Hydrostatics and Stability

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Module No. # 01

Lecture No. # 03

Archimedes Principle (Contd.)

This is the third lecture. In the last class, we talked about the Archimedes principle and we showed some slides showing the effect of Archimedes principle on floating bodies.

The principle states that the weight of the volume of water displaced by the floating body is equal to the floating body in the case of a freely floating body.

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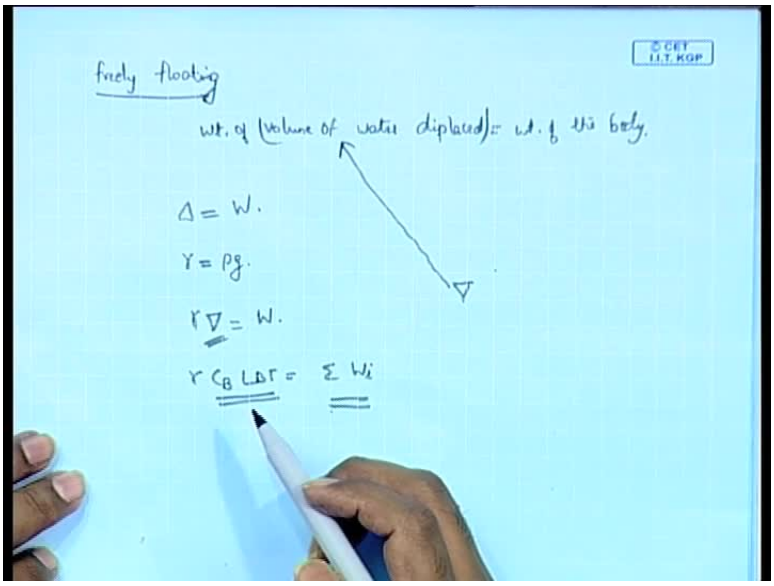

If you have a freely floating body, then principle says that the weight of the volume of water displaced equals the weight of the body. Now, the weight of the liquid displaced is called as The volume of the liquid displaced is known as the displacement of the ship.

What this says is that displacement delta which is the weight of the volume of water displaced is equal to W, the weight of the ship. (Refer Slide Time: 2:00) Then we also know that or you can say that gamma is equal to rho g. So, gamma into del equals W, where del is the del is this. It is the volume of water displaced by the floating body; that is del.

So, gamma into del is equal to W, which you can write as gamma into C B into LBT equals sigma weight i, where the right hand side sigma W i represents the net volume and it represents the sum total of the weights and each of them represent the different weights that are acting on the ships.

This will include the dead weight, include the lightweight, the hall weight, the dead weight, etcetera. So, this represents the total weight of the ship. This is the weight of the ship and this represents gamma C B L BT; we have already seen C B is the block coefficient; block coefficient into L BT gives you del which is the volume.

This is one first principle that we have to study in order to understand the principles of floating bodies. This is about the forces. Then another thing that exists along with this is the moments.

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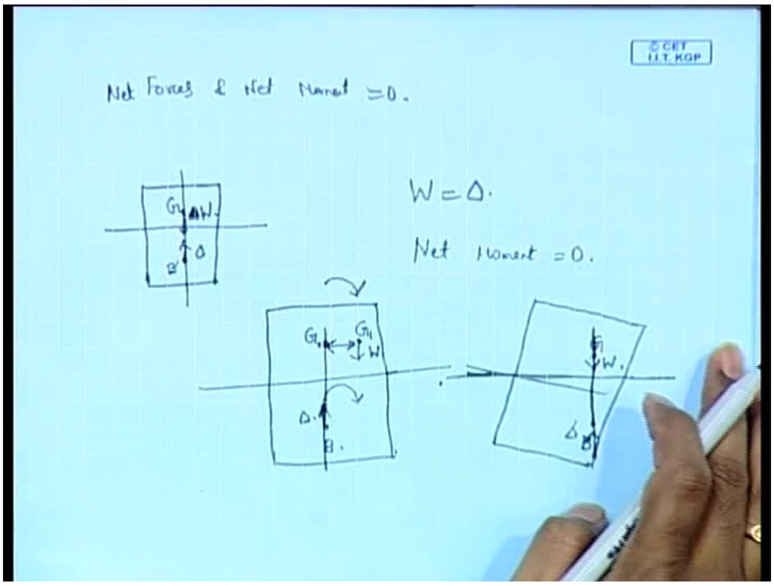

We have already seen, for equilibrium, the net forces and net moments should be equal to 0. This is the condition for equilibrium of a floating body. and if we consider the moment

Suppose, we have initially a body like this. Here you have the weight acting; this is G and the weight acting delta. This is B; B is the center of buoyancy; this is the centroid of the immersed volume of the liquid. From there, upward acts the delta and this is the weight. So, this is weight W of the body and from below the center of buoyancy upward acts the displacement of the ship.

As you can see, there are two forces; these two forces should be equal. W should be equal to the delta in the case of a freely floating body. This is the implication that the net forces should be 0 on a ship on a freely floating body.

The next one is that the net moment should be 0. How will the net moment be 0? Net moment can be 0, only if this weight W and delta which we are given going up, and W coming down - they act on a same straight line.

Like this, if they are on the same straight line then you do not have a net moment. Then net moment is equal to 0 in that case. Now, suppose that the same body is by some method, for example - by the shifting of weights - G initial has moved from G 0 to this new G 1 point. The center buoyancy is here at B; initially it was in the same straight line

But now, G 1 which is the new center of gravity has moved to G 1 and so here the weight acts W and here just as before, the displacement acts upwards. So, the buoyancy force and the weight W are separated by some distance. The buoyancy force and the weight are separated by some distance.

Now, if this happens then there is a net moment acting on the ship. There is a weight acting downward here and there is a weight acting upward which means there are two forces and there is a distance between them; it creates a moment. There is a tendency to turn the ship in this fashion like this. So, when this turns you will have

This is the water line again and then new G has moved here and because of this tilting B has also moved. As a result of it, you will get the new you will get this delta that is acting upwards and W that is acting downwards again coming on a same straight line. This is the condition for equilibrium. So, in this inclined position, you will see that the moments the net moment is 0 and the net force is also 0.

Then, in the case of naval architecture - in this case - instead of trying to draw all the figures - instead of drawing the inclined body bringing the inclined body into the picture more than once it is easier to draw the waterline itself

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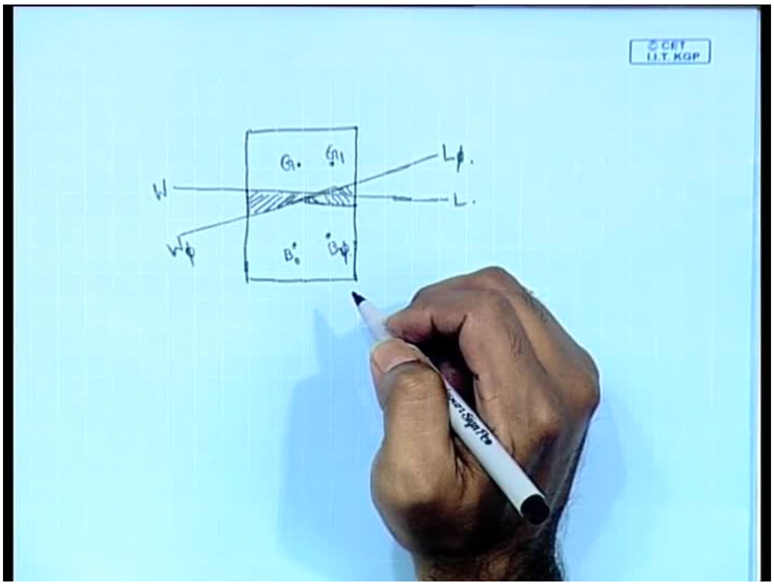

For example, instead of the body tilting, if this is the initial state and in the final state the this is the final state This is the initial state and this is the final state in which case, the body has rotated by an amount. more part has been submerged here so In this new water line, some more part is submerged on the right side; some more part is submerged on the left side. So, in that case, instead of drawing the body itself again in an inclined position, it is better, if we change the water line to meet this. For example, like this.

If I put a water line like this, the meaning of this is that this is the new waterline this much has immersed a lot of region like this much region has immersed and this much region has emerged. So, this is the new waterline.

We call the initial waterline as W L and then the new waterline as W phi and L phi. So, you have W L and W phi L phi. this is how you draw the inclined

This is how you treat the inclined waterlines, in the case of the naval architecture. This is how we do it. Initially, you have the G here and the B here; then here, G has shifted here to G 1 and this is B 1 or B phi. We usually write it as B phi. The book of Adrian Biran writes it as always B phi. So, B 0 and B phi and they will be in the same straight line.

Now, we already talked about the different kinds of equilibrium stable equilibrium, neutral equilibrium and unstable equilibrium. This is something, which we have already discussed. then so when you have a .suppose, the body inclines

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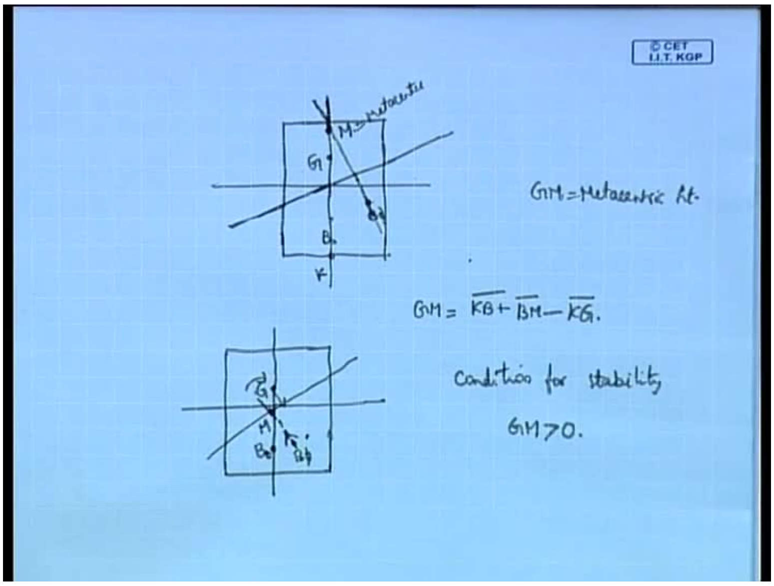

Suppose, the body has inclined with no change in G as such, but the body has inclined due to some other factor. For example, it could be a wind force; a wind force acting on the ship will produce an inclination heeling. So, the body is heeled. This is B 0 and this is B phi. This is the initial vertical and I draw a vertical from here. You have to remember, this is the waterline and this is horizontal. Even though, it looks inclined in this figure, the line is actually horizontal. It is the horizontal waterline that is drawn here and a perpendicular to it, gets is known as the it is always a vertical line. So, this is vertical and where this line meets this point, we call it as metacenter M and of this. GM is known as the metacentric height. As we showed in the last time, we have K, B, G and M. So, you will see that the GM is equal to KB plus BM minus KG and if this is M, another possibility exists.

That is, let us suppose the center of gravity has shifted here and this is the new waterline. Suppose, the vertical meets it here, M. G, B 0, M, G. (Refer Slide Time: 14:05) In this case, you see that M is below G and if M is below G, you can see that the moment is again like this.

There is no restoring moment. net moment is to The net tendency is to increase the tilting and this tilting will keep on increasing, till the body capsizes. That is this figure. So, what we see is that as long as M is above G, as you see here, as long as M is above G in this figure, the body is stable and if M goes below G then the body becomes unstable.

Therefore the condition for stability is that GM should be greater than 0.

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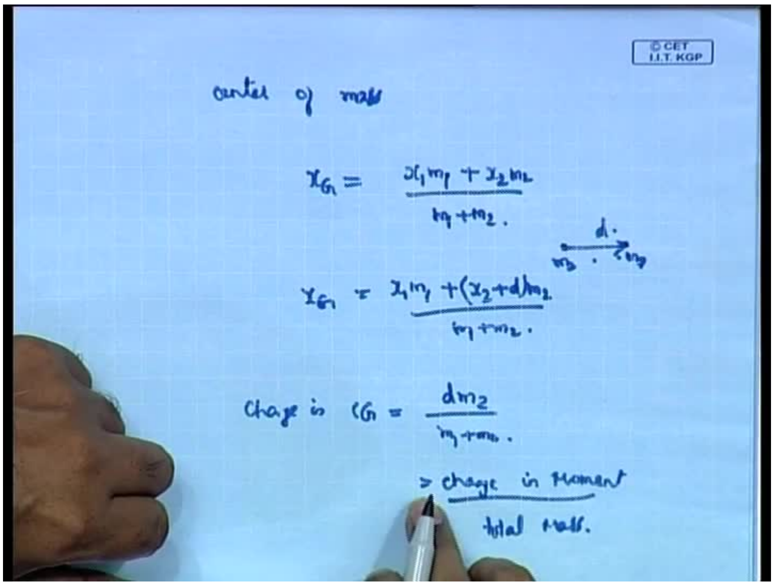

Now, we will have a small recap of something we already know. It is about the center of mass or even center of volume. Now, we know that the center of gravity of a body is given by: if there are two bodies, x1 m 1 plus x 2 m 2 divided by m 1 plus m 2. This is the position of the center of gravity of the body x G.

If we move the mass m 2, a distance d like this horizontally - m 2 has been shifted a distance d horizontally - then x G will be x 1 m 1 plus x 2 plus d m 2 by m 1 plus m 2 or the change in the center of gravity is equal to d m 2 by m 1 plus m 2 or it is equal to the change in moment divided by total mass.

If you are talking about a volume then the change in the centroid of that volume will be given by change in the moment of volume divided by the total volume. This gives you This is a lemma in center of gravity, a established result that you would all be familiar with. Just to remind, the change in moment divided by the total volume will give you the centroid of the volume. That is something that we will be using in many places.

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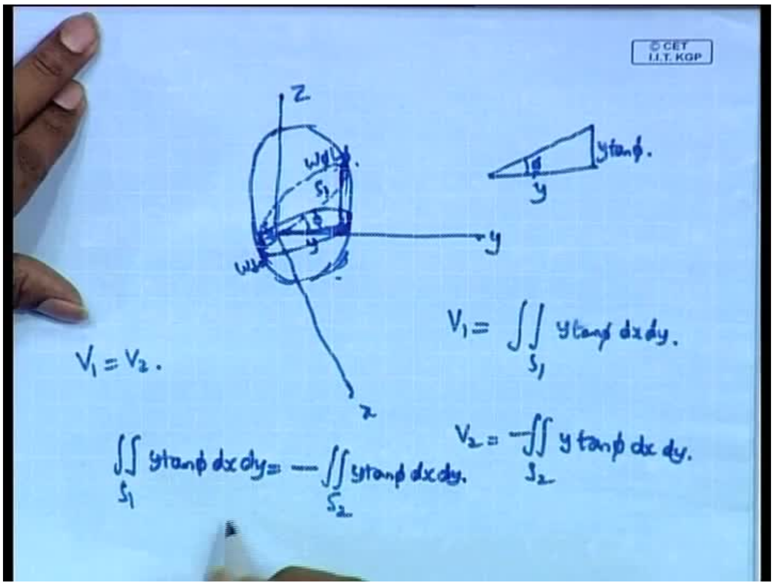

Now, we have a problem here. Let us suppose that we have a coordinate system like this and we have a body; this is z, this is y and this is x. Suppose, there is a body which is of whose surface and who let us Suppose, this is the body here; this is the water plane W 0 L 0 and then it tilts and it gets a new water plane area water plane this is W phi L phi. Initially it is here and then it is tilted into W phi L phi about the axis of inclination x. The body has tilted about x. Now, since there is no change in the volume of the body, we can say that the amount of volume that sinks in will be equal to the amount of volume that emerges out. This is true for, generally what we call as wall sided bodies. So, W phi L phi comes out and W 0 L 0 goes in. So, the volume that comes out is equal to integral, let us say that side is S 1, y tan phi that is because if this is y, this is phi and this is y tan phi.

Here, this is y and this is y tan phi. This is y; it is the y direction. If you put this and if it is heeled by an angle phi, this is y tan phi. So, the volume y tan phi into d x d y. This is the general expression for volume of any body. In this case, you have y tan phi d x d y.

The volume that has come up is equal to, This side is S 1 and this side is S 2. So, S 2 is volume that has emerged; S 1 is the volume that has submerged. So, it is the volume that goes in and S 2 is the volume that comes out. S 2 is equal to minus y tan phi d x d y.

Now, from our definition of the type of body, we have seen that V 1 is equal to V 2. That is, the amount of volume that has submerged is equal to the amount of volume that has emerged.

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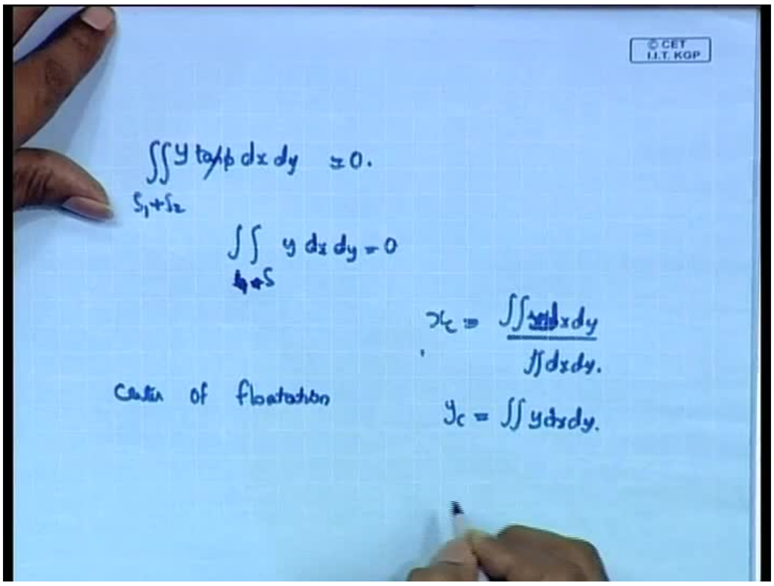

So V 1 is equal to V 2. Therefore, double integral y tan phi d x d y is equal to minus double integral y tan phi d x d y. This is s 1; this is s 2. If you bring this to the left hand side, you will get double integral y tan phi d x d y over S1 plus S 2 is equal to 0. Now, tan phi is constant because phi is a constant. So, that does not hold. So, S 1 plus S 2 is the total surface and let us call it as S.

y d x d y is equal to 0. Now, what is y d x d y? y d x d y represents the centroid of the surface S. y d x d y For example, x c will be x d x d y divided by double integral d x d y. What we see here is that over the total surface S, y d x d y is equal to 0. It just implies that this represents the centroid.

So y c, so from this what we get is that if there is a Let me read this. What it says is that if you assume the initial water line to be W 0 L 0, now after an inclination the intersection after the inclination along the axis of inclination and if this inclination is very small then this inclination will be a straight line that passes through the centroid of the body.

So, this inclination will be a thus axis of the inclination will pass through the centroid of the water plane area. Centroid of the water plane area in the case of a ship is known as center of floatation and therefore, we can say that the body inclines about the center of floatation or the axis of inclination. If you assume it to be the longitudinal axis, it will be passing through the centroid of floatation center of floatation.

This makes an assumption that the angle of inclination is very small. So, if you have a small angle of inclination, the body inclines such that the axis of inclination passes through the centroid of the water plane area.

This is a very important result and you should know that always inclination whether it is heeling or trimming occurs through the center of floatation or the centroid of the water plane area.

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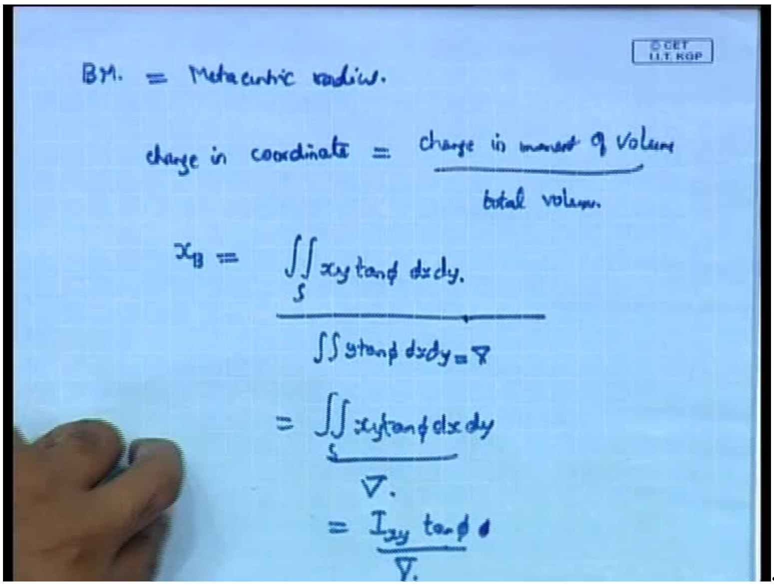

That is an important point in naval architecture. Then, let us consider something else - that is, BM. BM is known as the metacentric radius. Now, let us assume because G remains the same and because of some force acting or some wind acting, there is a heel and there is an angle of inclination and the center of buoyancy shifts from B 0 to B phi. If that has happened then the change in coordinate, how do you find the change in coordinate? It is given by the change in the moment of volume divided by the total volume. This, I write so confidently because of the lemma on the masses that I wrote in the previous paper. We saw that the change in coordinate is the change in the moment of volume divided by the total volume.

Therefore, x B the change in this is equal to The volume itself is double integral y tan phi d x d y and that, we saw in the previous section. y tan phi is the vertical distance and double integral over the whole surface, y tan phi d x d y will give you the volume of the body. If you are trying to get the moment then x into that y tan phi d x d y divided by double integral y tan phi d x d y which is equal to del. This is equal to del. So, this becomes double integral x y tan phi d x d y divided by del and this is equal to I x y tan phi by del, where I x y is the second moment of area or it is known as the moment of inertia; the second moment is known as moment of inertia.

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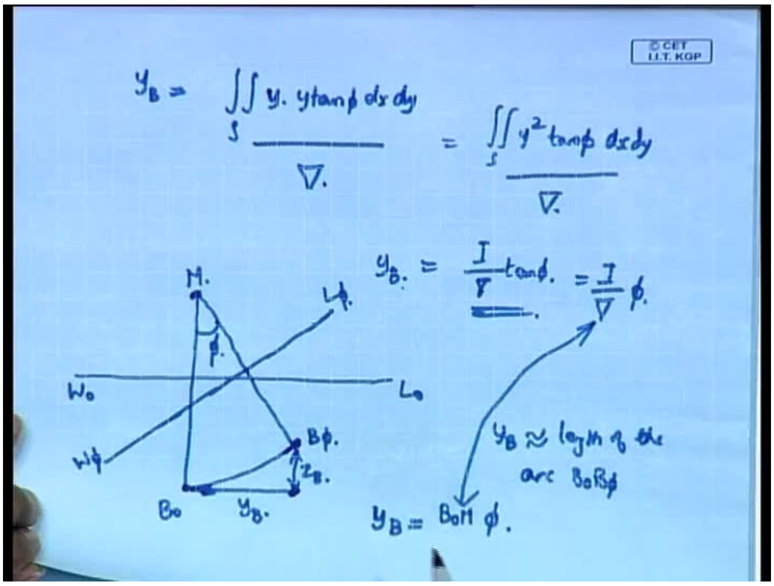

So, the I x y. Since it is x here, it is I x y about the x axis. I x y tan phi d x d y is I x y by del. So, this gives you the expression for x B. Then the expression for y B, the transverse moment is equal to double integral over S again y into y tan phi d x d y divided by del.

This gives you the shift in the centroid of the body in the transverse direction because of the heeling. Because of the heeling, there is a shift in the centroid of the water plane area and that shift is given by y into y tan phi.

So, this is equal to double integral over S, y squared tan phi d x d y by del. That is equal to I by del into tan phi, where double integral of y squared d x d y is the second moment of area about the longitudinal axis, this is about the x axis. About the x axis, I is the moment of inertia about the x axis and I by del into tan phi gives you the shift in the centroid.

Then, let us consider this. Initially, this is your metacenter; this is B 0. It has shifted to a new point B phi and this is y B; this is z B.

This is W 0 L 0, W phi L phi. We can see that the distance y B is really this distance, but since we have already said that our angle of inclination phi is 0, we can see that y B is almost approximately equal to the length of the arc B 0 B phi. This B 0 B phi is an arc and the length of that arc is almost equal to y B, if the angle of inclination phi is very small.

In that case, you get B 0 B phi equals y B. y B is equal to the length of the arc. Length of the arc as you know is given by r theta. So, B 0 M into phi and here we have that y B is equal to I by del tan phi. Now, if phi is very small, we can write this as I by del phi because tan phi approximates to phi in the case, phi is very small. (Refer Slide Time: 30:38) So, I by del phi and these two are same.

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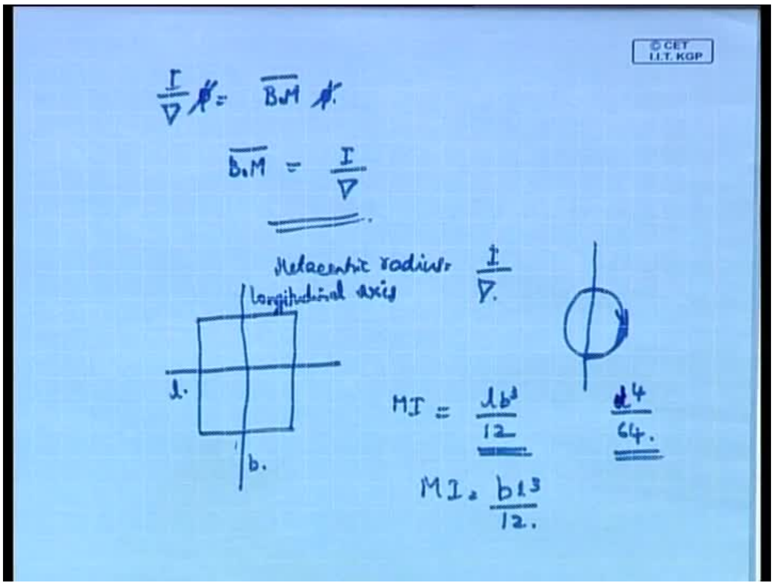

Therefore, you get I by del phi equals B 0 M phi. So, cancelling out phi, you get B 0 M equals I by del. Therefore, the Metacentric radius is given by I by del; that is a very important formula.

That is always that Meta centric radius. You can see that in a special case. For instance, when there is no water plane area. For example, in the case of a submarine, which is completely submerged? If the submarine is completely submerged then you know that I as you can imagine there is no I, there is no water plane area.

Water plane area, we have already said, is that area where the water line exists - the area at the point of water line. If there is no water plane area then there is no I and therefore, B M is equal to 0. B and M coincide in case of a submarine in a submerged condition. So, such different kinds of ideas you can get.

Then it is probably wise to remember the moments of inertia of various bodies various types of water plane area. For instance, if you have a rectangular water plane area and suppose that we are taking this is L, this length is L and this length is breadth.

If you are taking moment about this, then the moment of inertia of this system is given by l b cubed by 12 about this longitudinal axis, the length of the water plane. The moment of inertia is given by l b cubed by 12 and in case, you are taking the moment of inertia about this axis, then moment of inertia is given by b l cubed by 12. Moment of inertia about a transverse axis is b l cubed by 12.

Similarly, if you have a circle, the moment of inertia of this kind of area is equal to d power 4 by 64. this gives you the.

It gives you the moment of inertia of a circle about an axis that is passing through its center. So, if any axis passing through the center is considered, you will get the moment of inertia as d power 4 by 64. So, these are some important formulas and now, let us do some problems regarding this that comes in this chapter.

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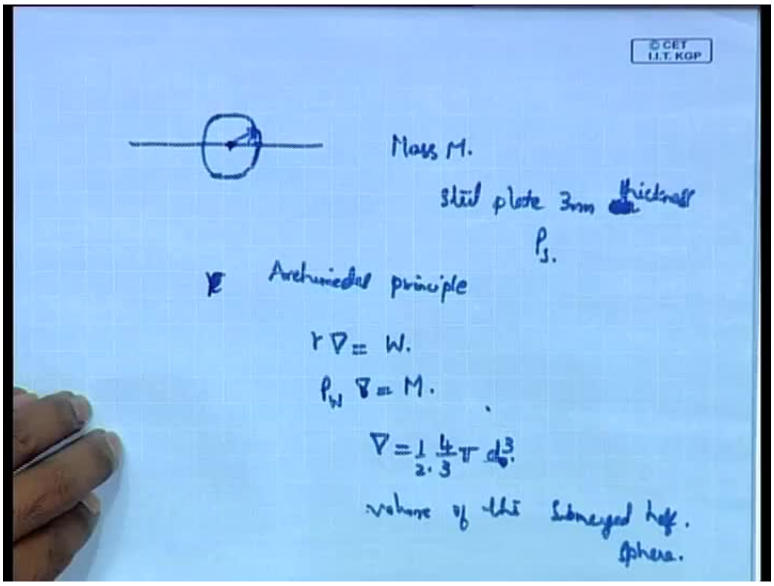

For instance, suppose, there is a buoy. The problem is as follows. There is a buoy and this buoy is of radius something, let us say. It is of mass M and it is a hollow sphere. So, there is an inside volume and an outside sphere and small amount of material of some thickness and then inside radius is a smaller radius.

So, it is a steel plate of 3 mm thickness. So, the thickness of the steel plate is 3 mm and the body itself has a mass capital M.

And the buoy now you are asked what and Let us suppose that the density of the material of the buoy is rho s. Your question is, under what condition will the body float?

So, the problem is you have a buoy which is actually a hollow spherical hollow shell. There is an outside surface and there is an inside surface; the thickness of the plate is 3 millimeter and the rest of it is hollow and this is placed on water. Under what conditions will the body float? That is your question.

First of all, we bring in the Archimedes principle. When you have the Archimedes principle, you say that gamma del is equal to W This is the first principle or rho of water into this. It is not the rho of the steel plate, this is the density of water into del, which equals the mass of the body M in this case. We have removed the acceleration due to gravity - the g term has been removed and therefore, you get the formula that rho W into del is equal to M. M is the mass of the floating body.

So this is the first rule. then the del of the body Let us assume that the volume of the submerged sphere and this is equal to 4 by 3 pi d cubed 4 by 3 pi d 0 cubed; this is the volume of the submerged half sphere. half of it.

So, it is floating like this; with the center that has to be mentioned. The body is floating with the center at the water line. So, half of it is inside and half of it is outside. The volume submerged is given by 4 by 3 pi d 0 cubed into half. This is the volume of submerged half sphere. Volume of the submerged half sphere L is equal to half 4 by 3 pi d 0 cubed.

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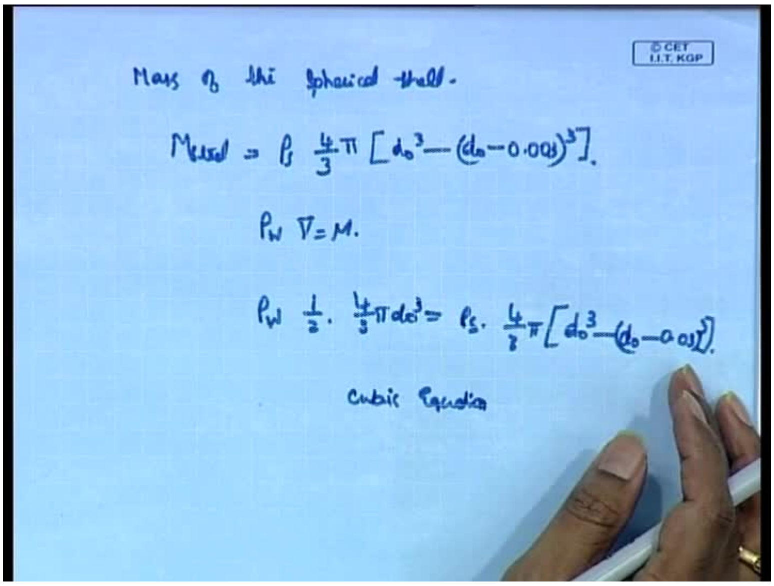

Then the mass of the spherical shell is given by M of steel is equal to rho s into 4 by 3 pi. Remember, this is a hollow sphere; it has an inside and outside and the mass comes only from the steel. The mass does not come from the hollow part inside; that does not add to the mass. When you are finding the mass of the body, you need to take only the steel volume into consideration. So, 4 by 3 pi into d 0 cubed minus d 0 minus 0.003 cubed.

Now, putting this formula rho W into del is equal to M, you get rho W into half into 4 by 3 pi d 0 cubed is equal to rho s into 4 by 3 pi into d 0 cubed minus d 0 minus point naught three cubed. So, this gives you the formula and from this, you will get when you do this you will get a cubic equation and that will that when you solve that, you will get the value of d 0.

So, that will give you the condition that is required for the body to float in this fashion, as we have just said.

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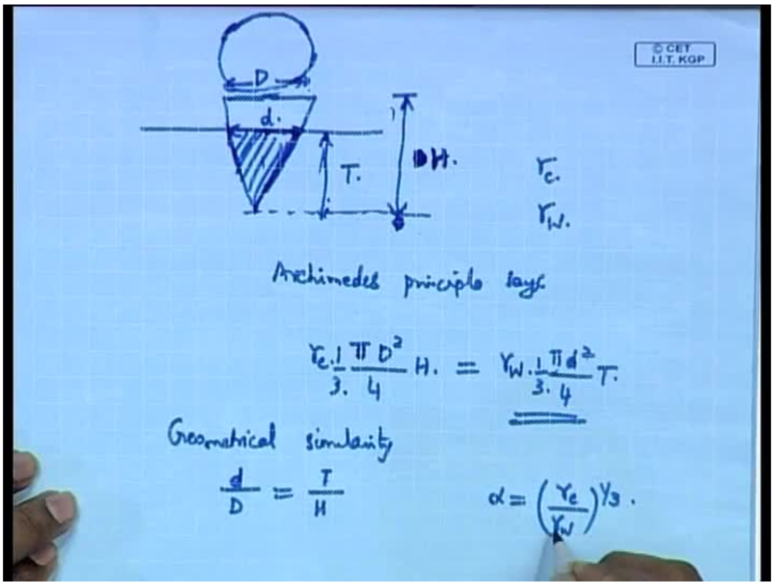

Then there is the second problem. Another problem states that there is a cone floating upside down in water and this is a spherical cone; this is a circle and it is a cone. This has a draft T and it has a depth H.

So, we are told these things; this is T and this is H and this distance this diameter is capital D; this diameter is small d. So, small d and capital D. These two are given. and then you are told also about the cone. The specific gravity is gamma c for the cone and gamma for the specific gravity of specific gravity is density into g. Specific gravity of the cone is given as gamma c, specific gravity of the water is given as gamma W. So, gamma c and gamma W and now, the question is under what condition will the cone float in this fashion.

For this kind of problem, we need to have two conditions. The first of it is the Archimedes principle: the volume of water is displaced is equals to the volume of the weight of the volume of water displaced is equal to the weight of the body itself.

That is number one principle. The second principle which we saw today is that GM should be greater than 0. If both these conditions are satisfied then the body will float up right as we have said in this problem.

First Archimedes principle says that gamma c into volume of the cone and this is equal to the gamma pi, this is the weight of the cone itself which is equal to the specific gravity of the cone into the total volume of the cone. So, pi D squared by 4 H into 1 by 3; the volume of a cone is equal to 1 by 3 pi r squared H - 1 by 3 into pi D square by 4 into H; this is the volume of any cone.

These are some common geometric terms that you should be familiar with; that is the volume of a cone, volume of a parallelepiped or a volume of a rectangular barge, volume of a sphere and similarly, the moment of inertia of a couple of things like moment of inertia of a spherical surface, moment of inertia of a rectangular surface, moment of inertia of square and parallelogram. These kinds of things, it is better to remember when you are studying this section

So, Archimedes principle says that the mass of the cone that is gamma c into 1 by 3 pi D squared by 4 H is equal to the weight of water displaced r w into pi, this much volume of water has been displaced, pi d squared by 4, 1 by 3 again, 1 by 3 pi d squared by 4 into T; in this case, the height of the cone is T.

So, the volume of water displaced is this. It is 1 by 3 pi d squared by 4 T. This is the volume of water displaced; into gamma w will give you the weight of the water displaced. So, the weight of the water displaced is equal to the weight of the cone here.

This just represents the weight of the cone which is equal to specific gravity of the cone into the volume of the whole cone. Just to stress on this again, there are two volumes: here, one is the volume of the whole cone and other is the volume of the area submerged.

When you are doing the mass of the cone, you take the whole volume of the cone and when you take the weight of liquid displaced, you take only the volume that is submerged. So, this gives you gamma W into This is the first principle.

We can put a kind of geometrical similarity because it is the same cone. Geometrical similarity says that d by D is equal to T by H; d by D is equal to T by this is not D, sorry this is H. I wrote wrongly; this is H. So, d by D is equal This is D; this is small d and this is H. So, d by D is equal to T by H is a geometrical similarity and let us say that alpha is equal to gamma c by gamma w power 1 by 3. So, gamma of the cone by gamma of water power 1 by 3 and we will write it as alpha.

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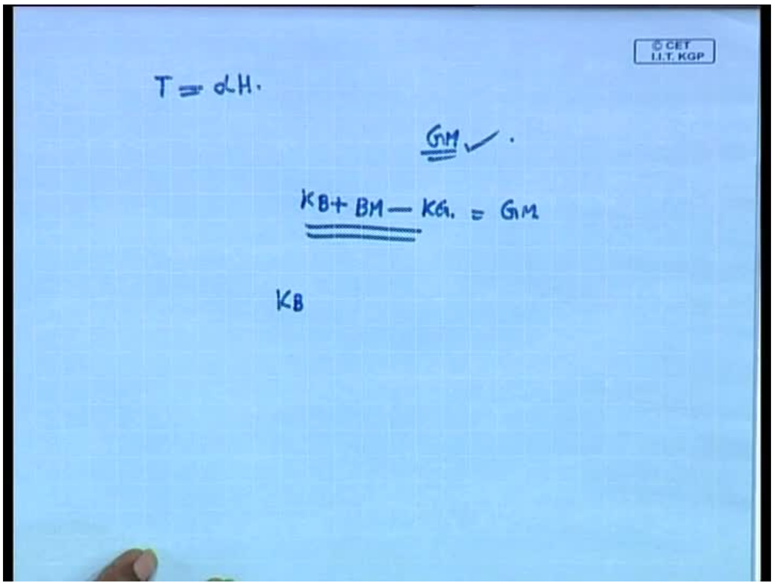

So, we obtain T is equal to therefore, alpha H from the previous principle. Now, we need to get GM. GM is our final goal and we need to say that GM of this system will be greater than 0 and that will imply stability. GM is greater than 0 will imply stability.

To get GM, we need to get KB plus BM minus KG. This is the way to get GM. This is the only way to get GM. KB plus BM minus KG will give you GM, the metacentric height. The metacentric height will be given by KB plus BM minus KG. So, we need to find for this cone, what is KB, what is BM and what is KG. We have formulas for everything.

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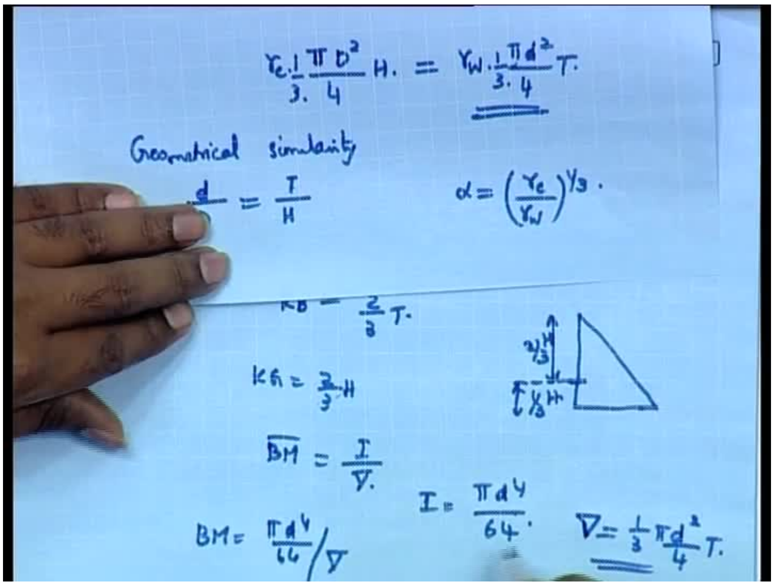

(Refer Slide Time: 48:10) For instance, if you look at this, KB is the centroid of this region of the cone - the cone inside. It is given by KB. So, KB is equal to 2 by 3 of T; this is another thing. If you have a right angled triangle, the centroid of this triangle is at 2 by 3 from here - 2 by 3 from this side or 1 by 3 from this side, of the total height H.

So, 1 by 3 H or 2 by 3 H. KB is equal to 2 by 3 T and KG is equal to 2 by 3 H. (Refer Slide Time: 49:07) KG is for the whole cone. Remember, we are talking about the center of gravity of the whole cone.

So, that will be at 2 by 3 of the H. 2 by 3 of this whole distance. The distance from here will be 2 by 3; the distance from here is 1 by 3. So, 2 by 3 H.

KG is equal to 2 by 3 H and BM we have seen is equal to I by del. I is equal to pi d power 4 by 64. Therefore, BM is equal to pi d power 4 by 64 divided by del, where del is equal to 1 by 3 pi d squared by 4 into T; this is the underwater volume. I do not have to repeat it. This is the underwater volume, divided by del. When you do that there, you will get BM for the whole thing.

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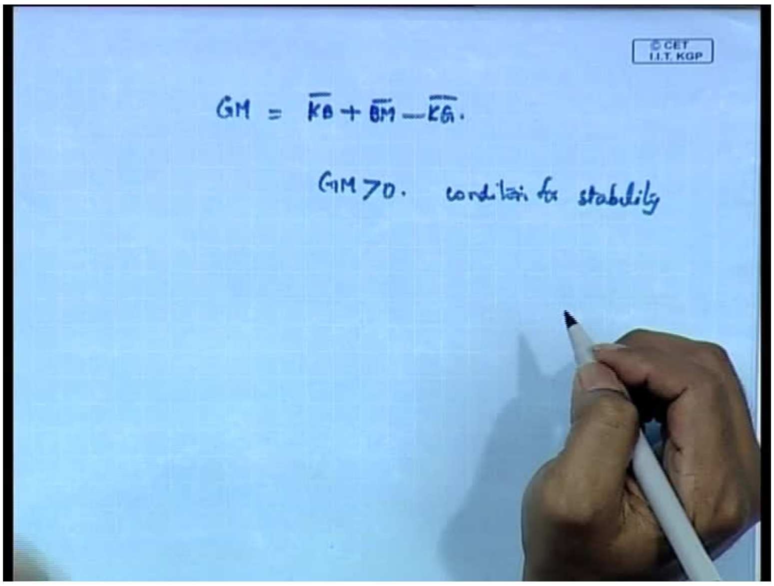

Once you have, all those things, you can get GM. You do KB plus BM minus KG and just putting all these things together, we give the condition that GM should be greater than 0; this is the condition for stability.

So, GM is greater than 0 and when this condition is met then you have the case of a cone exactly sitting in an upright condition like this. So, this is the problem. We will work on similar problems in the next class and we will stop here, today. Thank you.