to find pressure? If we go to Bernoulli’s equation, we can find the pressure distribution on the elliptic cylinder. This can be obtained.

What will we do? If I assume pi, this as pressure infinity, if they flow, if I say they have seen that this can flow elliptic cylinder fluid velocity infinity. This is because we all know when we have defined this control lapping, we have seen that from this, it is obviously that is equal to y z c square by 4 z for z. We have seen that z is at large list.

So, whether it is, so the point in the detach point in the z plane starts at infinity, the fluid pressure is fluid velocity remains u, in the cases of cylinder as well as in the case of elliptic cylinder, while the pressure at infinity and u the fluid velocity of infinity. This is because that we have started with that fluid velocity at infinity because flow inset is at any point on the fluid if you apply this.

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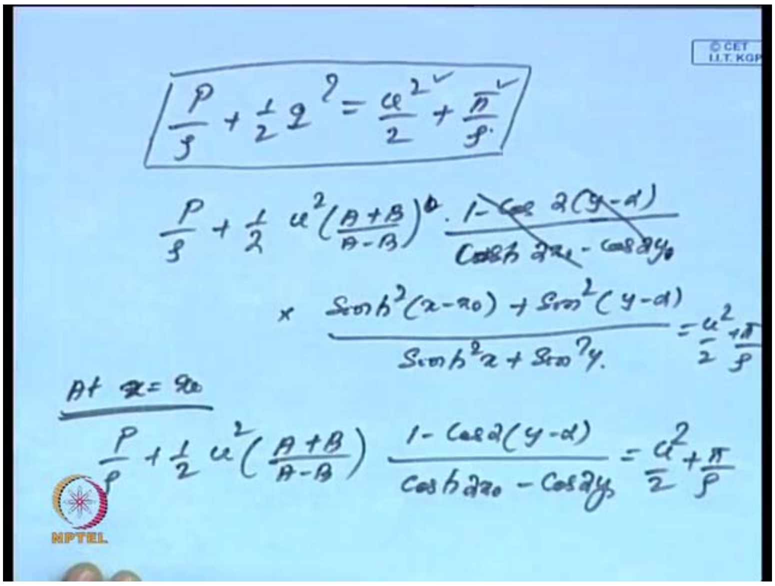

If we apply this, so I will have on this oppress of this cylinder P by rho plus half q square is equal to u square by 2 plus pie by rho. That is what we will get. We know u square. So, this is known. We can always get P by rho plus half. That is q square. It is nothing but we have seen that q square is A u square into A plus B by A minus B square. This is because c square is A minus B square. c is A plus B by A minus B because c square is A minus B square.

So, we have already that will give us A plus B minus A minus B 1 minus cos 2 y minus alpha by 1 plus cost hyperbolic 2 x naught minus cos 2 y naught cos y the other into sin hyperbolic square x minus x naught plus sine square y minus alpha divided by sine hyperbolic square x plus sin square y. So, this if I put at x is equal to x naught, then P by rho, then any point on this cylinder, P by rho plus half is equal to A square by 2 plus pi by rho.

So, that is u square into A plus B by A minus B. In alter tech algebra, this has been interesting. If I say x is equal to x naught, then I can easily get at x is equal to x naught means there should be 0. This is sin square. So, I can always put it 1 minus cos 2 into y minus alpha. Then, this will give me sin hyperbolic square x naught plus sin square y. So, I can put it in the hyperbolic.

This will make cos hyperbolic 2 x naught minus cos 2 y. x is equal is x naught. This is equal to A square by 2 plus pi by rho. Now, if you see the pressure at any point at x is equal to x naught, only cylinder, then if that is the case, then what will the minimum to obtain the minimum pressure?

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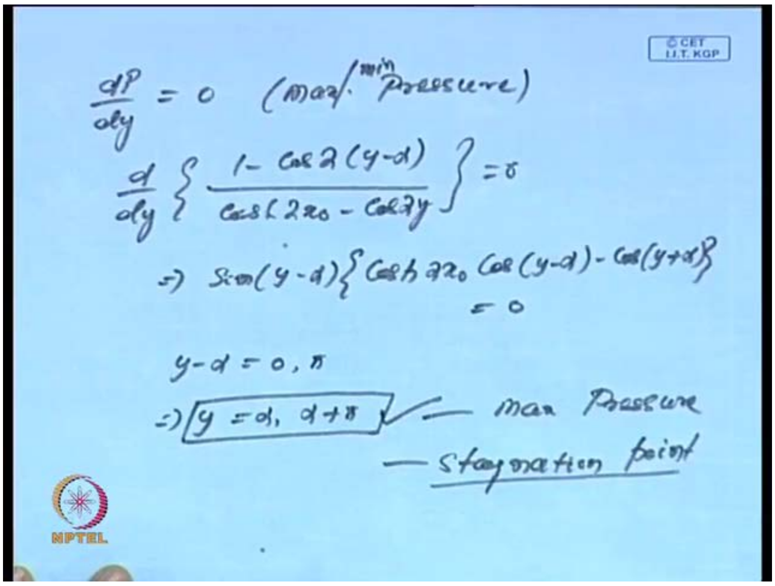

To obtain the minimum pressure, we have to say by d P by d y 0 because we have seen that the pressure is the function of y, so if we say d P by d y 0 means d by d y of 1 minus cos 2 into y minus alpha, all the things divided by cos hyperbolic 2 x naught minus cos 2 y. This is 0. Once this is 0, if we simplify, we can easily see that that will give us sin y minus alpha into cos hyperbolic 2 x naught into cos y minus alpha minus cos y plus alpha. This is called 0, which implies there is a chance that sin y minus alpha is equal to 0. So, y minus alpha is equal to 0 or pi 0. It can be 0 or pi. Another chance is 0. That means y is equal to alpha or alpha plus pi.

So, these are the full sides of the pressure can attain the minimum. Further, we have seen at these 2 points, agnail point in the flow, this gives me the maximum pressure. It can be said maximum plus minimum pressure. It can be checked that these points give me the maximum pressure. We have seen that these are the staginess points. So, this is the staginess point in the flow. Here, pressure has become more. Now, if you look at the right side as 0, so at this point, the pressure is 0 maximum.

You have seen that in case of a circular cylinder, near the staginess point, we have seen the maximum pressure. If we see the x axis, because my flow is making an angle with alpha, then we should deduction invasion when fluid is flowing. Now, with this, we will see what happen is that at this part is 0. The right side is 0.

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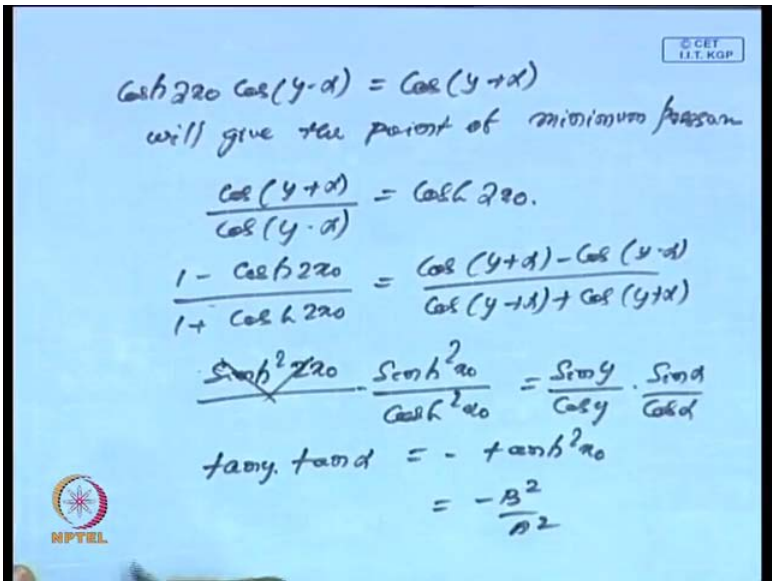

Then, we have cos hyperbolic 2 x naught into cos y minus alpha is equal to cos y plus alpha. If this becomes, this will give the point of minimum pressure. This will give the point of minimum pressure. We can see that if you simplify this, we can get cos y plus alpha by cos y minus alpha. That will give us cos hyperbolic 2 x naught. We can write 1 minus cos hyperbolic 2 x naught. It will all give zebra pi, 1 plus cos hyperbolic 2 x naught. It will all zebra pi, 1 plus cos hyperbolic 2 x naught.

I have to give us cos y plus alpha minus cos y minus alpha divided by cos y plus alpha plus cos y plus alpha. If you look at this again, it will give a sin hyperbolic square 2 x naught or sin hyper will square x naught or sin hyperbolic square x naught. It should be cos hyperbolic square x naught. That will give you sin y by cos y into sin alpha by cos alpha. Then, that each tan y, tan is equal to, there is a minus sign, minus tan hyperbolic square x naught. That we can see that A, it is minus B square by A square. So, we can see that tangent, which now further you put tan.

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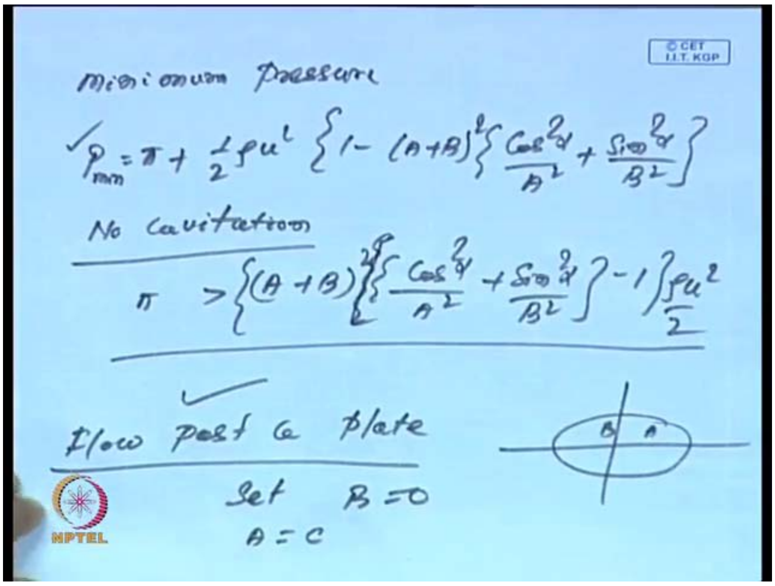

So, again by search tunnel alpha, if I call this 2 star or let me call it 3star, so if you substitute for this expression in my previous expression for the pressure, I lap tan. I lap tan by minimum pressure. This can be obtained pi plus 1 by 2 rho square into 1 minus A plus B square in a pressure distortion. If I substitute the value, it will give me cos square alpha by A square plus sign square alpha by B square.

Hence, if this is the minimum pressure P, it would be this P minimum. So, if I have to, I said that there is no cavitations, as I have mention if there is no cavitations, that means I should would have phi mist greater. Then, there is no cavitations means this should be greater than 0. This term P minus pi, this has to be greater than 0. That means phi has to be greater than A plus B square cos alpha by A square plus sign square alpha by B square minus 1.

There is rho square. This is somewhere A plus B square into rho u square by 2. Now, with this, I will get no cavitations greater. Once I have no cavitation that means I understand that I obtained the pressure. I obtained the minimum pressure. Also, I have obtained the cravat of no cavitations.

So, the flow past an electric cylinder is thoroughly understood. Now, what will happen if I have to analyze flow past a third plate? Then, what I will do because I know the flow past a cylinder? To understand the flow past a plate, I just put set B is equal to 0, if I put B is equal to 0 because I have led this is by major access A, major access B. So, if I put B is equal to 0, so these 2 flows past. Hence, in this case A will B and the flow past a cylinder. The flow past plate will complete, rather if I say that the plate flow is a uniform flow, which makes an angle, it makes an angle alpha with x axis. Then, I have a cylinder, a plate.

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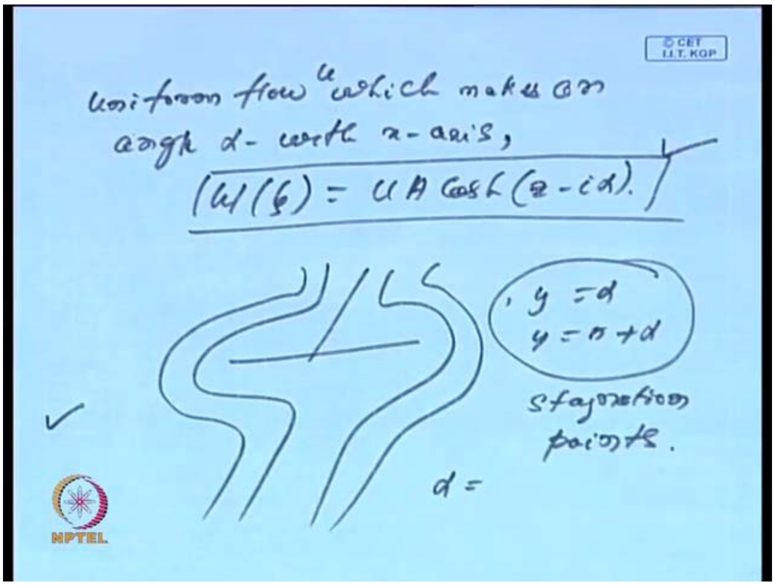

Then, the flow past a plate will be w eta and that will u A cos hyperbolic z minus i alpha because extra will be 0 here. z will be 0. So, this will be the flow past a past plate and here that principal by plate likes this. So, this deduction again in same way, here also, we will see that the point y is equal to alpha y is equal to pi plus alpha. They will represent the stagnation point. These are the stagnation points.

If alpha is pi by 2, in this 1, if alpha is pi by 2, so this we can write it in the form w eta with this. I think this is nothing. We will leave it here. Then, again if I had calculate, we can easily do that. I am not going to detail here today because we understood the flow past a cylinder and how to calculate the pressure and may be in a similar manner if I apply the theorem, I can calculate that is that is force blasices on the cylinder as well as well.

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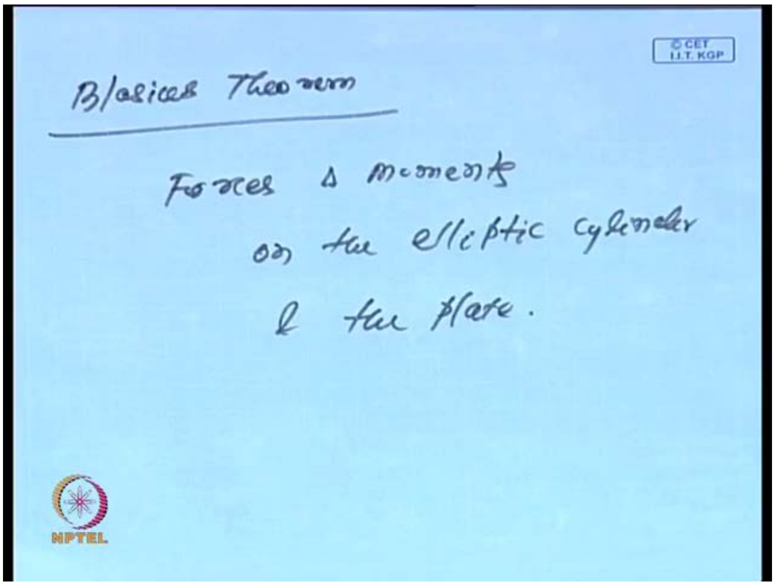

I can apply blasices theorem and calculate the forces movements on the elliptic cylinder and a plate. This is a very straight provider. So, we can find d w by d z. We can apply direct the blasices theorem and co residue theorem. So, we can calculate the force and moment distribution on the cylinder.

So, this we have although the technical detail littered d s, but we have that how a flow past an elliptic cylinder and flow past a finite plate even from the co basically can be the solution. It can be obtained. We can calculate the pressure, the velocity forces and movement acting the cylinder. This is important. The transformation is clear from this example. This is because otherwise it would have been very difficult to calculate the flow past an elliptic cylinder.

However, this is because of the Joukowski transformation were able to do it in a medicine prepared manner. There is a little complex. Basically, the algebra is tedious, but the method is very simple. This today I will stop. Next time, we will look at how we can calculate some more analyze more problems related the electric cylinder. Then, we will go to look into hydrophile and other complex flow patterns again best on this Joukowski transformation.

Thank you.