Hydrostatics and Stability

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Module No. # 01

Lecture No. #04

Numerical Integration

In the last lecture, we stopped with a problem of a cone floating. We had to find the condition for it, to remain in the upright position. So, we used two formulae; we used the formula for the Archimedes principle. Also, we used the formula that GM should be greater than 0, the meta centric height should be greater than 0 and use that as the condition for the cone to float upright.

Now, let us look at another problem, which is again similar. It is another cone, but it is floating in the opposite direction like this.

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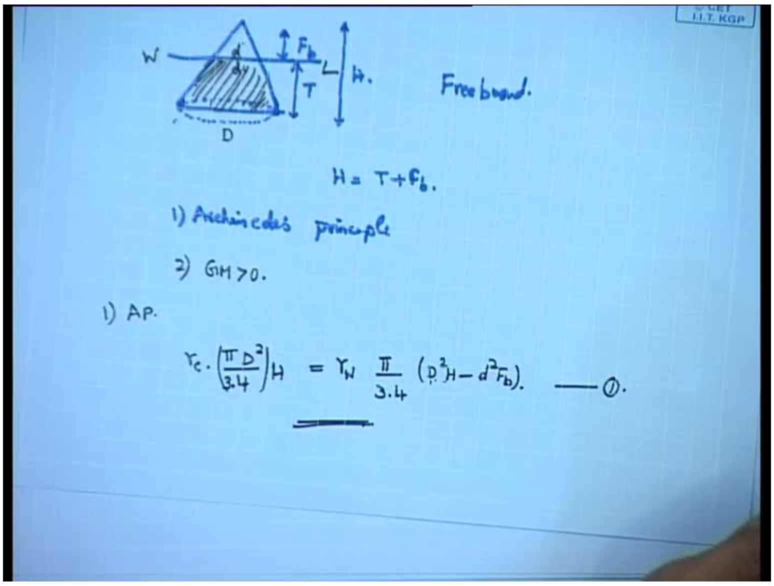

So, the cone is floating like this. The cone is floating in this condition; half part of the cone is outside, let us call the distance as D. This is one, this is other and this is obliviously what we called as the draft T. This distance is put as F b; usually we call the distance outside as freeboard.

The region outside the cone - region outside the water, is always called as a freeboard. So, this is the draft that is T, the whole thing is H - the height of the cone; therefore, you get H is equal to T plus F b. The part of the cone - inside plus outside - is the total height of the cone.

Now, first of all, the question is what is the condition under which this system will float in this? It is upright condition; so what we have to do is, solve for 1 - the Archimedes principle; 2 - we need to solve for GM greater than 0. We usually put GM equal to 0; GM greater than 0 gives you the condition for stability.

So, Archimedes principle given here would be; the first thing, the gamma c - the weight of a cone into pi - this distance is D, this distance is small d - pi D square by 4 into H. This will give you the total mass of the cone. This is the volume of the cone - pi D square by 4 is the surface area of the cone here into H -multiplied by H, gives the volume of the cone into gamma c - that is the density of the cone material, which gives you gamma is equal to weight of the water displaced is equal to pi by 3 into 4 D square H minus d square F b. This comes by - so this is equal to this volume.

This gives you this total volume - the volume of the cone. There is a 3 also here. 1 by 3 pi r square H is the volume of the cone, so 1 by 3 pi D square by 4 H is the volume of the cone. So, this is the total volume of the cone and this is this volume (Refer Slide Time: 04:19).

So, you get same thing, pi by 3 into 4 into D square H, this gives you this total volume minus this volume. This volume is equal to d square into F b, d square is this, d square into F b. I think it is obvious that how this equation has come about. This is the Archimedes principle, the mass of the cone is equal to the weight of the liquid displaced. If it is not clear, again I will repeat this; that is, the total volume under the water - this is the total volume under the waterline, let us call this as water line.

The total volume under the water line is equal to pi is equal to this total volume pi by 3 into1 by 3 into pi D square by 4 H, is the total volume minus the volume of the top portion will give this volume under water, which is what we call as displacement. So, pi by 3 into 4 d square F b, F b is this height - freeboard that is what we call as freeboard.

This is the term not only used for this problem, but freeboard is the common term used in naval architecture. It refers to the region of the ship that is outside the water, so the region that is visible outside the water that region we call it as the F b - the freeboard.

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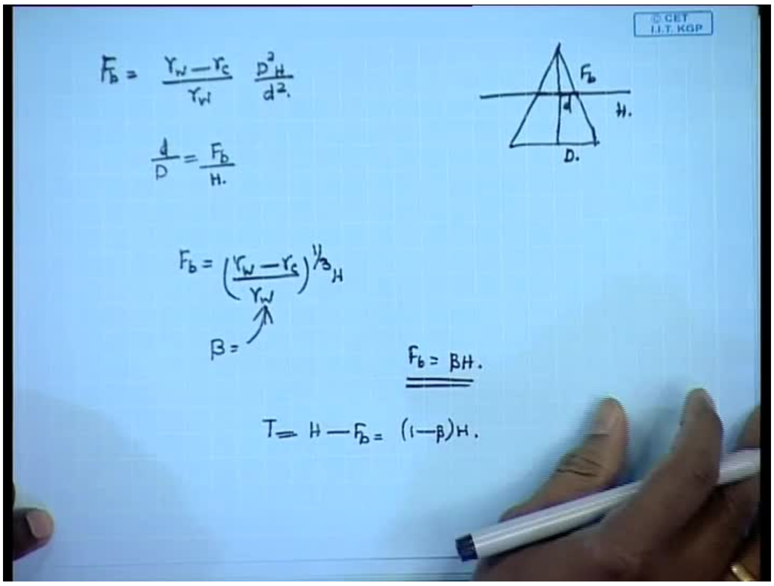

So, this is the Archimedes principle. This is the first equation that we have. So, from this we get F b is equal to - just by doing some arithmetic on the previous equation by gamma w into D square H by d square. Now, if you apply similarity theorem, so this how we have here. This is the water line, so we have here F b; we have here total distance H, d, D, this distance and this distance.

So, if you put similarity theorem, if you use the similarity of these two triangles; that is, this small triangle and this large triangle, then you get d by D is equal to F b by H. This gives you a similarity formula. Combining this equation with this, gives you F b is equal to gamma w minus gamma c by gamma w the whole power 1 by 3 into H. If you just do this, you will get this expression.

Let us put beta equal to this thing - this whole expression. So, this gives you F b is equal to beta H, where beta is given by this parameter that we have given here gamma w -

gamma w is the density of the water minus gamma c - the density of the cone divided by the gamma w hole power 1 by 3 H, so F b is equal to beta H.

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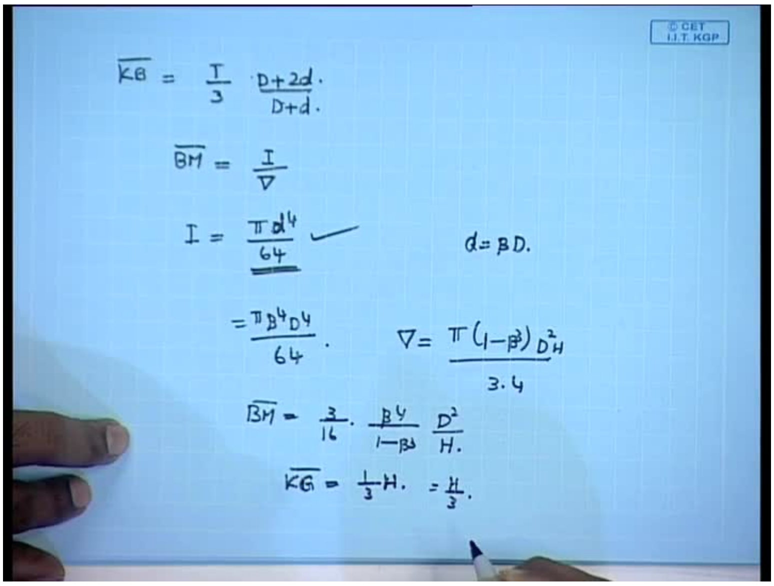

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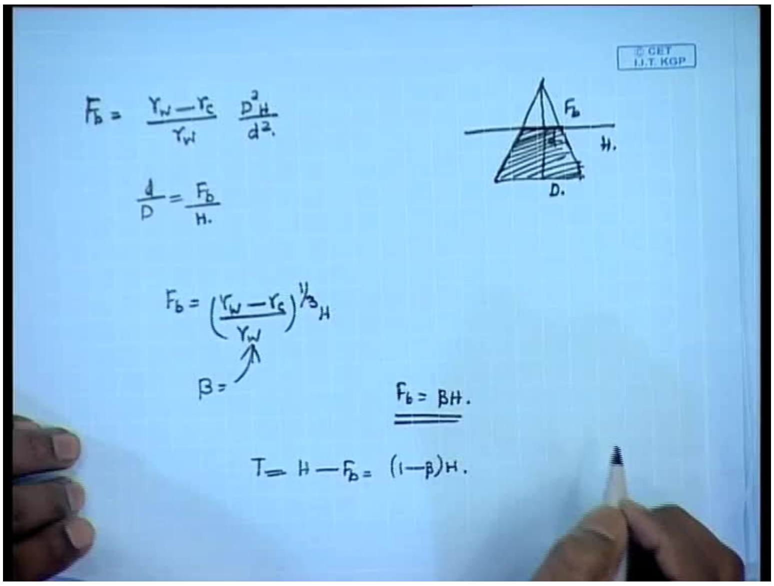

Then draft T is equal to H minus F b is equal to 1 minus beta into H that is the draft of the cone. Then, you will have K B of the cone - K B of the cone is given by (Refer Slide Time: 08:40). This gives you the diameter; this gives the center of the buoyancy. What you really have here is the centroid of this; what we want is the centroid of this region. So, the centroid of that is given by this formula D plus 2 d by D plus d. Now, this is the formula for the centroid of a trapezium. So, the centroid of the trapezium is given by this formula, there is no other way other than to just remember it as such, because centroid of a trapezium we hardly use anywhere, but this is how the trapezium centroid look like. So, you just have to by heart this expression for the centroid of trapezium, we may be using it again and again.

So, just memorize this. So, this gives you T by 3 into D plus d, this gives the K B - the centroid of the trapezoid (Refer Slide Time: 09:56). Then, BM is equal to I by del, this we know. I - what is it? It is a centroid of the water plane area about a transverse axis. Now, the centroid in this case, the water plane area is a circle, because we are taking about the cone. The water plane area - the centroid of the water plane area - the moment of the inertia of water plane area about any axis is given by pi d power 4 by 64. We use the small d and not a capital D, because we are talking about the water plane. Water plane area is the area at the region where the water line axis - at the point of the water line, the area that you have is what you called as water plane area. The water plane areas diameter is small d, so we have pi d power 4 by 64 as I, I is equal to pi d power 4 by 64.

Now, d is equal to beta D that comes from above, therefore this is equal to pi beta power 4 d power 4 by 64. Del is the underwater volume; we are taking about the volume of a part of the cone. You can take that as the total volume minus the volume of the cone at the top, will give you the volume underneath that is equal to pi into 1 minus beta cube into D square H divided by 3 into 4. It is 1 by 3 pi r square H always; that is what we have done here, 1 by 3 phi r square H. Then, we get BM, therefore equal to 3 by 16 beta power 4 by 1 minus beta cube into D square by H.

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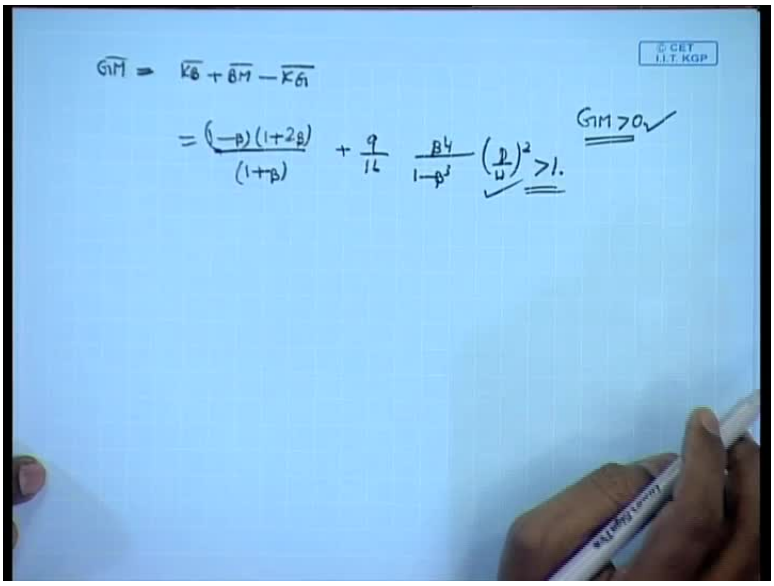

So, it comes like this for the BM, the meta centric radius BM becomes this. Now, the center of gravity of the cone as we know is a center of gravity of the triangle - this whole triangle. The center of the gravity of a triangle is 1 by 3 from the base - 1 by 3 H, 1 by 3 of the height from the base or 2 by 3 of the height from the top. Therefore, KG is equal to 1 by 3 of H or H by 3, so this is the center of the gravity. Now, the resulting meta centric height – therefore, the resulting meta centric height GM is, therefore given by KB plus BM minus KG, this the meta centric height is equal to 1 minus beta into 1 plus 2 beta divided by 1 plus beta plus 9 by 16 beta power 4 by 1 minus beta cube into D by H the whole square greater than 1.

So, this is the condition for GM (Refer Slide Time: 11:15). This is the cone is (( )). So, if this happens by putting the condition GM greater than 0, if you put in this condition, you get this expression. This is the condition for the cone to be stable - so this is the condition for the cone to be stable, so you get an expression like this. So, this is a problem, so whenever when you are asked a problem on how to calculate the problem, how to figure out if something is stable or not, what we need to do is first apply the Archimedes principle and then apply the conditions the GM greater than 0. By combining these two formulas, you will able to get the condition for the object remained in a stable condition.

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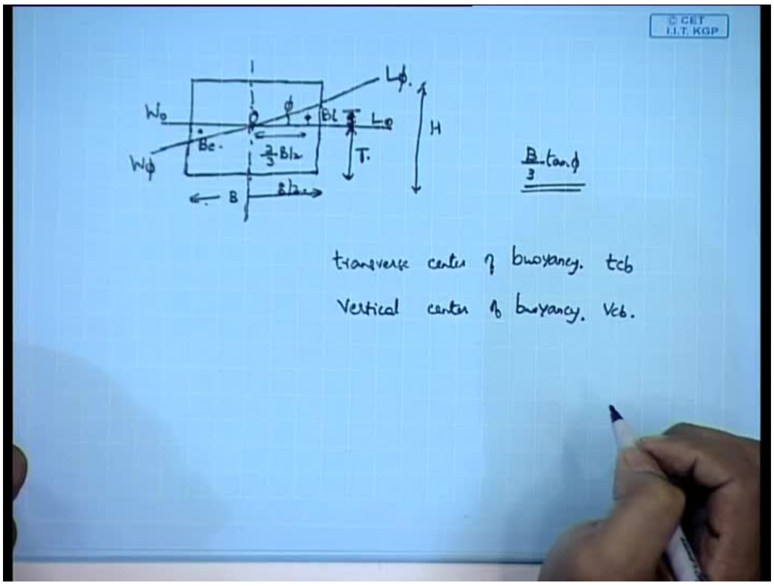

Now, let us consider another problem; that is, there is a parallello pipette - pipette barge - parallello pipette barge, which means this is a rectangle, all such barges are also called as box shape barges. It is like this, so your cross section is a rectangle and anywhere it cross the whole thing is a parallel pipette barge; means whole thing is a box, it is just a rectangular box.

So, initially, this is the water line – WL and W0L0 is the initial water line. When it tilts, it becomes like this - W phi L phi. So, here there is a region immersed, somewhere here. The center of the buoyancy that region B i be immersed. Here is the region emerged, it is known as B e - the emerged region; so, this is an emerged religion and this is an immersed region; so two regions.

Now, from our knowledge, we should know that this region is 2 by 3 B by 2. This whole thing is B, which will imply that this region is B by 2. So, 2 by 3 of B by 2 will give you this distance, which is equal to B by 3, so this distance is B by 3. This height is - this whole thing is H, this region is T, this region is H, B immersed - this height will be 2 by 3 of this whole distance - this distance.

So, this distance we do not know, but we know that is heeled through an angle phi. So, this distance is equal to B by 3 tan phi that much we know - this distance is equal to B by 3 tan phi. If this is B by 3, if you put a triangle here, this whole distance will be B by 3 tan phi. This is how you calculate the center of gravity.

Now, suppose you are asked to find the transverse center of gravity and the vertical center of gravity, the transverse center of buoyancy and the vertical center of buoyancy, which are known as transverse center of buoyancy and the vertical center of buoyancy.

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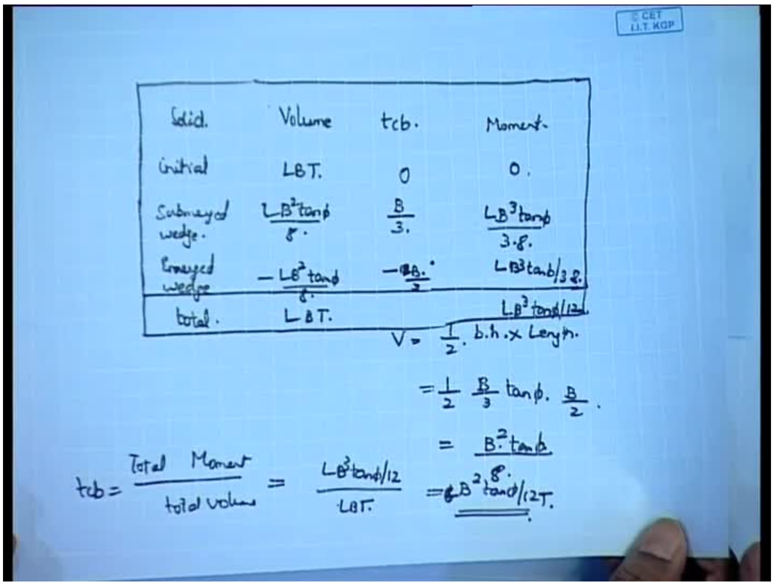

You are asked to find them. In short notation, we write it as tcb and vcb. This figure here will do it, so let us make this table. This is the solid we are talking about, then you will have the volume, then you will have tcb and you will have the moment.

Initially, we are talking about the initial condition that is without any tilting. Initial condition, the volume is LBT - the volume underneath the water line, which we call it as the displacement is LBT, because it is a rectangle, it is just length into breath into its depth, which is T - the draft, so LBT. The tcb, it is the transverse, everything is 0 here. This is considered to be origin, so this 0. Then, there is a submerged wedge, for the submerged wedge its volume is equal to half base into altitude, H will give you the area into length.

So, this will give you the volume of the submerged portion. This will be half base - base is given by B by 3, here you have seen the base is B by 3. Half base into altitude, this base is this; this distance is B by 3 tan phi. Half base into altitude, altitude is again this region - altitude which is again B by 2. So, you get B by half base into altitude - B by 2, B square tan phi by 8 - let us see how it become 8, half base B by 3 tan phi into B by 2.

Well, this problem means, I think they have made a mistake here. Anyway, it is B by 3B square tan phi by 8 LB square tan phi by 8. This is the tcb, is the transverse center of gravity - this distance is equal to B by 3 that is B by 3. Moment is the product of these two, which is LB cubed tan phi by 3 into 8. Similarly, there is an emerged wedge for that the volume is just the negative of this. Whatever is emerged is equal to whatever is emerged, so minus LB square tan phi by 8, this is equal to - the moment is the same LB cubed tan phi by 3 into 8. This is also same minus 2B into B by 3, which is minus B by 3.

For the total, we will have summing this sub, summing the submerged wedge, emerged wedge and the initial. We have the total is LBT, thus summing of the moments we get LB cube tan phi 12. The total moment divided by the total volume gives you the net tcb that is equal to LB cubed tan phi by 12 divided by LBT, which is equal to D square tan phi by 12 into L.

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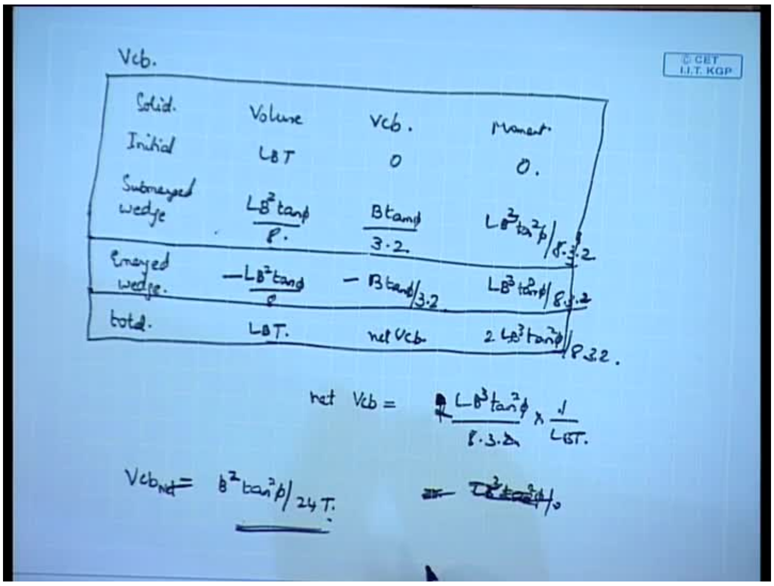

Now, L is cancelled, B square tan phi by 12 into T. So, B square tan phi by 12 into T will give you the net tcb of the whole system, after it submerges and emerges. Now, similarly, we need to find the vertical center of the buoyancy. This is again following the same procedure. This is to find the vertical center of the buoyancy, what we need to do is again solid, volume, vcb of that portion and the moment.

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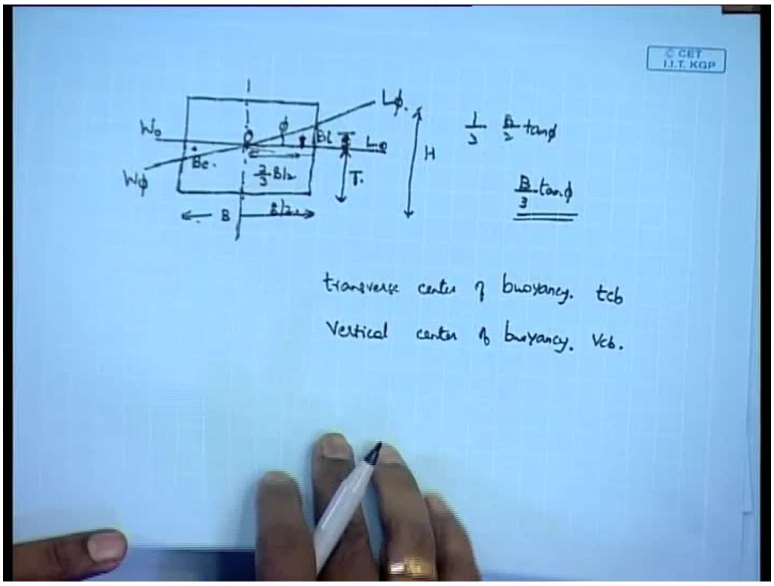

So, you have first the initial; that is always equal to LBT, this is 0, 0. Then you have the submerged wedge, volume is LB square tan phi by 8, then this is B tan phi by 3 into 2. This is the vertical center of buoyancy of the total cone of the submerged wedge; it is the vertical center of the buoyancy of the submerged portion. Vertical center of buoyancy means, this region, this height, this is the vertical center of the buoyancy of the emerged region, it is equal to, as you can see 1 by 3 of the distance from here, 1 by 3 of B by 2 tan phi, this whole distance is equal to B by 2 tan phi. So, this distance will be equal to B by tan phi 1 by 3.

So, 1 by 3, B by 2 tan phi that is what? This is B tan phi by 3 into 2. So, the moment is the product of these two, which is equal to LB cubed tan square phi by 8 into 3 into 2. So, this 8 into 3 into 2, just the product of these two will give you the submerged wedged moment. Then, we will have the emerged wedge; volume will be negative, LB square tan phi by 8. This will be negative B tan phi by 3 into 2, again nothing to explain, because it is just this region - this height. That region is equal to the negative, it is in opposite direction, so it is negative of that, so this moment is again positive LB cubed tan square phi by 8 into 3 into 2. For the total thing, for the total parallel pipette barge volume is equal to LBT. Moment is the some of these two, which is 2; this is twice LB cubed tan square phi by 8 into 3 into 2. The net vcb is given by this moment divided by this volume, which is LB cubed tan square phi by 8 into 3 into 2 into 1 divided by LBT.

This becomes equal to LB cubed tan square phi by, sorry it will become equal to B square tan square phi by 24 T, 8 3s are 24, there is the 2 here, there was a 2 here, so 2 into 2 will cancel. Therefore, this becomes B square tan square phi by 24T. So, this is equal to the vcb of the net system, is equal to B square tan square phi by 24T. So, this gives you the vcb for the entire system, therefore we have calculated the tcb and vcb of entire system.

Now, we have seen, as a result of the movement of that barge - the box shape barge, because of its movement, it is tilted in one side - one side as submerge, another side as emerged. The side that is submerged has a new center of buoyancy, not the center that is emerged, is the center that inside the water, the body that is inside the water. The amount of body inside the water has a center of buoyancy that is slightly displaced from its original position. As we have seen, initially, it was at the center, tcb was 0 and then this system sifted to the new point B immersed. Similarly, a small region came out on the other side, which is equal to B emerged.

So, the 2 Bi's they will be symmetrical, because we are talking about the box shape barges. All the systems we are talking about have this symmetrical inclining and symmetrical healing. As the result of which, what comes out is equal to the volume that goes in - volume that comes out is equal to the volume that goes in, so that is the problem.

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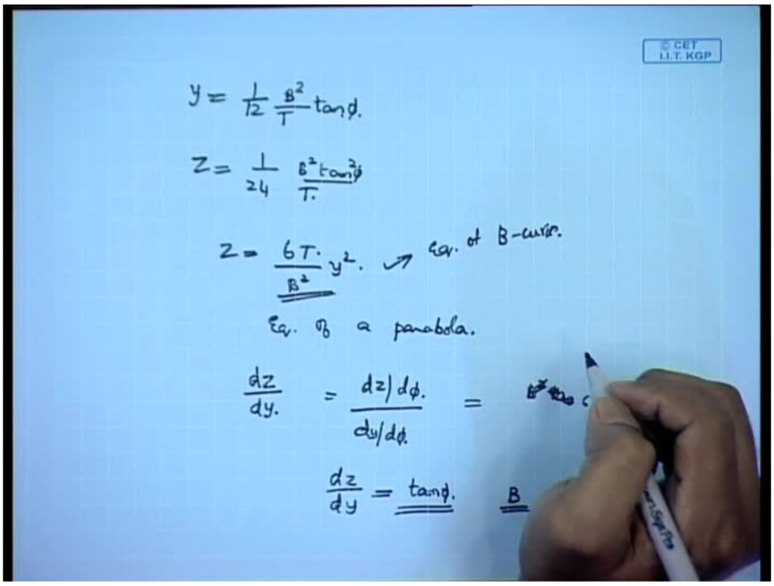

Now, you will get y is equal to 1 by 12 from previously what we have done. We get like this. y of the center of the buoyancy is equal to 1 by 12, the y of the entire system, the centroid of the entire system, the tcb is this; that is the y - tcb is the y and vcb is the z - z is equal to 1 by 24 B square tan square phi by T.

Now, from this if you get right, to get relationship between z and y, we get z is equal to 6 T by B square into y square. This is the equation of the parabola, so this is a relationship between z and y, and it is an equation for a parabola. Now, to get the slope of the equation, we need to get dz by dy, this is equal to dz by d phi divided by dy by d phi that is equal to dz by d phi is equal to B square tan phi - tan phi is cot square phi. So, just differentiate this, differentiate this and differentiate this with respect to phi; tan phi is cot square phi, so like that you differentiate. You will get dz by dy is equal to tan phi, just become equal to tan phi.

So, if this is the equation of the B - dz by dy gives the equation of the B. This is equation of B actually; we call it as the equation of the B curve. Similarly, there are more complicated equations for the M curve, which is the meta centric position. There is the position of the meta center that is given by the equation of the M curve, now we have the equation of the B curve.

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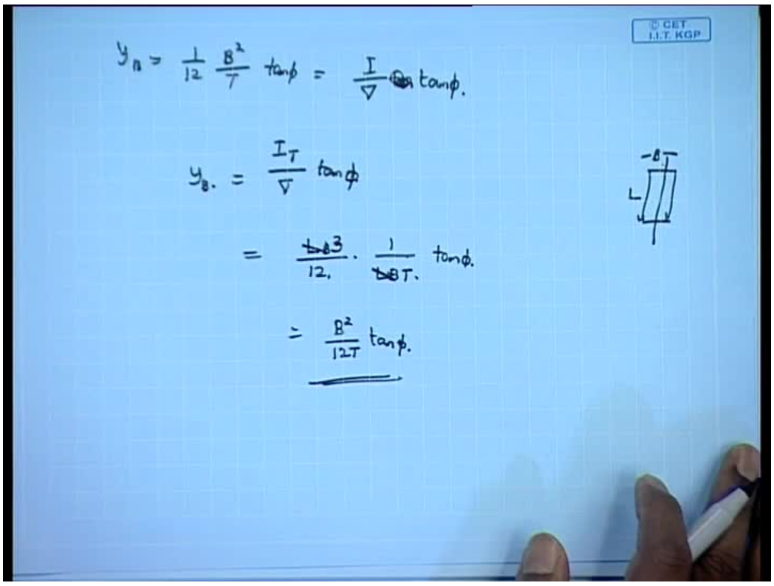

This can also be done using the original formulas that we had like BM is equal to I by del and then x b is equal to i x y by del tan phi, z b is equal to half i - for example, if you use the original equation, you will have y B is equal to 1 by 12 B square by T tan phi. This comes from I by del tan phi, if you have I by del, I mean y B is equal to I by del tan phi that we have seen in a some of our previous derivations. y B which is the transverse point of the center of buoyancy that is given by I by del. BM is equal to I by del and y B is equal to BM tan phi that is how we derived that expression for BM.

Using that y B is equal to I by del tan phi, what is I b, what is I? I is the centroid - I is the moment of the inertia of the system about the longitudinal axis; that gives you the transverse moment of inertia. So, this I transverse divided by del tan phi, this is y B. This is equal to I transverse is equal to LB cube by 12. For a rectangle, if this is B and this is L and if you need to find the moment of inertia about this line, then that equal to LB cube by 12. So, LB cube by 12 into 1 by delta is LBT into tan phi. That is equal to LL canceled 1B is cancelled, so B square by 12T tan phi.

This is what we got in the previous expression also. By the more complicated procedure of finding out the centroid and solving it, we can just use this expression of y B is equal to I by del tan phi. We get the same expression very quickly. This concludes this chapter in this section.

Now, we go in to something else, what we called as numerical integration in naval architecture. We will be talking about the different forms in which we can do integration.

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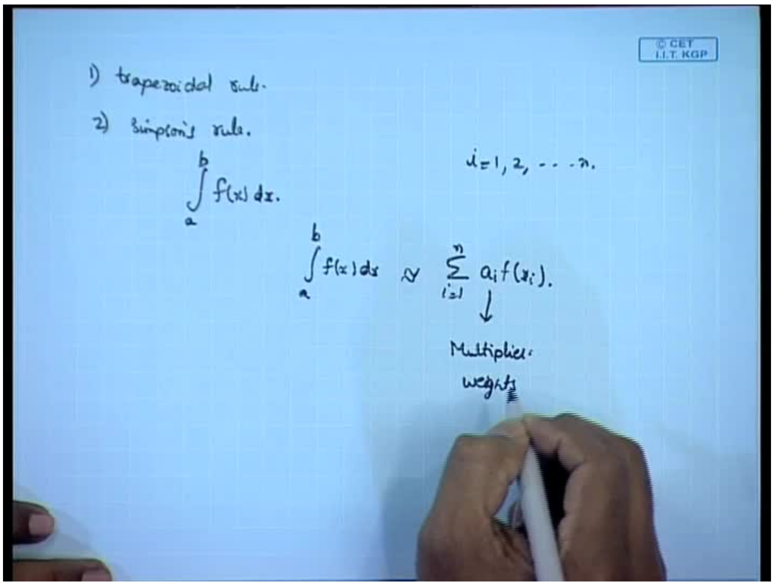

There are two forms generally used in naval architecture, they are known as the trapezoidal rule and the Simpsons rule. These are two methods used in naval architecture; the trapezoidal rule and Simpsons rule. Our idea is to find integral a to b f of x dx. This is our goal to find the integral f of x dx. Now, if you divide the region from a to b into n intervals; that is, i equals 1 2 up to n, then you will get integral a to b f of x dx can be approximated by sigma i equals 1 to n a i into f of x i.

We will see how this is done; that is, how do you get this a i f of x i. Here, a i is known as a multiplier, in some mathematics books they are called as weights. These are different weights that are used for different f of x i, f of x i is the value of f of x at the different i's; that is, i goes from 1 to n and f of xi goes from f of x 1 to f of x n, it is multiplied by a i, a 1, a 2, a 3 like that. These are known as the multipliers or the weights - a i's are known as multipliers or weights.

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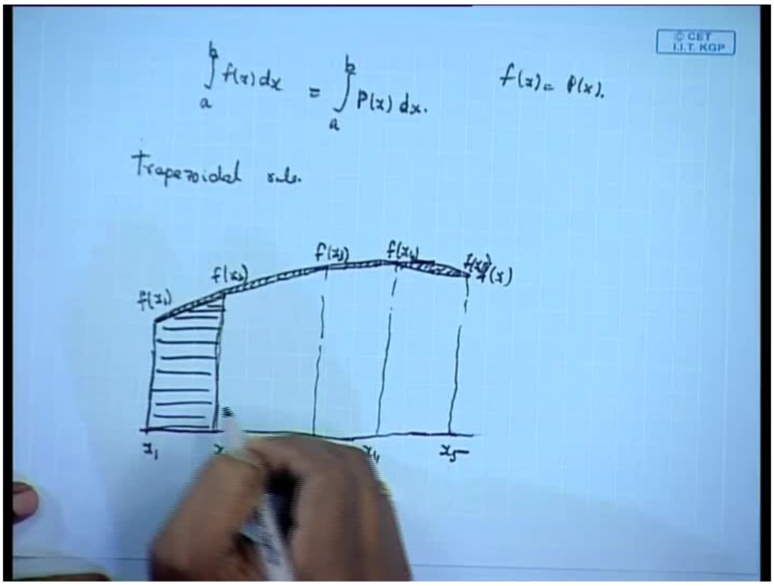

Then, in some cases, what they do is, when they do a to b f of x d x, you replace the f of x by a polynomial p of x. Therefore, f of x dx will become equal to p of x dx, where p of x is equal to the polynomial that substitutes for f of x, so this comes here. Now, let us do the trapezoidal rule. Consider this equation, f of x and these are different x as x 1, x 2, x 3, x 4 and x 5. These are the different x i values x 1, x 2, x 3, x 4, x 5 and it has the corresponding values f of x 1, f of x 2, f of x 3, f of x 4 and f of x 5. These are the different values of the f of x, suppose we replace this f of x by straight lines like this, like this, like this, like this and like this (Refer Slide Time: 39:40).

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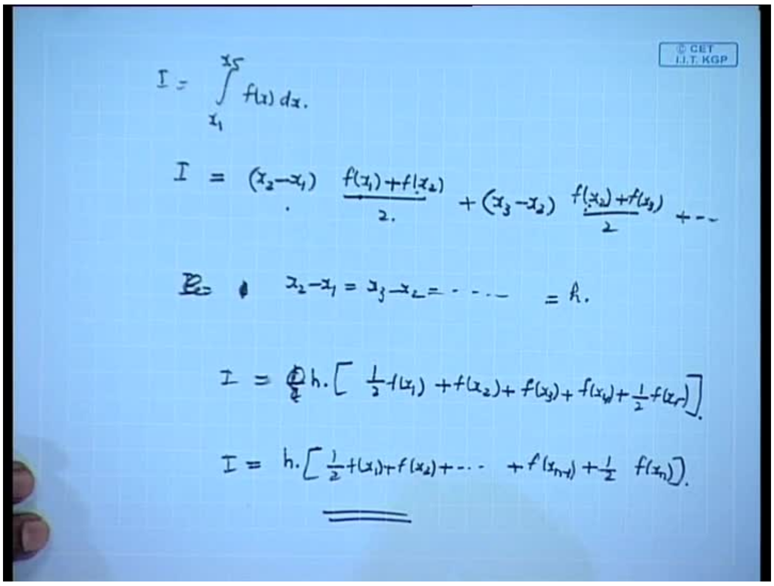

This f of x value have been replaced by straight lines and these straight lines represents the values of f of x; that is, this f of x is now replaced by these straight lines, so what you get? I, if it is an integral, we can always say that I is the integral from, what we are trying to do is integral from x 1 to x 5 f of x dx; this is what we are trying to do.

Now, this I is equal to - we always know that the moment of I integral is some of the area under the curve. We know that integral is always given by the area under the curve, so what we want is the area under this curve. As I said before, we have replaced f of x by these straight lines. Therefore, we can say that I is equal to the area under this curve, a small region here is left out, which is the error in this calculations. This whole area left out, this is the error in our simulations; that is, we try to minimize it, but becomes like this x 2 minus x 1(Refer Slide Time: 41:00).

So, we try to find area under the curve, it is the area of these one, two, three, four – four trapeziums. Our area under the curve is the area under these four trapezium; this is one trapezium, this area under this trapezium, so it is given by x 2 minus x 1 into plus x 3 minus x 2 into f of x 2 plus f of x 3 divided by 2 plus so on (Refer Slide Time: 41:35).

This gives you the area under the curve. This is the area of the first trapezium, this is the area of the second trapezium like that you have areas of four trapeziums. First of all, let us assume that x 2 minus x 1 is equal to x 3 minus x 2 is equal to - all of them are equal to h. Therefore, we get this whole expression, I is equal to half - coming from here half H into or H into half f of x 1, there is an f of x 1 here and f of x 2, there is f of x 2 here, so half f of x 2 plus half f of x 2 gives you one full f of x 2 plus f of x 2 plus f of f 3 plus f of x 4 plus half f of x 5 (Refer Slide Time: 42:15). This gives you the area under the curve.

Similarly, if you have n such numbers, then I is equal to H into half f of x 1 plus f of x 2 plus so on till f of x n minus 1 plus half f of x n. You will get an expression like this, this is an expression for trapezoidal rule, which is followed in naval architecture (Refer Slide Time: 43:30).

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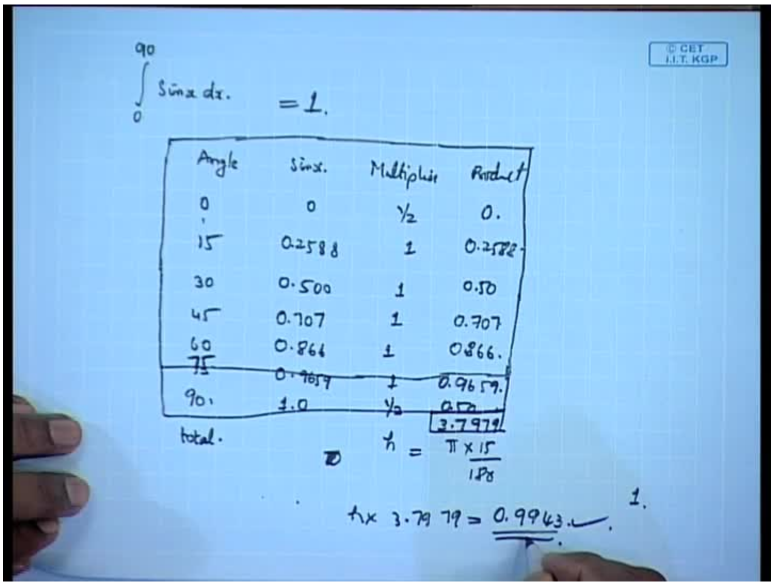

For instance, you might be having a problem like finding integral 0 to 90 sin x dx. Suppose, we have a problem like this, then this how we solve the problem according to the trapezoidal rule. The analytical form of the solution would be equal to 1. As you can see it is cos x, it will become minus cos x, it will become cos 90 that is 0 minus minus cos x, so 0 minus minus1, which is equal to plus 1. 1 is the answer to this problem; it is 0 to 1 x dx. If you have to do it using the trapezoidal rule, we do it like this angle; this is sin x multiplier product.

First of all we will divide this in terms of 15 degrees, so 0, 15, 30, 45, 60, 75 and 90. This will be the region in which we divide; so sin x, 0, multiplier is half product is 00.2, sin 15 is 0.2588, this is 1, the is 0.2588; sin 30 is 0.50, this is 1, so this is 0.50; sin 45 is equal to 0.707, this is 1, 0.707; sin 60 is 0.866, 1, 0.866, sin this is 0.9659, 1, 0.9659; sin 90 is equal to 1, this is half, so it is equal to 0.50. This gives you the whole list of terms, so what do you have? Your expression is - first of all how much is this H? We have to figure out H first, it is uniform; H is equal to pi into 15 by 180. We can see here we are doing it in terms of 15 degrees.

Pi is 180 degrees, so this 15 degree interval is equal to pi into 15 by 180. Once you have that H, you have the sum of the total. The total comes to 3.7979, so H into this total 3.7979 that comes to 0.9943. This is the expression for the integral of sin 90; for integral of sin x between 0 and sin 90 degrees, this is the expression, this is the answer. Comparing to one, we see that we got fairly very good answer; this is how you do it. Similarly, when you do it, you always be having an error in doing the calculation. There is always an error in it, but I will skip over the region of error, because that is not used mostly in our studies. In this course, we are not going to use that but understand that remember that definitely there is always some error associated with this.

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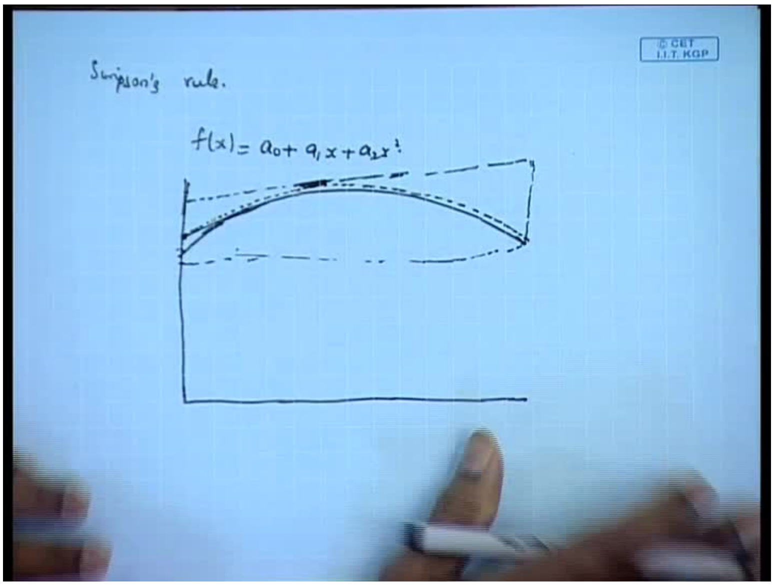

The next is another method that is known as the Simpsons rule. What we do is we replace f of x by a 0 plus a 1 x plus a 2 x square that is a parabola; so the situations will look like this. This is equal to your f of x, this you are replacing with the parabola - proper parabola. Therefore, we have here a parallelogram, it actually comes like this. Actually this figure has not come out properly, tried further.

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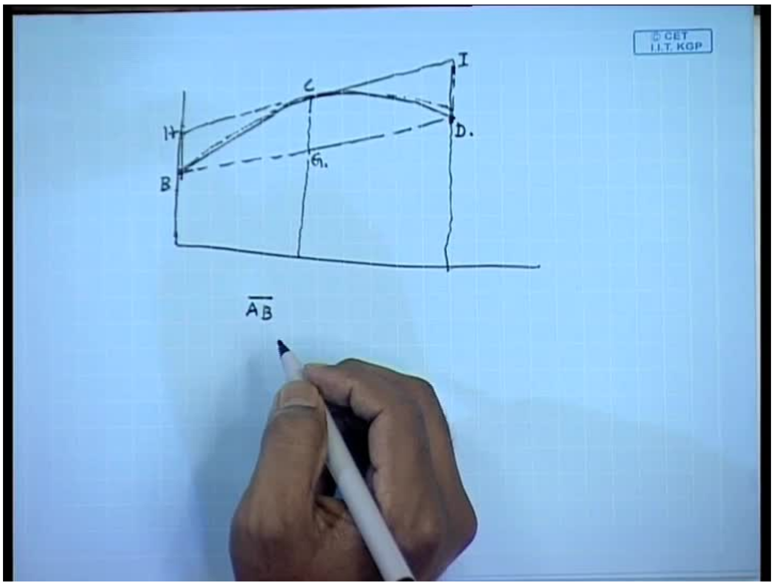

This is your f of x; we draw a parallelogram like this. This distance is equal to this distance; it is a parallelogram and the whole thing is replaced by parabola. The system is as if we have a parabola inscribed in a parallelogram; that is what this thing is about (Refer Slide Time: 50:20). Here, you have this B, H, C, G, I, D. You have AB. Actually this is the slightly long derivation. As we have fallen out of time, we have only 5 minutes left, I leave this derivation for next class; we will do this integration in next class. So, I will stop here now; thank you.