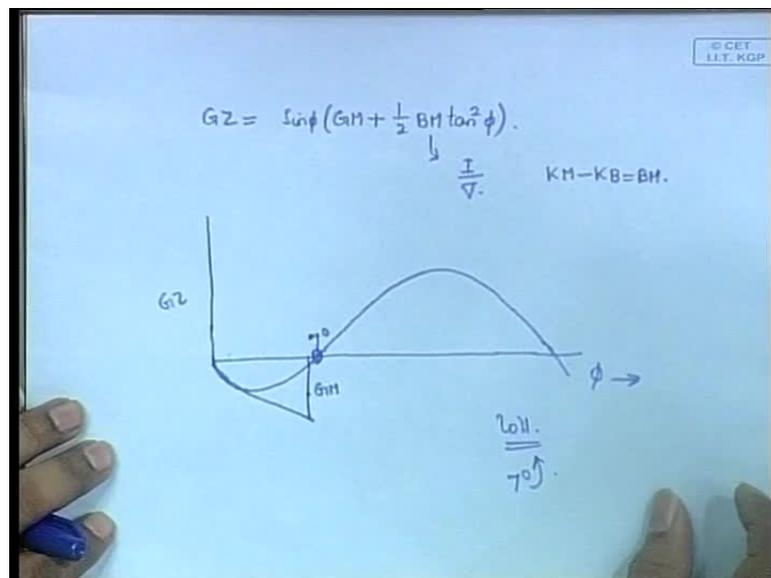


Hydrostatics and Stability
Prof. Dr. Hari V Warrior
Department of Ocean engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

Module No. # 01
Lecture No. # 21
Righting Stability -1

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In the last class, we started on righting stability that is what we talked about. We talked about a correction to the GZ formula that is GZ is equal to $GM \sin \phi$ is slightly modified to be written as this. I mean, at least for the purpose of this course you have to remember how to derive this and to know this (Refer slide time: 00:35-00:50). Now, it is not enough to know the derivation. You have to remember the formula as well because you will see from now on, I will be doing a lot of problems and you are sure to get at least one problem. At that time, you cannot derive things.

So, remember this formula GZ is not $GM \sin \phi$ but it is $G \sin \phi$ into GM plus thing half $BM \tan^2 \phi$. So, this implies that you need to know your metacentric radius BM . This is an additional quantity. The one way to calculate it is I by ∇ . Then the other way is using KM and KB means KM minus KB is equal to BM . So, these are two ways

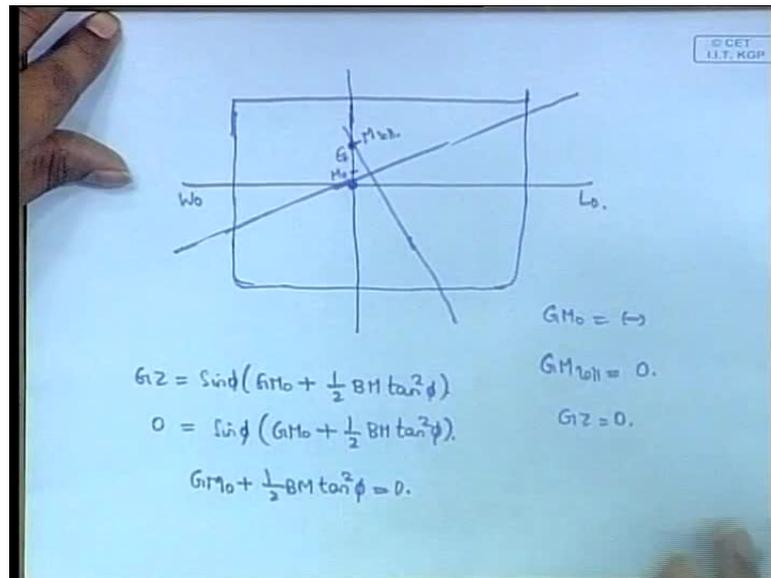
in which we will be calculating the metacentric radius. You will either be given KM and KB and then you have to calculate BM or the problem will be such that you can calculate the I. Means it will be like a box shaped or something. Of course, you have to remember that this is I about the longitudinal axis, not a transverse axis. So, this formula is important and as they are saying in case theta is small, not theta phi is small. In case phi is small, you can reduce this formula to $GM \sin \phi$ without much error. So, that is thing.

Now, let us suppose that we have a vessel with a negative metacentric height. Initial metacentric height is negative, so we have already discussed this kind of ship before. For example, I also said this is a usual question. So, in vivas and all that it is nice to remember. It will help you. That is we have seen that when you have a negative metacentric height, it goes like this. This is the statical stability curve.

So, for a ship with an initial, it is not total always but initial metacentric height. Initial negative metacentric height GM is negative initially means that ship was designed with some flaw such that the initial metacentric height became negative in the upright condition. As a result of which you will get a GZ phi curve in this fashion. This is how it will look like because here when you draw a tangent, you see let say at whatever angle you get GM, this GM will be negative. So, this is negative GM. That is the meaning of this negative metacentric height but such a ship what will it do? It will always be such that it will start from here. This is its upright condition for this ship means the ship will never be in an upright condition but it will always be like this. That also we have discussed. This is known as loll.

So, remember this thing. It is known as lolling and when you have loll, you will always have the righting arm curve or the GZ curve like this. Statical stability curve will look like this. Also, remember that so this is called loll and so, the ship always moves at the angle of loll. This angle is called this probably something like ϕ . Let us say 7 degree. Something like 7 degrees will be your angle of loll. So, the ship always moves at an angle of 7 degrees, instead of being in upright condition. It always moves in angle of 7 degree. Now, for such a ship, let us see what these correction, what it reduces to?

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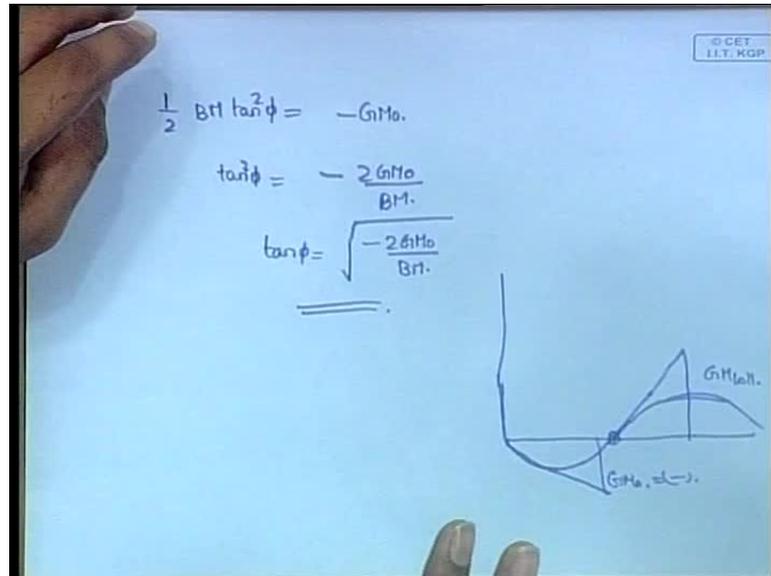
So, the ship has an initial negative metacentric height and as a result, the ship has heeled and as a result, it will be at an angle of loll. (Refer slide time: 5:28-7.00) Therefore, this is the center and then initially here. Then here now as we discuss the problem M is below G , it is a negative metacentric height problem. So, initially this is GM_0 . Now, what will happen is that the ship will loll as the result of which its M will keep rising. When we have already discussed M curves like that M keeps rising such that, it will finally come here at this stage, somewhere here. You have M phi or M loll. They are called this is at the angle of loll, G and M will coincide.

That is a condition of equilibrium. It is a stable equilibrium, neutral equilibrium. It is not stable equilibrium, so in that case of neutral equilibrium, it will be GM will be 0. GM loll, so the GM_0 is negative and GM loll will be 0 and as a result, you can see GZ for that particular case of angle of loll when it is moving at the GZ is 0. You can see that. Then if you have that we are going to substitute in this formula GZ is equal to $\sin \phi$ into GM_0 plus half $BM \tan^2 \phi$.

So, this is the expression we have derived for GZ , the corrected expression. Now, for this particular case, initially is unstable now. It is now at an angle of loll, it is moving in a neutral equilibrium where GZ is equal to 0. As a result, we put GZ is equal to 0. So, 0 is equal to $\sin \phi$ into GM_0 plus half $BM \tan^2 \phi$. Now, two possibilities are ϕ equal to 0. Directly you can see that or the second quantity 0. Now, ϕ equal to 0

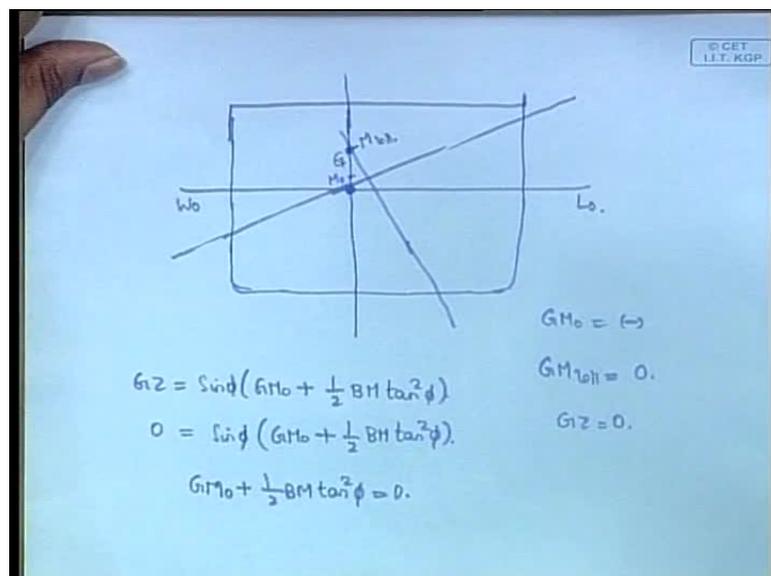
implies a state of uprightness means ϕ equal to 0 means ship is like this. Now, that is not a solution because we have already said that is not the solution, the ship is in a lolled condition.

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So, ϕ is not 0, so this is not 0. So, GM_0 plus half $BM \tan^2 \phi$ is equal to 0. Now, it is just a matter of solving it. It becomes half $BM \tan^2 \phi$ is equal to minus GM_0 or $\tan^2 \phi$ is equal to minus $2GM_0$ by BM . Therefore, $\tan \phi$ is equal to square root of minus $2GM_0$ by BM .

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Actually, there is one thing you should note that is when the ship is moving, when the ship is heeling here, GM keeps changing because M is changing. G is at the same point because weights are not moved but M keeps changing because of the heeling. This GM is GM0 in that equation that corrected equation GM is GM 0; it is not GM at any point. It does not vary; it is GM0 means it is in the upright condition. Whatever is GM that is the GM you are using in this equations, so this is tan phi. Tan phi is equal to this thing. As you can see, GM0 is negative and therefore, whatever is under the square root is positive. So, therefore this solution is true only if GM is negative, otherwise this has no root. This square has no meaning at all.

Therefore, this is the tan phi or this is the angle of loll at which it will move. If the initial GM is negative, you have to have some expression for the angle of loll. This is one way of calculating it. So, this gives you the angle of loll. Then they have just drawn this figure also, it will be like this. (Refer slide time: 11:10-11:40) That is the ships GZ curve will look like this. Now, here we will have GM 0, this is negative. Here, you will have at this point if you draw the GM tangent; you will get your GM loll somewhere here.

So, 2 GMs will come. One is GM 0 and another is GM loll. Now, the usual GMs that you plot in the statical stability is this GM loll. Now, this is GM 0 which is in the negative side. It is negative because it is an unstable ship and it becomes stable or it tries to become stable by moving at an angle of loll and the expression for the loll is given by tan phi is equal to these things. So, this is the expression. Phi is GM, not GM loll. For which one? Actually, we derive this expression yesterday. In that if you look at the derivation, let see where it is. In that expression, the derivation was done using GM naught is not that we are putting GM naught or anything. It is we have derived using geometric at that time, it was done using GM naught.

So, in that equation the value is GM naught. Now, there it is coming in. We will see how to get GM loll that is to be derived that we will do. That is the different thing but what we have done so far is everything is using GM naught. In fact, all the day we have studied also is all using GM naught. We have never used GM loll anywhere. It is a new thing that I have introduced now. We will go in to how to derive GM loll. I think in fact, there is a next thing problem.

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$$GZ = \sin \phi \left(GM_0 + \frac{1}{2} BM_0 \tan^2 \phi \right).$$

$$\frac{dGZ}{d\phi} = \cos \phi \left(GM_0 + \frac{1}{2} BM_0 \tan^2 \phi \right) + \sin \phi \left(BM_0 \tan \phi \sec^2 \phi \right).$$

at $\phi = \phi_1 = \phi_{loll}$.

$$\tan^2 \phi_{loll} = \tan^2 \phi_1 = \left\{ \begin{array}{l} -\frac{2GM_0}{BM_0} \end{array} \right.$$

$$\left. \frac{dGZ}{d\phi} \right|_{\phi = \phi_{loll}} = \cos \phi \left(GM_0 + \frac{1}{2} BM_0 - \frac{2GM_0}{BM_0} \right)$$

It is the next thing, we will do this then. Let us write that expression again. We have GZ is equal to sin phi. If you want to clarify, let see where that is GM coming. Actually, if you see that derivation which I did yesterday, in that there was only 1M that was when it was the upright condition. If you look at the figure, I cannot draw the figure again. If you look at the figure that I drew yesterday that is M when it was in the upright condition, you can see from the figure itself it is obvious, it is when the ship is in the upright condition. I told you so every and then something comes like GX is equal to GM 0 sin phi and all that.

So, from the figure you can see, it is GM 0. It is not when it is heeled. So, GM loll is different which is coming now, so GZ is equal to sin phi into GM 0 plus half B. This is also BM0, in fact not BM at any point. BM 0 tan square phi. Now, let us just differentiate this. So, this becomes d G this by differentiate this. With respect to phi, this becomes cos phi into GM 0 plus half BM 0 tan square phi plus sin phi into BM 0 tan phi sec phi. This is just the differentiation of that d GZ by d phi.

Now, suppose we find out this d GZ divided by d phi at the 0.5 equals 5 loll, actually 6.5. That means this is wrong. Now, let me see may be, it should be 6 square phi. Let us continue. We will do this ourselves. So, d GZ by d phi becomes this. Now, at phi equals let us call this phi 1 which is actually equal to phi loll. There is the angle at which the ship is lolling. We already have derived that tan square phi at loll which we will write it

as $\tan^2 \phi$ is equal to $\frac{1}{\cos^2 \phi}$, so $\tan^2 \phi$ is not $\frac{1}{\cos^2 \phi}$ is equal to $\frac{1}{\cos^2 \phi}$ minus $2GM_0$ by BM_0 . We have already derived this in the previous section $\tan^2 \phi$ is equal to this. Therefore, $\frac{dGZ}{d\phi}$ at $\phi = \phi_{roll}$ is given by $\cos \phi$ into this expression, I have to write GM_0 plus half BM_0 . At this point, I am replacing $\tan^2 \phi$ by $\frac{1}{\cos^2 \phi}$, it is replacing it with $\frac{1}{\cos^2 \phi}$.

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$$+ BM_0 \left(1 - \frac{2GM_0}{BM_0} \right) \sec \phi.$$

$$\left. \frac{dGZ}{d\phi} \right|_{\phi = \phi_{roll}} = GM_{roll} = -2GM_0 \sec \phi.$$

$$\underline{\underline{GM_{roll} = -2GM_0 \sec \phi.}}$$

By BM_0 plus BM_0 into $\frac{1}{\cos^2 \phi}$ minus $2GM_0$ by BM_0 into what is it becomes $\frac{1}{\cos^2 \phi}$ tan ϕ . I think they have mistaken here. What is it becomes in ϕ BM_0 sin ϕ into $\sec \phi$ is tan ϕ again. So, tan cube tan square ϕ into $\sec \phi$. So, tan square ϕ is replaced and that is why $\sec \phi$. So, tan square ϕ into $\sec \phi$. Then $\frac{dGZ}{d\phi}$ at $\phi = \phi_{roll}$ equals ϕ_{roll} . Now, for small ϕ 's what is $\frac{dGZ}{d\phi}$?

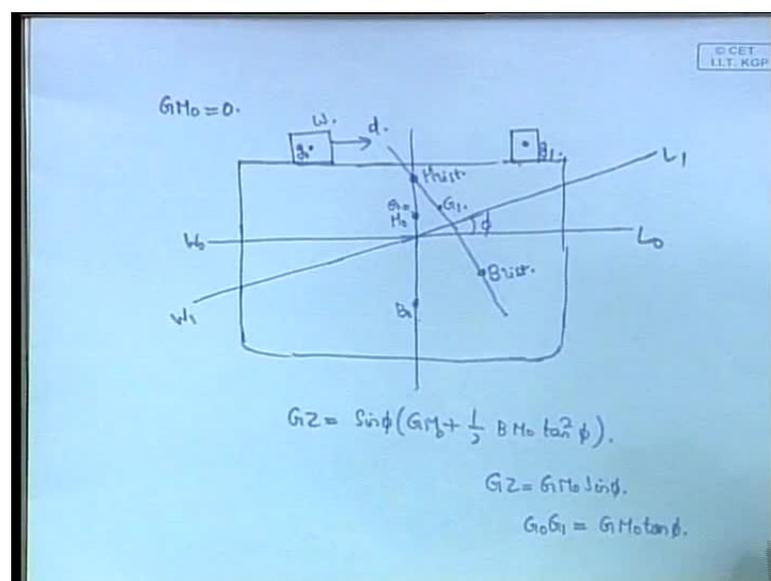
Now, what is GZ ? According to the simplest expression, GZ is equal to $GM \sin \phi$ which is $GM \phi$. Therefore, $\frac{dGZ}{d\phi}$ is GM . So, this is the simplified expression. If you can definitely find errors in it, what we have done. So, this can be replaced in general by GM_{roll} . Therefore, now look at this expression we have done here. This expression, this becomes BM_0 . BM_0 will cancel out 2. If 2 will cancel out, so this become minus GM_0 . GM_0 minus GM_0 cancels out. So, this is 0 here. Also, this cancels, this cancels out, so this becomes $\cos \phi$ into and $\cos \phi$ into $\sec \phi$ will cancel out. Therefore, this becomes $\frac{1}{\cos^2 \phi}$ minus $2GM_0$ by BM_0 into $\sec \phi$.

It reduce to this, therefore GM loll is given by minus 2 GM 0 sec phi. So, there are some errors in it, some kind of approximation. This is a kind of approximation because at many places like d GZ by d phi if you use that corrected expression, it will become slightly different. So, this is one formula you can use. It does not produce much error, it is fairly. So, GM loll so you are finding the value of GM at the angle of loll is given as the function of GM at 0 means upright condition. So, you are getting GM loll as the function of GM 0 and it just depends upon sec phi. So, this is the derivation.

Now, the thing is whatever I am doing today, there are lots of problems in it, so there are lots of formulas like this. For instants, this one I put it here. This is one of them, tan phi is equal to this or this one GM loll is equal to minus 2. There are formulas like this and in problems used have to straight away apply them. So, first of all of course you have to know how to derive it but you have to by heart it. There is no other way because it is difficult to derive it. In the exam, it takes your time and this is not and you will see as we keep going on it is just formulas only from now on.

That corrected expression is the big formula. This is a formula, lot of formulas you have to remember them. So, this is your value of GM loll then. Now, all these derivations have been done for wall sided ships. These are all for wall sided ships. They keep repeating that because it is different for the other kind of ships, these formulas but most of the ships are in fact, wall sided. So, more or less wall sided.

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Now, we do another case. Let us consider the case when $GM = 0$, this is the special case when $GM = 0$ is equal to 0. Therefore, it is in a state of neutral equilibrium, neither stable nor unstable. Then therefore, you have B_0 , then M_0 . This is your M_0 and this is as we said, this is the special case when GM is 0. Therefore, G_0 and M_0 coincide G and M_0 coincide both of them are this point. So, this is GM and then the ship heels and as a result into this W_1L_1 from W_0L_0 , it heels to W_1L_1 and this is the vertical at that point.

B shifts to, we call it B use the same thing. We are calling it as B list here. This is BOB list. Then as a result of this moment, M shifts. M as you know, this is M . This will be your M , M list and actually, it will explain in the whole thing. The ship is heeling because of this condition. In this case, actually this is just a particular case; this is not a general case. To use for any formula or anything, so if you have a problem of this type, we have to use this formula that is all. So, you just have to remember it. That is nothing. So, suppose that you have a particular case when GM_0 is negative that is the first condition and suppose that a weight is moved from this point to this point, other way heeling like this.

So, it is moved from here to here. So, the ship is heeling and the weight moved is w , small w . This weight is moved at distance d , small d from here to here. It is d distance. This position, it is center of gravity. We call it as a G_0 . This position we call as G_1 and what has happened is the ship is initially in upright condition with $GM = 0$ equals 0 with neutral stability. Since, it is not negative, it will not loll. $GM = 0$, so it will remain in the upright condition only just like an ordinary ship. Now, a weight is moved from one side to another. A small weight of w is moved, a distance of d and it is moved from one side to another side.

As a result of this, the ship heels by some angle ϕ and into W_1L_1 and these are the final condition. So, G as you can imagine because of the shift of weights, G will shift. G of the ship itself will shift from initially we will call this G_0 , will move to G_1 . Let us say, this one somewhere here will move on G_1 . It will move and it will heel to such an angle such that finally, G and B will be in a straight vertical line. That is always how it will come in rest. If G and B are not in the vertical line, as you know there will be moment acting and that is lw coming down and lw going up. That moment will act tending into turn and because of that it has now come to a final stage such that B and G are in the same vertical line.

M is also in that line, so this is the condition. Now, this formula says GZ is equal to sin phi into GM plus half GM 0 plus half BM 0 tan square phi. Now, let me see this GZ is equal to GM 0 sin phi. This is simple and I am just doing something GMZ is equal to GM 0 sin phi and if you have G0 G1, actually in an inclining test there was a relation between G0G1 and GM. G0G1 is equal to G0 GM0 tan phi. You should remember this is the formula. There is the formula called G0 and G1. This is from the inclining test experiment. If you remember long back G0G1 is equal to G, there is a formula like this.

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The image shows a whiteboard with the following handwritten equations:

$$GZ = G_0 G_1 \cos \phi$$

$$GZ = \frac{w \times d}{W} \cos \phi$$

$$GM = 0$$

$$GZ = \sin \phi \left(GM_0 + \frac{1}{2} BM_0 \tan^2 \phi \right)$$

$$\frac{w \times d}{W} \cos \phi = \sin \phi \cdot \frac{1}{2} BM_0 \tan^2 \phi$$

$$\frac{w \times d}{W} = \frac{1}{2} BM_0 \tan^3 \phi$$

$$\tan \phi = \sqrt[3]{\frac{2 \times w \times d}{BM_0 \times W}}$$

Now, from this I am writing GZ. These are all just some manipulation just to simplify things. G0 G1 cos phi will come like this. Now, G0G1 also you know it is the shift in the center of gravity of the whole ship due to a shift in the weight.

If a weight w, small w shifted a distance d that G0 G1 will be given by w into d by whole W into so into cos phi. This becomes, so this is GZ and we know that in this condition, GM is equal to 0. Now, therefore I put in this expression GZ is equal to sin phi into GM 0 plus half BM 0 tan square phi. Now, GM0 is 0, therefore this becomes GZ. We have said is equal to w into d by W into cos phi equals sin phi into half BM 0 tan square phi. Therefore, w into d by capital W equals half BM 0 tan cube phi. Therefore, tan phi equals 2 into w into d by BM 0 into w cube root. This derivation by itself does not have much value. I mean it is not very important formula or anything but for one particular

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Handwritten derivation on a blue background:

$$GZ = G_0 G_1 \cos \phi$$

$$G_2 = \frac{w \times d}{W} \cos \phi$$

$$G_H = 0$$

$$GZ = \sin \phi \left(G_{10} + \frac{1}{2} B M_0 \tan^2 \phi \right)$$

$$\frac{w \times d}{W} \cos \phi = \sin \phi \cdot \frac{1}{2} B M_0 \tan^2 \phi$$

$$\frac{w \times d}{W} = \frac{1}{2} B M_0 \tan^2 \phi$$

$$\tan \phi = \sqrt[3]{\frac{2 \times w \times d}{B M_0 W}}$$

$G_0 G_1$ is equal to $GM \tan \phi$. These two expressions you know, so that is all. Just combine them and you get the third one, GZ is equal to GM . So, that is why there is a big approximation. It is not exactly true; it is true only for, well it is just a bit of approximation because again though we have made one approximation, the answer would not be very far from the exact value because to be really correct, you have to of course use that corrected formula in the second one also.

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Handwritten calculation on a blue background:

draft = 8m. $KG_1 = 10m$.

$KM_1 = 11.6m$	$KM = 11.6m$
$KG_1 = 10.0m$	$KB = 4.2m$
$G_{10} = 1.6m$	$BM_0 = 7.4m$

disp = 28,200 tonnes.

$$GZ = \sin \phi \left(G_{10} + \frac{1}{2} B M_0 \tan^2 \phi \right)$$

$$GZ(\phi)$$

5° 12° 15° heel

Then let us do some one or two problems. Now, there is a ship which has a draft of 8 meter and a KG of 10 meter. Yesterday, now if you remember I did KN curves. Yesterday, I told you that if that is the way of calculating the GZ curve from the KN curve, so the question here is compare values from KN curves. So, we have what I have done yesterday and today, we have two ways of calculating GZ. First is using the KN curves from KN curves, $KN \text{ minus } KG \sin \phi$ will give you GZ. So, GZ there is one way of calculating GZ. So, once you are given the KN table, you just subtract that $KG \sin \phi$ from it and you will get your GZ.

So, that is one way of calculating your GZ. Now, that will be for different angles like discrete one after one, you will calculate the other ways using this formula $GZ \text{ is equal to } \sin \phi \text{ into } GM_0 \text{ plus half } BM_0 \tan^2 \phi$. So, using that expression also you can calculate GZ. So, the question here is compare the values using the two expressions provided you are given the KN table, you can compare the values.

Now, some more hydrostatic data are given for instance, I will tell you. We just see how the method of doing this. First of all KM will be given. Let us say in this case, it is 11.6 meters. Then you have KG is given as 10 meters. So, once you have KM and KG, you can get GM_0 . GM_0 is equal to 1.6 meter. Then some more hydrostatics data are there that is KM is there 11.6 meters and then KB is also given 4.2 meters.

Now, $KM \text{ minus } KB$ will give you BM_0 which is your BM_0 . So, that is BM_0 , that is equal to 7.4 meter and you are also told that the displacement is some 28200 tonnes. So, this much data we have. Now, once you have this, you can directly apply the formula that is $GZ \text{ is equal to } \sin \phi \text{ into } GM_0 \text{ plus half } BM_0 \tan^2 \phi$. So, we have this formula, we have everything. GM_0 is here BM_0 is there.

So, they are asked you to do GM_0 for different values of 5 degrees, 12 degrees and 15 degrees heel, so that is all you just told to do. So, in case you are not given the angle what you do is, you do is 51015 like that may be upto some 25 or something. So, you take ϕ , so this GZ you put ϕ equal to 5 degrees or 10 degrees put it here. GM_0 and BM_0 will be the same for all values of ϕ . It is not going to change; it is the constant for the ship because it is not GM, it is GM_0 . GM_0 is the value of GM and the upright condition, so it does not depend upon the heel angle of heel. So, you have this ϕ . You

have just substitute it here, you will get values of GZ at each angle of you will get GZ as a function of phi.

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from heel	KN Curves	KG sin phi	KN - KG sin phi = GZ
5°	1.02	0.87	0.15
12°	2.49	2.08	0.41
15°	3.06	2.57	0.49

KM	0.14
Kb	0.367
KM - Kb	0.483

So, that is the question here and you are also told that the KN curves are this. Also, I will just write how you calculate? You are given like this KN curves, so you are given the heel and you are given KN, you will be given like this KN. It is given 1.02, 2.49, 3.06, so you will be given KN like this. Now, what you need to do is, you need to make another table, another column which says KG sin phi KG, you should be knowing. Of course, in this problem we have values of hydrostatic data are given.

So, KG is given sin phi. You calculate for each value of phi and this you do it becomes I will just write this 2.08 like this. You will get your value of KG sin phi and then make another column which will say KN minus KG sin phi which is equal to GZ. This formula you have, then it will become 0.15, 0.41, 0.49. **Now when we did using the** so, there are two ways here we have done in the previous method. The values came out to be I will just write that here 0.14, 0.367, 0.483.

So, this represents the value of GZ obtained using the two methods. This is using the wall sided formula or the correction formula that is called the wall sided formula. So, using this corrected formula and this is using the KN curves they are close. It will never match; it will never be an exact but will be close. So, this you have to remember, two ways of calculating. The GZ means the GZ curve, so the next time, for the next exam

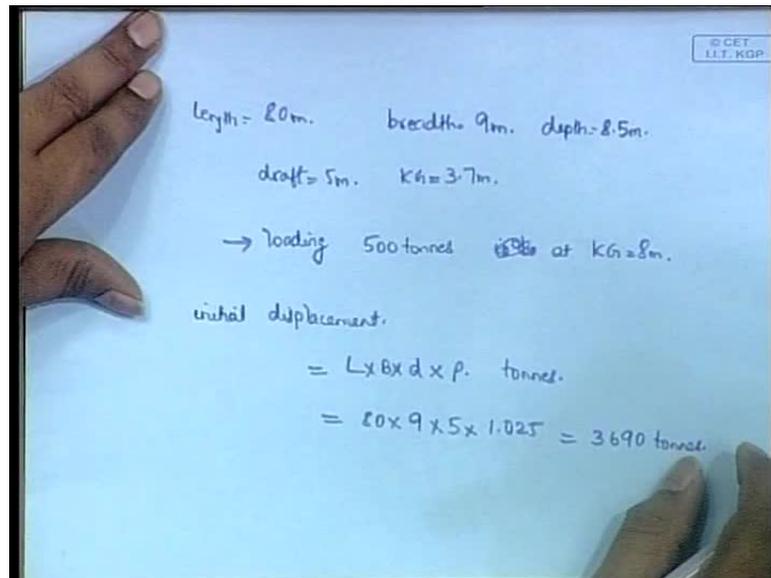
you would not be given the GZ curve at all. This time it was just GZ curve; you have to plot to get the GM that is very straight forward.

Next time you have to plot, you have to generate your own GZ curve using some other things. For instance, this is one possibility is your wall sided formula using that you calculate, so you will be just given the hydrostatic data. That means you will be given your KM, KB and KG that is all. Once you have that for each angle of heel, you have to generate your GZ using this formula that wall sided formula. Using that, you have to generate your GZ and then you have to plot it and then from that you can find your GM. Then here another way to find GM is KM minus KG directly. So, you can compare using the two methods mean what I am saying is, you will be just given KM, KG and KB. This is known as hydrostatic data.

So, once you are given this hydrostatic data, you just go ahead with the wall sided formula to calculate GZ and once you have GZ at different angles of phi, you will have to plot it. Once you plot it, you draw the tangent at 0 and you get GM. You will get one value of GM. Similarly, KM minus KG will give you another value of GM. These two GM should match exactly, both are GM 0. They should match that is the rule.

Now, another problem is now as I said before. If you are told box shaped vessel box shaped large box shaped vessel, nothing more will be given means that itself gives you some formulas you have to remember in the exam. I told you but usually, it should not be told. You **are you** should just be told. It is written on the problem that is the box shaped vessel that itself will give you that d by 6 plus v square by 12 d formula will give you should remember that. So, this is what it says.

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There is a box shaped vessel which has a length equals 80 meter, breadth equals 9 meter, depth equals 8.5 meter and is floating at a draft of 5 meter and it has a KG equals 3.7 meter in salt water. You are asked to find the angle of loll and the GM of loll. If a weight of 500 tonnes is loaded at KG of 8 meters, therefore your next step is that you are loading 500 tonnes is loaded 500 tonnes at KG equals 8 meters. Now, the first thing you can see from the problem itself is that they asked the angle of loll, so you should get your GM negative. First of all, when you do the first case itself without any, I have done this. So, whenever you do it first, you should get your GM negative. Otherwise, there is no loll. So, you are told that find the angle of loll and the GM of loll once a weight of 500 tonnes is added at a KG of 8 meters

So, you have a ship initially to it you are adding. You have a ship initially, which itself is unstable to it. You are adding some weight of 500 tonnes at a KG of 8 meter. Then you are asked at that state when it starts moving, it will be at an angle of loll. Then also, the GM is negative that means what will be that angle of loll and GM of loll is to be found. Now, from this data, first of all you need to calculate the displacement of the ship before the weight is added. So, let us call it initial displacement.

So, initial displacement is equal to length into breadth into draft into density of salt water. So, you can write it in so many tonnes length is given 80 into 9, into. Note you do not put the depth, you put the draft. I mean we have discussed so many times because we

are calculating the weight of the thing you are multiplying it with the density of salt water. Therefore, it is the underwater volume that is needed.

So, length into breadth into draft, draft is given to be 5 meters into density of salt water is 1.025 tonnes per meter cube. So, this will give you the displacement of the ship. This is the displacement of the ship before the weight has been added or this is the initial displacement of the ship.

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Weight	KG.	Moment.
3690	3.7	13653
500	8	4000
<hr/> 4190		<hr/> 17,653

final KG = $\frac{17,653}{4190} = 4.213m.$

final draft = $\frac{\text{displacement}}{L \times B \times D} = \frac{4190}{L \times B \times D} = 5.678m.$

Now, it is the first thing. Then now we are going to load the weight, so let us write like this weight KG and moment. So, we have already calculated the initial displacement of the ship 3690 tonnes. Now, see the problem itself it says its KG is 3.7 meter. So, the KG of the initial displacement ship without the weight added is given is 3.7 meter. Therefore, this product of these two will give you 13653 here.

Now, you are told that 500 tonnes is added, so you are going to add 500 tonnes and you are also told that 500 tonnes is loaded at KG equals 8. So, that is also given. So, it is loaded at 8, so this becomes 4000. Now, total moment becomes 17653, total weight becomes 4190, so our goal is to find the final KG after the weight has been added. Final KG will be the total moment divided by the weight. So, 17653 divided by 4190 that will give you 4.213 meters.

This is your final KG that is the first thing. Now, one more thing we have to calculate. Now, once the weight has been added, your draft is going to change. You have to remember that is one thing you might miss here that is when you are finding. For example, remember that this problem requires you to do box shaped vessel. So, it requires you to use that formula because you are not given KM here. If that formula gives you a value of KM, that box shaped barge gives you KM is equal to d by 2 plus b square by 12 d .

In that there is a d which is the draft. That draft is with the weight added, not without the weight. Therefore, you have to find the final draft here after the weight is added. That will be displacement divided by L into B into ρ . This is very straight forward displacement. We know initial displacement plus the weight added, so it is given. We have calculated here 3690 plus the weight added 4190 divided by L is everything. We know L , we know B , we know ρ , we know. So, you just do this and you will get your value for final draft as 5.678 meters and so, this is the first part we have calculated after the weight is added. We know what KG is and we know what the draft is.

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The image shows handwritten calculations on a whiteboard. The top part defines the formula for KM for a box-shaped vessel: $KM = \frac{d}{2} + \frac{b^2}{12d}$. It then substitutes values: $d = 5.678$ m and $b = 9$ m, resulting in $KM = 4.028$ m. Below this, the metacenter height is calculated as $GM = KM - KG = -0.185$ m. The final part of the calculation determines the heel angle θ using the formula $\tan \theta = \sqrt{\frac{-2GM}{BM}}$, yielding $\theta = 29.15^\circ$. On the left side, there are additional notes: $BMo = \frac{I}{\nabla}$ and a calculation for $\frac{b^2}{12}$ as $\frac{81}{12} = 6.75$, which is then used in the denominator of the KM formula as $(4190/1025)$.

Now, next is we are going to use formula for the box shaped vessel, box shaped barge, so you have to find the KM. KM, we are going to use the formula d by 2 plus B square by 12 d . Now, this d will be this 5.678 which we just calculated plus B is 9 squared by 12

into 5.678. This d is no longer your 5. It is a 5.678. So, this will give you your KM 4.028 meters.

Now, we have KG and we have KM, therefore we can straight away find, we have KG already. We get KG KM minus KG will give you your GM and in this case, it becomes (refer audio time: 47:56-47:59). Now, this it is a kind of check which will tell you whether you are at least in the right way of your calculations in the right track. You should get GM as negative because the problem itself says find the angle of loll. So, that means you definitely should get your GM as negative, so you have got your GM as negative.

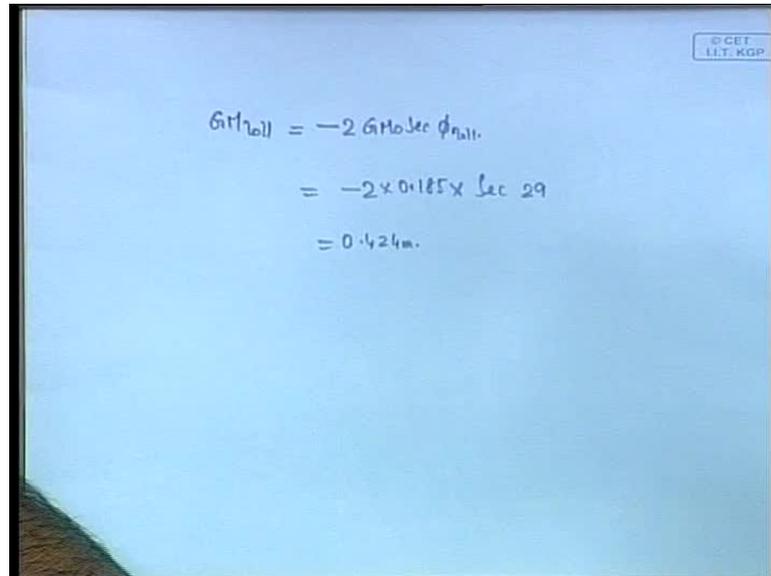
Now, this is why I said if you just remember the formula you just apply it or otherwise, you just have to do all these things again. Means this derivation which says how to find the angle of loll, you have to put that GZ equal to 0. All the derivation I did or you just memorize the formula, so $\tan \phi$ when this lolling will be I have already derived this minus 2 GM by BM. At any rate you would not be able to solve this if you get GM as positive. Anyway, this will become, so \tan of ϕ of loll will be. So, you just apply GM 0 you already have BM. BM we have not calculated. Is BM given? I by Δ is one way.

It is a box shaped barge. You have to calculate I by Δ that is why. They have not calculated it here but you have to do it. I by Δ you have to calculate. So, BM 0 I by Δ I is given by it is a rectangular $b^3 l$ by 12 will give you I divided by Δ we have, which is the displacement of the ship. There is a weight. We need to convert it to volume, so 4190 divided by 1.025. This will give you Δ because 4190 was weight divided by the density of sea water will give you the displacement volume and the I is $b^3 l$ by 12 that means b we know, b is 91 also. We know length is 80 meters, we know everything.

So, there are again to calculate BM also. There are two ways. You can calculate it from the hydrostatics data provided you are given KM and KB or if you are given nothing. You will be given something like this, like it is a box shape vessel. Then you can calculate I directly. So, it is so this you calculate BM. BM 0 GM 0 we have. So, once you have that you calculate $\tan \phi$ of loll.

So, actually this is very high but it can happen. So, you are having a ship that is lolling at an angle of 30 degrees. So, that much its ship is almost like this. So, it is always going like this. So, in a way nobody will be able to stand on the ship. It is highly inclined.

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The image shows a handwritten derivation on a blue background. In the top right corner, there is a small logo that reads "© CET LIT, KGP". The derivation consists of three lines of equations:

$$\begin{aligned}GM_{\text{loll}} &= -2 GM_0 \sec \phi_{\text{loll}} \\ &= -2 \times 0.185 \times \sec 29 \\ &= 0.424 \text{ m.}\end{aligned}$$

So, now next question is GM loll. Therefore, GM loll is equal to minus 2 GM 0 sec phi of loll. Therefore, I think you know all of these minus 2 GM 0. We know GM 0 comes to this into sec of this thing 29 degrees something. So, this becomes 0.424 meter, so this gives you the GM loll. So, two things you have GM 0 and GM loll. So this is one particular type of problem. So, while you are studying, you just have to study all the particular cases because if this one comes for the exam, it is not going to be possible to derive it at that time.

Similarly, I think the next problem actually it has the second one. We talked about the second derivation. I will do nothing. I will stop here because this problem, next one is another problem will continue after 5 minutes.

Thank you.