Hydrostatics and stability

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Module No. # 01

Lecture No. # 36

Ship Stability on Waves

We start with the lecture 36. In this lecture, as far as our lecture have been concerned, mostly we have been dealing with the different forms of stability, all associated with a still water line. Now, by still water line; I mean, a water line that is horizontal, it does not have to be horizontal, but it is straight. So, the water line it is always horizontal, so the water line when it is horizontal, we have already divide device, the set of equations that will deal with the stability of ships in a such of horizontal water line.

For instance, we deal with the heeling; we have dealt with all problem associated with heeling that is happening again in case of as till water line or what we called as a horizontal water line. Now, another possibility exists, which is actually the reality in seas - in a most kind of sea that is the presents of waves.

Now, waves – since you are in the beginning of the naval architecture, you might not have had much of exposure to waves. But, a wave is an important phenomenon when it associated with ship motion. Mostly, we are talking about waves that occur on the surface.

Now, you know that when we are dealing with the ocean, we have the interface between the ocean and the air; that is you have that surface part of the water, so that interface which divides the water from air that interface we call it as a free surface. Now, as you imagine, as you must have seen in the source and even in deep seas, there will always be some undulation of the free surface. It is not a horizontal plane, but it is the undulating time dependent, unsteady transient phenomenon that free surface is a transient surface, it varies with time, it is unsteady.

Now, the consequent of such an unsteady surface or a wavy surface – that waviness is due to the presence of these free surface waves. Almost all the waves - probably not all the waves, but most of the waves we are dealing with here or dealt with or associated with the free surface waves.

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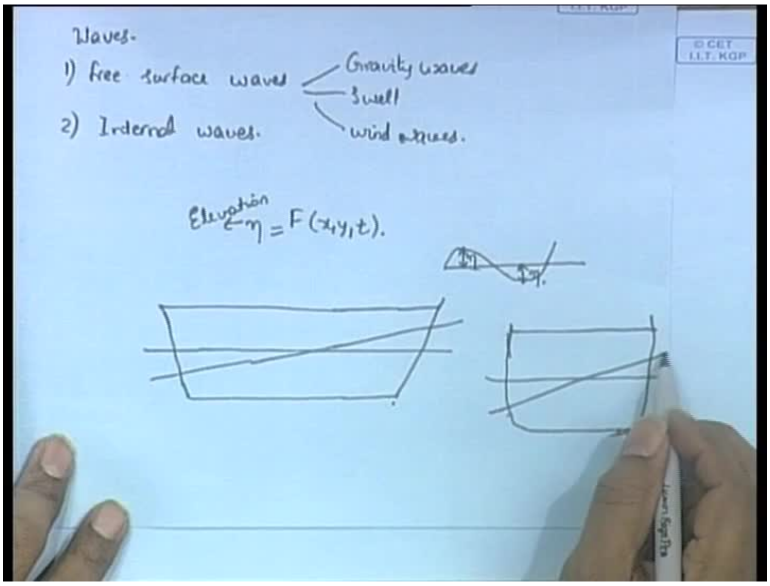

So, we deal with - so there is in naval architecture, a broad category of waves, in which we deal with free surface of the waves. These are the waves associated with the free surface. Then, this is not that important in naval architecture, but in oceanography we come across another types of waves that are associated that are called as internal waves.

Then, of course, even with this free surface waves, there are different kinds gravity waves, swell and wind waves. Of course, gravity waves are mostly generated by wind only, so these different kinds of waves exist. When you go down into the region below the surface, where there is a density gradient. This is not part of this course, but you will see that when you go down in the ocean, after a region - in the top region of the ocean there is the region very too close to the surface for about 50 meters. There is the region where the density is uniformly distributed, you have very little gradient intensity or there is no density gradient, so that we called it as a mix layer. There is the region below that were that is a not very strong, but a fairly good gradient in density that is in that region, the density continuously increases with depth. So, as you can imagine, density has to increase with depth, because only then the system will be stable.

If you have a less dense fluid lying above, if you have a high dense fluid lying above the less dense fluid, as you know the system will be unstable and it will produce circulation. The whole thing will actually become totally unstable that is the different system. So, inside this region, where there is a very strong density gradient, you can have internal waves; these are the waves that are generated due to this density gradient. Different kinds of waves exist.

We in this naval architecture only deal with what are called as free surface waves. Now, because of the presence of these waves on the free surface, you will see that the ocean surface is always not even – it is not a horizontal line - it is not a line as such, it is a curve; so that free surface is a curve - some function.

So, you get the free surface eta as a function of x y and t. This free surface - this is what we call as the elevation. Elevation - by elevation we mean, if this is the free surface, if they consider this to be a still water level, this or this are called as eta.

Here, eta is positive; therefore, the surface of the water is above the mean water line or the still water line. In the second case, the eta is negative, as a result of which it is below the water line. So, this eta is an elevation, which is the function of x y that is horizontal distance and time, it varies with time.

So, this is known as – this is the effect of waves on the free surface. Now, as a result, what will happen is that if you have a ship like this, now all problem that we consider till now, included a water line like this or like this. At any case, it is a horizontal water line. As we know even though I have drawn this as slanted, it is still a horizontal water line. It means that the ship as tilted, so the ship as tilted by some amount or trimmed by some amount, as a result of which you have got the - which is represented by a tilted waterline.

Now, this is the type of water lines we have been considering till now. The different type of water lines whatever we have considered, even if you are considering heeling also - so if you remember, we have consider different types of heeling problems. All those problems we consider so far, dealt with water lines like this and like this.

So, always it is a straight line, we never can dealt with any kind of problems, where we have water lines that are anything other than a straight line. So, this is the problem that we have considered till now.

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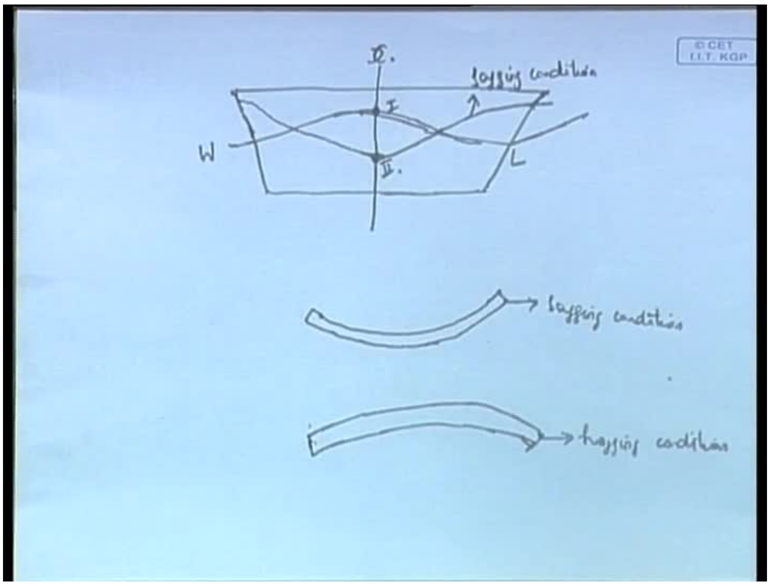

Now, we come to this new type of problems. As I just told you, the presence of waves makes it a curve instead of a straight line. So, it become like this - it become a curved surface instead of a straight line. Therefore, your water line really looks like this, in the presence of a wave, in a free or in the presence of the effect of - in the presence of the real free surface.

Please note here one thing; I have used the term free surface or the free surface effect. Actually I have used the term here two times, in hydrostatics there is the free surface, which have already talked about, which is the effect of free surfaces on free surfaces of liquids, means it is the effect of tanks.

So, if you have tanks inside the ship, you have a free surface; that means if the tank is not a fully filled, the surface is free. You know that the surface can be tilted, if the surface tills that effect, the effect on GM is called as a free surface effect.

This chapter - this lecture today - the free surface I am talking about has nothing to do with the free surface of tanks. What we called as free surface effect so far, they are totally unrelated and there is no relation between the two. This free surface effect or this is associated with marine hydrodynamics, the wave hydrodynamics. The hydrodynamics, call it so as a free surface effect, but please decent the different between two very clearly. There is no relation between the two free surfaces, the one is different and the other one is totally different.

This free surface, what we are saying is that as a result of this free surface effect, all the free surface effect we are talking about in today's lecture are concern with the air water interface and waves, it has nothing to do with tanks, do not bring tanks into this picture, no such thing. Because of the presence of this free surface effect, the shape of the water line is something like this. The water line WL is no longer as straight line as we will see in the case of all problems so far. Of course, if you have a very calm ocean where there is no wind, probably let us say that we talked about stability regulations, we saw that there are coastal regions where the winds can be as low as some not coastal, let us say that even in the open ocean, there are region where the winds can be as clam as probably 10 knots.

10 knots is 0.5 meter per second that is fairly low wind, even in such cases the elevations might be very small in terms of few centimeters it is very less that is almost a straight line. So, in those conditions we can consider the free surface to be a straight line, the water line to be a straight line, but in other cases, where in reality, when you go to open oceans, where the winds are in excess of 2030 knots. Even in coastal ocean, definitely coastal regions are highly more prone with waves.

When you study the wave height in coastal areas, it comes to1 meter or 1.5 meters, it can be very as high as that. In such - that is an ordinary case, you have above 1.5 meters wave height. Now, these waves - the presence of waves produces a water line like this. Now, this presence of water line, this is a new type of water line that we are going to address in this kind of problem. You will see that when you change the water line into such a wavy form, there are effects of that on the different stability calculations, the problems become different. In general, there are two types of waves - two types of scenarios associated with waves and ships, means the effect of waves on ships is two, there are two extremes that we are considered.

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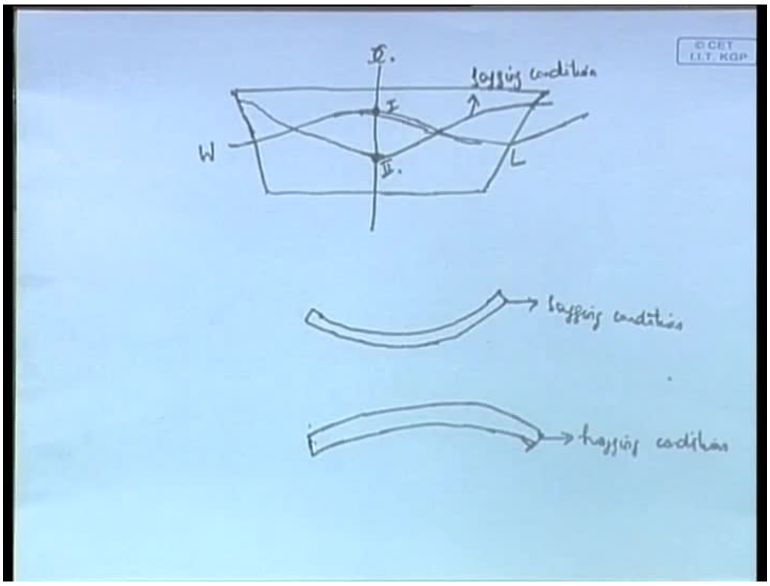

First is this, this is one case; that is we assume - I am going to assume here the basics of waves, by basic of waves I mean things like the crust of a wave, the trough of a wave, height wave, length wave, time period, wave frequency, etcetera, so these are very basic things of wave which are usually taught probably in the 12th standard. So, for may be in the first year of B tech, you should have some idea of what are waves, we are not going do that - I am not going to do that it is too elementary. So, this is the crust of the wave, now assume that the crust of a wave occurs at midship.

So, this is the mid ship, means the exact medal of the ship. At that point you have the crust of the wave, so the crust of the wave is occurring at mid ship; this is one possibility, one extreme scenario. Another possibility is when you have the trough of a wave; the other possibility is when you have the trough of wave occurring at midship.

This is case 1, this is case 2 and now two possibilities exist. When you have the crust of a wave existing at midship, as you can see, as you can imagine, the wave is like this, the ship is on top of that wave. So, the wave is like this, the ship lying on top of the wave, the midship of the ship, the midship portion of the ship, the center part of the ship, the exact midship section - that cross section, is exactly staying on top of the crust.

In the second case, you have the wave is like this, the midship section is lying on the trough, the wave is like this. Why wave occurring? The midship section is occurring on the trough. Now, these are two extreme cases, you will see that when you study these two extreme cases you are in fact considering all the possibilities, means you are considering the extremes effect of waves on ship.

If you assume that the presence of crust, there is some effect of stability, there is some effect of wave on stability. In the second case, when the ship is on a trough, you will see that the extreme opposite possibility of the effect of wave on ship occur. These are two extremes, so you study this two extremes every situation in between, where you have the wave occurring somewhere in the ship. Means, occurring along the ship you will have - it will be somewhere in between these two extremes.

So, between the crust and between the trough, if you have these kinds of wave were you have the trough, this is known as a sagging wave - sagging condition, the ship is in a sagging condition. As you can see the ship is like this now, the ship is lying on the trough, so ship is trying to - ship is being bent like this. If you consider ship to be a beam - simple beam, ship is being bent like this, produces a sagging condition. In the other condition, when the ship is - the center of the ship or the mid ship section is lying on the crust, the ship is like this.

If you considered the ship as a simple beam, the ship becomes like this. We call it in a hogging condition, so these are two extremes of the waves, so the ship can either be in a sagging condition or in a hogging condition. So, the ship is bent like this or the ship is bent like this, two particular cases - when the between - because of the shape of a wave; so these are the effects of the waves on ship.

Now, we will have to see how this will change, the different parameters of the ship how will affect the GM, for instance the Meta centric height, may be the BM, how it will affect the virtual weight of the ship itself. You will see that the virtual weight of the ship, which we will see again, it is coming. The virtual weight of the ship, which is the apparent weight of the ship, feels in the presences of a wave. Means, on a top of a wave, for instance the ship in fact has a tendency to feel less weight than its real displacement, it feels less weight and there is some extra force acting, which tends to reduce its weight. In the presence of the crust, in the trough for instance, the ship will be producing some extra forces, as a result of which it will tend to become heavier than it really is.

So, it will appear to be regular, it is not heavier than it is, but because of an extra force acting, it will add to the displacement and the total displacement, seems to become more than its real weight. These different conditions exist because of the presence of waves, so these have to be studied in detail, because as you can imagine, the ship is mostly subjected only to such wavy waterline.

Ship is very rarely subjected to very straight horizontal waterlines; of course, it is a fact that this wavy waterline or the curved waterline is important only in terms of the length of the ship, means only in the longitudinal direction; so it is only the longitudinal.

This longitudinal analysis that is mainly affected by waves, why because the wavelength is usually of the order of - it can vary of course, wavelength varies a lot. I mean, for a tsunami, it can be as high as 100 kilometers, but we are talking about ordinary free surface waves, they usually have a wavelength ranging of the order of the length of the ship, 100 to 200 meters.

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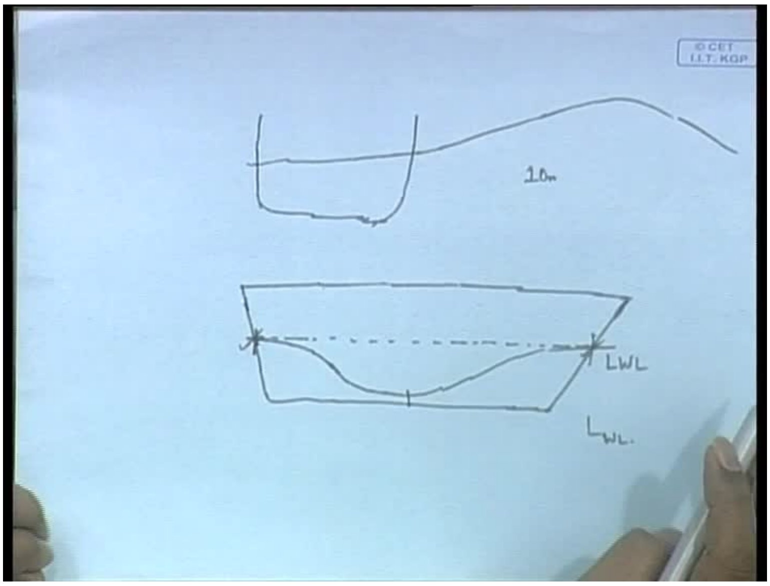

So, if that is the wavelength of the ship, when you are considering the transverse direction like this, which is the breadth of the ship, let us say it is 5 meters or may be even 10 meters. Even if it is a large ship, the breadth might be even 10 meters, so even at distance of 10 meters this waviness may be - it will go like this. It is like this and then slowly goes, so this variation is much larger than, can be seen in a breadth as such.

So, the breadth really not descent that difference, so wavelength difference is not descent so much, so we do not bother so much about waves in the transverse stability analysis, like GM KM etcetera, so it is not that important. But, it is very important in the longitudinal analysis, because longitudinally you will see that the wavelength is not at all a straight line, wavelength is a curved surface. It is very important that you consider the curve, curved nature of the surface, if you are considering the longitudinal section of the - if you are considering the longitudinal or the shear profile of the ship, in that section, when you are considering the longitudinal bending, it is very important that you consider the wavy profile.

So, what are going to do next is, we are going to consider the longitudinal analysis. Before that when you have this longitudinal, in the longitudinal, we say that the wavelength of the wave is the same as the load waterline -length of the load water line. So, what we say? The meaning is this.

So, what we are saying is that this length, let us assume that this is the load water line, this is the horizontal load waterline or the plane load waterline without the wave. Now, let us consider a sagging wave, therefore in a sagging wave what we have here, this distance, you know that - you have to know that, the distance this is the sagging wave, where the wave trough is exactly at the midship, so this is the crust. So, this is one crust, this is another crust, two crust are occurring at two ends of the ship of the load water line. This is the load water line; this is known as length of load waterline or LWL length of waterline.

Now, so what we are going to assume in our analysis is that the length of the ship. Length of the waterline is equal to the wave length, so you know that wavelength is defined as the distance between two successive crusts; this is a crust, this is a crust. So, this distance LWL - the distance of the waterline; the length of the waterline is the distance between two successive crusts.

You have trough at the midship section, so this gives you the - this is the condition that we are going to study. Now, you will see that when you are trying to analysis waves, we can have different types of waves. I am not talking about physically, mathematically when you trying to simulate waves, you can define different types of waves. The simplest of those waves would be a sinusoidal wave, a sin wave or a cosine wave, so these are sinusoidal wave.

Another type of waves which are usually used is known as a trochoidal wave. These kinds of waves have found much application in naval architecture, mainly because of their resemblance to the exact type of waves that are found in the ocean.

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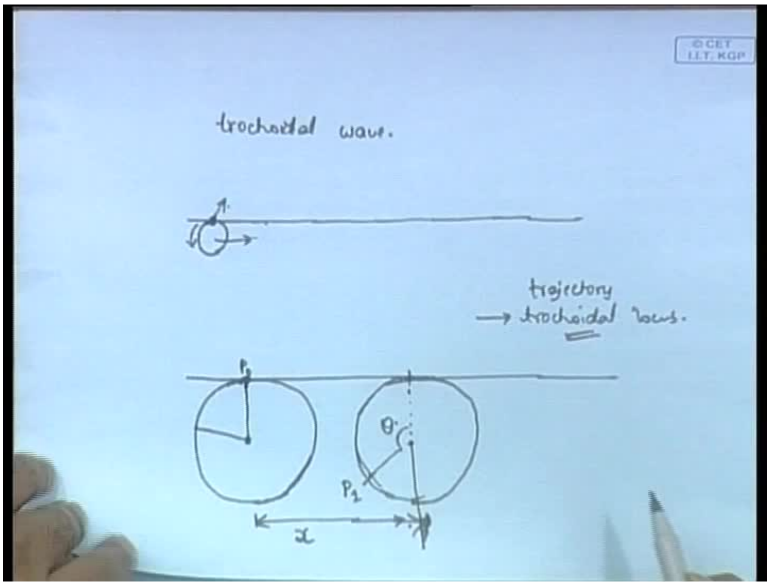

It is mostly in the ocean we consider trochoidal waves. Trochoidal waves; first, before we go into the analysis of the longitudinal section of the ship, we will see what is meant by a trochoidal wave and what is the mathematics associated with the trochoidal wave. This is the - we have what is called as a trochoidal wave, now we will have to define what is the trochoidal wave?

Now, trochoidal wave is defined like this. Suppose, you have a straight line, I have a circle here, you consider a point on the circle, the circle rotates like this - let us assume the circle rotates like this. It just rotates and it moves so moves on the underline - on this under part of this straight line. It is like you are considering a ground and a ring, it is rotating on the ground and just rolling like this, it is moving like this and it is rotating. It is rotating like this and it is moving like this, so it keeps going like this, in this direction, it moves in the longitudinal direction along the underside of the straight line. When this happens, the trajectory of this particle it traces out a path, so the trajectory of this particle traces out a path. This is the particle; we are considering about it traces at path and that path is what produce a trochoidal wave.

It that path, is a trochoidal that is known as that is the trajectory or the locus. Locus is what we call as a trochoidal, so that is what we are going to define now. Let us assume, it is like this, I draw it slightly bigger, so this is this and this is the circle. Now, we are considering a particle here, so this moves through a distance x, so it comes here, it becomes here. The same ring or the circle has rotated or rolled by this much distance x, the center. The center as moved by the distance, this is the center, this center has moved by distance x. In the time that the center moved a distance x, this particle is initially here; P0 is the initial position of the particle and the particle is now at P 1. In the time, the circle moved a distance; circle horizontally moved or translated a distance of x in the horizontal direction along the underside of this.

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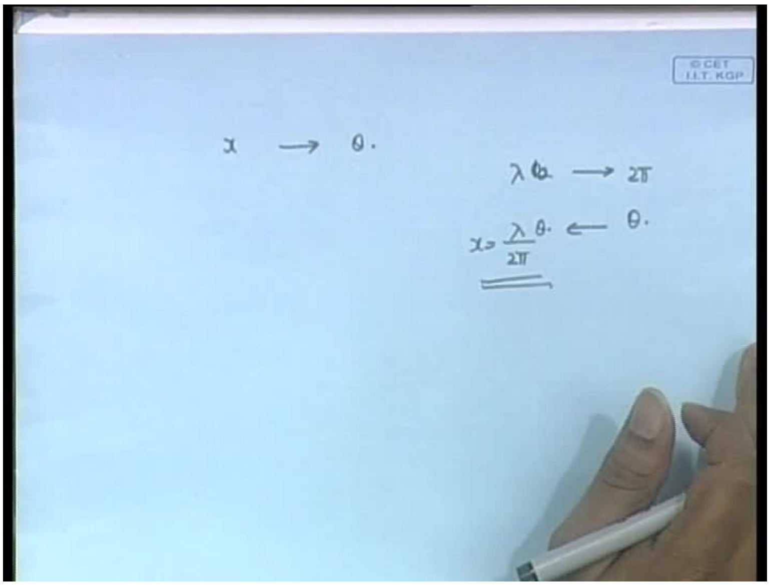

The particle P0 reached a point P1, it moved an angle let say theta. What we have is that as the particle moved a distance x, it moved through a - particle rotated to an angel theta. Now, what we say is, let us suppose now we are going to define the problem such that a trochoidal wave is define like this, so a trochoidal wave - in a trochoidal wave, we say that as the circle moves through a distance. As the center of the circle moves a distance of lambda, which is the wavelength of the wave - wavelength of the trochoidal wave is lambda, as the circle rotates through a distance of lambda, as the circle translates through a distance of lambda, the particle rotates through an angel of 2 pi. By the time, it moves lambda here, it rotates through 2 pi completely; it starts from here and reaches back there that is 2 pi.

When it moves a distance L or in the case we called it as lambda as the wavelength, it rotates through an angel 2 pi. Therefore, when it rotates through an angle theta, it must have translated by a distance of lambda by 2 pi into theta. This is the distance through which it would have moved, x will be equal to lambda into theta by 2 pi. This is first relation that governs how the particle tills. Now, in this, note that the distance between the heights of the wave - what we means of the height of the wave is the distance between the crust and the trough.

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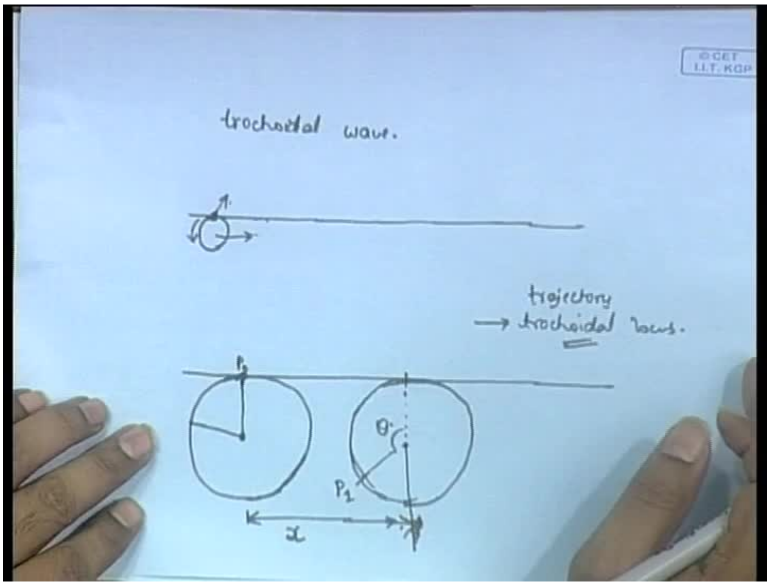

So, if you consider the crust of a wave, in case of the sinusoidal wave, you will see that the crust of the wave is at an equal distance, then the trough in this case also it is true, in trochoidal wave also its true. You will have the crust trough at equal distances from the still line - still waterline.

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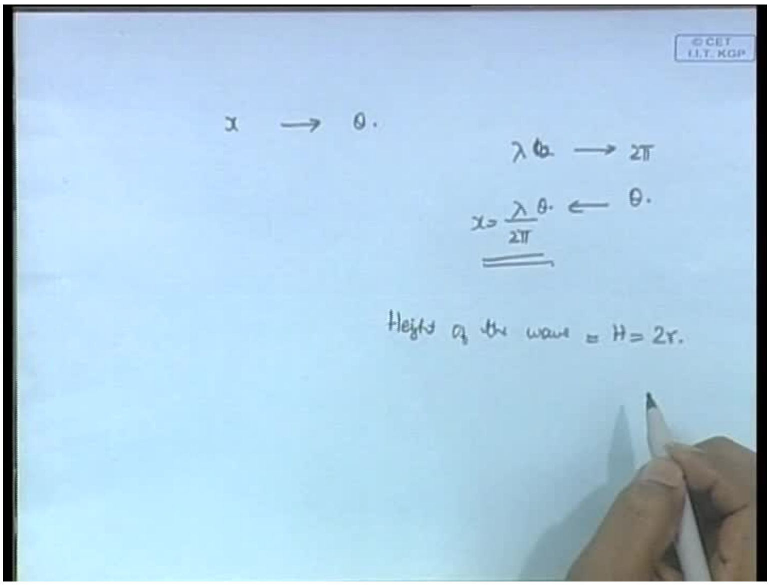

In this case, you will see that the height of the wave, you see here is equal to H, which we call it as it is equal to 2 r, where r is the radius of circle - this circle.

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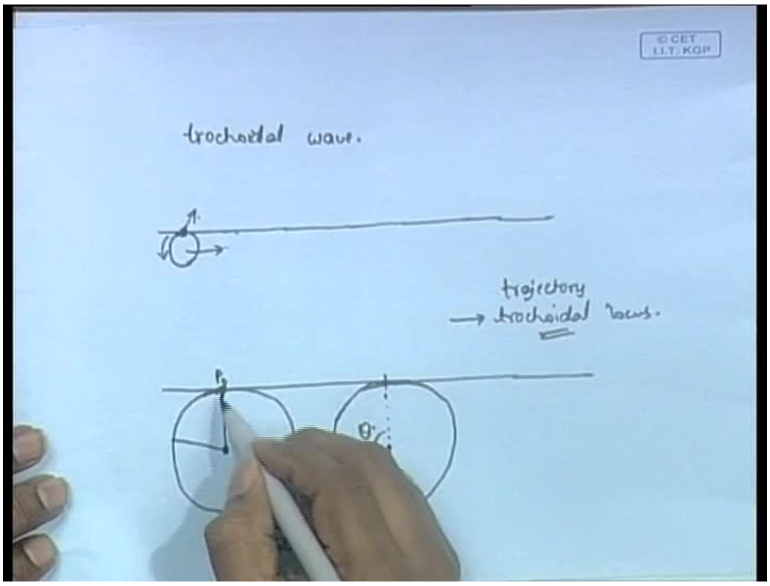

So, this is the circle, this distance is r, therefore this is r, another r, because the particle always goes like this, particle can never move a distance greater than r, because the particle is moving along the circle, therefore the particle can always move a maximum distance of only r from the center.

This is r, so r plus r 2 r gives you the height of the wave that is the distance through which the height of the wave moves maximum.

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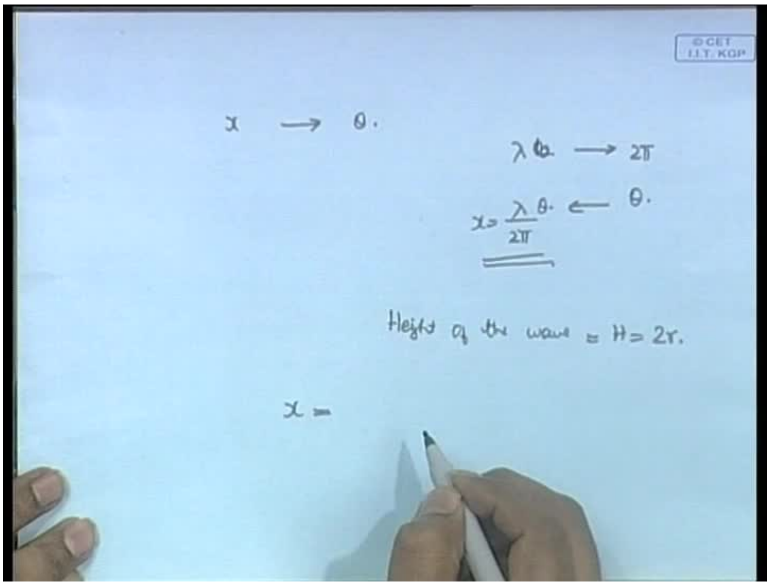

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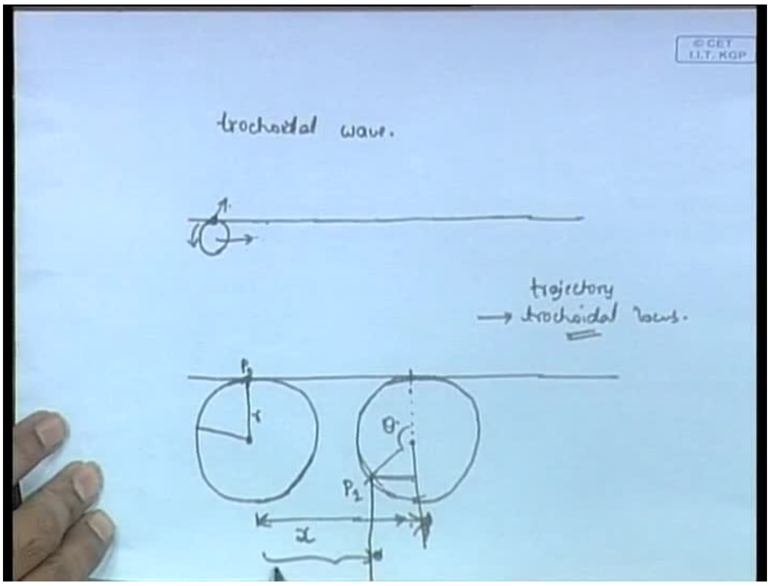

How do you find the position of the - how will you get the x coordinates? x coordinates at any center is given by - I mean the x coordinate of the particle at any point is given by - this x, which is the distance through which the center of the circle as moved x minus this distance. We are talking about the position of the particle, note a particle is actually this P; it is the particle that is moving along the circumference.

We are not bother what happens in the center of the circle, the particle is moving along the circumference of the trochoidal wave, therefore the distance through which the particle has moved is x - at any instance is x, which is the distance through which the center has moved minus this distance. So that will give you the distance through which - this distance through which the particle has moved this distance, so you get this distance through which the particle has moved.

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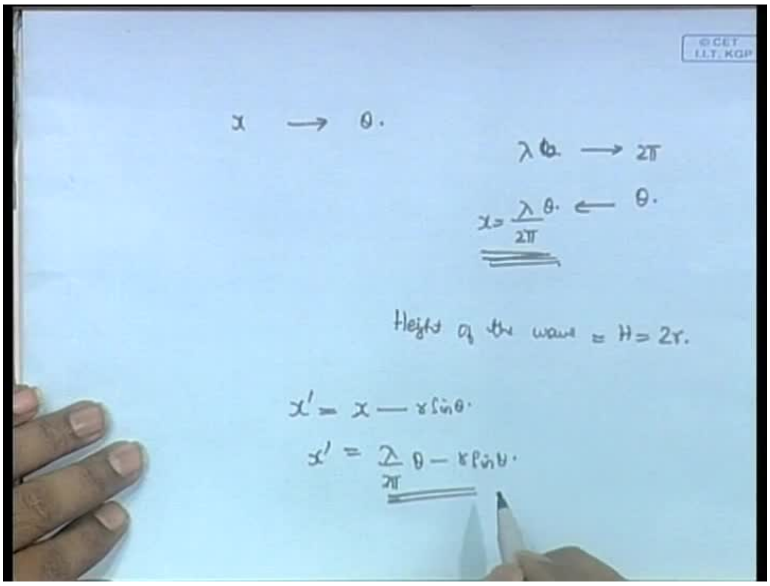

So that is x, is equal to - let us call it as x prime, is equal to x that is the distance moved by the center minus r sin theta is this. So, r sin theta minus - so this will be - so if this is theta, this distance will be r sin theta and this distance will be r cos theta. You can look at it like this, it is probably easier. So, this is theta, this is therefore 90 minus theta. This is r sin theta, therefore this distance becomes r sin theta and this distance becomes r cos theta. Oh sorry, this distance becomes r sin theta and this distance becomes r cos theta.

So, therefore, center of the distance through which the particle as moved horizontally is x minus r sin theta. Now, x we have seen here is lambda by 2 pi theta minus r sin theta, so this gives you x prime - the horizontal distance through which the - so this gives you the x at any instant of time for the particle. The x coordinate of the particle at any instant of time, provide you to fix some origin. The best way to fix origin is, suppose you are considering the ship in this case.

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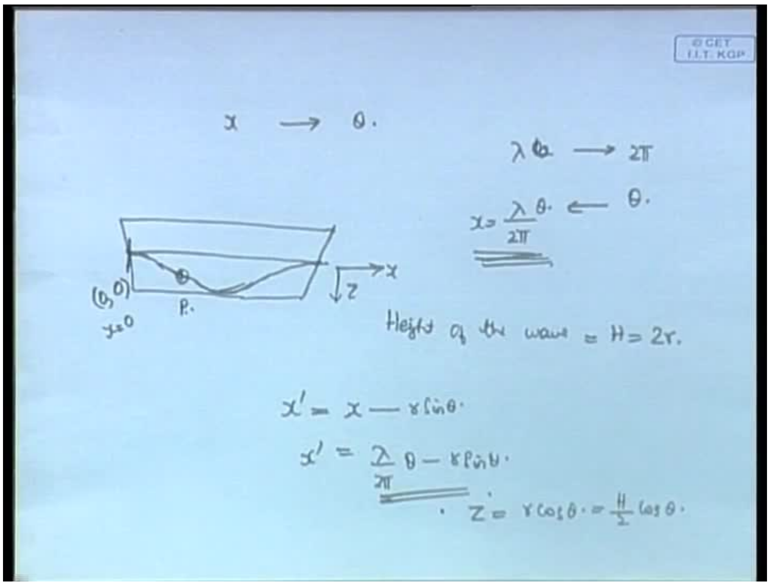

Here, you have the ship, so this is the water line, you know that you are - let us consider a sagging wave in all cases, so this distance let us fix at the origin always - this is always fixed as 00, therefore here x is equal to 0. You are talking about this position at any instant of time P, which is we are talking about the P's. Right now we derive the relation for the x coordinate, it is very easy to get the – let us call this is z coordinate, this is z and this is x. You know y is in the transverse direction that does not come in the picture as such, because we are talk about waves in the longitudinal direction, transverse direction, let us assume it is horizontal for that matter, for the reason I described before. So, this is x prime, this will give you the x coordinate. Now, the z coordinate is very simple, it is r cos theta, I already have shown you that it becomes H by 2 cos theta.

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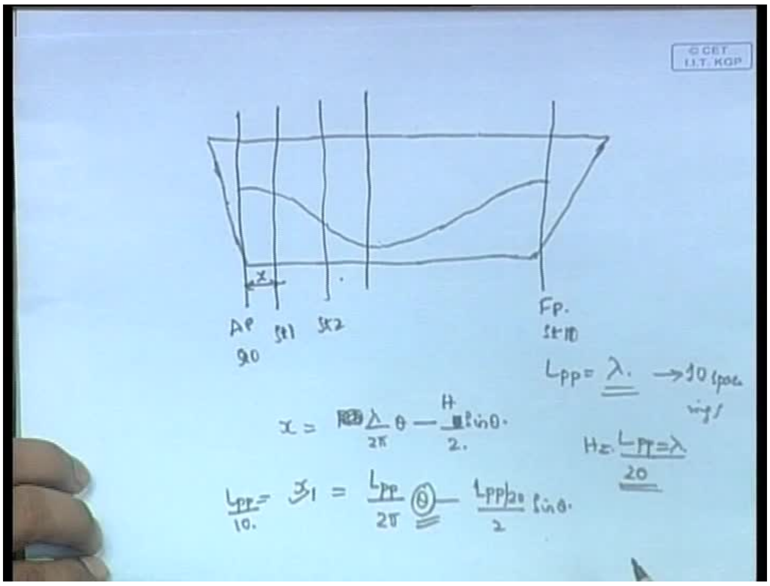

Now, once you have this x and z, you have - you are getting at any x and z, with any particular value of x and z, you can draw the curve. Means at any point if you have the x and z, how will you solve this as such? I will tell you; that is, what we could do first is, you take the ship and first of all you draw the stations. So, let us assume that this is the aft perpendicular, this is the forward perpendicular. What we will do here is I make a slight difference from the previous statement. What we could do is, let us assume that instead of LWL equal to lambda, we will this time - we will assume that L PP is equal to lambda, L PP as you know is the length between perpendiculars.

So, we are again going to consider a sagging wave that means you will have the trough of the wave at the midship. Now what that implies is, we will first set the crust - one crust of the wave at the aft perpendicular. The wave will proceed; the center of the trough will come at the midship. Again, the next crust of the wave will come at the forward perpendicular, so like this. So, this is the length between perpendiculars, is equal to lambda, the wave length of the wave.

Now, what we do next is divide the whole ship into a fixed number of stations. Let us assume we divide it into 10fix number, let us say 10 stations. Like this you divide it into stations, so this we will call it as station 0, station 1, station 2 like that it becomes station 11. So, you have 10 fixed - 10 intervals or 11 stations, so your 11 stations and 10 fixed intervals; this will be station 10, because 0, 0 to 10.

You have 11 stations and 10 spacing's, so what we have done is, the whole region, so the whole lambda, the wavelength of the wave, we have split in into 10 spacing's. So, 10 spacing, we have split the whole ship into 10 spacing. Now, the next step is, we have already got this expression x is equal to r theta, I mean x is equal to lambda by 2 pi theta minus r sin theta. We have this expression, where is equal to H by 2, we have already seen r is equal to H by 2. This H that is the wave height is a property of a wave that is being studied.

In general, you will see that when you are doing ship analysis, you come across wave that have of an order of LWL or L PP divided by 20 that is roughly the range of H, the height of the wave. Of course, if you are really doing an analysis, you really need to know what is the height of the wave, real - the exact value of the height of the wave, but since right now we are doing - we are just expanding on the theory, we will say that the height of the wave, remember that is something you have to be given, you cannot guess the height of the wave, no such thing. Whatever method you have, you run some numerical wave models - some model and you develop the height of the wave as a result or you measure the height of the wave using some tide guage or something. Once you have the height of the wave, you will get this H - the wave of height.

So here, what we have is x is equal to H by 2 in this case. We can assume for an instance it is L PP, by roughly this is the order in which you will be seeing. This wave height will be about the wave length of the wave. As you can see L PP is equal to lambda here, the length between perpendiculars we have set it equal to lambda, so roughly H is equal to lambda by 20.

So, x is equal to lambda by 2 pi, again we have ever thing here, now x, what is x? x is this, now x is this distance, so let us consider this station 1 now. Station 1, you know what is x, this is your x. It is the position of your first station from the 0 water line, 0 stations that is from the aft perpendicular. From the aft perpendicular what is the position of your first station that distance is this x. Let us call it x 1, x of station 1, is equal to lambda again, is the distance between perpendiculars. You know the length of the ship, I mean, of course, you have to know what ship you are dealing with it, so you know the length of the ship, this divided by 2 pi into theta minus H, H is L PP by 20 by 2 into sin theta.

You end up with this, x 1 we know, this is some value, the distance which is actually equal to in this case L PP by 10. Means, the total length of the ship we have divided into 10 spacing's by 11station; we have divided into 10 spacing's. So, L PP by 10 is equal to x 1 is equal to this equation you get. Now, in this equation, the only unknown that you have is actually theta, so of course it is a slightly transcendental equation, it is not directly solvable. Because, you have theta here and sin theta here, but it can be solved using iterative method like Newton traps on method, all that you can use to solve it and you will get theta. You solve for theta, once you have theta, therefore remember this is the value of theta or this is the angle through which the particle that is travelling along the wave has rotated when it as moved some distance x. Now, how do we find the vertical distance z though which it has moved if it as rotated though an angle theta?

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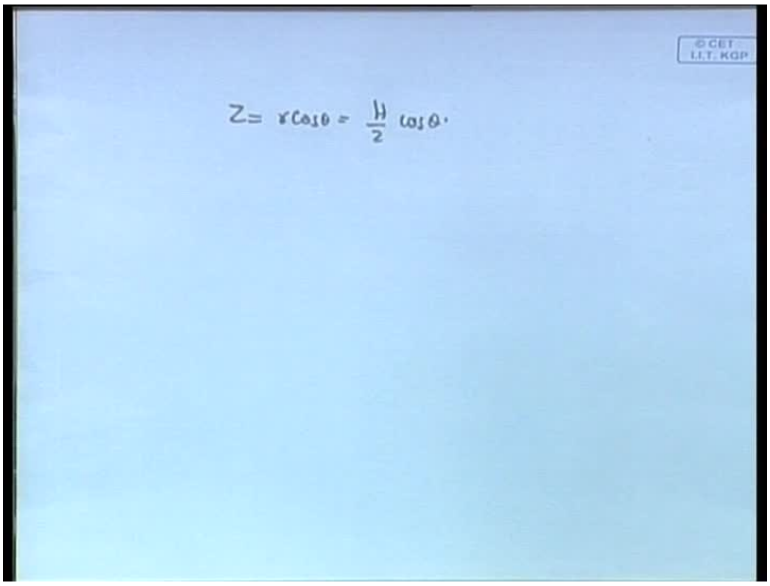

It is very simple, we have the formula z is equal to r cos theta equals H by 2 cos theta, therefore you have z equals H by 2 cos theta. Now, H is known, L PP by 20 into cos theta, theta you just measured using - the theta you just calculated in the previous section, so there you have z.

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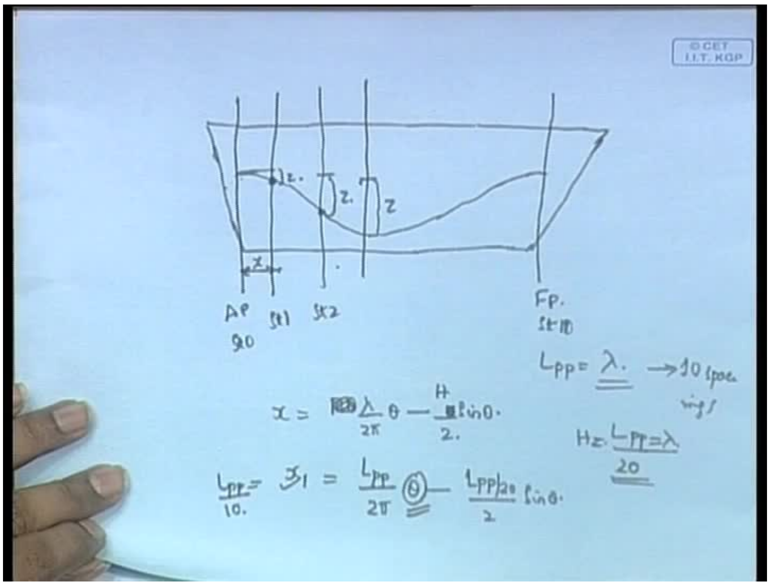

So, like this we have for the first station. Like now, this one, for this particular x, you have this position, so you know what z that we have measured is. This z is actually this; this is the z that you have just calculated. This z means, it is the vertical distance through which the particle has moved, as the result the wave translating a distance x.

So, this gives you the x and z of this point of the wave, similarly you take the next point x 2 - station 2, you will calculate the z, like this here you will calculate the z. With these difference values of x and z, for different station, you can calculate the wave height or the wave profiles. As the result of this calculation, you will now end up with the proper wave profile.

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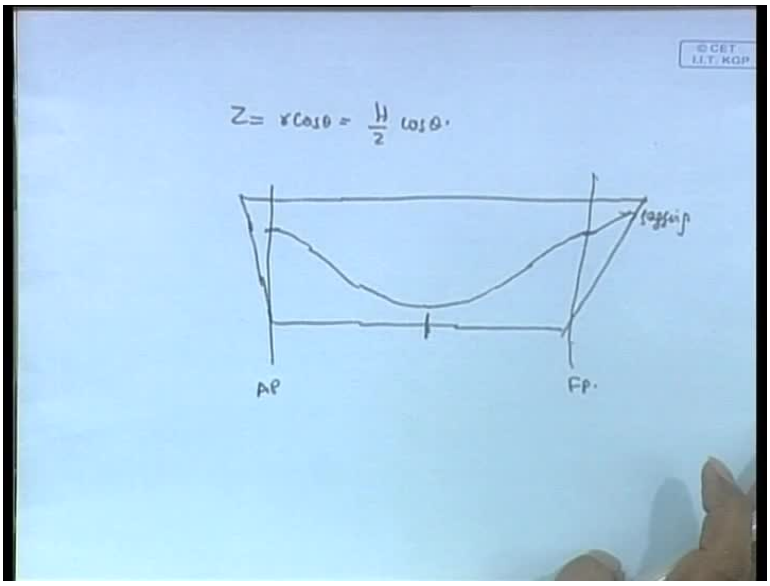

The wave, remember we have define such that you will get - it is defined such that the center of the ship, the midship will have the trough on it. This is L PP, this is the aft perpendicular and this is the forward perpendicular. You have the aft perpendicular and the forward perpendicular, here you will have a crust, here you will have a crust and here you will have the trough.

You have all these z, you have the z at each station, as a result you have the complete wave profile, you have a complete sagging wave profile, so you have the sagging wave profile. Now, this will give you the trochoidal wave, you will get the trochoidal wave on the ship. We have, now the profile of the wave super imposed on the total length of the ship that is in fact the one of very important way of - this is one of the important things. Doing this, now you have the wave profile, based on this we can calculate that longitudinal position. Right now we have the different - we have the wave profile, now how do you find the mean position about which - of course, the circle is rotating about that center, as a result you are getting the wave.

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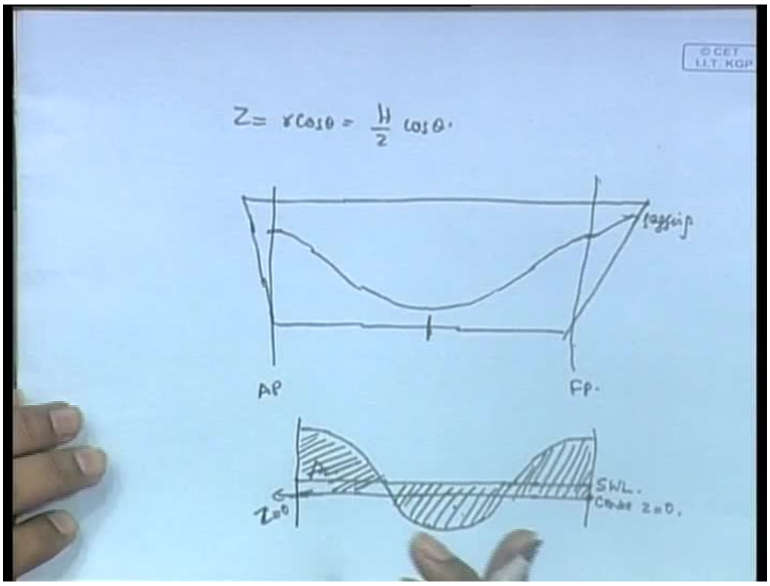

Now, there is a region, there is a line, in between the trough and the crust of the trochoidal wave such that if you take the integration of one wavelength of the wave, if you integrate the wave over one wavelength, you will see that the total area comes to 0. Means, if you consider this to be that line, in the case of a trochoidal wave, it does not - in the case of the sinusoidal for instance - so first of all we are considering a trochoidal wave here, what I am saying is that if this is the point, which will I call it as some still water line, this is the center over which the circle rotates, this center is given by z equal to 0. z equal to 0 is the point over which the center of the circle moves. The circle I am talking about is the circle over which the particle is rotating, so the particle is rotating like this and the center is moving that line, which is traced out by the center is this line, it is z equal to 0.

Now, in case of a trochoidal wave, you will see that if you take an area, area means this is the positive area, this is a negative area, I am talking about area about this point, about this z equal to 0, if you take area about this specific area. If you sum up this area and this area, I mean this area and over one whole water length. If I sum up this area, this area and area, it does not come to 0, because it is not symmetrical about this z equal to 0, the trochoidal wave.

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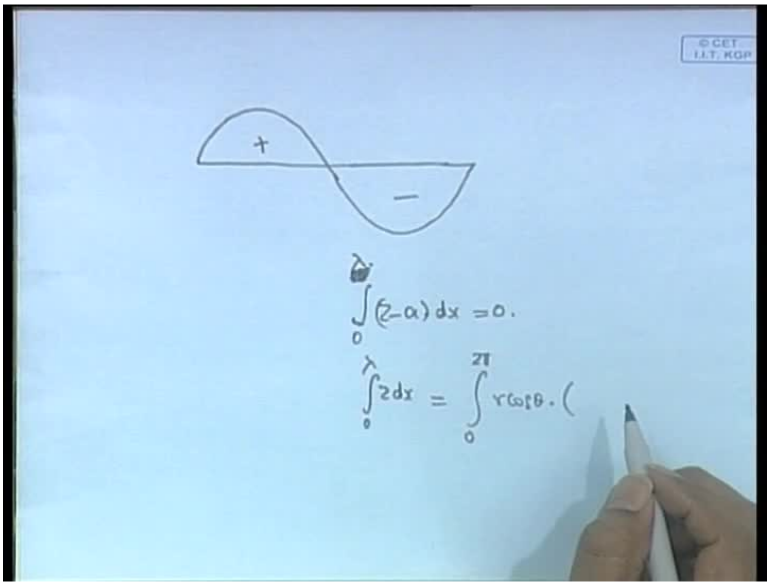

On the other hand, if you have the sinusoidal wave, then it will be - if you take an area, let us consider a sinusoidal wave like this. This is one full wavelength, so it is started from the center and ended at the center, now if I take the whole area, this area will be positive, this area will be negative, this area and this area will cancel out each other and therefore the total area comes to 0. Now, in trochoidal wave it is not at the center, but it is at a distance - in trochoidal wave it is at some point z equal to a, some distance away from the z equal to 0. How will you find this? The easiest way is you use z minus a the total integral between 0 to 2 phi of dx is equal to 0, where dx, this thing is x varies from 0 to lambda, the wavelength of the ship, wavelength of the wave, so z minus a dx between 0 to lambda is equal to 0.

Now, the first one 0 to lambda z dx can be written as that is you know that by the time that the wave center moves through a distance of lambda, center when it was moved lambda, the theta as moved through 2 phi. Therefore, integrating between 0 to lambda of dx, is the same as integrating d theta from 0 to2 phi, it is the same time, it is the same instant.

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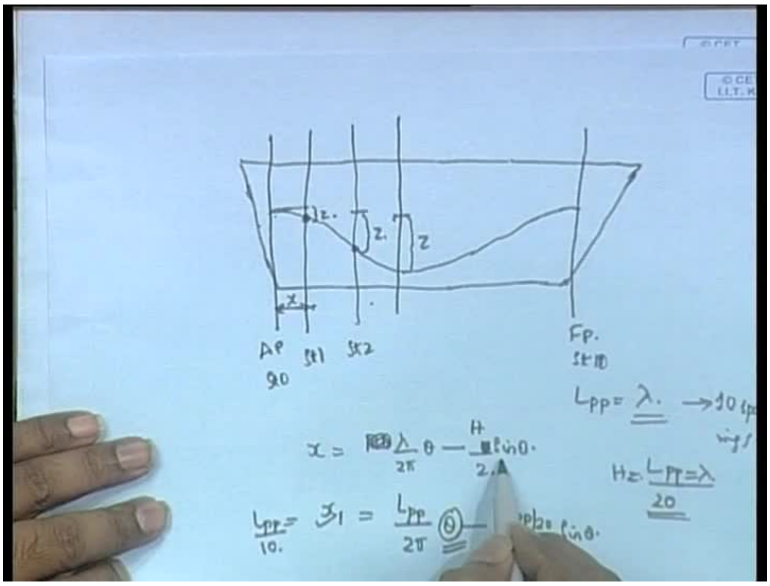

So, 0 to 2 phi, so z we have seen is r cos theta into what is dx? We have already seen that what is x? Remember, this is the expression for x, you just differentiate this, dx will give you lambda by 2 phi into d theta minus H by 2 into cos theta d theta, therefore r into lambda by 2 pi minus r cos theta into d theta.

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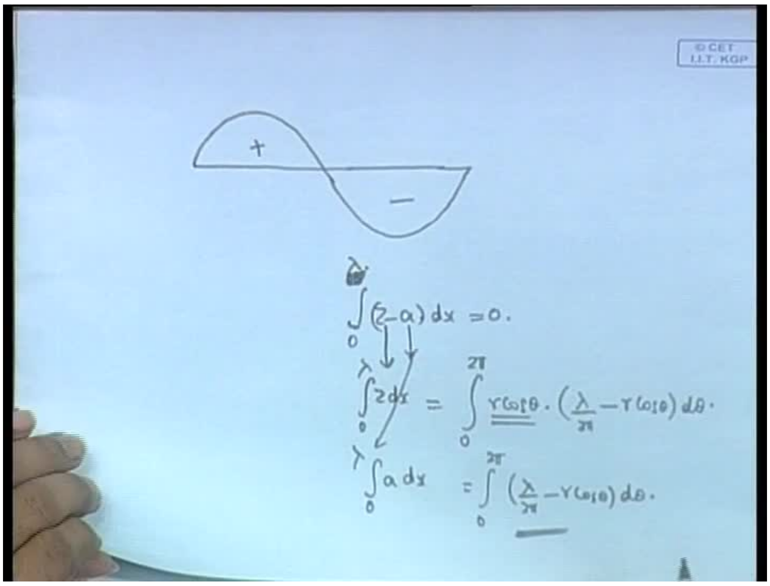

Now, just integrate this whole expression, so this is z and the second integration, this is this one. Second one is just this one, is just 0 to lambda a dx, it is very simple, you would not have this term, so 0 to 2 pi, r cos theta is not there just lambda by 2 pi minus r cos theta into d theta.

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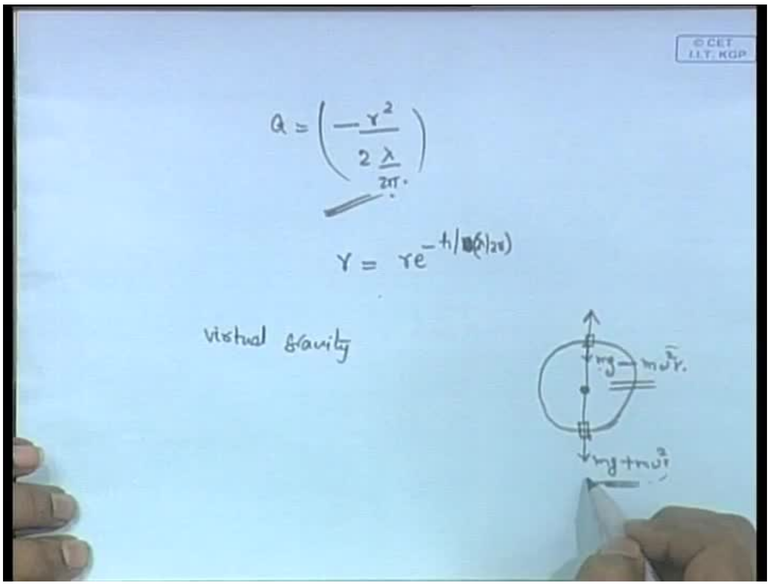

Now, if you integrate the whole thing, if you said this, means the total area equal to 0, you will see that you end up with a equal to minus r square by2 into lambda by 2 pi. So this is your value of x. This is the center about which the total area under the curve will become 0, so this is the expression for trochoidal wave. So, this is the trochoidal wave that is mostly adopted in naval architecture studies, because of its proximity to the real type of the wave found in the ocean that free surface waves are very close to trochoidal waves. Therefore, the concept is, it is easy to do this calculation as well, it is more accurate also than the sinusoidal waves, therefore we follow this.

Then, couple of points is that just like any type of waves found in the any other type of the wave; you will see that in a trochoidal wave. First of all, when you have the surface, there will be some waves, they are travelling in this circular direction in this, and we have seen it moving in the circle, this particle is actually moving in a circle. It is translating like this, it is moving in a circle, below that that is the first circle and this wave does not really completely stop here. Just below this also there will be another wave, there be another thing moving in another circle, another particle moving in another circle, here another circle it moving like this. Below that another circle, like this it keeps continuing going down. The end result is that you have the serious of circles going down, you will see that as it progressively go down the radius of the circle will keep decreasing.

So, the radius actually exponentially decreases with depth, you will see that the radius is given by h by lambda by 2 pi. So, h is the water depth through which the center of the circle. If h equal to 0, then you have the first one that is are surface, h keeps increasing, you have the water particles as you going in a circle, as you go down. So, you will see that as you keep going down, the radius of this circle through which the water goes that water rotates, radius of that circle actually keeps coming down exponentially. So that is the first thing.

Then that is another concept which we call as virtual gravity, I already mention this. That is, as you can see, as the particle is going in a circle, that particle has two forces acting on it. First one is the weight of it MG, the mass into gravitational acceleration; M into G will be acting on the particle that is directly downwards. Since it is moving in a circle, there will always be centrifugal force acting on that particle. Now, centrifugal force is given by m omega square r, where omega is the angle of frequency of this rotation. So, r is the radius of the circle, m omega square small r.

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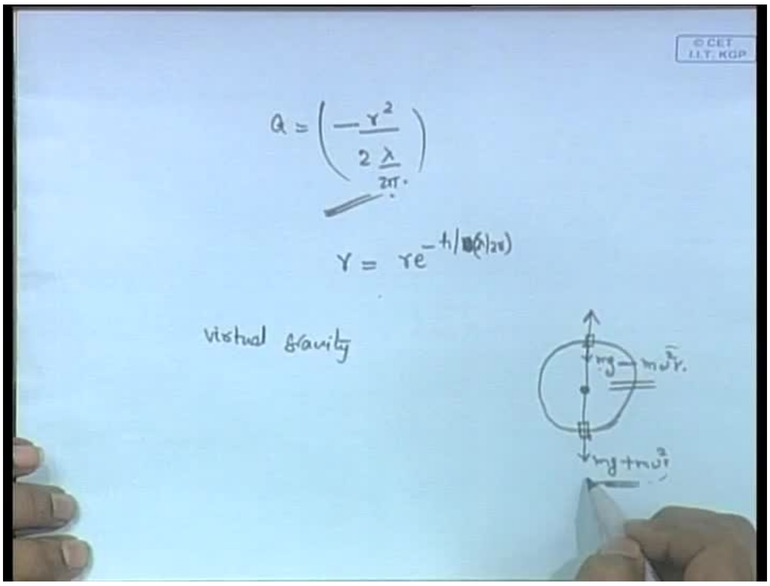

Now, this will be like this, so this particle here it will be subjected into mg minus m omega square r, because this will be acting outwards like this. If the particle is at the bottom or other trough, it will be subjected to mg plus m omega square r. I hope it is clear why its plus and negative, because here always centrifugal acts outwards, here when it is acting outward is against mg, here when it acting outwards, it is augmenting mg, so mg plus m omega square r. These are - what you see here is this, is the additional thing actually seems to augment the gravitational force or it either decreases or increases the gravitational force that is what we called as virtual gravity.

There is the slight increase or decreases in weight from what is normally seen, what its original. If you have a body floating on that it will be subjected to this mg plus this centrifugal force, will either increase or decrease its weight, it is seem to increase or decrease its weight. Therefore, on the surface, when it is on the wave crust, when the particle rights on the wave crust, we will see that the - so when the particle rights on the wave crust, the weight of the ship is seen to be decreased.

As you can see that minus comes, its weight is decreased. When the ship - the ship will seem to be tender that is the word, the ship becomes tender at the wave crust. On the other hand, when the ship is on the wave trough, the weight is augmented and the ship is seems to become heavier and then it really is. On a trough, the ship seems to become heavier and then really because of this additional centrifugal force, which augments the weight and this. There are two principle mainly associated with trocheidal.

First thing we have to remember is that the ship analysis mostly done using trochoidal waves. Of course, the lot of application using sinusoidal waves are also there, not excluding that most of them are done using the sinusoidal waves only. But, trochoidal waves are the very important phenomenon that is use mostly in these longitudinal studies. This is what we really mean by a trochoidal wave, these are the some of the properties of trochoidal waves. In the next class, I will talk about some of the applications of the trochoidal waves, longitudinal bending on the properties of the ship on how we calculate it; thank you, I will stop here.