Hydrostatics and Stability

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Lecture No. # 39

Wave Theory

So, welcome to the lecture thirty-nine of the series. We are winding up the series of lectures now, so this is the 2nd last lecture on the series. We were talking in the last class about wave effects, we are, we started this, the topic of today's lecture we did start in the previous lecture, we discussed some preliminaries of waves; we were discussing how we deal with waves.

The main effect of waves on ships is this, that is, the waves have the effect on ships, why are we studying waves? First of all, mean, what is the purpose of waves as such, I mean, we do not study so much currents on the, on the, on, on this hydrodynamics as much as waves because waves have the property of producing a force and the movement on the ship. So, when, when a ship encounters a wave, it is subjected to a force due to the wave. And an encounter due to the wave, mostly the force due to the pressure distribution around the hull is varying because of the presence of waves and this, and this force and moment produces its own motion on the ship. As you know, any force or any movement will produce motion force, will produce a translatory motion and moment will produce a rotational motion. Now, this is seen to happen in the ships. So, because of this we study waves.

So, in the last class we have seen, we have seen some of the basic definitions of wave. Basic assumptions, first of all we saw, that the fluids are all incompressible, invisit, irrotational fluids, fluids with 0 vorticity; there is no rotation for the fluids. Then, then we, also, what is known as the kinematic free surface boundary condition, the boundary condition, which says, that the velocity of, velocity with which the curve is changing. So, this curve, it is actually moving up or down the velocity, which that curve is changing the rate of change is equal to the velocity, which with the fluid particle is also moving because the fluid particle is moving because of change in the curve, because the wave, because of the wave of fluid particle is moving and therefore, dou psi by dou t, where psi is surface elevation. So, dou psi by dou t is equal to dou phi by dou z, where phi is the velocity potential and dou 5 by dou z represents w, the velocity in the vertical direction. So, they are balanced and this is the primary condition known as a kinematic free surface boundary condition applied.

Now, the 2nd boundary condition, that is usually applied to the free surface is known as the dynamic, remember that, note that now itself, that there are 2 types of boundary conditions usually applied on any problem, we call them as the dynamic or the kinematic.

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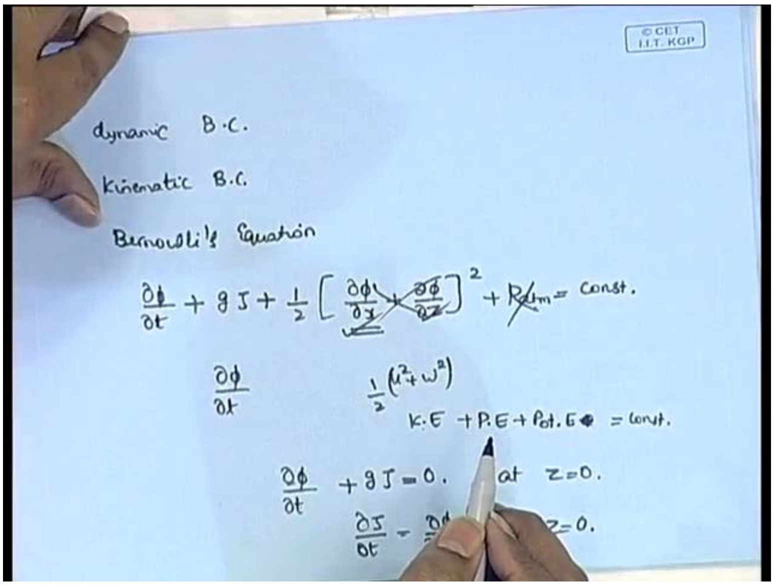

So, you can have dynamic boundary conditions or you can have kinematic boundary conditions. These are 2 types of boundary conditions, that are seen to occur, that has found to happen in a real ship and these are the, kinematic boundary condition is the, kinematic boundary condition… Now, the word kinematic always refers to velocity, that means, we, when, whenever we say some, something about kinematics, it is the description of velocity, kinematics the word is description of velocity, it is the study of velocity. So, kinematic, kinematical boundary condition is a boundary condition on velocity.

The other is the dynamic boundary condition. Dynamic boundary condition - that word dynamic stands for force or pressure. Pressure is force per unit area, so it is also equivalent pressure in this sense. So, the dynamic boundary condition is a boundary condition on pressure. So, there are 2 types of boundary conditions. We give boundary condition on pressure and we say, on the free surface and boundary condition on the velocity, these are these are 2 parameters we give, we have to solve.

See, at the end of the day we have to solve for 2 main things, one is the velocity distribution around the anything, any, around a ship or anybody or a floating structure. The 2nd is to study the pressure distribution around the hull. So, these are 2 problems that finally, we end up solving. So, the boundary conditions given are the pressure boundary conditions and the velocity boundary conditions. So, you have the dynamic and the kinematic boundary conditions.

Now, the dynamic boundary condition is usually given in terms of, you, I am sure you must have heard by now what we call as the Bernoulli's equation. So, there is a very famous and very widely used equation, which is known as the Bernoulli's equation. Again, note that Bernoulli's equation is assumed for inviscid flow along a streamline, where laminar inviscid flow. So, whenever, the moment the flow turns into turbulence or it becomes the turbulent flow, there is no, there is no more Bernoulli's equation. So, the Bernoulli's equation states, that dou phi by dou t plus g psi plus half…

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Now, this is the simplest formula this is Bernoulli's equation. Bernoulli's equation states, that the pressure plus dou phi by dou t plus g psi, we are talking about. Now, this is applied to the free surface now. Why is it P atmosphere? Because at the free surface, the water is exposed to the atmosphere, so it is a, pressure becomes atmospheric pressure and in many cases, and in most cases we said, the atmospheric pressure equal to 0, which is known as the gauge pressure; we said it equal to 0 atmosphere has a gauge pressure of 0. So, this is does not exists, so this becomes 0. Therefore, our equation becomes just dou phi by dou t. Now, what is this? dou phi by dou x is used, this is just u square plus w square, half of u square plus w square, so this is the kinetic energy of the particle.

So, what we are saying is, that the different forms of energy, so the kinetic energy plus pressure energy plus potential energy plus, this is also some kind of a dynamical energy, this, the total energy is a constant, it is, it is actually a kind of energy conservation (( )).

Now, pressure is, we call it as pressure energy, so thus sum total becomes constant, which is the Bernoulli's equation.

So, in this case, we, as I said before, we, we are, we do most of the problems in the small amplitude wave theory. There is case where you have the amplitudes much smaller than the wave length, in those cases, in the small amplitude wave theory, we say, that the velocities are fairly small, these are velocities u and v, are fairly small. So, u square and v square are, u square and w square are much smaller. So, this also, can be in general neglected. So, the equation becomes dou phi by dou t plus g psi equal to 0 becomes the equation. This becomes, the, at z equal to 0, this becomes the, this becomes the Bernoulli's equation on the free surface.

Now, what we can do is, we can combine this equation, dynamic boundary condition with the kinematic boundary condition, which we just derived. So, what we do have, we have now 2 equations. So, we will do one thing, we will differentiate this with respect to time, dou psi by dou t comes dou phi by dou dou square phi by dou t square comes and so this dou phi by dou t here, we will replace it by dou phi by dou z.

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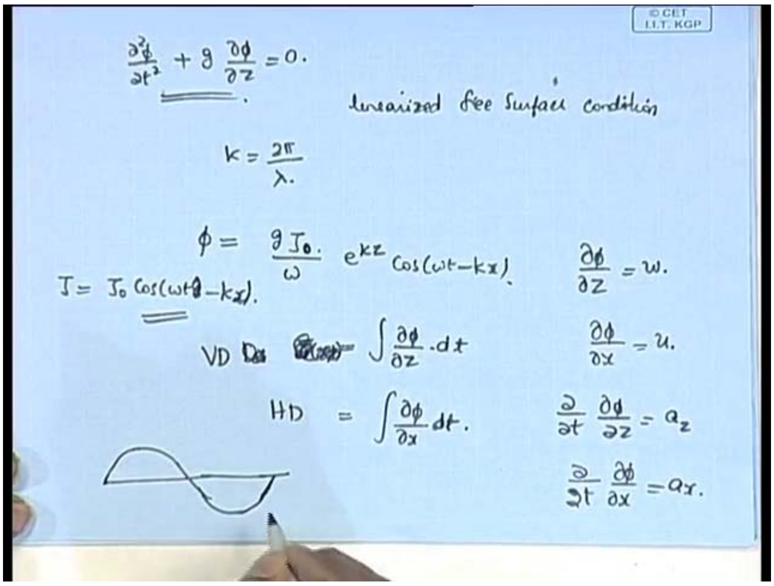

And therefore, you come up with the equation, which says dou square phi by dou t square plus g dou phi by dou z equal to 0. This is known as the linearized free surface boundary condition. So, this we call it as a linearized free surface condition. So, this is the, this is the very important equation, it is, it is important. And just for information sake we call something as the wave number; we defined as 2 pi by lambda. If lambda is known as the wave length, it is the wave length of the wave, then k equal to the wave number is equal to, defined as 2 phi by lambda, it is the wave number of the wave.

Then, the solution, now suppose you solve the Laplace equation with the different boundary conditions. So, if you solve the Laplace equation with the kinematic boundary condition, dynamic boundary condition. Now, if you can also apply a bottom boundary condition, a bottom boundary condition will whole, that the normal velocity across the bottom is 0, you know that when there is a fluid, when there a solid, that is, when there is a fluid flowing over a solid, the velocity normal to the body will be 0, at the body.

So, because it cannot penetrate, it is the impenetrable condition. So, that condition states that. So, at the boundary, bottom boundary, that condition holds, that is one then. So, that is known as a bottom boundary condition.

Then, there is a dynamic boundary condition, free surface boundary condition; there is a kinematic free surface boundary condition. Now, with these 3 boundary conditions and the Laplace equation, if you solve, you will end with a solution for phi, which says, that phi is equal to g psi by omega psi 0 by omega e per k z cos omega t minus k x. So, this is the equation of phi for a wave, which is travelling in the x direction and it has a frequency of omega. So, this is the expression for phi of that wave, the potential of that wave, provided these boundary conditions hold.

So, what have we done? We have just solved the Laplace equation subject to the boundary conditions, dynamics free surface, kinematic free surface, bottom boundary. So, when you solve this equation, you get an equation for phi as this. So, what you see? First of all we see, we can get many things, like when you do for instance dou phi by dou z you will get the vertical velocity of the particle inside that wave; dou phi by dou x will give the horizontal part velocity of the particle inside that wave.

Now, dou by dou t of dou phi by dou z will give the vertical acceleration, acceleration in the z direction. dou by dou t of dou phi by dou x will give you the horizontal acceleration integral of dou phi by dou z dt, it will give you the, the vertical displacement psi at any point. psi at any incident of x and time will give you the dou phi w, it is integral of w dt, which is the displacement in the, displacement of the particle inside. It is not this psi, I will just let us call it as displacement in the vertical direction or I will call it V, vertical displacement. So, the vertical displacement will be given by dou phi by rho z dt.

Similarly, horizontal displacement will be given by integral of dou phi by dou x dt, this gives you the horizontal displacement, vertical displacement. So, different things can be calculated, as you can see, just from this equation for phi, psi 0 is the maximum amplitude. So, that means, how does psi varies? Psi, so psi varies as psi 0 cos omega t minus kx.

So, this is the elevation, how the elevation varies? So, this is like this. So, this represents a wave form elevation, which is varying in the sinusoidal fashion and it is travelling in the x direction. So, psi is equal to psi 0 cos omega t minus kx, this is the equation of the sea surface and phi is given by this expression.

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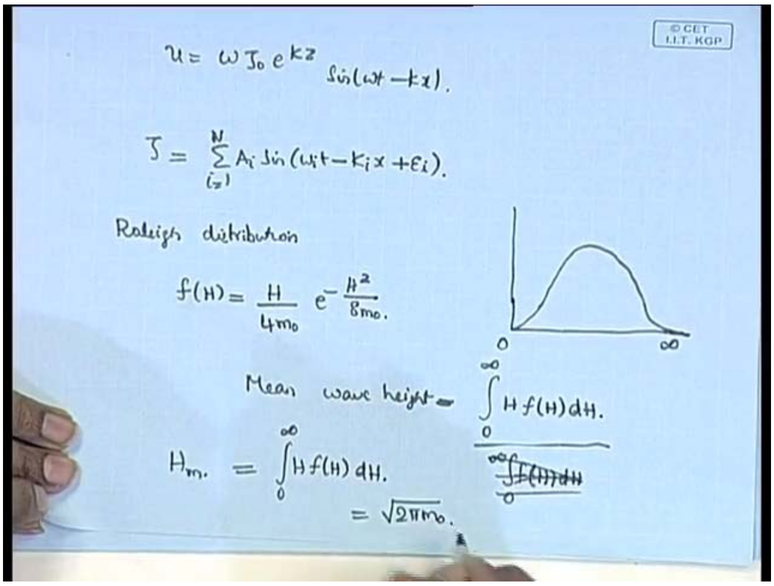

So, this represents the wave as such and if you, the same things, which I said, if you do you will get, that u is equal to, anyway this is not that important. So, u becomes this, you will get it by doing dou phi by dou x will give you the particle velocity. Now, you will also see, that the speed of the wave or the, what we called as a celerity of the wave. In this case, when you do here, you will see that it becomes a function of the wavelength of the wave. So, the wave celerity or the velocity with which crust is travelling will be seen into, be a function of the wave.

So, these waves are, will be a seen to be a function of the wavelength of the wave and these kinds of waves are called as dispersive waves. And on the other hand, there are some types of waves, like the sound waves, which are acoustic waves, which are not really dispersive and these waves, which are the surface waves are all dispersive. These gravity waves or free surface waves, they are all dispersive waves. Then, this is the some basic introduction to waves.

Then, now we have in the ocean, what we call as real sea. When we come to the real seas, we have combination of waves, we do not have a wave with what we had, that phi was for one wave with one frequency, one wavelength. So, what we end up with in the sea at any point in space is actually a collection of different waves, it is the, it is a sum total of different waves of different frequencies and wave lengths. And therefore, we can write them, the net, the net psi will be actually a sum total of different waves, means, they, these waves.

Now, we come to another term, which is known as phase of the wave. So, if the waves can be in the same phase, they can be in different phases with respect to each other, so it is usually when you talk about different waves, that the concept of phase becomes important, that is, the phase difference between the waves. It affects the waves in which the two waves interact. So, this total psi, the total displacement becomes the sum total of the displacement due to the different individual waves, which have different, which has different frequencies and different, different wavelength.

So, in general, in the case of an ocean we can think, we can talk of different types of distribution for usually we say that, we say the distance between the crusts of a wave, we call as the distance between the crust of a wave and the trough of a wave as the height of a wave. So, there are different formulations for the height of the wave, we can represent the height of a wave as a probability spectrum like this. So, if you represent the, it is as a Rayleigh function, it will go like this, this is how a Rayleigh, Rayleigh probability distribution looks like, so Rayleigh distribution looks like this. You can say, that the heights of waves in, in general we are talking about a sea, in open sea in general, the heights of the waves are usually represented by this, H by 4 m0 into e per minus H square by 8 m0. So, this represents the distribution of waves as the function.

Now, this is the probability distribution, so this distribution is known as a Rayleigh distribution; it is a mathematical distribution. So, H is the wave height. So, the mean wave height, if you want to find in case of, if you have a distribution like this and if you are fond to find the mean wave height, we say, that it is given by, mean wave height is equal to, so this is the real waves from 0 to infinity, so 0 to infinity, 0 to infinity H f of H dH, it will be like this.

Now, you know, that this is the integration of the probability density function between 0 and infinity and what is it? It is equal to 1 because the probability is, it has some value H between 0 to infinity is 1. So, between 0 to infinity, so it is integrated over this whole thing. The area under the curve is equal to 1 and between 0 and infinity. So, this is, so you remove this, so this becomes, the mean wave height is given by H f of H where f of H is the probability density function into dH, integral of this between 0 to infinity will give you the mean wave height, which we can call as H mean. This is a mean wave height of the, of a general ocean.

So, the mean wave height is given by this function and this by some method, it will come down to this. If you just do this for that particular Rayleigh distribution, this will come down to root 2 pi m0.

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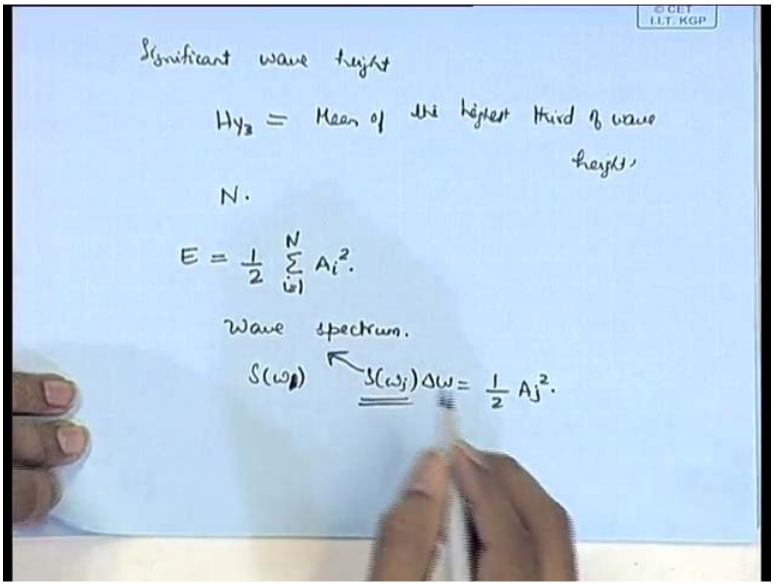

Now, there is also something defined as a significant wave height. Significant wave height is defined as, is root, is usually written as H 1 by 3, it is written as the mean of the highest 3rd of wave heights. So, as this definition itself says, you take the top 3 wave heights, you take a mean of that; you call that as a significant wave height. So, that, that is the very important term used in used in this wave hydrodynamics, you will use the, you will see the term significant wave height used very frequently. So, that is an, one important term.

Then, now suppose you have, suppose you have a wave, which has N components, means you have a total of N wave of N frequencies, N waves of different frequencies and corresponding wave heights and different wavelength, everything different.

Now, the energy of these, the total energy is given by, total energy of these wave heights, waves, the N waves, these N waves, the net is given by i to N A i square. I mean, what you need to know is that the energy of a wave is proportional to the amplitude square. Amplitude is the, it is like the wave height, it is half the wave height, that is the meaning, that is the definition of amplitude. So, the distance between the mean to the crust or from mean to trough is called is an amplitude. So, half A i square will give you the energy of that wave per unit area and therefore, you sum them up for all the N components, you will get the total energy of the wave.

Now, we also defined something known as the wave spectrum. It is a, wave spectrum is usually defined like this as the function of omega. So, you are now having a number of waves, let us say N number of waves, this is the, so the energy, energy spectrum is given by wave spectrum S, which is the function of omega, S of omega will be, S of omega into delta omega is equal to half A j square. Therefore, a wave spectrum is in fact, a kind of the, is that a, it is really a spectrum of the wave energies. So, what we are actually floating is wave energy and this, this S of omega is known as a wave spectrum; this is known as a wave spectrum.

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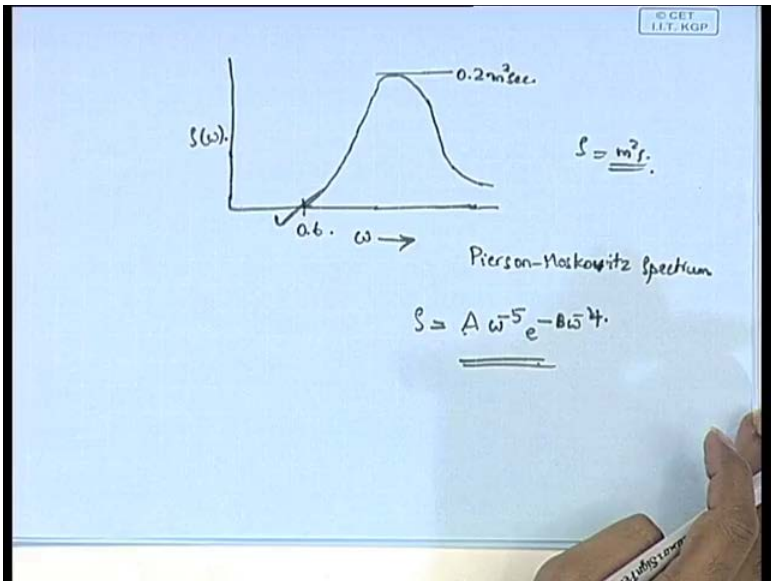

And there are different, so you, usually you will see wave spectrum plotted in this way. So, there are different types of wave spectrum plotted in this wave omega, it will be plotted as omega versus S of omega.

So, this is the wave spectrum, will be a function of an omega, you will have a spectrum like this; you will have spectrum like this. So, this is a, this is a wave spectrum, possibly there are different types of wave spectrum for the real seas, the, for the seas, that are found in real practice. We, there are different types of wave spectrums, that is, energy density functions.

Now, one of, some of the famous one or one of the most commonly used one is known as a Pierson-Moskovitz spectrum and there is also one as the Johnson spectrum. We will see here one example of; this is a Pierson-Moskovitz, Pierson-Moskovitz spectrum. So, this is an example of one of the spectrums, Pierson-Moskovitz, it is v, Moskovitz, so v, Pierson-Moskovitz spectrum. This is one of the wave spectrums that are seen in practice in the ocean. It is usually given by the wave spectrum, means A omega per minus 5 e per minus B omega per minus 4.

So, this is the, so this is the expression for the Pierson-Moskovitz spectrum, where A is a constant, B is the constant, omega is the frequency of the wave. And so, as the function of this wave frequency you will have S, the wave spectrum and therefore, you have the total S, where this omega, you will have a curve like this, this will give you this Pierson-

Moskovitz. This occurs at round 0.6 and maximum comes to around in or is comes to around 0.22 meters, 0.2 meters square seconds. The unit of S is meter square seconds.

And so, this is one example of commonly found seas, sea state in the ocean. Now, this is some exposure to, so here we have given some exposure to the waves, very basic concept of waves, the different types of wave spectrums found. What is a wave spectrum? Then, then about the different principles of wave, that is, what is potential Laplace equation. We have seen some basics of waves; this should help you in doing some stability analysis. Since the course is on hydrostatics, we are, and stability concepts, these wave concepts, with these wave concepts will be helpful for you in analyzing the stability of ships on waves. So, you apply, the ships, the ship is now seen to float on the ocean and we apply these concepts there now.

So, actually, before winding, we are just mentioning some of the alight topics, means, the topics, that are close to stability or those are the factors, that effects stability and so in addition to the factors, that affects stability, we will also mention some of the effects of stability, like how does the stability calculation, then translate into some other computations in the ship, that is another thing we, we will take a look at. Since we have only 1 more class left, we will, we will be looking a little bit into that next class.

Right down we have seen what is the, what is the waves? Now, these are, waves are some phenomena, that definitely effect the stability, it is not affecting hydrostatics as such, it is more to do hydrodynamics, that is, the study of the movement of ships in response to or the, how the hydrostatic parameters of the ship, like the GM or even the different KM, it, how, how all those things vary as the response of the ship to waves; as the response to waves, how these parameters vary, these are all important things.

Now, let us go into some, something else, means, now we have seen what are waves; we have basically seen some, something about waves. Then, let us see, what is the effect of waves on ship before we go into stability itself. There is something else, that is, how does a wave directly affect a ship? Now, we have seen waves produce motion of the ship, we have already said, that waves produce forces, wave produce moments; as a result of forces translation occurs, as a result of moments rotation occurs. So, two types of activities occur here as the result of the wave on, effects of waves on a ship and the effect the different wave forces, you know, the force, types of effect of, I mean, the different types of forces exerted by waves on ships. We come to different types of forces (( )) that is outside the scope of this course, we will not go into that, we will just, we are just winding up here with some, some of the properties of wave induced moments on ships.

Now, as a result of these wave motions, it does not have to be as a result of wave motions (( )). So, some moments are there in the ships. We say that a ship has 6 degrees of freedom. Now, by degree of freedom the name, it is the, as the name itself suggest it as the freedom moved in that direction. So, there are 6 degrees of freedom, we, it can move like this, it can move like this, these are all translation motion. So, can move like this. So, you have 3; 3 translational degrees of freedom. Then it can rotate like this, it can rotate like this, it can rotate like this. So, it can rotate about the x-axis, it can rotate about the y-axis, it can rotate about the z-axis. So, there are become 3 rotational degrees of motion. So, 3 translation plus 3 rotational, there are 6 degrees of freedom for a ship. These motions have, these are hydrodynamic terms and these motions have their own names: surge, sway, heave.

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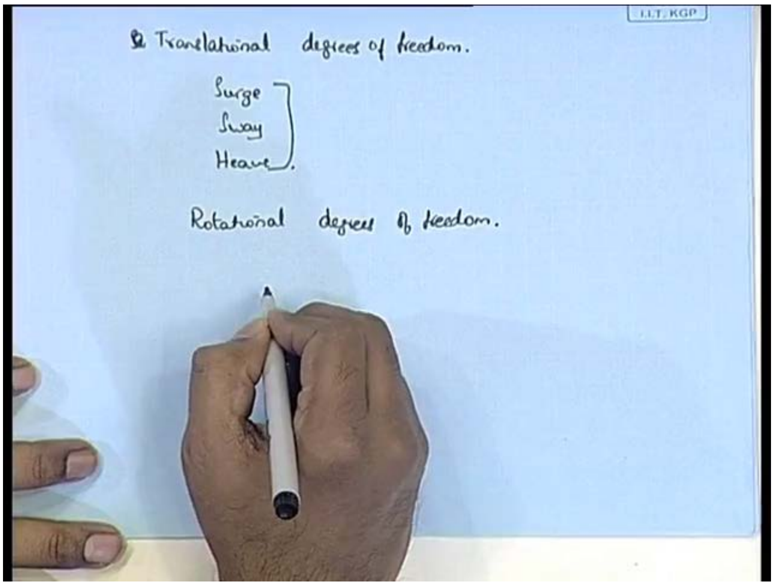

So, we call that as the translational motions as, so you have the translational degree of, degrees of freedom; translational degrees of freedom are: surge, sway and heave. So, these are, all 3 are translation, these, these things, these are the movement in the x, in the translational direction.

And then, you can have rotational degrees of freedom. Now, these rotational degrees of freedom are defined as roll, pitch and yaw. Now, we have already defined a lot about roll in a static condition. When the ship is static we say, that the ship is either heeled or listed. When that is happening as the function of time, we say that it is rolling. So, rolling is a dynamic counterpart of heeling. So, you either have heeling, which is static or you can have the dynamic counterpart of it, which is rolling. So, these are, this is the meaning of rolling, that is one degree of rotational degree of freedom.

Then, you can, we have already talked about what is called trimming, which is another degree of freedom. So, that is one, again it is a static phenomenon, it is, the trimming means, it just trims and stays at direction, stays at that draft, so that is called trimming and when it is occurring as a function of time, you call it as pitching, that, that moment like this, it is known as pitching. So, that is another degree of freedom.

Then, as you can imagine, there can be a 3rd degree of freedom about z-axis, we call that as yawing, yaw, that movement is called yaw.

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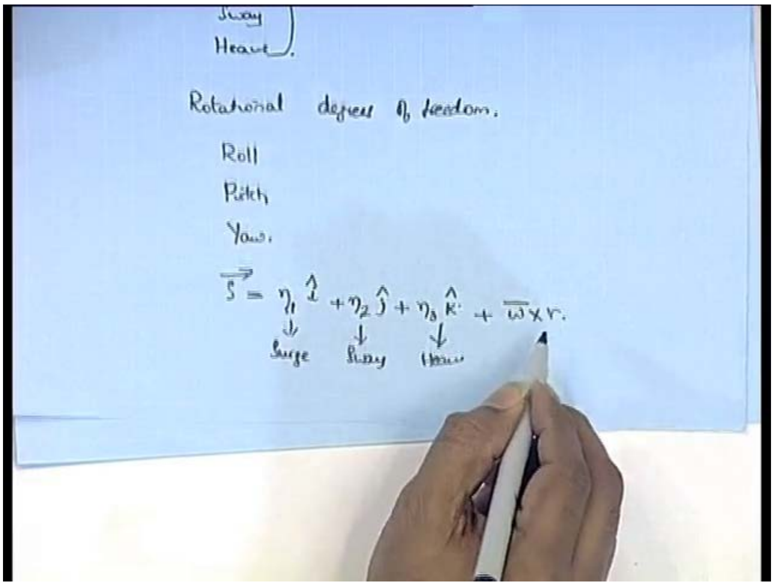

Therefore, the net displacement of a ship can always be written as S is equal to eta 1 I plus eta 2 J plus eta 3 k; so, surge, sway, heave. So, surge, sway, heave, eta 1, eta 2, eta 3 plus omega cross r, where omega represents different types of heel motions. Now, r the, this is the vector product, r is the, you can consider as the vector, the displacement vector and here, means this is about some axis, omega is about some axis.

We will see for instance, it, this axis is different, it is, it cannot, in all the cases it cannot be defined precisely. For instance, for instance in the case of pitching, if you are dealing with pitching, this motion, that is, the dynamic trimming, you know, that pitching always occurs about the center of flotation. Therefore, pitching, the axis we are talking about is the center of flotation. So, for pitching the axis is the center of flotation. So, that is pitching. So, that is very well defined.

And r represents the distance from their axis to any point where we are trying to find the displacement. Suppose, we are trying to find the displacement, the trim displacement or pitch displacement, not trim displacement, if you are trying to find the pitch displacement at a distance, let us say 10 meters from the center of flotation, that r is the 10 meter and omega, which is the pitch frequency into r, will give you the displacement of that pitch or it gives the pitch displacement at that point of 10 meters from the center of flotation.

So, this is, so this gives you the different displacement value, displacement, this gives the total displacement vector S, the displacement as the function of eta 1, eta 2, eta 3 and omega cross r.

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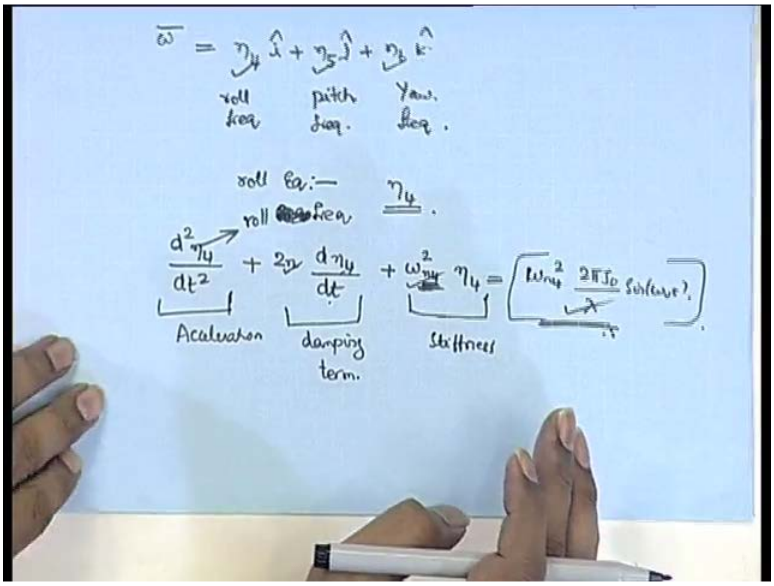

And omega, the total frequency, there are 3 types of frequencies. We have seen 3 rotational degrees of freedom: eta 4 I, eta 5 J, eta 6 k, roll frequency, pitch frequency, yaw frequency. Remember, these etas are all frequencies: eta 4, eta 5, eta 6 represents frequencies; eta 1, eta 2, eta 3 represents displacements. And therefore, this is the pitch frequency and this is the yaw frequency. So, this represents that different or different, display this, this S, as we have seen before, represents the total displacements of the fluid particle.

Now, what do we say is the general equation of motion of the body? Now, first of all, there are 2 ways, which you can study these motions, the 1st we call it as uncoupled motions and the 2nd is the coupled motions. In uncoupled motions we have some motions in, let us, let, suppose, that we have a pitch motion, we assume, that the motion in the, in uncoupled, we say, that the motion in the pitch direction does not later affect the motion in the, let us say, the roll direction. That means, roll is, had affected by pitch, pitch is not effected by a roll, yaw is not affected by pitch, surge is not affected by yaw, totally uncoupled. One is not affected by the other.

The 6 degrees of freedom, 6 degrees of displacement or the displacement in the 6 degrees of freedom are all in, independent, interdependent in, not dependent upon each other. It is completely independent or mutually exclusive, so those are uncoupled equations. We develop the uncoupled equations and later, when it becomes more complicated, you will see that they depend upon each other actually and they are called as coupled equations.

So, the uncoupled equation, you know, that the general equation of motion can be written as, if you consider for instance a roll equation, remember roll is always associated with eta 4, it is the equation for eta 4, therefore the equation is this.

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Now, we would not expand, this is known as, this is the complete roll equation for a ship, that is exposed to this forcing function. The right side represents the forcing function, the external force acting, this equation is something like 4 s equals m a, F equals MA, we are finding out the total force, the total, we are finding out the total external force acting. So, if this is the net external force acting, this is acting, it is, it is balanced by the net motion of the, it is like F equals MA. Suppose, an external force F acts on a body, the body is subjected to an acceleration a, such that MA equals F, that is how we say, that, that is we, how we put the force balance.

Just like that if this external force acts, this force acts, we will not discuss too much about this when this external force acts, this produces a motion like this. This 1st term is an acceleration term as you can see here, it is d square by dt square, it is of course, it is an angular acceleration. If this is angular motion, we are discussing this whole equation is for angular and this is actually a damping term.

Now, there are many derivations and vary many very good derivations of this equation as such, the whole equation for, but we are not going do it for the lack of time. And this is actually a 3rd term, which actually represents the stiffness of the, this actually represents the stiffness of the, stiffness of the ship. So, these are three terms that come in the left side, which balances the net force on the right side. So, this is the equation for, here this N, this is the damping term as such, this N is therefore, the damping coefficient.

Now, this N 4 is the natural frequency of oscillation of the ship in roll, so ship's natural roll frequency. So, this represents the ship's natural roll frequency N 4, the omega N 4 represents the ship's natural frequency in roll and again it is, something it is, that responds, it is the net force acting and this we defined, all the terms. So, this is acceleration, remember eta 4 is the roll frequency, I mean, roll not roll frequency, it is eta 4 is the roll, yes, roll frequency, sorry, yes, eta 4 represents the roll frequency. So, d square…

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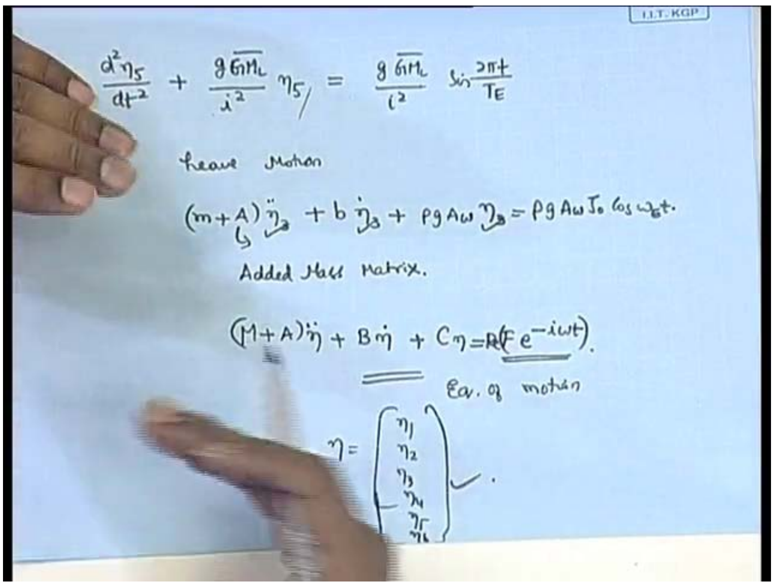

Alright, then you can also have a pitch, you can also have this equation in the pitch direction, that is, you can have, write the equation of the motion in the pitch direction remember the pitch frequency is always is defined as eta 5.

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So, in this case of pitch we have not used any damping. So, there is no damping in this equation and therefore, this, this is the stiffness term again and therefore, this becomes your, you can have a similar equation for the pitch, this is the equation for pitch. These GM L and G R coming in because of that omega, remember, we have already defined omega for ship, the natural frequency of roll; like the natural frequency of roll, you have the natural frequency of pitch. They are all, they all depend upon GM L and I is the radius of gyration and that, from that you have, if you remember their expression and time period, I think we did it in the previous class and we did the derivation in some 10 or 12 classes before. So, when you did that, we saw how that time period of roll and time period of pitch comes, means you have to derive the expression for time period of roll. If you apply to pitch, it becomes the time period of pitch.

So, like that, this becomes, this is, this is an undamped, this is an undamped equation, there can be damped pitch, we, this is just an undamped pitch equation given. So, this is the equation in the pitch direction. So, in general, you will see that or let us consider heave motion. So, so, as we have said before, the heave, the heave motion, the heave displacement is always represented as eta 3. So, M plus A, this I will tell you what.

No audio 42:08 to 42:21

So, this is the wave acting, psi 0 cos forcing, wave forcing function acting is this and Aw is the water plane area and eta 3 is the heave displacement. At any instant of time, eta 3 dot is d dou eta 3 by dou t, which is the rate of change of displacement. This is acceleration in that heave direction in the vertical direction. What is the acceleration is eta 3.

Now, this M is the mass matrix and this A is something, is known as added mass matrix. So, this in turn says, that, then it will be seen when you are doing the heaving and some other things as well, that the net mass of the ship is seen to be not just mass of the ship, but some amount of water there is carried by the ship as well. So, that is what we call as added mass, added mass term. And therefore, M plus A becomes the total mass of ship. The mass, total mass in this equation, that is, the mass of the ship plus the added mass. So, that into it, eta 3 is the heave. So, this will give you...

So, always there is the acceleration term, there is the damping term, there is the stiffness term and there is the, is equal to the final forcing, forcing function. So, this is the, this is the equation. So, this gives you the equation for like. So, likewise we have got the equations for roll, pitch, heave. You can have it for the other types of motion as well.

Now, so in general you will see, that equation can be written as…

No audio 44:03 to 44:18

The real part of this. So, if this is the forcing on the right side, it is given, the real part of that is equal to this whole term. This is the equation of motion, net equation of motion of any kind of body subjected to different kinds of roll, pitch. So, what we see is that.

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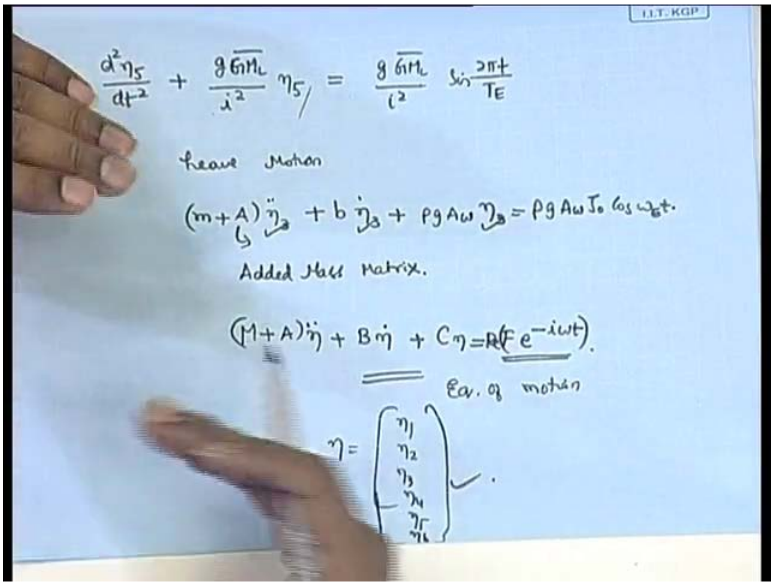

Now, this eta in this equation, eta is the, eta has different components, you will have different types of eta, different waves, the vectors of motions, except motion.

So, this is the exciting force or it can also be the moment. So, F is F e power minus i omega t, is the exciting force and moment and this is the real part of that force. It is balanced by the net move, movement of the ship, movement of the ship affected by acceleration; it is in turn reduced by damping and stiffness. So, this total, total force, so this represents the, this represents the equation of motion of a ship.

Now, so we have defined right now some basics of waves, we have defined on, we also defined different types of motion that we have seen on ships. You can have this kind; of course, the simplest roll equation is the Mathieu equation, which we derived. Remember, there, there was no damping or anything, it was just a very a big simple equation.

The real equation will involve damping and stiffness. So, it is a, it is, it is the complete equation. That is one aspect of, one aspect of hydrodynamics, which is, I mean, these are some basic things and probably this will come more in sea keeping and maneuvering, that is this or what we call as motion. Ship motions, ship, analysis of ship motion is, even you do the analysis of ship motions, you will end up these different degrees of freedom and this, this equation of motion and finally, the, you know, the equation, which involves the mass, added mass, etcetera, all that comes in.

So, what we will do is finally, this equation for eta will be written as a matrix. So, it will be the mass matrix. There will be an added mass matrix, then there will be a damping coefficient matrix like that, there will be different terms and it will become a matrix and you solve the matrix equation to get the motion of the ship. So, this is, this is some basics of ship motions.

Now, another thing, that is also important here is something we defined as, means, another important effect of waves on ships is, we have already mentioned how the wave affect the stability. Now, waves will also in bring in some kind of a, the effect of that is, waves will also bring some kind of forces on it. Now, the, even if waves are not there, first of all, let us see in the, in the absence of waves also a ship is subjected to different kinds of forces over the entire length of the ship. The ship is subjected to different kinds of, ship is subjected to a continuous force, force and moment throughout, there will be a force and moment acing in different parts of the ship, we call the main, main one, which we call as the shear force acting throughout the, acting along the length of the ship. And the bending moment, that is, the moment tending to bend the ship, these are all longitudinal forces and I mean, these shear force and bending moments are to be analyzed in a longitudinal direction. So, the, we are talking about longitudinal bending and shear force.

Now, these shear forces, how do we calculate the shear force? For instance, now first of all I told you, that in the case of a ship when it is built, so it will be initially, there will be a weight distribution on the ship, that is, first of all the ship will be subjected to the, ship will be, lot of weights will be distributed on the ship. First of all, there will be the hull weight, that the weight of the hull itself, which is just the, just the weight of that structure. It will include the weight of the, what you call that, keel, the side walls, the side hull, the deck, the platings, the bulk heads, stiffeners, all the different types of panels, everything, it will, it will be the weight of the hull. So, that is known as the hull weight of the ship.

Now what? Then, you add the propulsion machinery to the ship. So, the, when you add the propulsion machinery to the ship, remember you are adding it at different places. So, when you add the propulsion machinery to the ship, you will, mean, machinery is somewhere near the aft to leads. The machinery is kept somewhere near the aft of the ship and from their onwards, even going further out and jetting out from the aft of the ship, you have the propeller. So, you have a distribution of weight there, continuous distribution of weight.

So, this distribution of weight will add to the hull weight to produce what we call as the light weight of the ship, that is, the weight of the ship after the propulsion machinery is put. Then, we, this, of course, the propulsion machinery and pipings are put on the ship. So, we include all the different pipings on the ship and so in addition to the hull, what we call as the bare structure of the ship, pipings are put, machinery is put, then everything is, everything else, like even the HVAC system or the fuel system everything is in place. So, the weight of all this together we call it as the light weight of the ship that produces some distribution on the ship. On top of this you have the dead weight of the ship, dead weight on the ship.

The dead weight, again we have defined, it is the net, the rest of the weight, everything else, this will include the weight of the crew, it will include the weight of passengers, it will include the weight of fuel, weight of food, weight of grains, weight of cargo containers, ballast water, lubricating oil, anything else, everything else, that we can think of, all types of consumables. So, all this will end up together produce the dead weight of the ship, now and of course, the belongings, everything else together, comes together, adds up to become the dead weight of the ship. So, this dead weight is also distributed. It is very important, when you are designing the ship that you have to design the distribution of all the weights. Of course, we can, there are some weights, that will move in the ship, for instance at least the passengers or even, but or even the fuel, not fuel, the fresh water, which tank it is, but you have to make all the, first you have to make a calculation using the different distributions of waves.

So, you have, you should have all the weight, you put the hull weight, light weight, the dead weight, every kind of weight, you distribute all over the length of the ship and then you calculate what is known as a weight diagram.

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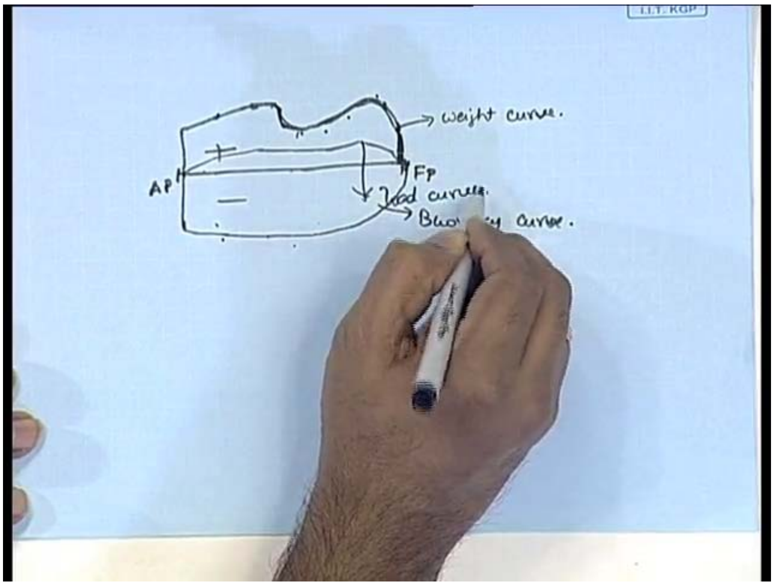

So, weight diagram will tell you like this. So, a weight diagram will tell you, like you put the different weights, you have weights all over, this is the length of the ship. So, this is the aft perpendicular and this is the forward perpendicular, so you have different weights distributed all over the ship. So, now I am going to join them, this is the net weight, the sum total of the light weight, hull weight, dead weight, everything. So, you join them, I am not showing, it should be like this, but whatever it is, you have some weight distribution. This is the net weight distribution on the ship. Now, this will, this is known as a weight curve.

Now, another thing we will have in addition to the weight curve is known as a buoyancy curve. Now, what do you mean by a buoyancy curve? That is, we know, that the ship has an underwater portion in it, which gives rise to the buoyancy on the ship, that is, rho into

V, where V is the underwater volume and you know, that once you have the (( )), you know how, before that. So, at each region, in this region, this region, if you consider this is to be the total length of the ship, this, this region, there will be a fixed amount of buoyancy force associated. This region, this is the buoyancy force, buoyancy force buoyancy force. So, amount of buoyancy force affect, acting in this different region. So, so therefore, we have, we can make a curve of what we call as a buoyancy force.

Why did I put it on the negative direction? If this is 0, if this is positive, I have defined this as negative because the buoyancy force acts opposite to the weight curve, I mean, weight acts downwards, buoyancy acts upwards. So, one of them is positive, the other is negative. So, you have the buoyancy curve, it is actually the, this, it is actually the curve, which gives the distribution of volume on the, it gives the distribution of volume on the, along the length of the ship, from the aft perpendicular to the forward perpendicular.

What is the distribution of volume? So, this, so what you, this is known as buoyancy curve. Now, now the difference between these two curves, the, this at each point, so this minus this; so, this minus this, this minus this at each point you do. Weight minus buoyancy curve ends up with some another curve, some curve here, which we call as load curve. This actually represents the net load; this curve is actually representing the net load acting per unit length on the entire length of the ship.

So, it keeps varying, it is acting per unit length. The force acting per unit, the load vertical force acting per unit length, which is the resultant of the weight and the buoyancy acting per unit length in the, over the length of the ship, starting from the aft perpendicular to the forward perpendicular, is called as a load curve.

Now, an integral of this load curve, the area under the load curve actually, this needs a little bit more explanation, we will, since the time is up today we will stop here, we will, we will do it in the next class. We will actually, we will be wrapping up in the next class completely; next class is the final lecture on this series. So, we will just end up with some detailed explanation, what is the sheer force and the bending moment and we, we will stop. So, right now we have seen what is the load curve and from this we will, how to derive the sheer force curve, for that time being I will stop here.

Thank you.