Hydrostatics and Stability

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Module No. # 01
Lecture No. # 38
Ship Stability on Waves (Contd.)

So, we continue with the lecture 38 of the series. In the last class, in the last lecture, I mentioned about some of the basic stability criteria dealing with waves; we are continuing here with the same. We are continuing here on the same topic of waves and some effects of effects on stability due to waves or some changes in the stability of the ship due to waves.

So, the main thing we have seen is due to the presence of a, there are two cases as we have seen in the last class, there can be a case of a sagging wave or a case of a hogging wave. Now, these two represent a case; when a sagging wave represents the case of a trough existing at the ships midship or the midship of the ship is really riding on a trough that is called as a sagging wave condition. When the midship of the vessel is riding on a crust, we call it as the hogging wave condition.

So, in these two conditions, we have seen that the stability of the wave can be enhanced or decreased, as a result of the change in GM. The change in GM occurs because of the shift in the metacenter or the position of the metacenter and this occurs because of a change in the breadth of the vessel. So, when the effective breadth of the vessel rather, there is no change, not much of a change in the breadth of the vessel at the center ship; that is, midship where you have a more or less wall sided ship. So, you see that the breadth of the ship is more or less constant with draft, but when you go towards the aft of the ship or to the forward or the front part of the ship or the bow of the ship, you see that the breadth is very much dependent upon the draft. As the draft keeps increasing, the breadth of the ship keeps increasing and as a consequence of this, you see that when the draft is higher in the forward side, which means that it is a sagging wave condition.

If it is in the trough, therefore the breadth is more, M is higher, GM is more and therefore the stability is more. When the ship is riding on a crust, the stability is less; so, GM now becomes a function of time.

So, this whole problem of response or the movement of the ship in the wave or the movement of the ship in the sea, that is whether we are talking about the different degrees of freedom; we will see what are the different degrees of freedom now. Before that, we know that we can represent any, we have already done this. We have seen that the response of the ship, the heeling of the ship, the heel angle can always be represented by a simple formula: phi double dot plus.

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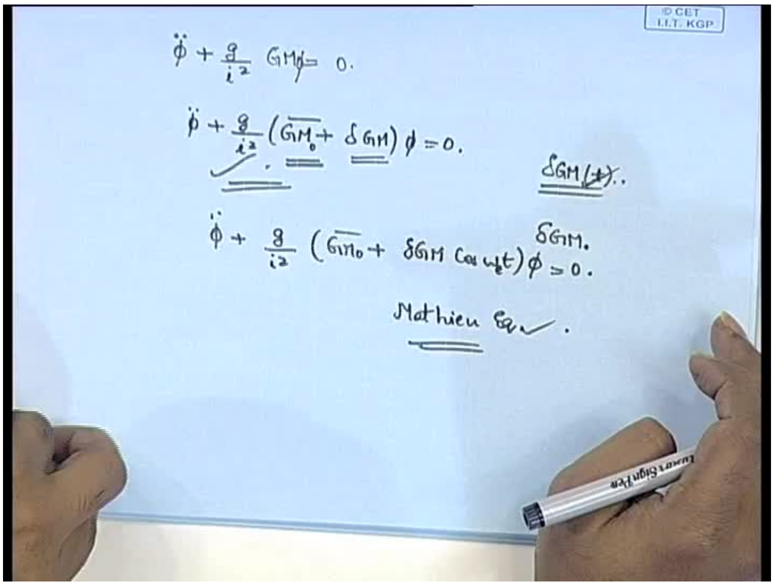

Therefore, we have seen that the heeling of the ship or the heeling response of the ship can always be represented by this (Refer Slide Time 03:15). This is a very simple formula of simple harmonic motion where you have the angular acceleration phi double dot directly proportional to minus of or directly proportional to minus of the displacement, that is heel - phi, in this case. Here, GM is the metacentric height, I is known as the radius of gyration and we know that g is the acceleration due to gravity. So, g by i square into.

Now, what does this problem become in the case of a ship? That is, now on a wave crust, so now the ship is riding on a wave. So, as we have seen the ship and finally its end up with a wave. Finally, in most of the seas the ship ends up being in a wave condition, in which the waves will come in the direction of ship, what we call as 0 degree angle and we call it as following waves.

The ship can come; the waves can come 180 degrees to the ship, the direction which we call as head waves. So, when any of these waves come, the waves will then propagate across the ship, not across, propagate along the ship. So, when it propagates along the ship, it will alternatively be on a wave crust, a trough, and a crust like that and its GM varies with time.

Now, if you write this same equation, this is the simple harmonic motion, we can modify it in the presence of a time varying GM. Now, as we have seen, here heel acceleration is now proportional to, and it is equal to a constant time the heel angle and this constant is not GM, something into GM.

So, if GM now becomes a constant variable with respect to time, this equation can be written as (Refer Slide Time 03:15). So, what this equation shows is, that the equation that this is now a GM 0, which is a constant, which is the presence of that, it is in the absence of waves; so, it is GM in the absence of waves plus delta GM. Now, this delta GM is a function of time and this is one important point. The delta GM is the function of point that is the GM; it is the perturbative part of the GM.

So, the GM, the net metacentric height is now a sum of a stable component, which is a fixed component, which is basically for a still waterline and it is and on that it is superimpose a perturbative component of GM which varies with time which is actually a function of the wave, the position of the wave or the location of the wave and the position of the ship on the wave. Decides whether the decides what the GM is? And consequently what this delta GM is?

So, we get this as the new equation, this becomes our final equation (Refer Slide Time03:15). And now what we can, very simple to do is remember the wave. It is a wave; so, it is varying with time in a periodic fashion; so, one possibility the simplest way of dealing with this problem is, if we assume that the variation of GM is sinusoidal, it is of course, not probably true, but they can always assume it to be correct. It is some delta GM some value into cos omega t omega e t; let us call it into phi equal to 0.

Now, therefore, what we have done here is we have just assume that the variation of GM with time. There is the ship is going over the wave; now, there is a wave like this and as the ship is going over the wave like this; it is going in a the GM is varying in a cosine fashion; it is varying in a sinusoidal fashion, in a cosine fashion.

And once you assume this, you get this equation and this equation is known as the Mathieu equation; it is slightly important. It is known as the Mathieu equation. Now, this is the equation that is mostly used to solve the response of ship in seas.

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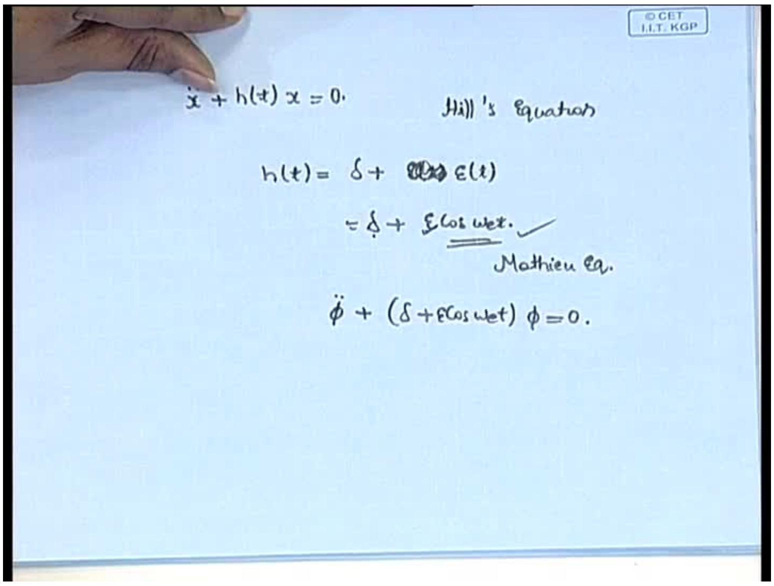

Now, let us write, first of all suppose you have an equation like this, the main equation that we started with this (Refer Slide Time 08:31). Suppose, it is something like this, x double dot plus h. Suppose, there is a mathematical equation of this form that is you have x, which is the horizontal possession of some particle varies like this; it is a kind of simple harmonic motion, but note that this h is a function of time; so, it is not purely simple harmonic. Now, this is a differential equation and this kind of equation is known as a Hill's equation; so, in this equation, where the coefficient of the differential equation is a function of time is known as the Hill's equation.

And in the particular case, if we are if we are able to write h of t as a delta plus epsilon of t, epsilon is also a constant, and yes this you write it epsilon of t, which is equal to delta plus epsilon cos omega e t.

Now, suppose you are able to write it like this, then this equation is called as a Mathieu equation. This is the equation that we just mentioned (Refer Slide Time 08:31). This is the equation that represents the ships motion or the heel of the ship on the wave. So, if you have a wave propagating, you have a ship on it and then the equation of the motion of the ship is given as the heel motion phi is always the heel phi. The heel motion is always given by the Mathieu's equation.

In this the important point note is, there are two components, there are two constants, one is delta and the other is epsilon. What you see is that, this delta in Mathieu's equation applied to ships. Note, Mathieu's equation was not generally defined for ships, as such he was a mathematician and it was applied for just mathematical problems. It is now going, this equation is now applied for ships as such that is all; so, this is Mathieu's equation applied to ship motion.

The differential equation applied to ship motion. In the case of ship motion, the Mathieu equation will have the following constant (Refer Slide Time08:31). This delta will be equal to GM and epsilon will be delta GM t. So, what you can understand right from looking it; this delta represents a stable component, the natural component. GM natural means, GM natural is the property of the ship. Now, actually GM natural is the property of the ship and it is a property that can be modified as the result of probably a wave.

So, if you have a wave, it can modify GM. There are other properties also that can modify GM; external properties that can also modify GM now. But in general, GM is the property of the ship and that natural GM is what we call as GM 0. The natural GM not subjected to the wave and that GM 0 is a natural. It is the property of the ship, it is the natural component of the GM and that is given as delta. This delta represents that GM 0 - it is a constant, it is a natural and epsilon. Here represents the delta GM which is a perturbation mean, which is a fluctuation about the still value.

So, if you have GM 0 to be the still or the mean value of GM. The fixed value of GM on a still water land then epsilon is a perturbation mean over it. Therefore, epsilon represents, if you want to call it as the perturbation mean or the exciting component or the fluctuation acting on delta. So, epsilon is perturbation acting on delta which is the mean and cos, this is how it varies; epsilon is a value and cos - this is the direct. This is the way in which it varies in a sinusoidal fashion, it varies.

Now, therefore, we have the equation like this , phi double dot plus this delta plus epsilon into phi equal to 0. So, this equation comes as the result of the Mathieu's. This is the Mathieu's equation for ships. Again, delta is a natural component and epsilon is a perturbative component and this is a sinusoidal variation of the perturbation.

Now, once you have this equation, any differential equation it can either have stable solutions or unstable solutions. What do we mean by stable solution? Suppose, you have phi I mean, suppose you have this equation that is, phi double dot plus something and suppose you solve the equation and you get an expression for phi as such, which is means you get an expression for phi whatever it be. It could be in the like, you know that it depends upon the type of the real and imaginary part that is depending on the type of equation that is differential equation. It cans you might end up with the solution that is like a cos something or it might be an E power something.

So, these are general differential equations. These are the two kinds of solutions you end up with, you get either a cos function or E power function. Now, supposes you end up with a function that is bounded. By bounded we mean, it does not increase exponentially or it does not increase or it does not keep increasing mostly, exponentially, then such solutions are called as bounded solutions and those solutions which increase without a limit or it increases in an exponential fashion.

So, once there is phi, when it is phi, it is a function of some parameter; when phi suddenly increases without any limit then such, if the variation of phi is in that fashion then you say that phi is unbounded; so, the solutions of phi which are bounded, we call it as stable solutions. Solutions of phi which are unbounded which increase an exponential fashion, very rapid increase we call it as unbounded solutions.

So, you have the bound solutions and the unbounded solution. The unbound solutions which represents the stable and the unstable regime, that is what phi depends on.

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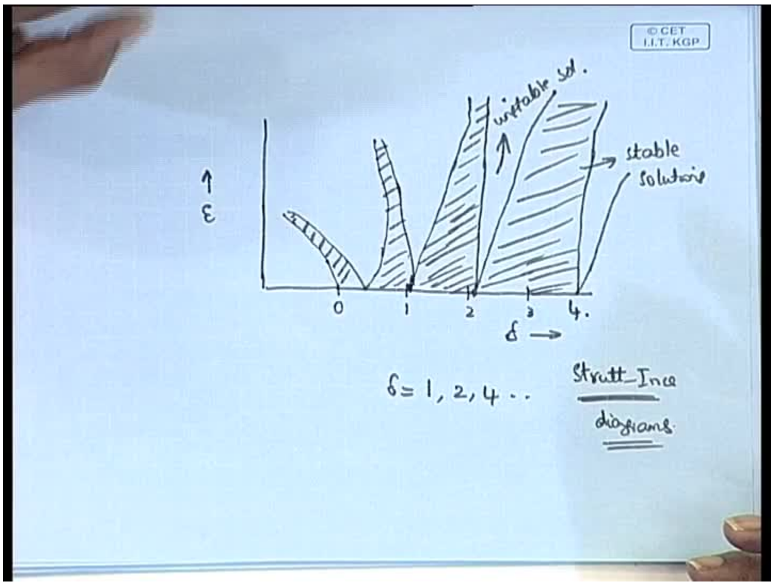

So, if you draw the variation of delta for a ship case. This is for a ships case; if you draw the variation of delta with means, if I draw delta here and I draw epsilon here (Refer Slide Time 15:23). Note, one represents GM, the other represents delta GM. If I draw a case like this (Refer Slide Time 15:23), you will see that. So, when you draw the curve between delta and epsilon and you see the values of delta and epsilon; correspondingly, you will have values solutions for phi.

Now, you will see that some regions of delta and epsilon will give stable solutions, whereas some regions will give unstable solutions. So, these regions, where I have shaded are represents the regions of stable solutions; the regions where it is unshaded like here, it represents unstable solutions.

So, what you see is that you have the different solutions to this Mathieu equation. You end up with two regions of delta and epsilon; you end up with some regions, which are a region of stable solution and some regions, which are region of unstable solution.

Therefore, you see that most of the solutions occur when delta is equal to means. This bifurcation region occurs, when delta equal to 1 then it occurs; when delta equal to 2 then it occurs; when delta equal to 4 like that, these are roughly the regions where these bifurcations come; so, this is one point that is to be noted.

So, these diagrams which draw the variation between delta and epsilon are known as Strutt-Ince diagrams; it will be helpful to remember this name, this is known as the Strutt-Ince diagrams. These are the diagrams, which represent the regions of stable and unstable solutions for this Mathieu's equation.

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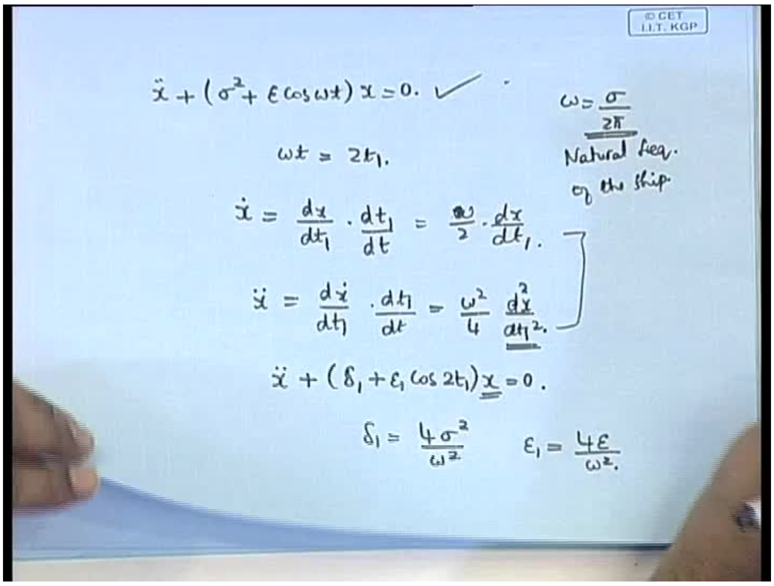

Now, similarly, consider this equation; this is the same equation, I have just change the changed from phi to x that is all. Suppose, you have the equation it does not obviously in this case, represent a heel. It represents some form of translatory motion it could be in surge way. But we will come to know what is surge way and all that, but it represents some form of translatory motion.

Now, let us suppose that we make the assumption, omega t is equal to 2t 1, we define like this. We define some non dimensional t 1 such that, omega t is equal to 2t 1. Therefore, we see that x dot can be written as, dx by dt dt 1 dt 1 by dt. Now, what is dt 1 by dt? You get it here omega by 2; so, it is equal to omega by 2 into dx by dt 1. Similarly, x double dot becomes, dx dot by dt 1 dt 1 by dt, which again becomes omega square by 4 d square x by dt 1 square.

So, from this we are just changing the equation into (Refer Slide Time 18:42). Now, if you substitute these two in this equation, if you put it there, you will end up with, x double dot plus delta 1 plus epsilon 1 cos 2t 1 into x equal to 0; where, x double dot is equal to d square x by dt 1 square x. It is just been transformed from one system of coordinates to another system of coordinates from t to t 1, it has been shifted.

And similarly, you will get a if you were, here delta 1 that will represent, 4 sigma square by omega square epsilon 1 equals 4 epsilon by omega square. This is just from this simple manipulation; you will get these two values.

Now, what you seen here is, here look at these equation, what we mean by. The first of all, we have the case of an undisturbed solution; we can have a case of undisturbed solution. The undisturbed means, when the perturbation component is 0, it will be like saying that is the case, when you have your, there is no wave. There will be the case in the ship; in this case, it will be a case where there is no wave. So, if you put this to 0 you get, x double dot plus sigma square x equal to 0; your omega will become sigma by 2 pi, this is known as the natural frequency of oscillation of the ship or natural frequency of the ship.

This is the natural frequency of the ship; this we have already given in the expression. Remember we have given some class back; we have given an expression for the natural frequency of the ship, heel frequency. There is different frequencies - there can be pitch, there can be heel, there can be yaw, different kinds of things. But we are talking about the natural frequency of heel for a ship; It depends upon GM, it depends upon i that is the radius of gyration and some route of i by GM something like that it comes to an expression for omega.

So, this is actually sigma by 2 pi, where sigma in this case becomes GM and not GM, GM by i square or something, if you just check back and you will see that. So, omega is equal to sigma by 2 pi, this is known as the natural frequency and then there is a frequency of the perturbation itself, it is this.

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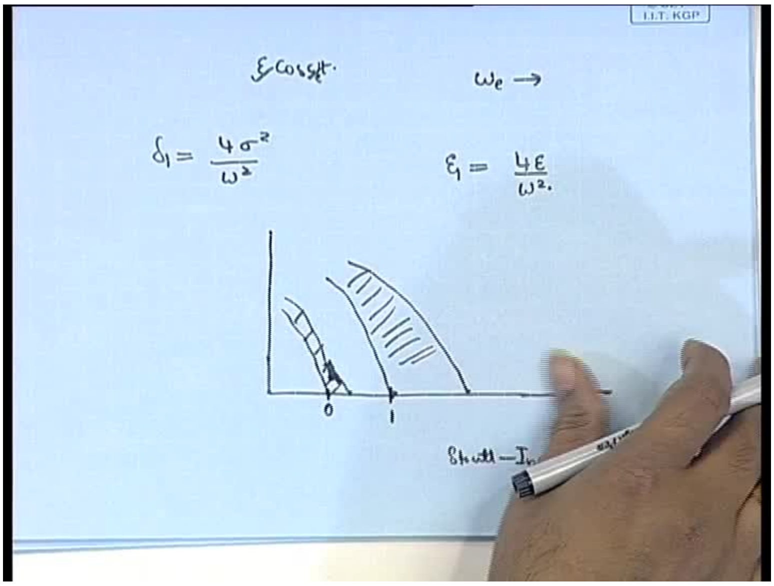

Now, we have this epsilon cos omega t. So, I just want you to know, what are the two types of frequencies we talk about here? One is the natural frequency of heel, if this equation, I mean in this case, this does not represent the heel motion, it is x double dot; so, it is not heel motion; it is represents the natural frequency of whatever translatory motion. We are discussing here the natural frequency of that motion of the ship natural frequency.

And this, epsilon cos omega t represents the perturbative oscillation of the ship. Epsilon is the perturbative value and omega - here we can call it omega, e - it is the perturbative frequency.

Now, therefore, we have so, we ended up with solution which said that - delta 1 is equal to 4 sigma square by omega square. Here, we have seen what is sigma and this omega is the perturbative frequency; epsilon 1 is 4 epsilon by omega square; again these are all perturbative frequency.

Then, you will similarly have a Strutt-Ince diagram, it can also be drawn for the delta 1, epsilon 1. Remember all we have done in the delta 1, epsilon 1 is (Refer Slide Time: 22:26) and we have converted from a t domain that is the time domain into another non dimensional t 1 domain.

It is almost the same thing, we are still talking about; it is just a domain which makes it is easier to do some calculations that's all it, but it is basically the same Mathieu's equation.

So, we have similarly a Strutt-Ince diagram for, this is also known as the Strutt-Ince diagram only. So, you will have at 0, 1 and similarly you will have drawings like this, I believe it becomes here (Refer Slide Time 22:26); so, what you see is that. Like this you have similar drawings; some regions which are shaded, some regions which are not shaded. So, these unshaded regions represent the regions of unstable unshaded or unstable shaded regions represents stable regions; so, these are two types, these are two regions in this. In this Strutt-Ince diagram which represents the stable and unstable regions.

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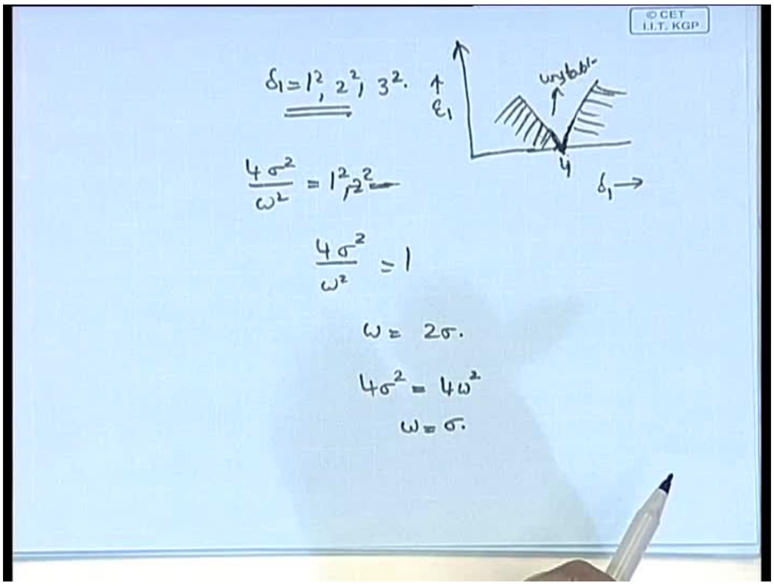

Now, you see that if you draw the diagram properly, I am not going to draw in any once more. So, when you draw the diagram properly, you will see that when delta 1 equals 1 square, 2 square, 3 square; so, in these regions in these regions close to 1, 4, 9, etcetera. You come at one point like this, where you have (Refer Slide Time 25:25). Here an unstable region like, I will take for example 2 squares at 4 in the Strutt-Ince diagram. This is delta 1 in the Strutt-Ince diagram, there will be like this epsilon 1 at when you reach when delta 1 becomes 4. This is a region of unstable; so, here will be shaded. So, here it is stable, here also it is stable.

So, at this point you reach a region, where there is a boundary between the unstable and the stable region; rather we can say that these are point of these values of delta 1 or regions where the solution becomes unstable. So, it changes from stable to unstable or these are the dangerous regions in the ship, where delta 1 equals all these values are the dangerous regions of the ship; and then these represents are 1, 2 square, etcetera represent that dangerous areas of the ship.

Let us take the first solution equals 1, it is dangerous. Therefore, what do we see? We see that when the perturbative oscillation is twice the natural frequency of oscillation. Remember, sigma was the natural frequency of oscillation, when the perturbative frequency omega becomes equal to twice the twice the natural frequency of oscillation of the ship, for that particular kind of motion whether it is heel, or pitch or roll or whatever it is. So, when omega is equal to 2 sigma, in this case you come to know one of the unstable solutions of the ship is the unstable dangerous situations of the ship. So, when omega equals 2 sigma, there is an unstable or a there is a dangerous condition.

Now, next, when omega equals or next case when 4 sigma square is equal to 4 omega square or when omega equals sigma, this is another dangerous case; so, like this (Refer Slide Time 25:25).

So, this is some kind of resonance we call it as parametric resonance. You know that resonance is a condition, when the frequency of vibration or oscillation of the perturbrating forces or whatever is causing the perturbation exciting force. If the frequency of oscillation of the exciting or the force that is generating the noise of vibration is equal to the natural frequency of oscillation of the system itself. of whether in this case a ship

So, if the frequency of oscillation of the perturbation is the same, it is twice the angular frequency of natural frequency of oscillation of the ship, then you have a parametric resonance occurring and this produces a very dangerous conditions. So, the ship can means phi can go in an unbounded region and you know that when you say that phi goes unbounded, the meaning is that the phi can increase means, if this is phi, if phi keeps increasing it means that the ship will capsize.

So, it is a dangerous situation; that is so, this phenomenon is known as parametric resonance and it is an interaction between the perturbrating. Remember the perturbrating force, here is the wave, the perturbrating frequency or the frequency which is causing the excitation on the ship in this case; it is the wave.

So, when there is a relation between the wave frequency and the natural frequency of oscillation of the ship, let us consider heel, if the natural frequency of oscillation of the heel and the frequency of the ship has a relation, if they are equal or if related in such a fashion in, if they are in some ratio, then you will end up with parametric resonance. Where is the region? When there is very high vibration and very high phi, phi becomes unstable or it goes into a capsizing mode. So, these are different types of parametric resonance that you come across in ships.

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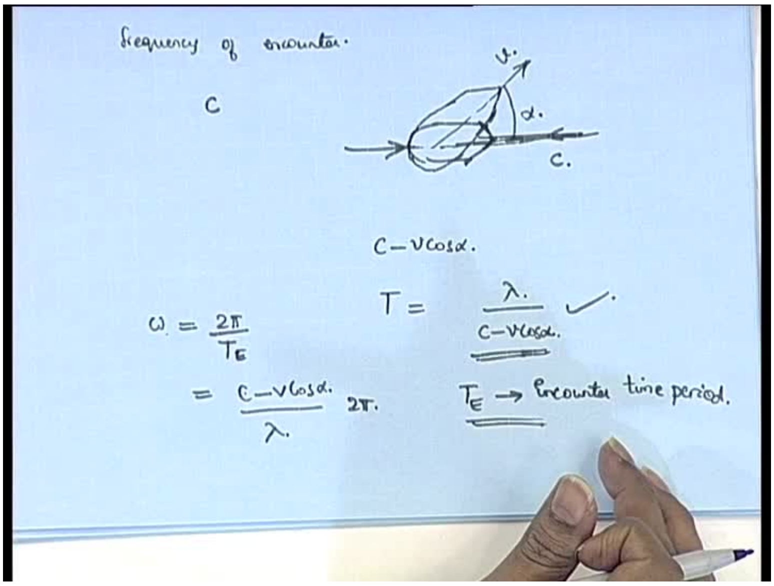

Then, there is another term, which we talk about; we call it as the frequency of encounter. This is a term that is used commonly is known as the frequency of encounter; it means it is a kind of relative motion between the ship and the coming wave.

So, if, let us assume that a wave is again as I told. Let us, assume that a wave is coming directly on to the ship; we call that as a head wave. So, when a wave comes and hits the wave, directly the ship goes over the wave. And, let us, suppose, that we are measuring the time taken between the time period or the time elapse between two successive position means, suppose that the ship is measuring two successive position of the crust; so, it measures one crust coming then it waits, it undergoes some oscillations and then it sees the next crust coming there. So, the time between elapse, between these two positions of the crust reaching there; so, this is some time period, it is elapse between the two; so, that time period is called as the time period of encounter and the frequency associated with it is 2 pi by T E, in which T E is the encounter time period; so, 2 pi by that T E will give you the encounter frequency.

Now, we will see what the method is. Now, I am going to assume some basics of waves; when I say basics of waves, it is the absolute basics of waves. We are not talking about even slightly engineering concepts like, we are not. I am not assuming that you know even things like the dispersion relation because there is a bit of wave that we will cover in the next lecture, very little bit that is a little bit of its engineering application, which is it is actually the beginning of wave study, wave theory. But right now at least things like wave crust, trough, the wave time period frequency then what we call as. There is also wave speed such things will be known to you; we assume that it is known to you.

So, let us assume that the speed of the wave, we also call it as the wave celerity which is the speed with which the waves propagate or let us assume that the crust of a wave propagates. So, that is C, and let us assumes that the ship is moving at an angle alpha like this (Refer Slide Time 32:07). So, there is a ship here and there is a wave coming, C. It can be in either way, you mean you if it can either be coming like this or the wave is coming like this, either way this wave is coming, the ship is there and this is a very simple case.

Now, let us assume a case, when the ship is like this means, it is at an angle to the wave. This is the wave, this is at an angle, the ship is propagating with the velocity v. So, the ship is now moving with the velocity v at an angle alpha relative to the wave; so, C that is v. Now, we can see that the relative velocity between the two is - C minus v cos alpha or C plus v cos alpha. Depending upon the sign of C, whether the ship is, whether the wave is following wave or a head wave depending upon that, we will write it as - C minus v cos alpha or C plus v cos alpha, at any rate it will be v cos alpha.

Now, we can say that how much time it takes for the wave. First of all, let us see this wave how much, what do we say? We say that the time period between the seeing of two wave crust. Now, if you see two wave crust that means you have travelled a distance of lambda that is a wavelength. So, to see two wave crust, you must have travelled a distance of lambda and the time it takes. Right now, we have seen that the relative velocity with which the ship is moving in C minus v cos or the wave is C minus v cos alpha.

So, the distance moved is lambda and the relative velocity is, C minus v cos alpha. So, the time taken to see to move that distance or which is also the time taken to see two successive wave crusts is equal to the time period is, lambda by C minus v cos alpha, this is straight forward. So, this is what we call as the encounter time period; this is known as the encounter time period.

This is the time period between two successive encounters of a wave by a ship. So, if you take the frequency of this, which we call as, omega 2 pi by T of this T E. This becomes, anyway you just put it as, C minus v cos alpha by lambda into 2 pi. So, this will give you the value of the encounter frequency; this is what we call as encounter frequency.

Then, this is, now you can always get a relation between the frequency of the wave and the frequency of the encounter frequency. You know that the frequency of a wave can in general be written as, C by lambda; so, you can get the omega of the wave that is the angle of frequency of the wave by C by lambda, and the angle of frequency of encounter by this expression, C minus v cos alpha by lambda into 2 pi.

Now, so from these two expressions we will get the relationship between the frequency of the wave and the encounter frequency. Then this is some basics of parametric resonance. Now, we will also, before we wind up the course, we will discuss some details about waves; some basic formulations of waves, some concepts of wave theory, some very basics of wave theory.

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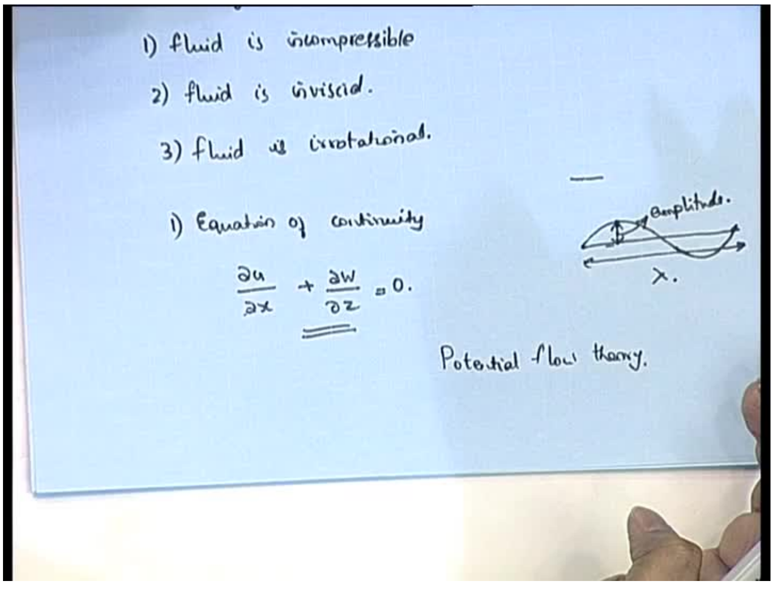

Now, in wave theory, first of all a few assumptions are always made. The first assumptions of which are that the first assumptions is that the fluid is incompressible. The meaning of the fluid is incompressible is that the density of the fluid does not change.

So, the density of the fluid does not change with time. What do we mean by incompressible or compressible? There are some cases very rarely found in oceans as such but more found in atmosphere. These are the cases, when you end up with the air, when you change up with a very large change in the density of the fluid whether it be the air or the water, when you encounter a sudden drastic increase change in the density, this can happen. It is mainly happens when you for instance, if you have a aircraft instead of a ship and when you are studying aircrafts, suppose that those are you know that when the speed of the aircraft gets up to the speed of sound when it reaches, when it equals the speed of sound, which is something like 350 meter per second. In air, we say that the aircraft has reached as mac 1 speed and when any velocity greater than that we call it as supersonic velocities and one plus is all supersonic.

In fact, I believe some of the fighter jets go at even beyond the mac 2, which is twice the speed of sound, at any rate. When you have the aircraft travelling at speeds greater than the speed of sound; when you reach a situation where density is highly variable.

In supersonic flows and transonic flows you end up with flows that are where you end up with situation, where the density of the fluid changes where rapidly; so, that is a case. That such a case does not arise in the ocean in ships because ships go at very slow speeds; so, that is one possibility.

When you have drastic changes in or when you have sudden changes in the temperature or possibly in the atmosphere, the moisture content of the atmosphere or in the case of ocean, the salinity of the ocean; so, either of this, I mean if you are studying air and airplanes then you see that when your moisture or temperature changes with rapidly, your density can change; so, the air becomes compressible, in that sense there is a dou rho by dou t, there is a compressibility effect.

Then, and in the other case, in the ocean if you have a change in temperature or a change in the salinity, you will have a change in density as well the change in temperature is such that, if you have an increase in temperature the density decreases; if you have an increase in salinity the density increases.

Salinity, by the way it is defined as salt content of the ocean; so, if the salt content increases the density increase. So, these possibilities removing all these possibilities in a simple case. We says that fluid is incompressible, that is how wave theories are developed all the cases are, and for the case when the fluid is incompressible that is number one.

Number two, we are assuming that the fluid is in viscid. Now, the meaning of the term in viscid is that the fluid has no viscosity. Now, viscosity is a concept in fluid mechanics which might not be familiar, it is viscosity. It is a property of all kinds of fluids, whether it is gasses or liquids.

So, water also has a viscosity. Because of the viscosity property of water there is acting always on the fluid, a force called a viscous force; it is like a frictional force. So, when you have a body is travelling through the water or an air; for example, if you have a plane travelling in the air, if you have a ship travelling in the water. This viscous force is a kind of force that will act on the ship or the plane and prevent it from moving; so it is a kind of drag forced produced it is a kind of drag produced and this drag is called as a viscous drag.

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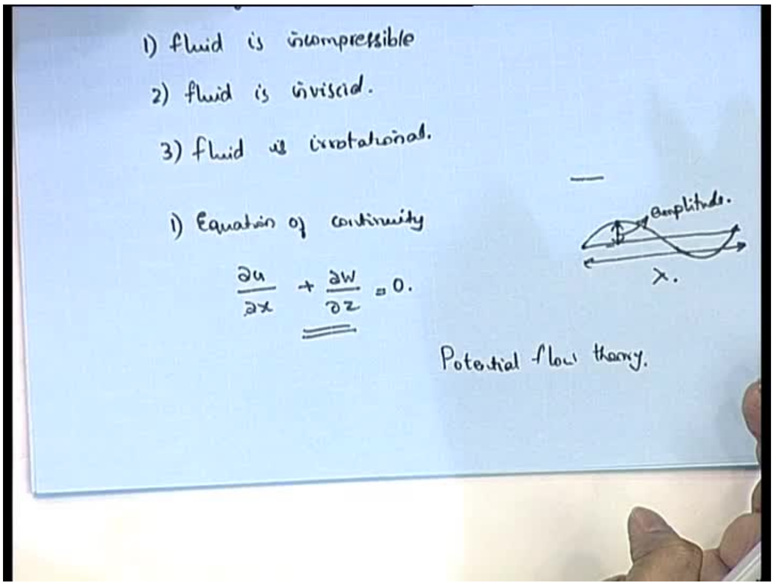

And another effect of viscosity is the generation of heat; so, because of viscosity there is a generation of heat and dissipation of heat. So, production of drag and dissipation as heat, these are two properties of viscous fluids. Now, we assume that there is no such thing as viscosity and that the fluid is absolutely in viscid.

So, there is the next assumption. Then we assume that, again in fluid mechanics, there is a term that is known as rotational property or rotation of the fluid; so, we say that if the fluid is either say a fluid is rotational or a fluid is irrotational.

A more detailed analysis or more detailed discussion of rotation and irrotation is beyond the scope of this lecture. So, it is not appropriate for this hydrostatic and stability but we say that a fluid is irrotational, the meaning is that it has 0 vorticity. The property of fluid excess called vorticity, which is a rotational property of a fluid and fluid parcel Therefore, in this waves we assume that the fluid is irrotational.

So, now we are always talking of small amplitudes; by amplitude of a wave we mean, this means if we have a wave like this, its fluctuation or its displacement away from the mean this is what we call as amplitude.

Now, what we are assuming is that this amplitude, this distance is much smaller than this distance this is known as a wavelength. As you know lambda, we say that the amplitude of the wave is much smaller than the wave length of the wave.

So, this brings us to a small amplitude wave theory and this is the simplest form of wave theories. Of course, these are theories which do not make this assumption, but in general here we are going to deal with this and so we have small amplitude waves.

Then the main equations, which we come across in the wave theory are - first one is we use the equation of continuity. The equation of continuity is written as, let us consider a two dimensional flow, where you have (Refer Slide Time: 37:23). we are We are not considering the transverse motion of the ship or a transverse direction of the ship; therefore, we have only the x and the z directions; so, you have only the longitudinal direction of the ship and the vertical direction of the ship. It is just a simplification; we are not saying that there is the case but it does not make a big difference, whether you consider the trans, because it is a small. First of all the main properties which is, which we want to study occur in the longitudinal direction that is, what we are mostly interested in, so and depth it is needed definitely because of the weight criteria. So, dou u by dou x plus dou w by dou z equal to 0, this equation is what we call as a equation of continuity.

This equation states that it is basically a restatement of the basic principle of mass conservation. We say that the mass of the fluid is never destroyed means a mass is never destroyed as such, water as it is always remains as water. So, it is not going anywhere; so, whatever mass of water is there, it is there; even after, whatever is there before, it is there after also. This concept comes as, dou u by dou x derivation none of these we are going to do any derivation

So, dou u by dou x plus dou w by dou z equals 0 becomes the continuity equation; in this case of analysis of waves which is usually done, what is known as a potential method.

We call the study is known as the potential flow. The methods of potential flow theory, why is it called a potential flow theory? Because we deal with the potentials, we will deal with what is known as a potential. I will tell you, what is potential.

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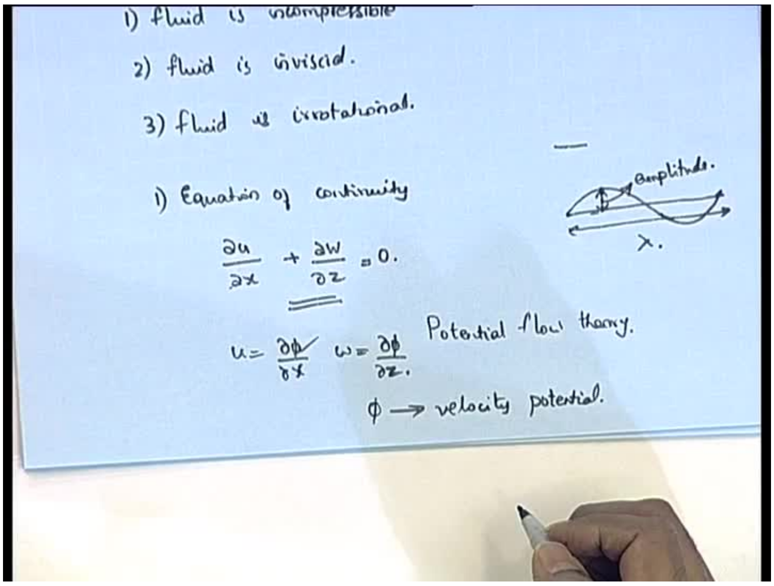

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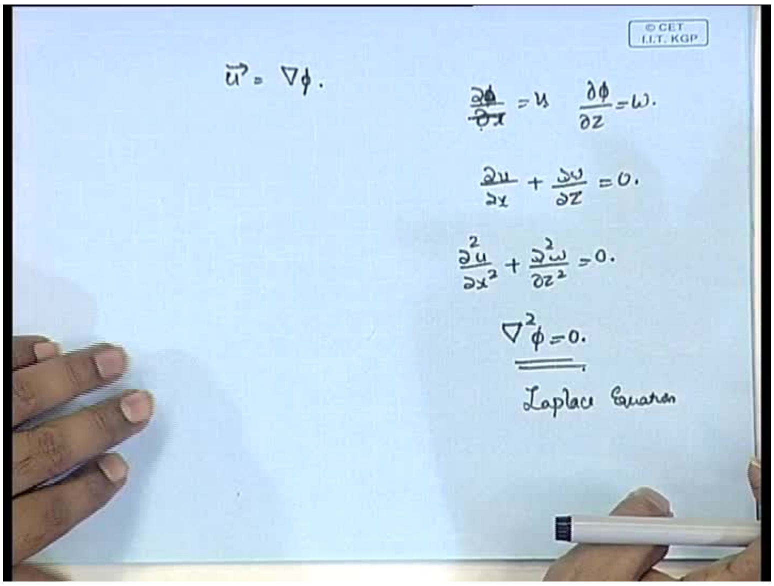

We say that, u is equal to a dou phi by dou x and w is equal to a dou phi by dou z, we make a definition like this; so, what we say as phi is known as a velocity potential. We define it as a velocity potential phi; so, this phi is, what we call as a velocity potential. Therefore, what is the simplest formula that comes from, what is the two equation that I wrote before that is, u is equal to dou phi by dou x and w is equal to dou phi by dou z, can be combined together, in a way as, u vector is equal to actually del phi. So, it can be written like this and when you combine it with the now so this equation of for phi these two equations ,when combined with the continuity equation. This is the continuity equation. You put this value for u, here you will see that the equation reduces to this (Refer Slide Time: 47:35). In vectorial notations or on this notation of del can be written as, del square phi is equal to 0.

Now, this equation is known as a Laplace equation and it is the basic equation for waves. This is the equation from which all the this is the equation that describes a waves; so, we say that when a when a fluid particle of base Laplace equation, there are some basic assumptions that go as you can. As you have seen already, first of which is that potential excess. The meaning of potential excess is that vorticity is 0; so, again these are allied concepts, the concept is, you get the idea of Laplace equation.

So, this is the equation of a wave; so, a wave follows this equation, del square phi is equal to 0; where, phi is the potential velocity potential; so, phi is the velocity potential. Now, this equation of Laplace equation is solved, subject to some boundary conditions, because you cannot get a solution for phi; unless you have some boundary conditions, you have some conditions which say what is the value of phi at some points.

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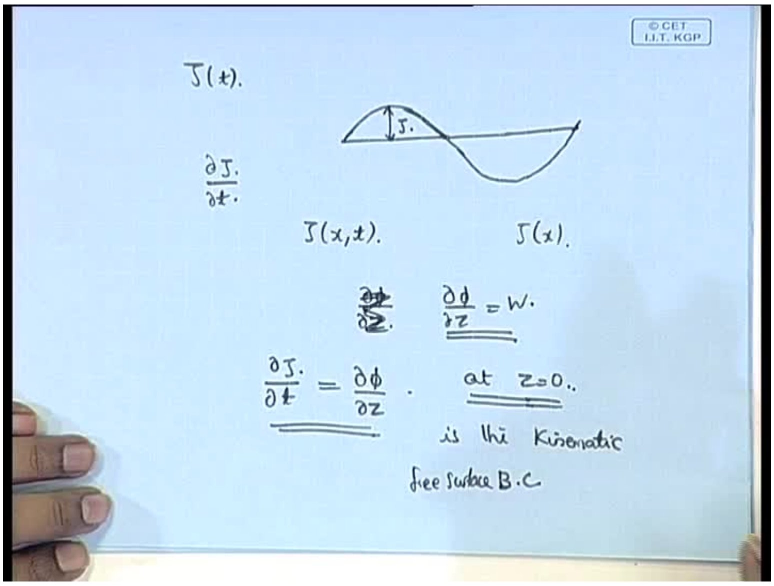

So, first of all remember a wave is like this (Refer Slide Time: 49:46). Let us consider the free surface of liquids; so, there is a wave like this, if you have a wave like this on the free surface of, let say ocean. For instance, if you have the free surface like this, now if you say that this is psi, which is the displacement of the free surface; what we call as elevation. The psi at any point represents the shape of this curve; so, here psi is positive, here psi is negative.

At any rate the variation of psi represents the position of this curve; so, position of psi represents the position of this curve at any point. Now, this psi therefore what is called as elevation represents the shape of the curve; psi of x which mean this curve, actually this curve usually which we write as f of x. The function of x is written as this psi of x, that is the curve.

Now, what is at this curve, what is the movement of what is the velocity of the curve in this what is the variation of this curve? Now, as you can see this curve is in the z direction, this size is in the z direction. Therefore, if you want to get the vertical velocity of the particle, the movement of this (Refer Slide Time 49:46). This is a psi and this psi is a function of time.

So, because of various factors acting on the surface, the psi varies with time. How do we get its variation with respect to time? This is got by, so just think of it, it is the variation of displacement with time it gives the velocity; so, it gives elevation is a vertical displacement and variation with time represents, it actually represents the velocity vertical velocity of this curve.

So, this curves what is rated, which it is moving in the various. It is the rate at which it is changing, it is a vertical direction; and how it is varying like this, at any instance; psi at any instance of x is given. Actually, psi is a function of x and time, not just time; psi is a function of x and time and this psi how it varies with time? It will be varying like this, at some x.

If you fix the x, psi will vary like this (Refer Slide Time: 49:46). Now you can see it is a variation in the w direction or the z direction. Now this is the variation of the curve; now, what can you know about the particle that is residing inside that curve? You will see that a particle will be moving up and down with the curve, it is a wave and it is made up of this particle only.

So, when this wave goes up and down, the particle also goes up and down and what is the velocity of the fluid particle? The vertical velocity of the fluid particle is dou phi by dou z, this gives the vertical velocity. We have already seen that w is given by dou phi by dou z, this gives the vertical velocity of the fluid particle; dou phi by dou z is equal to w and at z equal to 0. We will see that, dou psi by dou t is equal to dou phi by dou z at z equal to 0.

Now, that is what we have eventually said here is that the rate of movement of the curve, rate of fluctuation of the curve is equal to the water particle velocity. There is water particle inside that curve; so, the rate at which the curve moves is equal to the water particle; velocity rate at which the water particle also oscillates or moves up and down. So, that therefore this expression holds x z equal to 0 and is known as is the kinematic free surface boundary condition; it is known as the kinematic boundary condition. So, this is the free surface condition kinematic free surface; so, this is known as the kinematic free surface boundary condition that is one of the conditions on which the wave theory holds.

So, since, the time is up today; we will stop here now and we will continue in the next lecture, thank you.