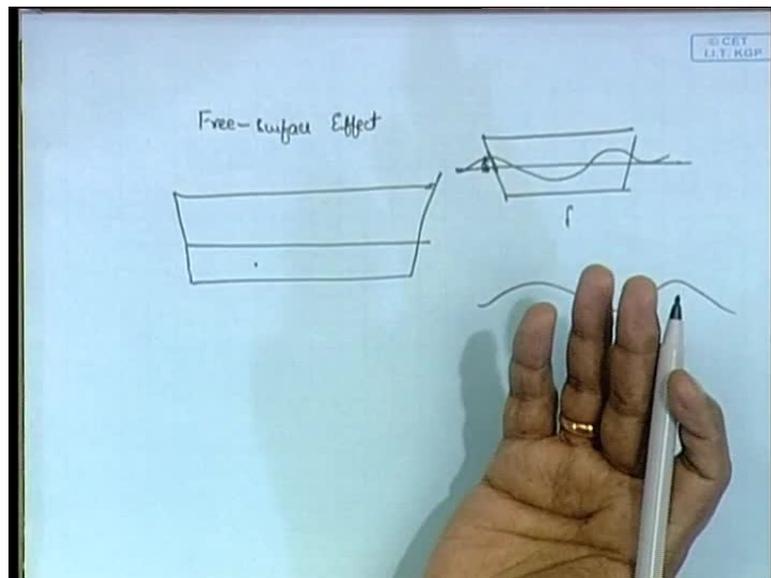


Hydrostatics and Stability
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Module No. # 01
Lecture No. # 09
Free Surface Effect

In the last class, we have been talking about the shifting of weights from one side of the vessel to another by a small shift. If there is a small shift of w of d of a small mass, what will be the distance through which the center of gravity of whole ship will shift?

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Now, we come to a new type of effect that we call as the free surface effect; in your course of study here you will see that free surface effect is used to denote to two things - two different things - both are called by free surface effect.

One is free surface effect as you see it in hydrostatics here - that we are going to do; then, you will see another free surface effect that will come in your marine hydrodynamics which Professor Sen will be teaching you; he will be teaching you and Professor Sahu - they will be teaching about waves and all that, there another free surface effect comes; I will tell you what the two things are.

First of all, let me tell about what they are talking about - free surface effect means the effect of the air-sea interface; we have talked about a water line - so when you have a ship - we talked about a water line...so, in general this has some movement of its own; this is the water line, this is the surface of water that has a movement of its own; for example, when a wave passes through it the free surface will become like this; this will become like this, it is not a straight line.

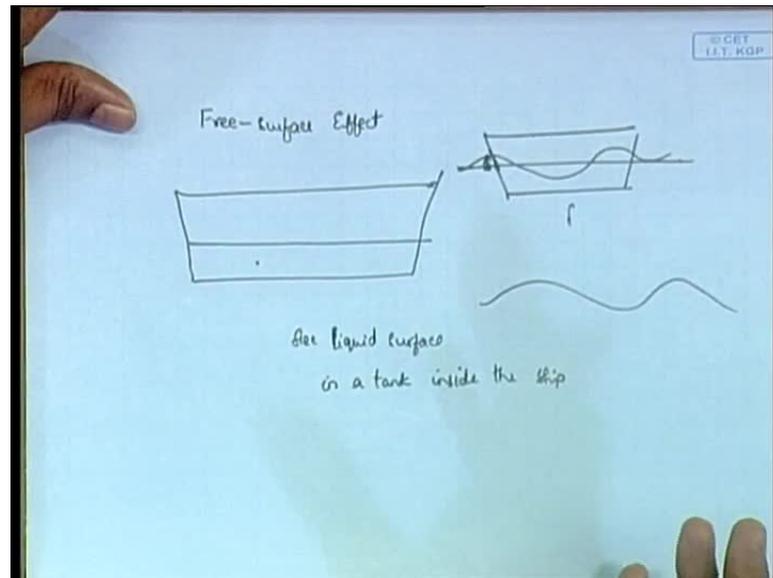
When you are studying the effect of this - what is the speed with which the wave is moving, what is the displacement of this free surface from the mean position - like that, that study is called free surface effect in their terminology; that is their free surface - that actually comes in waves and hydrodynamics - marine hydrodynamics.

Now, what we are doing is a free surface effect that we talk about in hydrostatics which is different from that; but, not very different - that is, the meaning here is when you are studying the free surface effect what you are doing here is - suppose you have a tank in a ship, let us draw a ship like this.

Suppose, there is a tank in the ship - this much is a tank; now, in that tank let us suppose there is some liquid, for example, you can have oil tankers which carry oil; they will have huge tanks - in fact, only tanks and they carry oil in it or ordinary ships itself will have in it huge tanks which will carry ballast water - I have already told you what is ballast water - taking in and leaving out of water for going up and down.

That is also having a tank of its own; these things you will have something...that is, you will have a tank which is not fully filled with water, that means, you will have a free surface of liquid; that situation where you have a free surface of liquid in a tank inside a ship is known as free surface effect in hydrostatics.

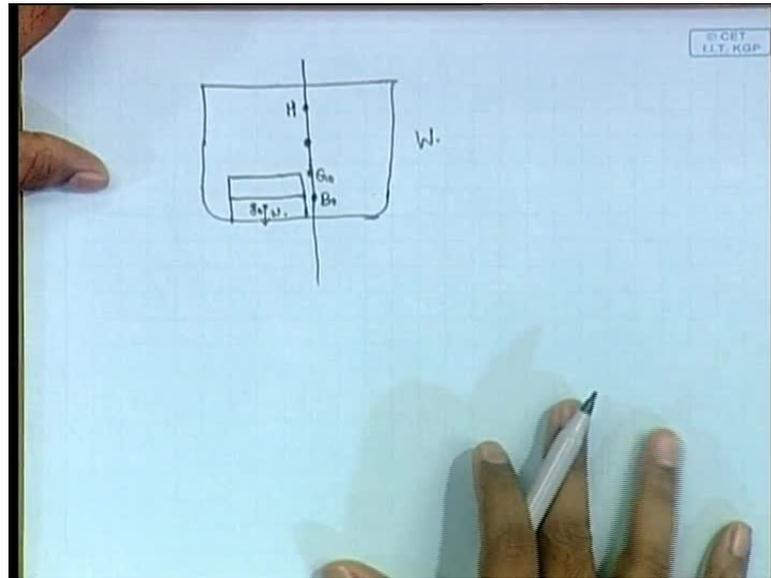
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It is actually a free liquid surface or a liquid surface in a tank inside the ship; you have to note that if the tank is completely filled with a fluid what is the difference? The difference is that you would not have this movement of free surface - it will be fixed; it will be just like a solid it does not make any difference - that liquid become just like a solid and it moves with the ship; there is no separate motion of its own; but, if it is not full, this free surface will move like this and it has an effect of its own on the ship's stability.

That is what we are going to look at next; that is what we are meaning by free surface effect; let us see what that is.

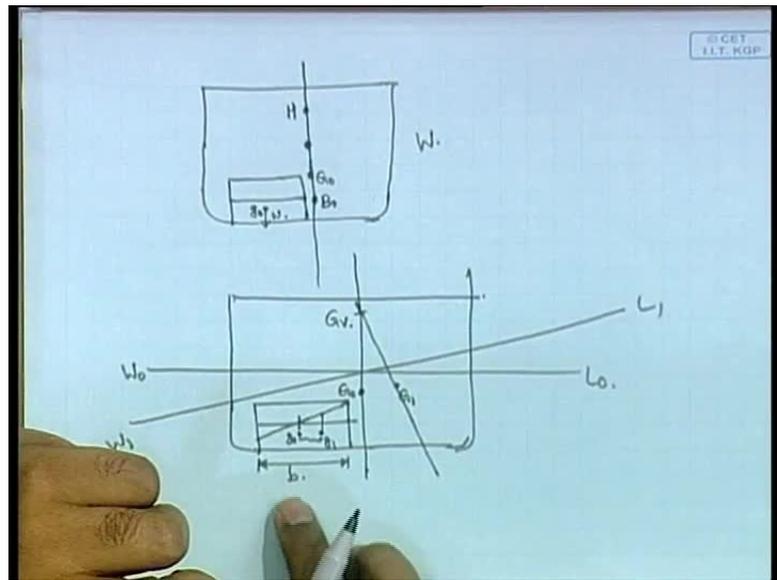
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Let us look at a cross section of the ship - this is a cross section; let us suppose that in this ship there is a tank like this - it does not matter where it is, but let us put it here - and this is the center of the ship and you have here G_0 - the initial center of gravity; this tank is now filled up to this height with some liquid, and let us say that the center of gravity of this liquid is at small g_0 ; it has a small weight it has a weight equal to small w and the capital W represents the whole displacement of the ship which is tank plus everything in the ship - the whole displacement of the ship.

Small w represents the weight of the water alone - let us call it water for a time being - it can be oil also it does not make any difference in that row, that we will see; then, the center of gravity of that small water body is g_0 - small g_0 - and capital G ... then this center of buoyancy is B_0 and let us put the Metacenter here M ; so, these are the... now what happens? Let us suppose that the ship heels, ship inclines.

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This will also move on its own; first of all, the ship will incline; let us say, like this, now this body will also... this surface of water will no longer be horizontal, it will start to move on its own.

Now let us draw that figure - $W_1 L_1$ this is $W_0 L_0$; here you have this tank, it has its G_0 and now it is because of that inclination this is like this now; the water surface is like this and because of this you see that more water is there on the right side, so G of that water body will shift to the right, so this will be G_1 let us say.

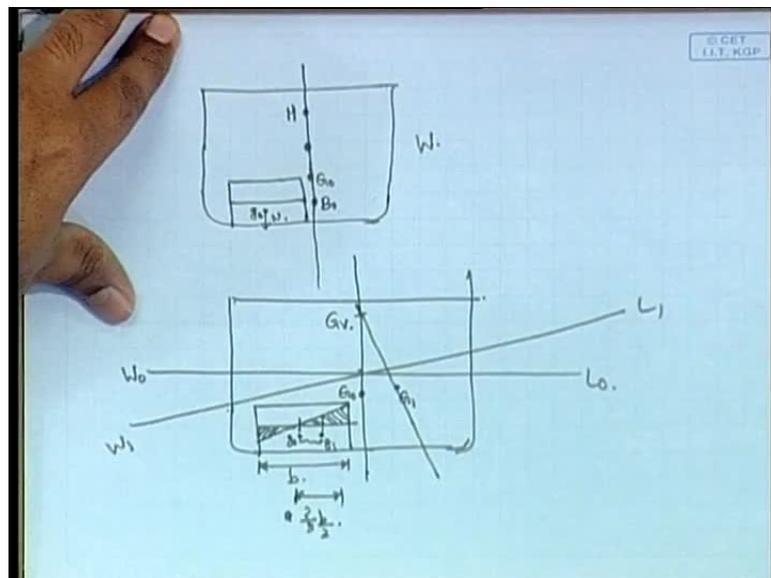
This is just the G of that water body, that is, small G always represents the G of that small water mass alone - so that is small G ; let us say that... so, this becomes... if you think of it, it has really become similar to that problem we did of shifting of mass - this is small weight w , that is, the weight of that water has actually shifted from G_0 to G_1 ; it is almost similar to that problem.

Now what has happened? Let us say the G_0 is - capital G_0 - is the initial center of gravity of the whole ship, now because of a weight shifting G_0 this will shift to here G_1 ; here this will be the vertical this will be the vertical and let this be G_v .

So, this figure is very much like that of a center of buoyancy shifting and the Metacenter you have G_1 and G_v ; there is similarly... that figure explains it; now, let us derive the expression

Let us do one thing, let us assume that in this derivation this is the box - it makes the derivation simple; we will do it for general later, but first of all let this is be a box - we have already said box shape barges - so this is one of those and because of that let us call the distances as... let us say that the this distance is b - small b ; so what will be this distance? This distance - I mean this g_0 is b by 2 will be at the center - g_0 is the center of gravity of their water body so b_0 is the b is the breadth of that tank.

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Now, g_0 is the position of that - we are talking about the horizontal position only - we are talking about... right now, forget the vertical moment; so, that g_0 is initially at the center because it is a rectangular box and water is like this means it will be at the center only, so this is g_0 so it is initially, let us say, 0 or it will be, in this case, b by 2 exactly at the center; what will be g_1 ? The distance of g_1 from the center?

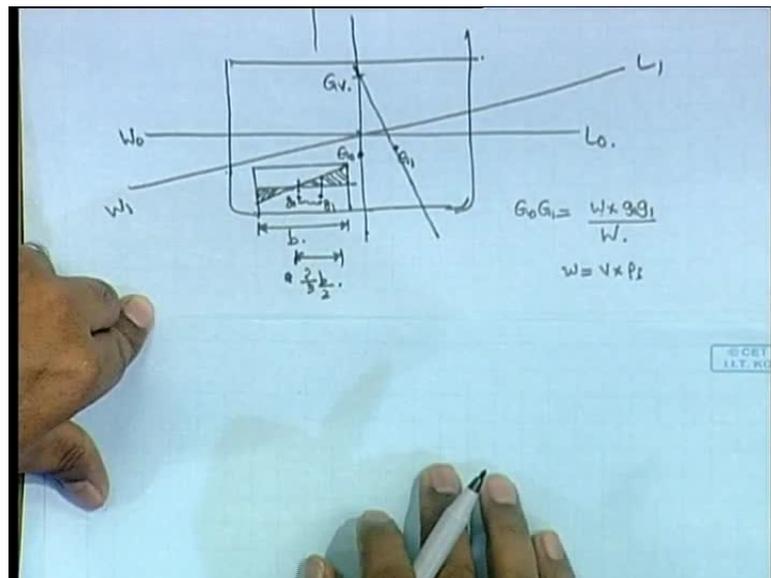
What do we have? We have this small wedge that has move to the right side; now, what we have to find? We have to find the center of gravity of this triangular wedge therefore; it is 2 by 3 b by 2 ; is it clear or should I... **however, it is anyway.**

It is the same problem that we did; like this, see we took this whole wedge - we took this wedge - we found the center of gravity shifting, the only thing is in this case we are just

taking the small region - this small region - so, it is this wedge that has shifted to this side.

One side has come down one side as gone up; so, g has shifted - this distance is 2 by 3 b by 2 , that is that distance; now, with this figure here - this figure we need, let me derive things here.

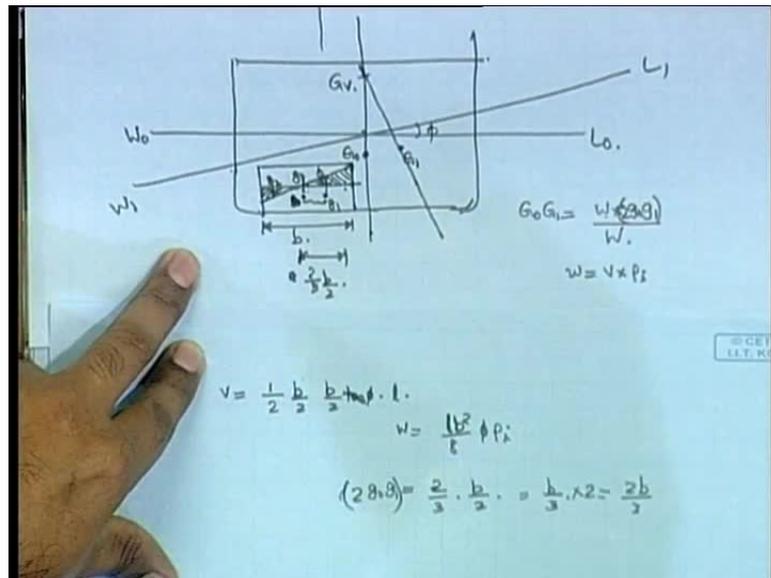
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First of all, what is $G_0 G_1$? I will just write the value then if you want I can explain it. Is that clear? Because, this we have explained many times; there is g , this is the value of $G_0 G_1$ which is the movement of... that is what do we have here? We have a small mass, small w inside a huge... the whole mass capital W ; when that small mass is moved a distance d we have seen that the movement of the center of gravity of the whole system is small w into d divided by whole weight, that is what this is.

$G_0 G_1$ is the distance through which the center of gravity of that w has shifted; w into g $0 g 1$ by capital W ; that is now... we can write this... this is obvious - volume of that wedge into row of that water; weight of that is... actually, they are talking about mass not weight, so w is equal to v into row i .

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What is v ? v is this volume of this wedge; what is the volume of the wedge? Volume of the wedge is area of the triangle into the length; what is the area of triangle? It is half base into altitude - base is b by 2 into altitude is b by $2 \tan \phi$.

Note one thing what will happen is that the ship is like this and at the surface of that there is a tank here and there is a liquid inside that, that liquid - that surface is also horizontal, it will be horizontal because if the ship is horizontal this will also be horizontal; then, it tilts like this, in this case - yes it tilts like this - then what will happen is that this will also tilt in the same fashion; therefore, it will tilt by the same degree as the surface as ship - you can just look at the figure, this angle is ϕ similarly this angle will also be ϕ .

Both are similar; therefore, what I am saying is that if this distance is b by 2 this distance is b by $2 \tan \phi$ because this angle is ϕ this is b by $2 \tan \phi$ b by $2 \tan \phi$ divided by b by 2 that is $\tan \phi$ into the length l .

Therefore, weight and $\tan \phi$... because we are talking about small angles we replace it with ϕ ; therefore, the weight small w is equal to l into b square by 8ϕ into row i - row i is the density of that fluid, let us call it water its therefore equal to 1000 kilogram per meter cube; that is the weight of that fluid that is being transferred; then you have g_0 g_1 is already given **it is equal to...** now, what is g_0 g_1 ? **g_0 g_1 is 2 by 3 into b by 2 by 3**

What is actually... here we are trying to find out the movement of the mass; here, actually I have made a small mistake here, that is, this is g_0 - actually let us call this g_0 , let us call this g_{small} .

Now what is g_{small} ? I will tell you; that is, see initially you had... so, what are we saying? We are saying that initially this wedge - initially the system is at g_0 because the fluid is at g_0 , the center of gravity is at 0; now, because of this shift it looks as if a small weight - it is not looks as if - actually a small weight has been lost here and a small weight has been gained here, that is how it is; the weight lost is at a distance g_1 to the left the weight gain is at a distance g_1 to the right - the center of gravity of that, is it clear or should I repeat that? That is, g_0 is a center where you have the center of gravity of the initial water body, which is - there is center.

Because of that something comes down here, something goes up here - water comes down here water goes up here; a small wedge is lost here and a small wedge is gained here - it is like this weight has been shifted there.

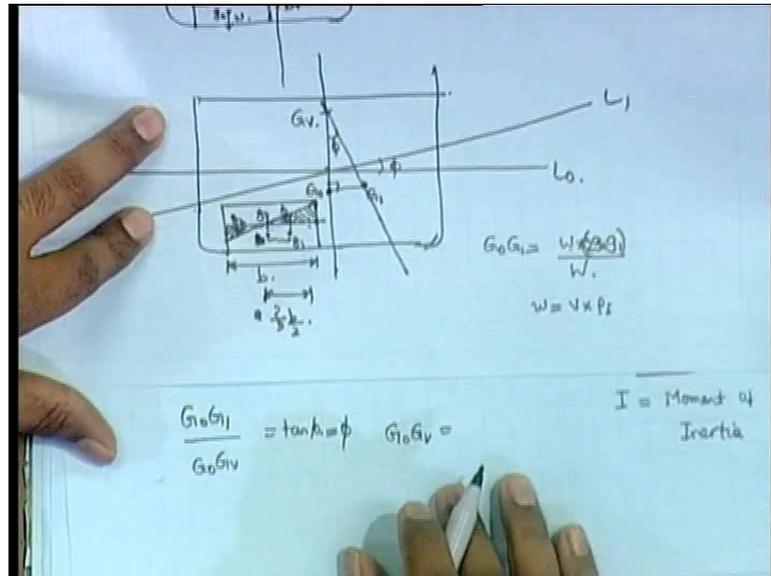
From the center of gravity of this point it is moved to center of gravity there, that is the distance through which the weight has shifted, right? So, g_1 to g_1 is the... or g_0 minus g_1 to g_0 g_1 is the distance through which the body has the weight has shifted. **this is not clear you would not understand this so if you want I can repeat it it is not a problem you understand it**

It will be twice this; this is the distance through which the... this is not the g_0 g_1 it should be $2 g_0 g_1$, you have to clearly understand; this will be twice this into 2 so this will be $2 b$ by 3 - twice $g_0 g_1$; what you are checking is - one weight is lost at one point and one weight is added at one point it is almost as if one weight has been transferred from one point to the another point, it is the same thing; minus w here plus w so this weight is transferred from here to there.

Because of this weight, the weight transferred is...both the volumes are the same otherwise you cannot do this problem - it becomes very complicated; one volume is different from the other volume means you have to do different other things we are not bother about that right now, this is the simpler case.

plane about that longitudinal axis; therefore, $G_0 G_1$ is that... this is $G_0 G_1$ is equal to this, that is once you have that now you can do one more thing $G_0 G_1$ let us look at this figure.

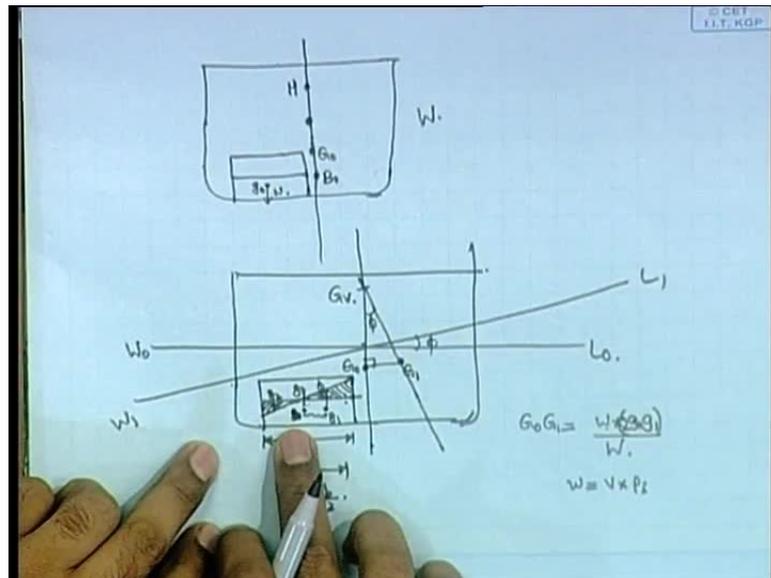
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$G_0 G_1$ - this is a horizontal moment of G so this angle is 90 degree; this is vertical and this is therefore horizontal - 90 degrees; now, we can... this angle will be phi, if this angle is phi - these are all geometry - if this is phi you will see the this becomes phi therefore $G_0 G_1$ divided by $G_0 G_v$ is equal to $\tan \phi$.

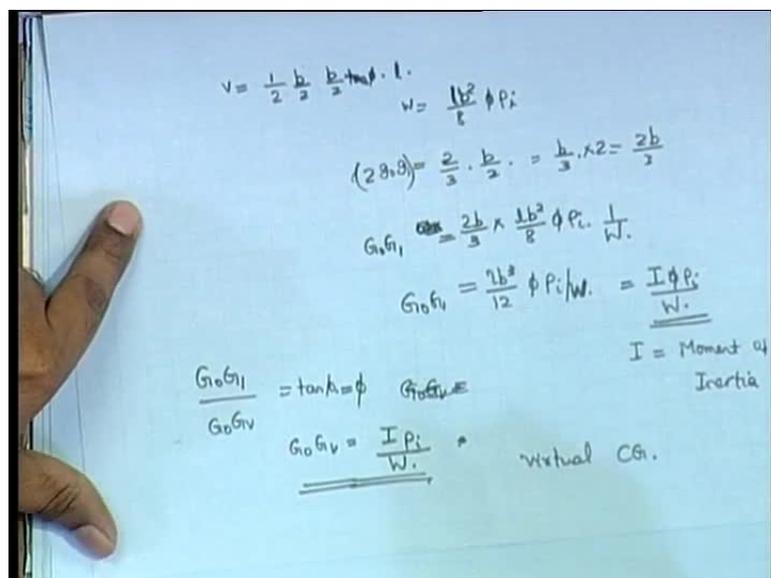
Therefore, $G_0 G_v$ is equal to... and which this is - let us write it as phi; then, $G_0 G_v$ is therefore equal to $G_0 G_1$ by phi, therefore this is $G_0 G_1$ divided by phi so you remove that phi and this becomes i row i divided by W ; actually, why this i is written as I am not sure - it is its row, the density of that fluid - so you have this is $G_0 G_v$. I do not think that derivation is given, but $G_0 G_v$ will actually be the distance through which the center of gravity of the ship moves in the vertical direction of the ship.

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Now, this liquid, right... what we are saying is now... looks confusing, but I will tell you what this is - you see this liquid is moving like this, when that happens the center of gravity shifts both in the vertical and horizontal direction.

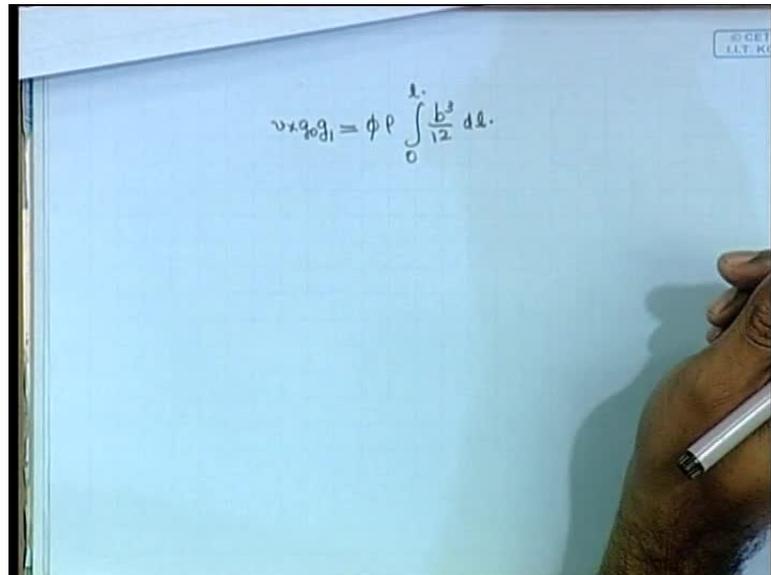
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What we derive - $G_0 G_1$, is the shift **in the vertical direction** in the horizontal direction; therefore, we are finding out that shift in the vertical... that we will come to later; just remember this expression - $G_0 G_v$ is equal to $I \rho$ by W , we will derive it.

Actually, why is it called G_v ? It is called G_v because it is called a virtual center of gravity; that is, when the free surface of the fluid... when the ship is like this and the free surface is moving up and down like this; that is, the body is tilting like this - inclining like this - and because of this the water inside the tank is moving up and down like this; because, of this the center of gravity of this is going up and down. When that is happening, the center of gravity of ship also goes up and down; so, that movement can actually be represented by this G_v ; actually, they have just written it here.

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If you do the whole set of derivations, for what is not a rectangular body and just a arbitrary type of body - actually you will end up like this; 0 to 1 right ya how does it move up? Because, the center of gravity of that free surface is moving up and down; no, this is like this right this - free surface is like this - it moves like this, so its center of gravity moves up; it has moved horizontally and it has moved up also, because something has come down here something as gone up there, so its center of gravity moves up; because, the center of gravity of the free surface of the liquid moves the center of gravity of the ship moves.

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Handwritten derivation on a whiteboard:

$$\frac{1}{2} \cdot \frac{b}{3} \cdot \frac{b}{3} \cdot \phi \cdot l.$$

$$W = \frac{1}{12} b^2 \phi \rho l$$

$$(2B, 0) = \frac{2}{3} \cdot \frac{b}{3} \cdot \phi = \frac{2}{9} \cdot \phi \cdot 2 = \frac{2b}{9}$$

$$G_1 G_2 = \frac{2b}{9} \times \frac{1}{12} b^2 \phi \rho \cdot \frac{1}{W}$$

$$G_0 G_1 = \frac{2b^3}{12} \phi \rho / W = \frac{I \phi \rho}{W}$$

$I = \text{Moment of Inertia}$

$$= \tan \theta = \phi \quad G_0 G_2$$

$$G_0 G_1 = \frac{I \rho}{W} \quad \text{Virtual CG.}$$

Because it has to move no ya; so, in this case you get v into... that derivation if you want we do the whole... that is, instead of that l b cube by 12 will become like this.

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Handwritten derivation on a whiteboard:

$$v \times G_0 G_1 = \phi \rho \int_0^l \frac{b^3}{12} dx$$

$$G_0 G_1 = \frac{I \rho}{W}$$

$$G_0 G_1 = \frac{I \rho \phi}{W}$$

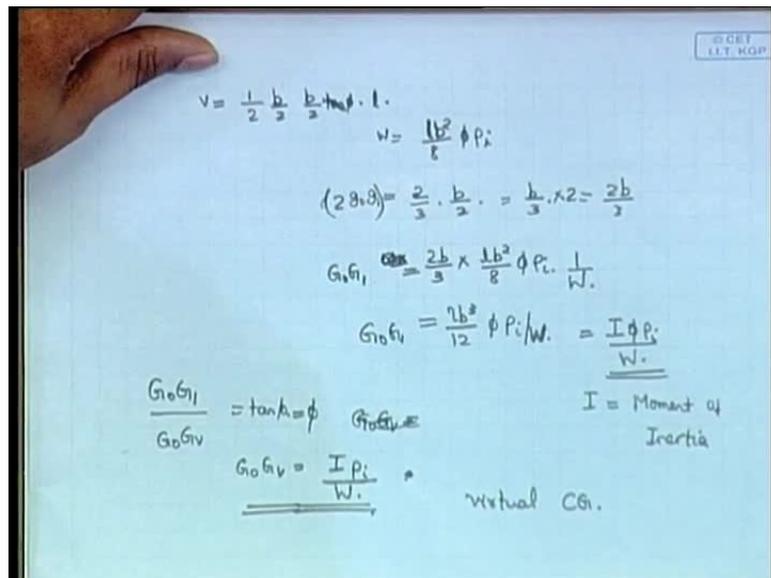
$$\frac{G_0 G_1}{G_0 G_2} = \phi$$

It does not matter, you can take this formula for granted; that is, $G_0 G_1$ is equal to I capital I into row divided by W ; so, this gives you the distance through which the center of gravity moves in the vertical direction and we have, before, an expression that gives how the center of gravity moves in the horizontal direction - $G_0 G_1$ I row phi by w -

that is obvious because $G_0 G_1$ by $G_0 G_v$ is just ϕ or $\tan \phi$ - $\tan \phi$ is ϕ in this case **so this is one thing alright.**

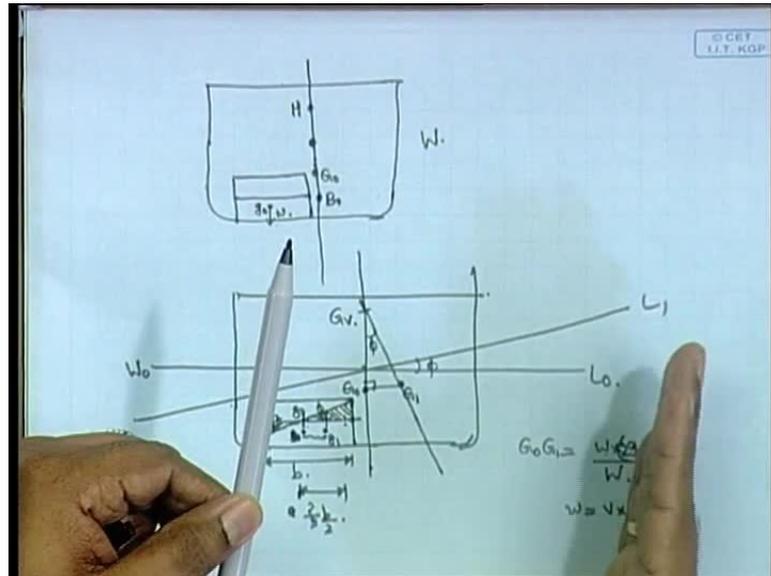
We can do sums - simplified cases, we will do it here. Two things you have to notice, its interesting actually, because this movement of the center of gravity actually depends upon I - note one thing this is not I of the ship.

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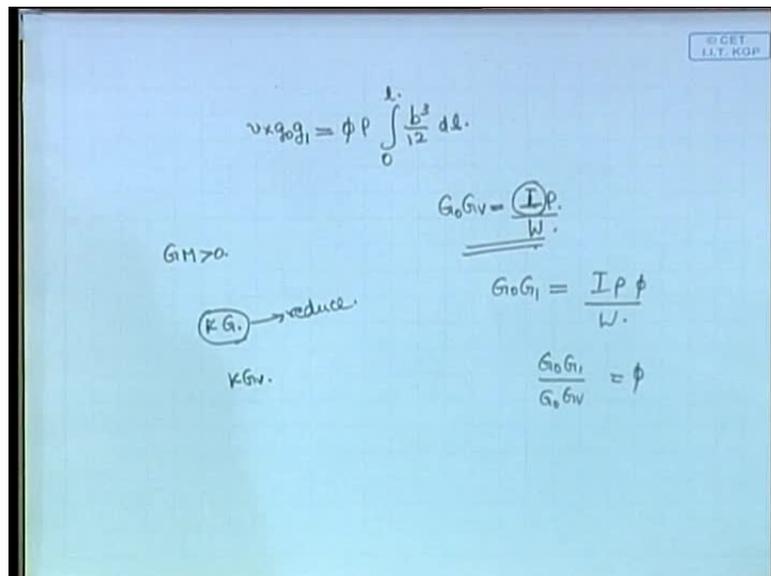
Let me see where I derived it - here; now, see this is $l b^3$ by 12 where l is the length of the tank - it can be the length of the ship also, if it is the whole length of the ship the tank extends the whole length of the ship it could be the length; but, remember this b is actually the breadth of the tank not the breadth of the ship; so, this $l b^3$ by 12 represents the moment of inertia of the tank. Here, if you have the whole ship like this - if you have the whole ship here - the tank is just here, this is the moment of inertia of this tank about this center axis.

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This center axis you have the tank here; about this center axis what is the moment of inertia? This is the center line of the ship and this is the whole ship; so, you have a whole ship, you have a small tank here, you have the center line of the ship - this is the center line of the tank itself; this is the moment of inertia of this small tank about its center line.

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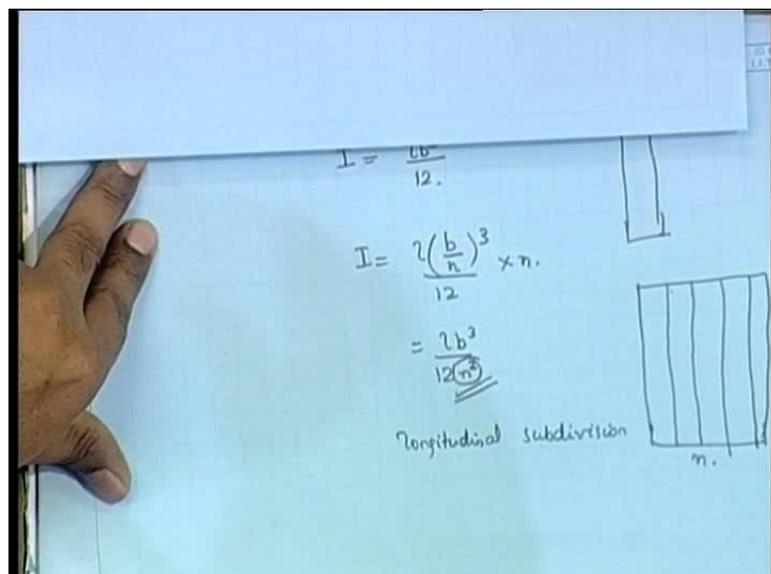
Now, we can see one thing - what are the different ways, for instance, to change this G_0 ; now, we have already seen GM is affecting the stability, we know that; because, GM - we know that GM has to be greater than 0 - which implies that... what is the best way?

You should have G as low as possible that means GM will be large; let us suppose that M is fixed, if g is as far down as possible then GM as large as possible and you will have maximum stability; therefore, our idea is to reduce G - KG - as such; we are trying to reduce KG. Now, in this figure if... so I said according to this figure we have to reduce KGv - means, the distance between G and Gv should be as minimum as possible that is our goal; that should be one of our goals.

What are the different ways to do that? $G \ O \ G \ v$ is now depending upon I, therefore what are we trying to see? We are trying to find out different ways to reduce I; because, we want to maintain the stability of the ship. What we are actually seeing, is that if you have a ship you have a tank in it you have a free surface in that tank, if the ship heels while it is moving - while the ship heels of course there is a loss of stability because of the heeling itself; but, because of the tank and that free surface effect there is an additional loss of stability than if the tank is not there.

So, if the tank is there, there is an additional loss of stability and that loss of stability - let us see how we can minimize. We cannot do away with the tank, we need the water or we need the oil so we cannot do away with the tank.

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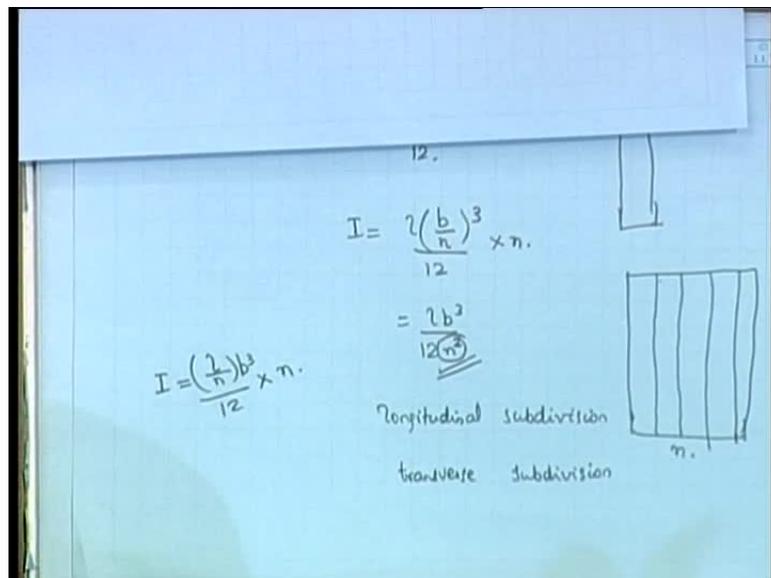


What are the different ways of reducing that reduction in stability? How to improve that stability? What we can do is - now, what is I? In some ways it can be written as $I = \frac{b^3}{12}$; what we do is... suppose... we are looking from the top now, let us suppose that

this is the ship and let us say the tank is like this - let me draw a big tank; therefore, you have a ship and on that ship you have a large tank; what do you think is a **partitionate** exactly? What is the best way to do the...that is, what we do what we can do is we can partition these tanks - exactly that is what they do.

So, instead of having one large tank you can partition that tank. What you see here is how much the I will change; I will become l into - what is b now? It will be, let us say, that it has divided into n compartments, it becomes b by n; so, it becomes b by n cube by 12; you have to sum up the I (s) to get the total I so you have to multiply with an n; still, it reduces many times - it is b by n cube into n so it is l b cube by 12 and there is an n square - it is reduced n times - n square times; it is reduced n square times; this is known as longitudinal...you know that this direction is called longitudinal - so longitudinal subdivision. Longitudinal subdivision of the whole tank - if you divide it like this you end up with a very low value of I which implies that it does not affect your stability so much.

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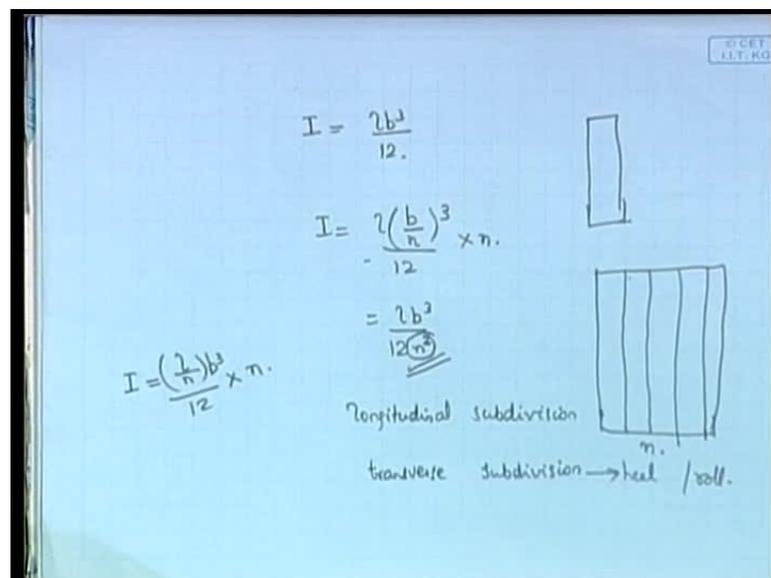
What will be the case if I divide the tank like this - transverse, what will happen? There is no change, exactly; it is because, I will become what? It will become L by n into b cube by 12 into n - it becomes...same thing; so, there is no point in transverse subdivision, this is called transverse subdivision.

Transverse we are talking only of the transverse swing of the ship what is longitudinal swing no that we are not at all we haven't talked about or in fact we would not be talking mostly about longitudinal movement at all by a longitudinal mov[ement]- no it is just that it is not in that course that is all

This is more in...one of the reasons is that...for one thing... that is, the moment is not affecting stability so much; see this length is very large, a small movement like this or like this would not affect the ship, it would not cause it to capsize and all; you are talk... about...we are talking about the capsize like this, it is a very rare chance - this is more important; when it tills in a transverse section, that is, as I told you... what is this called? We already mention that... trim - we called it trim.

This movement we called it as heel or list; heel is more important, because since this distance is very less very strong wind, for instance, when it hits like this it would not capsize it, but it would cause it to move very large distances.

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We are only discussing transverse stability; of course, there is a separate section on longitudinal stability as well, but it is there in the very end and if we have time we will do it. This is very important so we are only talking about transverse movements or these movements are called heel and such dynamic movements are called roll which I have already mentioned.

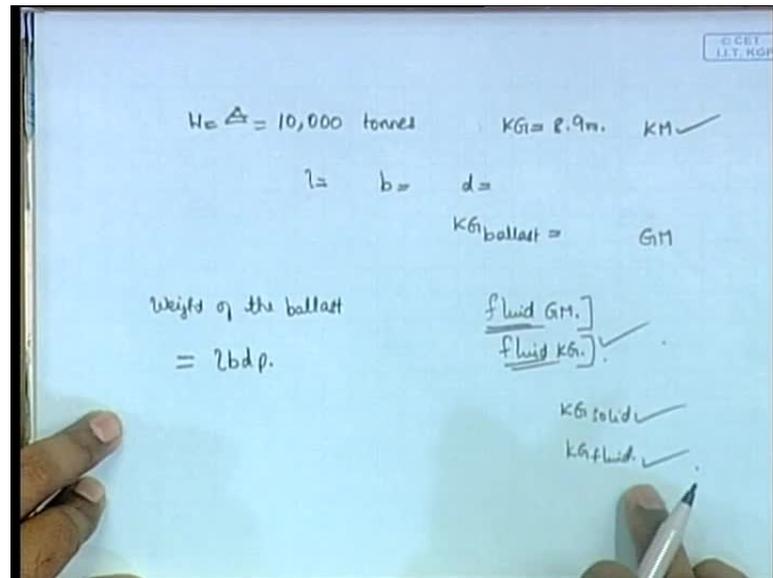
Dynamic movement means it keeps moving like this - vibration mode is called roll and just that static moment is called heel, if it just moves like this and stays there it is called heel but if it just moves like this it is called rolling; that process is called rolling. That is what we are talking about, we are talking about heeling right now - hydrostatics

Rolling actually comes in hydrodynamics; you know that the difference between hydrostatics and hydrodynamics? Dynamics is about motion, it is about fluid motion and the result of that on the ship and hydrostatics is about the effect of pressure, forces - it is a static case - what happens... just movement, that is all; it is not about continuous movement.

This is a very important point actually, because in your vivas they will ask you this; first question is what is free surface effect but more important than that... these are kind of trick questions, these show how much you have understood the subject; this question - why is it that you do longitudinal partitioning and not transverse partitioning? What is the reason for it? That is a very common question in your job interviews and your...

Another question similar to this is - they say, for a submarine what will be your b m? That is a common question again, I told you about that; that is, b m we have seen that I becomes 0 in that case so that - when it is under the water - and therefore that is again 0. That is one such questions; this is the another one - why is the longitudinal partitioning? This is the reason - you should remember this expression for I and this is how it becomes by n square in this case by n into n cancels out there is no effect. Now, we have two or three questions here - problems here - that give some understanding actually, these kind of problems.

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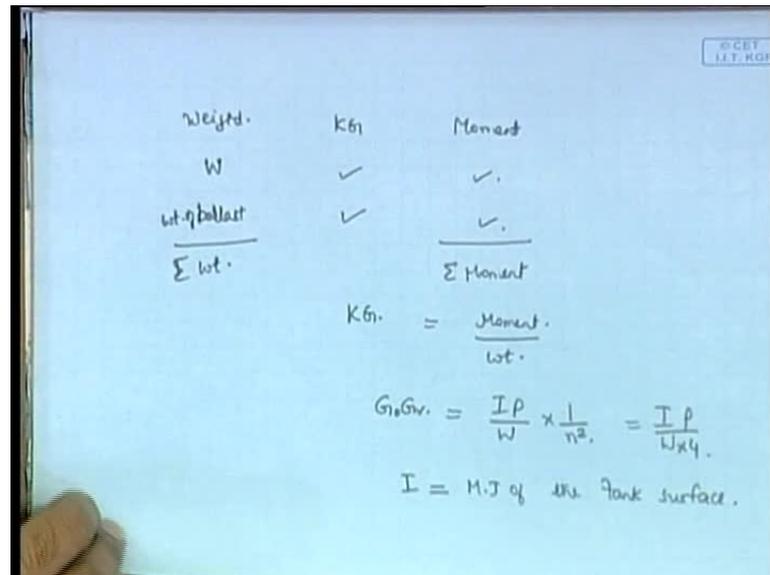


It says that a vessel has a displacement of 10000 tons. Or, you are given KG is equal to 8.9 meters and these are okay...and KM is also given, now you are told that the vessel is now going to load ballast water of some relative density into a rectangular tank and the length of the tank is given and the breadth is given and the depth is given - these three things are given; now, they are sent... this tank is now divided into two or rather it is a double tank so it has a center line division it is divided into 2 tanks in the middle.

You are told that the KG of ballast is equal to something and you are asked to find the GM; now, it is sometimes mentioned like this - they say find the fluid GM of the vessel the meaning of that is...or fluid KG; it is any of these, the meaning is that in the presence of such a free surface effect provided that the tank has a fluid, what is the GM? What is the KG? This is the meaning of this fluid; so, the question is to find the fluid GM of the vessel now.

First of all, we can find the weight of the ballast - is given by the length of the tank into breadth of the tank into depth of the tank into the density of water; you do this, you will get the weight of the ballast; then, you are given the KG of the ballast; now, what we can do is...so when you do the problem you will see in the book it is written like this.

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KG without the fluid, just with the ship - that is this, and this is KG with the fluid; so, water is added, that is ballast is added to the tank and then what you have is the KG of the fluid; now, what we can do is make a small table, there is...note that we need find the KG of the ship with the fluid; we have already done many times how to find the KG of a complete system when you have 2 or 3 weights; first, you have the weight of the ship itself - let us call this weight of the ship, and it has some KG you do not need to do the problem, I will just explain it.

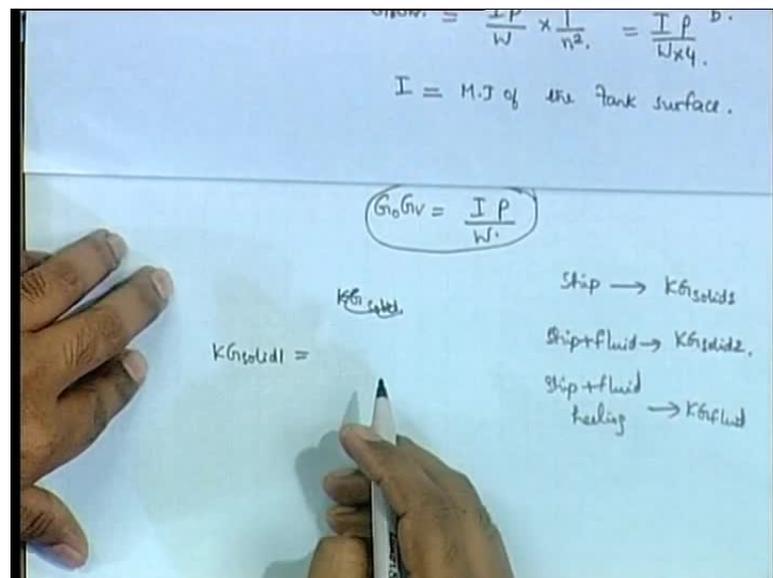
You find the moment of that weight, that is this - just w into KG, so that gives you this moment; then, you are given the weight of ballast; you put here the weight of ballast and you are given the KG of the ballast - the problem says you are given the KG of the ballast; then, you find the moment due to the ballast; find the total moment; find the total weight and that will give you KG as the moment divided by weight; actually, this is the initial moment before the fluid has heeled **as it this is given make.**

It is not specifically told, but you have to assume that otherwise there is nothing in this problem - you have to assume that the ship has heeled; now, this is the KG, in this case the fluid is just added because of it the KG has shifted and now the body is tilted - inclined - and because of because it is inclined, **its KG will move or** I mean its G will move; the free surface effect is happening - of course free surface effect was there before

also - but, it inclined now. Because of this free surface effect what $G_0 G_v$ will be is the distance through which the center of gravity of the ship seems have shifted.

This n square represents the tank - we have already seen how it comes; it is longitudinally divided. We have already told in this example you have a whole tank it is divided into half in the middle, so you have n equals 2 here; so, this is I into row by w into 4 because n is 2 here it is divided into two sections; of course; these are the regions where you have to be careful; you do not find the I of the whole ship, you do not find I of the water plane area of the ship - that is not what this I represents, this represents the moment of inertia of the tank surface; this is the moment of inertia of the tank surface - I will keep it here.

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That is just equal to... we will just do that - I is equal to $l b^3$ by 12, so this will give you the I of that surface; note that b is the breadth of the tank - it is the total breadth of the tank; because, you have a tank like this and it is divided like this and if this is b then this should be b - you have to be careful; all these things... just remember the derivation b by n comes because the whole b has been divided into n , that is why b by n came; therefore, b is the total tank width not the width of the each compartment, b is the total tank width.

Or, you have to do one thing you have to find I of each tank and you sum it up; you multiply with... in this case there are two tanks you find the I of 1 tank you multiplied

with 2 you will get i of whole thing; either way, you do but just remember that this is the I of the tank; so this is I b cube by 12 into 1 by n square; this will give you the I of that system; therefore, $G_0 G_v$ will become this into this value - I into row by W , so this will give you the distance through which the center of gravity seems to have shifted, the vertical distance through which that shifted.

Here, I will make a little change to what I have said before, that is, I said that K_G solid represents the weight of the ship without the fluid - I will make a slight change, it is not without the fluid completely; it is actually K_G with the fluid but without inclining - means, there are the other things also called a K_G solid; I will write it here so that there is a no confusion.

First, you have the ship alone, that is also K_G - it is 1 K_G ; these values will be different, this is also a K_G solid only but let us call it one; then, there will be ship plus the fluid, that is also written here as K_G solid - let us call it 2; its value will change, why? Because, the new weight has been added so K_G will shift down it will shift towards the water - it will shift towards the keel. Then, this is ship plus fluid - heeling, means it is no longer in an upright condition, it has heeled; in that condition, what you get is what you call as K_G fluid.

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The whiteboard contains the following content:

Weight.	K_G	Moment
W	$\checkmark K_G \text{ solid 1}$	\checkmark
wt. of ballast	$\checkmark K_G \text{ solid 2}$	\checkmark
$\frac{\text{wt. of ballast}}{\Sigma \text{ wt.}}$		$\frac{\Sigma \text{ Moment}}{\Sigma \text{ Moment}}$

$$K_{G_{\text{solid}}} = \frac{\text{Moment}}{\text{wt.}}$$

$$G_0 G_v = \frac{I \rho}{W} \times \frac{1}{n^2} = \frac{I \rho}{W \times 4} b.$$

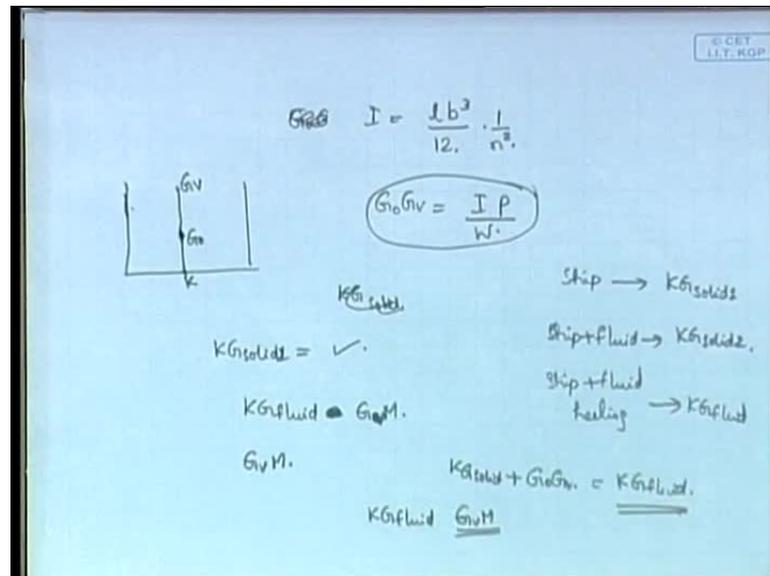
$$I = \text{M.I of the tank surface.}$$

A diagram of a rectangular tank is shown on the right side of the whiteboard, with a vertical line representing the centerline and a horizontal line representing the width b .

Here, you have your initial K_G and now you have your K_G solid 1 - that we have already calculated using that table - using this table we have calculated this K_G this is

KG solid 1; that is calculated. now you need to find KG no this is KG solid 2 sorry I am confusing KG solid 1 is actually just this this is it is just the ship alone the KG of the ship alone without the liquid being added to it is what we calling as KG solid 1 - that is this.

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To it fluid has been added or water has been added at the bottom and it still remains upright - it is not heeling or anything it is remaining upright, but water has been added; then, what happens? Therefore, you have KG solid 2 and then it heels and it becomes a KG fluid; so, KG solid 2 we have already calculated, how will you get KG fluid? Instead of this new G 0 I mean the old G 0 we replace it with G v; you get your... instead of G 0 M which we have - which was initial, it is actually G v M, this will...from this what KG you get is the KG fluid it is not equal to j k j so you use your G v M we have to get.

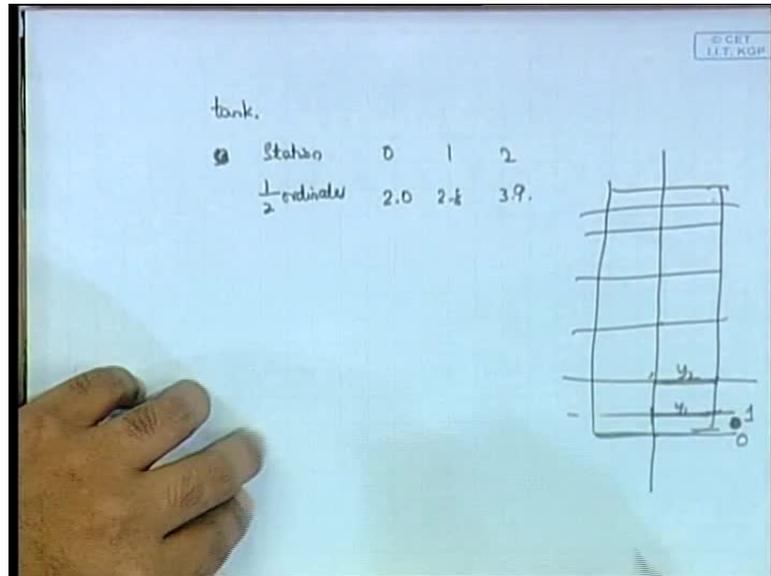
You have G 0 G v, so it is very simple; actually, to that KG solid if you add this G 0 G v that will give you the distance through which KG has shifted up - that will give your KG fluid; it is very obvious.

If this is the ship, if this is your G 0 this is your G v, this is K - to KG 0 if I add G 0 G v you will get KG v which will give you the new position of the center of gravity of the ship, by the presence of the heeling of the fluid.

This will give you a KG fluid, and from KG fluid if you want - if the question is there - you can find G v M; the only thing you have to know that this is the final position of

your center of gravity because of the heeling of the fluid; once you know that, then you just think what is your new GM? What is your new KG? Your new KG is $KG \times$ your new GM is $G \times M$; once you know that you can do this.

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Now, these are the kind of problems that you really come across in when you are doing a real design - you are given that **there is just a double bottom tank** there is a tank; they have given you the half ordinates of this tank - this is the second problem, you are given that **right now we are not having a**; you are given the half ordinates of the tank, so the difference in this problem is that this is not a rectangular tank, so it becomes a little difficult to find the I of the tank probably and it is just the problem in finding the I of the tank; so what they have given is this - for this tank your given the let us divide this in same way as stations and a half ordinates.

That means, you have this whole tank - long tank from the...let us say this is the aft of the ship and this is the forward of the ship this is divided into stations; you are going to divide these into stations; let us start from here itself - station 0 to 1, 2, 3 and it is not a rectangular it is some shape; it is a tank of some arbitrary shape and you are given the half ordinates - the meaning of half ordinates is this these distances are given.

From the center line, you are told what these distances are; remember, it is not a rectangular as this figure shows, but it is a different... y (s) are given y_1 y_2 like this;

these are the half ordinates, so station 0, 1, 2 like that - you are given the different half ordinates.

Your problem is to find the same thing - to find KG fluid; the only additional thing you have to do is you have to find the I of this tank, that is the only the additional problem; actually, I probably need not do this because we have already done a problem before to find the I of a water plane area.

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Str.	$\frac{1}{2}$ ord.	S.M.	Fun. of Area	$(\frac{1}{2} \text{ ord})^3$	S.M.	Fun. (second Moment)
0	✓	1			1	
1	✓	4			4	
2	✓	2			2	
3	✓	4			4	
4						
5						
6						

$I = \frac{b}{3} \sum \text{Fun. } 8j$
 KG fluid.
 $(4) = (2) \times (3)$
 $(7) = (5) \times (6)$
 $\sum \text{Fun. } 4 = I$

That was for a ship, but this... you must be knowing how to find the I of this tank because I have done half ordinate cube, if you remember; I have done a half ordinate cube, then I did a summation of the area of half ordinate cube; if you want I will write the table, but actually we have done this.

This is the way in which we can make the table; this is the function of the area, this becomes a half ordinate cube, **Simpson's multiplier is not get it again** but function of the moment - second moment - you will have stations like this; half ordinates are given for each station - you are given the these values; Simpson's multiplier we know 1 4 2 4 we are using - this is the first rule, Simpson's first rule; now, what is the function of area? Function of area will be just this into this - I will write it if you want 1 2 3 4 this is 5 6 and 7; the fourth column of the table will be second column of the table into third column of the table; we have explained this too many times, I think I will leave it; so, 4 is equal to 2 into 3; similarly, we have also derived the moment of the inertia so I will

say that 7 is going to be a half ordinate cube - that is 5 into 6 - this is again $1\ 4\ 2\ 4$ this will be this thing cube - it is actually 2 cubed; so, your column 5 will be column 2 cube to which is half ordinate cube.

When you do 7 is equal to 5 into 6 and then this will give you some $I(s)$ - function of $I(s)$ then you will have to sum up that I - you will have to sum up that whole thing - and finally here you will get the I to be h by 3 into summation of function of I .

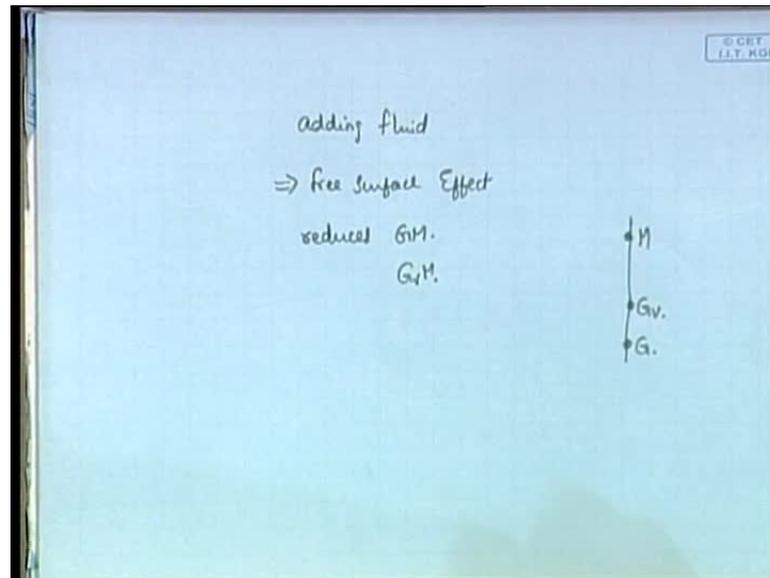
I think h by 3 is the distance between the stations; what you are doing is you are actually summing up the $I(s)$ l b cube by 12 in fact; or, integral of **l b integral** of b cube d l that you are doing for each - you are actually finding each l b cube into d l and summing it up that d l is fixed as h here so that is that h by 3 and that is of course the summation h by 3 into summation of function of this I this will give you the I of the section - I of that tank; other than it is exactly similar as the previous problem, so it is ok, I would not repeat that.

This is the thing you have to do - we can kind of combine the previous section with this section and make it look very complicated - the problem look complicated - but, there are couples of things if you know - like these formulas, whatever I have derived about the free surface effect has to be very clear; that formula should be clear and you should know what each terms stand for and once you know that then, of course... and another thing is you have to know to all this - you have to know how to find $I(s)$; this way of finding is using... not just l b cube by 12 , but finding I in case you are given in an irregular shape.

You have to be able to find the I , you have to be able to find the area, in case of volume - volume, or the displacement; these three things that is all - area, volume and moment of inertia; these 3 $I(s)$ you should be able to calculate it, provided you are given the station and the half ordinate; once you are given the station and the half ordinate you should be able to calculate the I , like this - using this table; once you have the I note that this I goes into that equation that is all.

This final value - it is just one value of I for that whole tank about that central axis; once you have that you just substitute it into the previous equation, that is, I by row I into row by w and the second that is the second part that is of course very straightforward and you get KG for the fluid.

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What you are going to see is that when there is such a case of adding a fluid - adding a fluid will reduce... actually what is going to do is - this is the net result, that is, when you add a fluid to a body in the ship in the form of a tank in such a way that you do not completely fill the tank and leave part of it free, then you end up with the free surface effect and this effectively ends up reducing your GM because your G moves up; it was initially like this - you had your M here, you had your G here; because, of this free surface effect G move to this point G v as a result your GM is reduced.

It is now equal to just G v M, which is less than your GM; so your initial GM... so you have a lesser GM and there is a condition for stability to be lost; one way to change that is partitioning the tanks which will reduce your I which will reduce your reduction in G preventing your G from going too far up.

Just know these things; in a ship you should basically have your center of gravity as low as possible; of course, there is another side to it - actually, that might come in the next class, you cannot have the G too far down there is another problem that will come up; but, in general, from our studies so far you have to conclude that G should be as low as possible; when G is as low as possible GM will be as high as possible and your stability will be very high and you will be you will be in good condition; the ship will be safe.

Thank you.