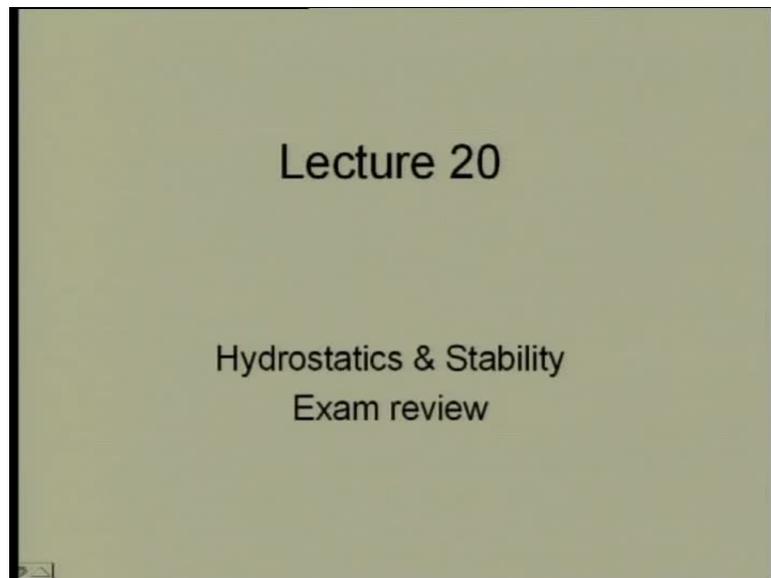


Hydrostatics and Stability
Prof. Dr. Hari V Warrior
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

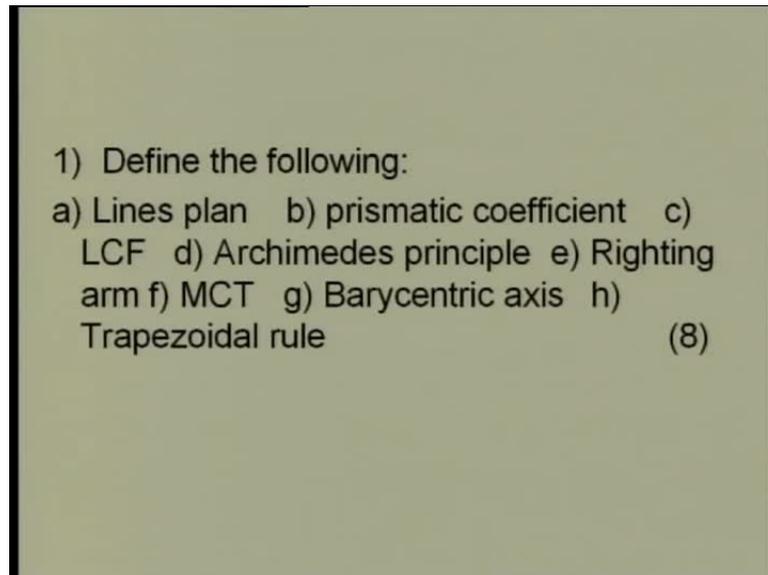
Lecture No. # 20
Discussion

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So, today we will start off with a, look at question paper what you had for the exam and the answers. As I said, sometime back there is not much to discuss for you because there is when the questions were very straight forward, and you have answered fairly well, but for the sake of this lecture we will do that.

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So, of course this question was, there are not really many people who have been answered, it like, lines plan you have. You are basically expected to write some definition of lines plan that it has, and write that you have these three, mainly that it has three views, **the all**, the profile plan and body plan.

So and if you can just draw a small figure something with looks like, that is more or less is enough. Then, prismatic coefficient, **I forget the exact definition, is the ratio of, what are the two the ratio volume to something delta, what is it? Midship area to length into, no breadth into draft, length into draft, probably length into breadth, probably most likely have to check again, I forget, anyway that midship coefficient.** That is very straight forward definition only prismatic coefficient. Then, some people have not written LCF - longitudinal center of flotation that is in little more, I mean, lot of people, I left that. So, LCF is, you know what is it? Is the centroid of the water plane area that is all you have to write really, that if you have written a lot of things and not, that then does not make any sense. So, it is the centroid of the water plane area, there is or the longitudinal position of the centroid of the longitude, of the centroid, of the water plane area.

So, that is Archimedes principle is very straight forward, downward weight is equal to upward weight or weight displaced is equal to the weight of the floating. **What is something like that, that is ok.**

Righting arm is also, we have righting arm is basically GZ. So, if you know that then we can draw the GZ curve that is all right. MCT is the moment to change the trim by either 1 centimeter or by 1 inch whichever you write is ok. That also many people have left, they have not written that. Then, Barycentric axis, what is it? It is the axis through the transverse, axis through the center of floatation. So, that is enough that one sentence would I have been enough. Then, trapezoidal rule you just write it is to sum up the areas and I think is 1 2 4 2 right, like that right 1 2 4 2.

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3) Generate and describe the statical stability curve. Using the graph sheet given, calculate GM for the ship with the following GZ table.

- Heel 0 15 30 45 60 75 90
- GZ 0 0.391 1.00 1.138 0.774 0.129 -0.584

(5)

So, that is also, that is the trapezoidal rule. Then, so this is, this is also that is straight forward only, but there is I mean, the second part of it, like you, the first of course you have to explain what is the statically, that curve should be there; otherwise, it there is no point in this explanation, in this question.

So, what you have to do is to draw that curve, and of course in that curve you have to mention that the tangent and that description should be there. When you at 1 radians, if you draw a vertical that is equal to GM; that definition should be there, and then same thing you have to draw that is all. And I am not going to exactly measure if it is a tangent or anything, but it should look like a tangent, at that origin it should look like a tangent.

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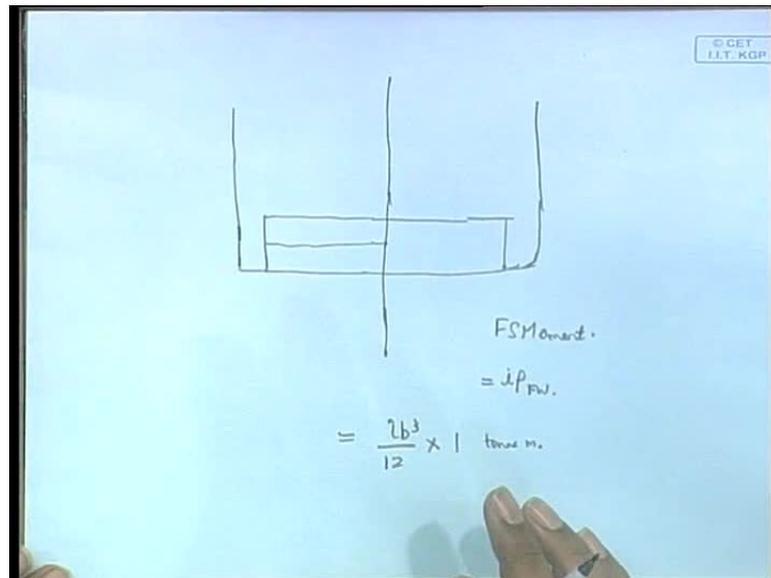
4) A vessel displacing 14,000 tonnes has $KG=11$ m; $KM =12$ m and is listed 3° to port. A tank of length 10m, breadth 5 m and 1 m deep with center of gravity 7 m to the port of the centerline is full of fresh water at $Kg=0.5$ m. What will be the list if half of this water is transferred to a similar tank on the starboard side of the vessel? The tanks are rectangular.

(9)

So, that is the important thing. Some people have drawn some strange things but mostly 90 percent 95 percent have drawn, so this is all right; I guess this is a little, it is not at all complicated but some mistakes are there, if you want I will just explain this 108.

So, the question is, so, you are told that the weight of the vessel is given, the once, there are 2 or 3 slight confusions in this problem; one thing, I myself made confusion in the exam because I told you, told some people that it is given 14000 tonnes is the weight of the ship; now, some many people ask me is in the weight of the ship plus water as the weight of the ship is without water like that.

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So, I told differently I think to some people, actually it is the weight of the ship plus water, **so but since**, I told like that both have given full marks, it does not matter whichever way you take. So, but really is speaking in the problem if you think itself it is said that the ship has water net and total weight is 14000 thousand tones; so, obviously it is the weight of the ship plus water, any way it does not matter.

So, the question is, so it is like this, you have a ship; so, here you are having a fluid and this fluid is transfer from the left side to the right side of the ship. It is listed in some angle initially, and so, you have to find the final list.

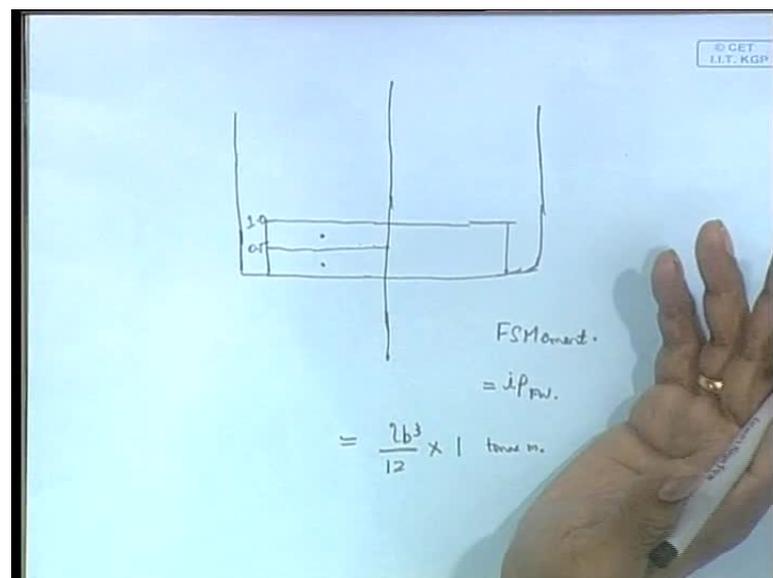
So, the first thing you have to find, as I told you before, someone ask me very valid question, it is said that the tank is fully filled, where is, it is full of fresh water that is actually a good point; the problem is defined like that it is, from the book also defined like that; that it is actually not, if it is full of fresh water there is no free surface, but let us assume something means, it is not full slightly means the tank is lightly bigger, so there is free surface effect.

So, that is problem is a free surface effect problem, so we have to consider that. So, first of all you have to find the free surface effect or free surface moment. So, you will have to do i into ρ of fresh water, that is this, the free surface moment, I mean, I do not think, I will do the derivation, you will just write the final expression. So, this becomes like $1 b^3$ by 12 into density of water is 1000 or 1 tonne.

So, that you have to do this is the first thing. Then what you have to do is make the table and you have to find the shift in G. So, first of all what should you calculate? You have to find the weight of water transfer first thing, volume into rho that and all you can very easily calculate, then shift in the center of gravity is $g_0 - g_1$ is equal to initial G_0 ; so, initial $g_0 - g_1$ is said to be initial $\tan \theta$ or θ is said to be 3 degrees, it is listed 3 degrees to port; so, $\tan 3$ degrees into GM. Is GM given? KG and KM are given, so you can find GM. So, $g_0 - g_1$ you find that is your initial $g_0 - g_1$.

So, that you have to find that you should have gotten as $1 \tan 3$, that we will get point naught 54. Then, this is one important part of it - that is weight. You have to find two things, first KG, then moment of that weight above and multiply with KG; that is, the weight multiplied with KG that moment. Then, you have to find also this horizontal moment, which is basically the free surface moment that will come there.

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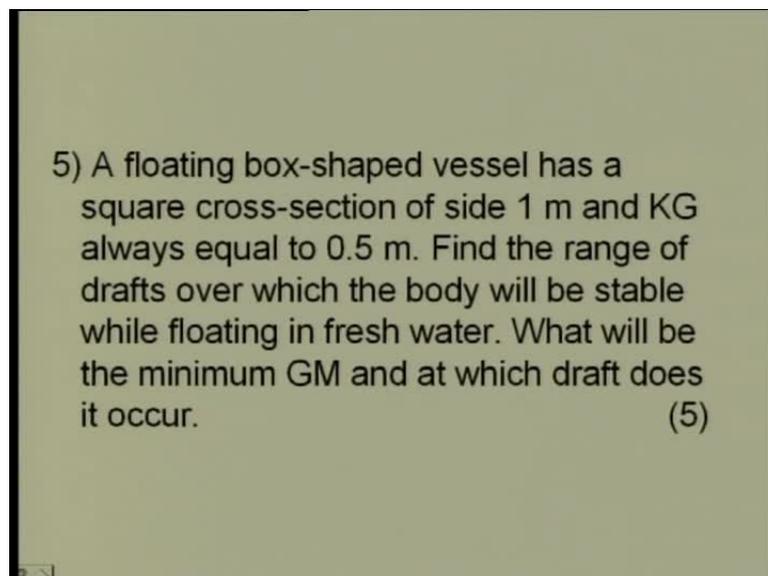
Only thing in that is one slide thing that is if you are finding the KG for the water transferred, that most people have done that wrongly, in fact, that is left side, note see that this is the water right, so water is removed from here, not from here. So, almost everybody has taken $c g$ of the water removed to be at 0.25, this is 0.5, this is 1. So, everybody has taken it to be at 0.25; that is, $c g$ of the water or at 0.5, either way means, this water is removed right. What are you doing here, subtracting one water removed, adding one water, add it; when you removing the water, what, where is the center of

gravity of the water removed? It is at actually 0.75 not at 0.25. Is it clear? **Right**. It is not means, you are not removing water from bottom; you cannot do that. So, you are removing water from top and it is transferred here. So, it is not at 0.25 actually, it is at 0.75.

So, that is the mistake in almost all, it was for except for very few, so that is one thing. Then, of course, you have to find then other thing that is free surface moment. Then, you find the moment port moment star board; you find the net G_0 , G_1 , then, other than that there is nothing in it.

So, what did you in general get as answer most of you final list, you remember, you do not remember. Anyway, you should get it as almost 1.5, 1.6 degrees, at least close should be; that 0.75 mistake is there with most; still it you will get some 2 or 3 degrees you should get I think, should not be getting very strange answers like, 10, 20 degrees, and if you get you have something really wrong.

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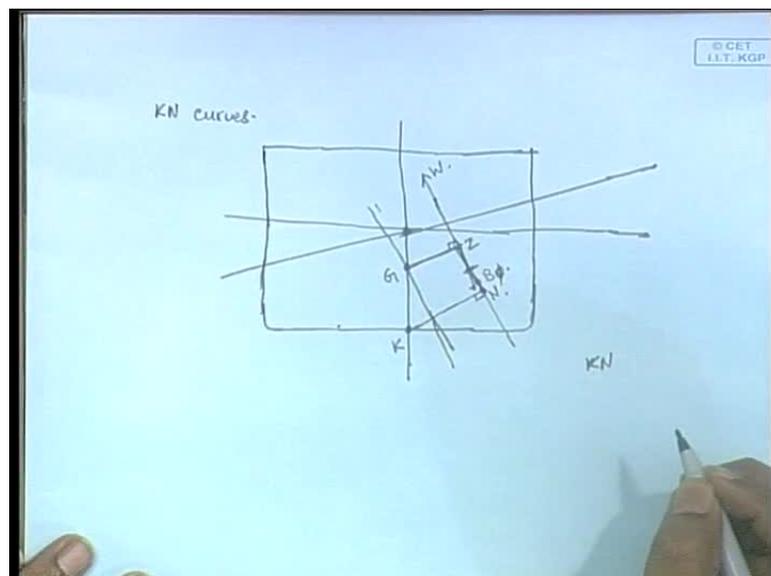
It is not tough at all I would not think that is anything. The only one problem is that is basically a free surface effect problem; you just have to how to apply then this is the problem. One more problem is that there, in this I told you, that is, one of the important things you have to see here is that it is a box shape vessel; that is, the important thing means, if you try to used the usual formulas, that is like GM, using GM, KG and all that you would not able to solve its big. That is why, that box shape vessel is there; box shape

vessel, I have already done set of formulas; that is, these things like this KM is equal to d by 2 plus b square by 12 d right? So, KM is equal to d by 2 plus b square by 12. If you know this formula, the problem is very just straight forward, this just substitution. So, you do KM is equal to this and you know that for the vessel to be stable KM should be greater than KG ; you put that condition and then you solve for it. You should get d should be less than 0.211 or draft should be less than 0.211 or draft should be greater than 0.789.

So, that is the answer and the other answer is though some people have asked me also it is a little weird; you get GM as negative, you are ask the minimum GM , but no, it is the minimum GM , it is not the optimal GM or the operating GM or any it is the minimum GM . Therefore, it is ok. Therefore, you get minus point naught 92 is the minimum GM ; anyway, from differentiation you can get only the minimum or the maximum; so, that is why that is asked.

So, that is the second part of the problem. So, the only thing here is, you should know this KM is equal to d by 2 plus b square by 12 d , once you know that then it is very straight forward something, all right, that is all for the exam.

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Now, we go into a continuation, it is mostly continuation only, what are known as something known as KN curves; it is not, of course not now whereas, as important as the statical stability curve or the cross curves of stability or anything, but still it is used, so

we will discuss it; it is known as a KN curves. The thing is you used the KN curves to find out the GZ curve of the vessel.

So, in that way, it is important. So, KN, from KN curves, once you have design the ship you can use KN curves, what I will tell you what it is, KN curves, you can use it to get the GZ curves; and of course, GZ curves are the most important. That righting arm curve and the statical stability curve are most important, in this course itself.

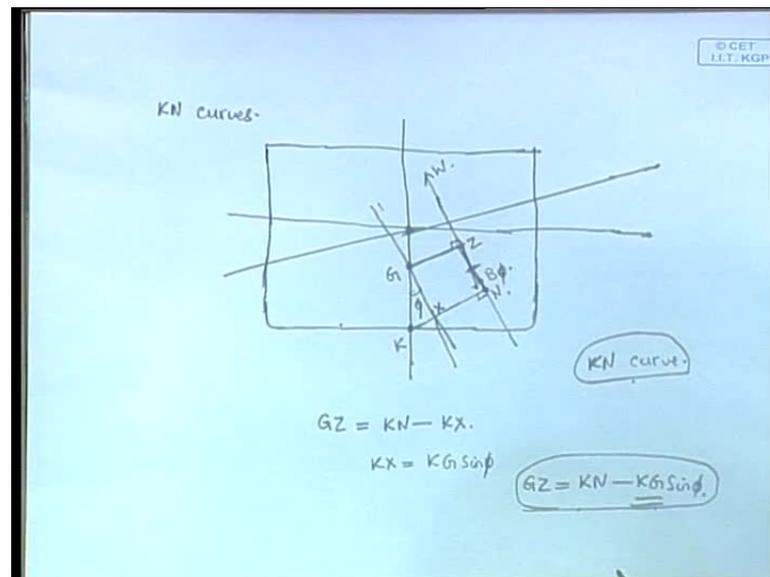
And we used a simple definition of a kind of ship, we call wall sided ships. It just means straight, the sides are straight, they are call wall sided ships. Of course, everything we have defined so far, all the derivations I have done so far, is all for wall side issue. I have never defined anything for, so it does not matter, but for **the** you should know what is the wall sided ship? that is, you because in general, you will see that ship, is not like this, it is very rarely like this, it is like this; it will have some bulging, it will come like this or little bit.

So, none of that is considered in wall sided ship, because it is for mathematics, it is all for straight lines. So, consider this ship, consider this, these are two parallel lines, GZ is the perpendicular to that line; so, this is the righting arm. So, here somewhere you have B theta, so B phi, initially we had the ship is riding, so heel; and as a result as moved into the new position B phi, and we have drawn the vertical there, and GZ is the righting arm, so the weight W is acting here upwards.

Now, this is K, this is the keel, the bottom most point of the ship is the keel. Now, to this line; if you draw a perpendicular, like this, this point we call it as N; this point, it is not B phi; B phi is somewhere up here. Let us say, this is B phi, B phi is up there and GZ is there. And so we see that KN is parallel to GZ, both of them are perpendicular to that vertical through B phi.

So, this KN, which is drawn as a perpendicular to this vertical at B phi, is what we are calling as KN curve. And as phi keeps changing means, this is phi; as phi keeps changing, you know that B phi keep going upwards, and similarly, the N as also you can see will go keep going outwards.

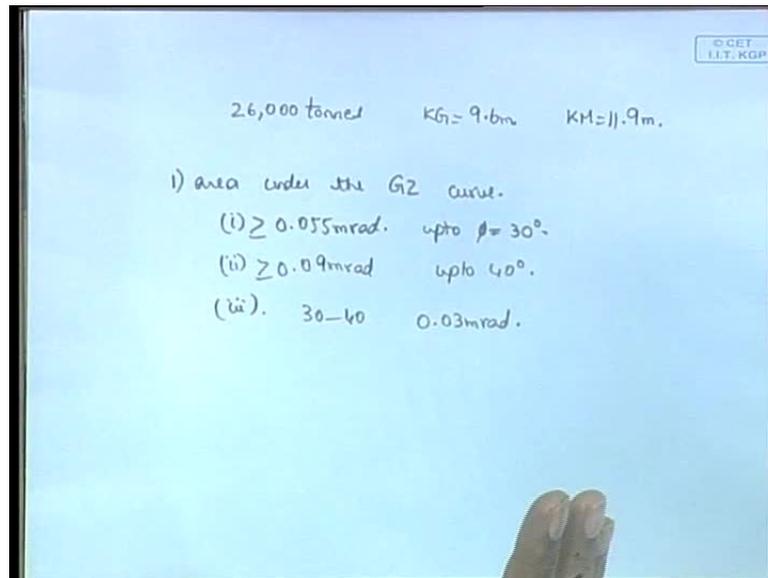
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So, K will be fixed; K is the keel of the ship. Now, this curve which is traced out by N is known as the KN curve. So, this is what we call as a KN curve. Now, just a little bit of GZ is equal to, let us call this point X, KN minus KX; GZ is equal to KN minus KX, and KX, from this, you can see directly is equal to KG sin phi, from the figure you can see that KG sin phi. Therefore, GZ is equal to KN minus KG sin phi. Now, that is what I said so first we used this KN curve to find the GZ curve.

So, that is the purpose of the KN curve. There is a way of finding a KN curve for a particular ship; and once you have the KN, using this formula that is KN minus KG sin phi, of course, we need to know the KG of the ship. And once you know the KG of the ship, for different angles of phi, means, you start from 5, 10, 15 like that you keep increasing phi, and as phi keeps increasing, you find the different values of GZ. And once you have the difference GZ, you can draw the GZ curve. So, this is the purpose of the KN curve. So, and this is the formula used for calculating the GZ from the KN curve. For instance, we will see this problem.

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Now, they are seeing, there is a ship that ways that has the displacement of the 26000 tonnes, and its KG is equal to 9.6 meter, and KM is equal to 11.9 meter. Now, you are given the KN table. Now, you are asked, does the ship comply with the load line regulations; that means, especially for the end semester, you have to remember these load line regulations, that is, this is the most important part, there are lot of them for grains have set.

So, if you have to do the problem, to do one problem itself will take you definitely half hour to 45 minutes, it is that kind of problem; and if you do not know the regulations then you cannot. So, this is the most important one is that you should know that the area under the GZ curve, just write the most important of it.

So, area under the GZ curve should be greater than or equal to 0.055 meter radians up to an angle of 30 degrees, up to phi equal to 30 degrees, this is the first rule; that is, the area under the GZ curve should be greater than or equal to 0.055 meter radians up to 30 degrees, this is very important; and in fact, there are three more very importance one. Then, it should be greater than or equal to 0.09 meter radians up to 40 degrees; this also is very important. Then, third one is that between 30 to 40 degrees, you should have an area of 0.03 meter radians.

These are three rules that you definitely have to remember, it should remember it always, it is very useful, I mean, like interview and all they will ask something load line regulations are very common.

There are of course many specifications this keeps changing. This is general which can be applied mostly for all ships, for example, for each type of ship it differs. We have seen for grains, it is different, we mentioned a couple of different rules; it is not different, but it is slightly more modified for different ships. Similarly, for container ship, there are slightly different rules like that, but this in, even if you do not know any of them, at least, if you remember this, you have something to say; you are not far off from the mark, you can say something. So, this better remember, so if you do not know anything else, just used this, these rule this for all the ships. Even if you do not know, remember the grains at least if you apply this you have something.

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Heel.	0	5	12	---	---	---	90
KN.	0	1.05	2.67				7.63
KG Ship	✓	✓	✓	✓			---
$G_2 = KM - KG_{Ship}$	✓	✓	✓				

$$KM = 11.9$$

$$KG = 9.6$$

$$GM = 2.3m.$$

So, your question is that does this ship comply with the load line regulations? So, you will be given the problem like this; that is, you will be given heel and KN; heel 0, KN 0, just like that problem, I give you the GZ just like that 1.05, and this 12 2.67; like this you will have till 90 have 7.63.

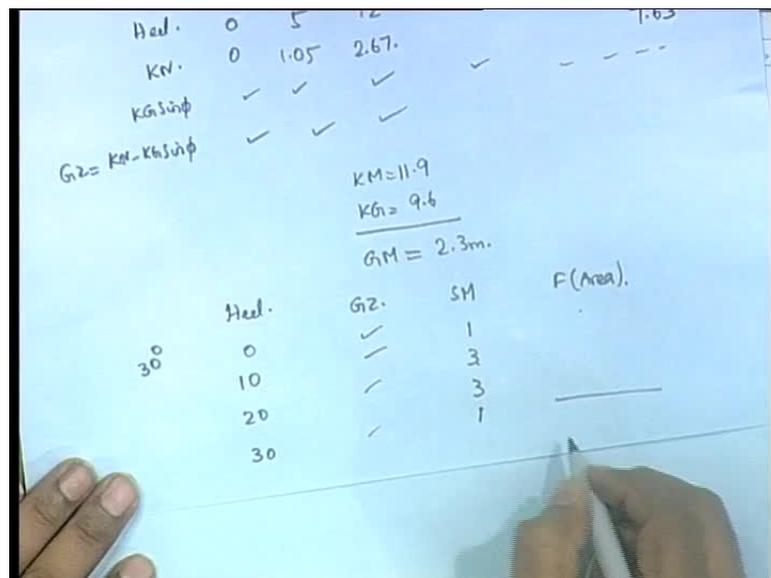
So, you are given the KN table; instead of giving you the GZ table. Now, we will be given the KN table from which you have to find the GZ. So, what you do is, first thing

you have to do is you make the first one more line here, you put $KG \sin \phi$. You have to know KG of course, it is given; 9.6 is given.

So, $KG \sin \phi$ for each of this ϕ 's you find the KG and put it here. Then, you do KN minus $KG \sin \phi$ GZ is equal to KN minus $KG \sin \phi$, you put that here. So, this set of table you have to make; this gives you a final GZ curve anyway. Then, your given KM is equal to 11.9, then KG is equal to 9.6; therefore, you find GM , KM minus KG , 2.3. Actually, I think there is a rule regarding GM also, GM should be, GM should be greater than 0.15 meter that is also important.

Now, so, first rule you can check that GM is 2.3; it is definitely much better than the minimum GM that is all right. Then, you have to check the other rules, does it satisfied the load line regulations.

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So, you have to find the area. So, that is like this only, heel, GZ , you write the Simpson's multiplier then function of area; 0, 10, 20, 30, so you find the area up to 30 degrees. You have to in fact to do two things; you have to find the area up to 30 degrees then you have to find the area up to 40 degrees, and you have to find area between 30 and 40 degrees both, all three things you have to find.

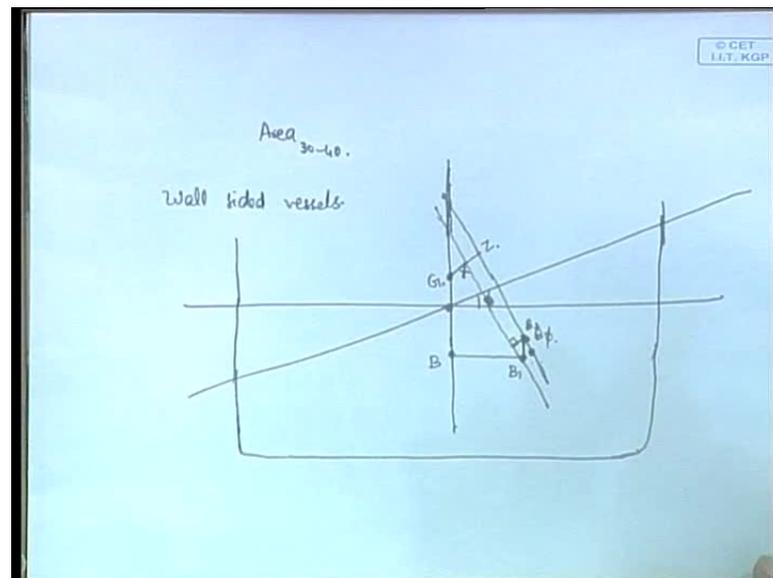
So, first you do this GZ is given you put that, then Simpson's multiplier you can use any of the Simpson's multipliers and or you can use a Trapezoidal rule or anything and

then, you find, so you put that here then the function of area, multiplication of this, summation and then, finally the rule here, they have used a 1, 3, 3, 1 that is a like this. So, 5, 12, actually, 0, 5, 12, 15, 30; so 30, is not a problem, but 40 is not there, next one is 45, let me see, how they have done; so, till 30 there is no problem. The only way when you are given a problem like this is you have to draw the GZ curve.

So, you have to do some kind of approximation. What they have done here is, you take up to 30 that is no problem here, from 30 to 40, you have to make some kind of parallelogram, make some kind of rectangle and make an assumption like that.

So, that is what they have done here, they have made rectangle and roughly said this much be the area and subtracted that from 45; that is all right. So, at any right you have to be ready to draw the GZ curve. So, similarly, you do this from 30 degrees, then next one you have to do till 40 degrees, you have to find the area.

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So, once you have that M you have to find the area between 30 to 40. Now, each of those you check, area up to 30 degrees, you find, you see if it satisfies the regulations, in this case it does; similarly 40 is satisfies, between 30 and 40 it satisfies. So, three things we have to check, if all three of satisfied then the problem is correct, the ship is ok.

Then, let us look at some effects of wall sided vessels. So, this as G here and it is now heel by an angle theta of phi; then, now from this G, I draw vertical to this Z. Now, we

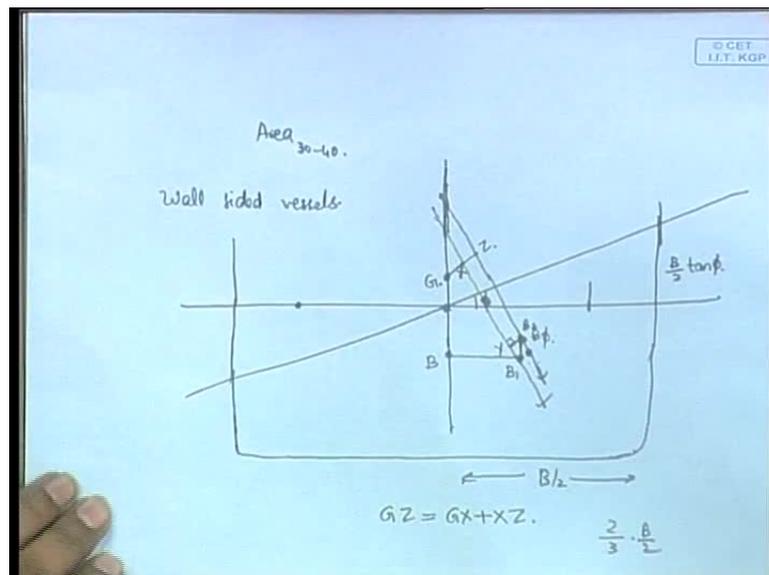
can do in this problem, we do it in a slightly different form, that is, initially it is at B and then it moves to B phi; let us say it is here.

Now, let us split this, it goes means let us assume, that the heeling is done in two steps, one like this and then like this, means, heeling can be anyway, heeling is we cannot change, heeling happens. Let us assume, that the moment of B is like this; B moves horizontally and then vertically or rather the total moment you split into horizontal and moment vertical; you just split it like this.

Now, here we will draw like, here I draw a vertical and you get B phi; now, I will call this B, this I will call B 1 and this will call B 2; from here I draw a perpendicular. Note that this perpendicular will be parallel to this GZ. Similarly, GZ also will have a component like this, let us call this point X, the intersection with the other line is X.

Now, as you can see, GX will be due to the moment of B to B 1; and the moment from X to Z will be due to the moment from B 1 to B 2; that is, all we have done is, in totality what is happened is B has moved to B 2; that is, the new position of B phi is B 2. Therefore, B has moved to B 2 and it has moved like this, B has moved to B 1 first, and from B 1 it has moved to B 2; B 1 is a horizontal movement and B 2 is a purely vertical movement.

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So, $B \sin \phi$ is just a general term, $B \cos \phi$ is the final $B \sin \phi$, $B \cos \phi$ is the final B . Therefore, you have this GZ , note that this we are calling it as Y ; this point is as Y ; so this is Y .

So, you have $B \cos \phi$; $B \cos \phi$ will be a line, which is parallel to GZ because both of them are perpendicular to this line, because this line which have tick mark is parallel to this line, these lines are parallel, its drawn like that I am drawing like that.

So, now, we need this figure; now, GZ is equal to GX plus XZ , it is straight forward from that GZ is equal to GX plus XZ . Now, note we are doing similar thing, couple of thing we have done many times. So, I will not repeat, that is, we have done that there is shift of wedge from one side to another, we have seen talked about that the movement of one volume to another side. And we have also talked about the coordinates of the center of that volume, which you we can call it center of buoyancy or center of gravity of that volume.

So, that is we have said if this distance is B ; this is B the whole thing is B ; therefore, this is $B \cos \phi$. Now, its center of gravity will be here, which is $B \cos \phi$, this will be $2 B \cos \phi$; $2 B \cos \phi$ of $B \cos \phi$; that will be this distance, those things we are said. Similarly, there is a height here, which will be, note that this height will be, if this is $B \sin \phi$; this will be $B \sin \phi$.

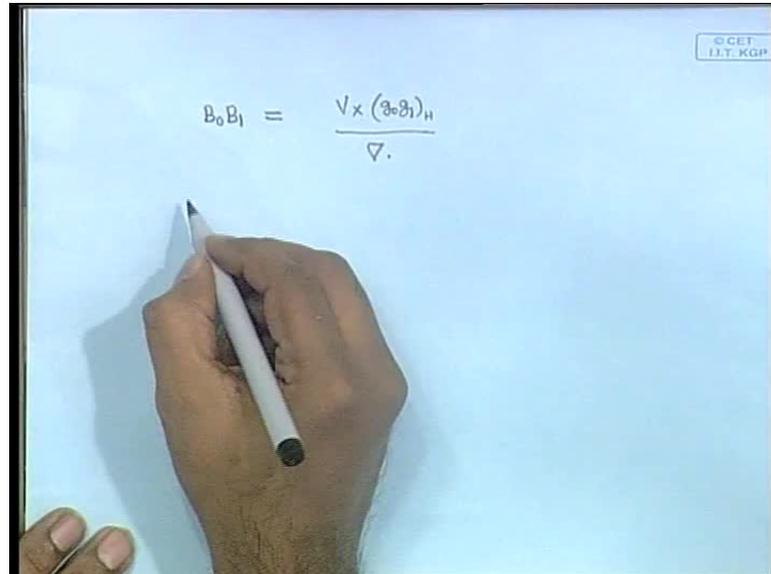
So, this is equal to $B \sin \phi$; $B \sin \phi$; $B \sin \phi$. Then, I am just writing the dimensions, because no need to repeat it. So, here there will be a, there will be a $B \sin \phi$; this is $B \sin \phi$. And the center of gravity is therefore to find there are couple of things, for instance, you will need to find the volume of the wedge that has transferred, which we write it as half base into altitude into length.

So, half base into altitude, this is the base half base, either way half base into altitude; so, will be base will be $B \cos \phi$, and your altitude will be $B \sin \phi$. So, half of that there will be give your area into volume into length, will give you the volume; there will give you the volume, then another thing we need is G_0 , G_1 , means, we have to find distance through which the here, we have a center of gravity somewhere; this center of gravity is shifted to here.

So, we have to find the shift in the center of gravity of that wedge. This also we have done many times, that shift, this for that you need this distance; this distance is $2 B \sin \phi$

by 2 which is B by 3 twice that 2 b by 3. So, that is the distance through which G of the volume has shifted.

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So, once you know these two things, I will just write the expression; that is, you will have the shift center of buoyancy of the whole ship, not for the volume wedge alone; for the whole ship the shift in $B_0 B_1$ is given by, V is the volume of the wedge into $g_0 g_1$ let us assume to be **the, shift in the**, I will explain. So, what we have here? We have here, a volume or which we called as a wedge. We shifting from one side to another side, that is, from one point it is moved to another point, in the whole ship. So, there is a shift in the B of that wedge itself, and because of this movement of the wedge, there is the shift in the B of the ship itself. There are two things, one is wedge itself which is shifting from one side to another; there is a movement, because of its movement that volume which is contained inside the ship that B of the ship itself shifts.

We know that if a mass inside a whole big mass moves, small mass will move, by distance X , the G of that whole big mass also moves by some distance. So, that is what, this is, $B_0 B_1$ is that of the ship is given by V into $g_0 g_1$ of that a small wedge distance through which the center of gravity or the center of buoyancy; it is the same, I am not differentiating here, center of gravity of the wedge shifts divided by the delta of the ship.

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$$B_0B_1 = \frac{V_x(\theta_0\theta_1)_H}{\nabla}$$

$$V_x(\theta_0\theta_1)_H = \int_0^L \frac{1}{2} \frac{B}{2} \frac{B}{2} \times \tan\phi \times dl \cdot \frac{2}{3} \times \frac{B}{2} \times 2$$

$$= \tan\phi \int_0^L \frac{B^3 dl}{12}$$

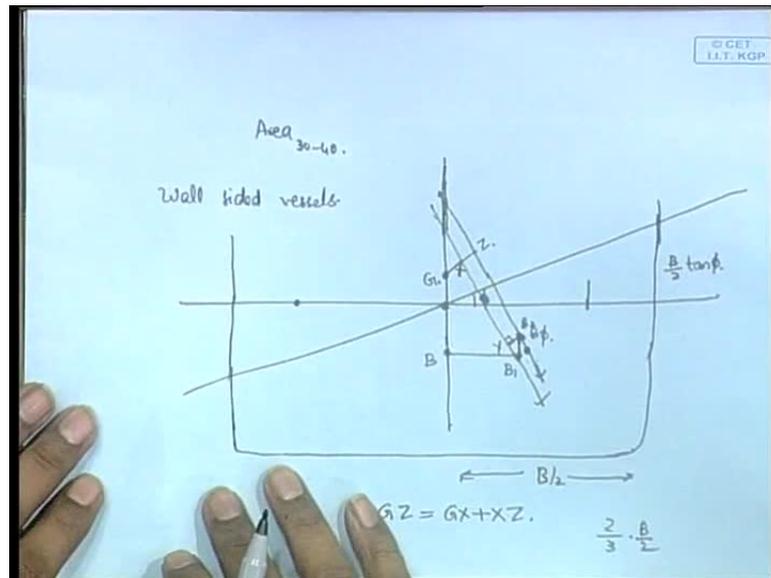
$$= I \tan\phi$$

$$B_0B_1 = \frac{I}{\nabla} \tan\phi = B_0M \tan\phi$$

H means horizontal, we are talking about the first, remember, I told you that the whole moment of the B like this; I have shifted into a horizontal plus a vertical. Let us, consider the first thing alone, which is the horizontal shift; so, we are finding B_0B_1 , the total horizontal movement of the whole ship. Now, the volume of the wedge; now, I will just write this, remember its half base into altitude, then you have to multiply with the dl ; this dl is, remember, you are finding the volume, dl is integrated. Small length, it is integrated over the whole length of the ship, you will get the total value dl into 2 by 3 into B by 2 into 2 . Actually, there is a B by 2 , one more B by 2 , actually there is a mistake in the book also, there is one more B by 2 , because finally becomes B cube; anyway there is a B half base into altitude.

So, its half B by 2 into B by 2 into $\tan\phi$ into dl , that is correct; that is, half base into altitude into 2 by 3 into this thing. So, this will be your that V into, this is equal to $\tan\theta$ into or $\tan\phi$ into integral 0 to L , which is the total length of the ship B cube into dl by 12 , it will become like this, which is actually equal to here $I \tan\phi$. Now, what is it tell you? Therefore, B_0B_1 is this divided by this V into this, so I by $\nabla \tan\phi$. Now, you know that I by ∇ is equal to B_0M .

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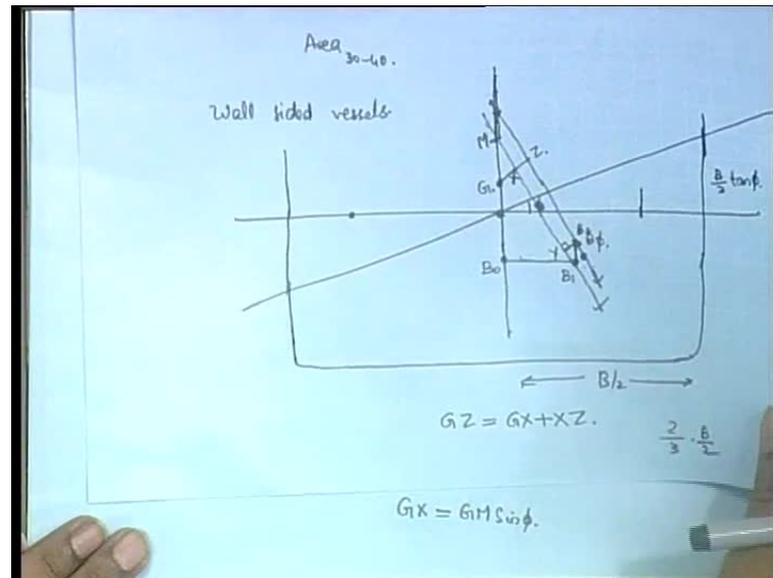
So, it is equal to $B_0 M$ or $B_0 M \tan \phi$. So, this is the first relationship you are getting, $B_0 B_1$ which is the horizontal shift in the position of B , is $B_0 M \tan \phi$. Then, now from this, we look at this figure again. What we just got is that $B_0 B_1$, this distance $B_0 B_1$ is equal to $B_0 M \tan \phi$; this is B_0 , so $B_0 B_1$ is equal to $B_0 M \tan \phi$.

So, directly what will you get? That will directly give you this is M . What actually we are doing here is, till now we have derived some expressions, for instance, we derived an expression that GZ is equal to $GM \sin \phi$, we have already derived that; it is a very basic expression, I mean, we have done that many times.

Now, what we are doing is we are in this derivation. We are actually giving it a small correction, means, it is not exactly G_0 ; everything is slightly different, means, GZ is not $G_0 \sin \phi$; what you will get it as, it will become something like $G_0 \sin \phi$ plus half $B_0 M \tan^2 \phi \sin \phi$; it becomes a little more complicated expression. This is the real expression, and to first order the other one is ok.

So, it does not matter means whatever you are using, you are mostly going to use only the other one; only for this purpose of this course, I think you will be using this way, very rarely will go to the other one into this one; you will be using mostly when you are doing any kind of calculation, you are going to use GZ is equal to $G_0 \sin \phi$; that is the very common expression, but we are doing the first correction, and this is the real expression, I mean, for this course you have to know this.

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So, you get that this is M. So, this is the correction we are adding. So, from this you can get one more things, from the same figure, you get that GX is equal to GM sin phi. Note that we have actually, what we get till now, we have already always defined that GM sin phi is actually GZ, means, this thing, GZ.

So, we see that in you really cut it down when you keep going deeper and deeper, GM sin phi is not really GZ but according to this it is only GX. And I will explain one more thing; for instance, if you how do we say means we initially say a ship is initially in the upright condition then we tilted the ship a little, what happens? When you tilted the ship a little, B goes to B phi and how did we defined? At B phi, if you draw a perpendicular or a vertical, where it heels the initial vertical, you get M, you see it is not exactly true here, it is this which hits M but you have to notice one more thing, look at the figure itself the distance is so small, it is almost 0, there is, for this, for the purpose of this derivation its important. What I am trying to say, is that for instance, in a viva or if someone asks you know what is anything related to this, you do not need to go into this thing because in a general sense is not relevant.

So, you do the previous thing only, but this is just a very detail calculation. In reality, this is correct but it does not matter you used other one always; for the exam only you used this.

So, the thing is when we always say, you will never here, anywhere in naval architecture that is when you draw a vertical at B phi, when you draw it where it hits is the original vertical, that is called as it should be here actually. So, it is not there but it is actually here; it is actually only the horizontal moment of B that matters in the movement of M.

So, in the position of M, the only thing that really matters is the horizontal moment of B, but when it shifts, the vertical moment of B, in fact is so small compared to horizontal moment, we never bother about it, that is all.

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$$B_0B_1 = \frac{V \times (\theta_0\theta_1)_H}{\nabla}$$

$$V \times (\theta_0\theta_1)_H = \int_0^L \frac{1}{2} \times \frac{B}{2} \times \tan\phi \times dl \times \frac{2}{3} \times \frac{B}{2} \times 2.$$

$$= \tan\phi \int_0^L \frac{B^2 dl}{12}$$

$$= I \tan\phi.$$

$$B_0B_1 = \frac{I}{\nabla} \tan\phi = B_0M \tan\phi.$$

$$\underline{\underline{G_1K = G_1M \sin\phi}}$$

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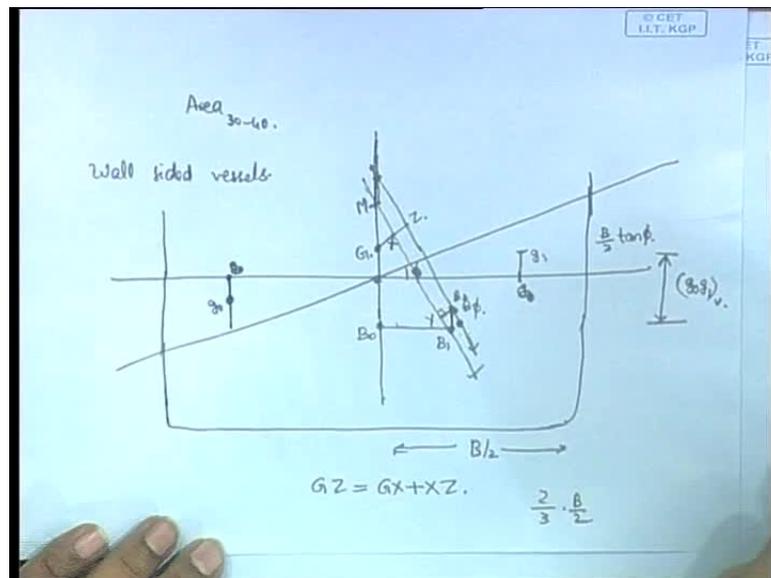
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vertical shift.

$$B_1B_2 = \frac{V \times (\theta_0\theta_1)_V}{\nabla}$$

But you should know, so you get GX is equal to $GM \sin \phi$, that is one expression. Now, next what you have? You have the vertical shift, so first we moved from B_0 to B_1 , now it will move from B_1 to B_2 , now we are moving into like that. Let us, consider the vertical shift. So, that is vertical shift is $B_1 B_2$, it will be given by V into $g_0 g_1$ vertical divided by Δ . So, the expression is same, the formula is the same, V is the volume of the wedge, this thing. So, this $g_0 g_1$ that I am talking about, small $g_0 g_1$, this one; this $g_0 g_1$ that I keep writing here is this, this is g_0 , and this is g_1 .

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So, I am talk about this $g_0 g_1$. It is the shift in the center of gravity, center of, either of those centers of gravity of the wedge or the center of volume of the wedge, either way you can look at it. So, this volume, one volume goes in, one volume comes out. So, this volume goes in, this volume comes out; as a result of which this wedge is coming out, this wedge is going in. Again, as a result of which one volume is it is like a one volume shifted from one side to another side.

So, g_0 gets shifted to g_1 . In fact, it will be here, not here, it will be here, actually sorry g_1 will be here, and this g_0 will be somewhere here; this is just the horizontal component of it. So, g is, in this previous case, I wrote $g_0 g_1$ horizontal which is actually this distance. And now we are taking about the vertical shift in g_0 ; that is, $g_0 g_1$ vertical; so, this distance is what I am calling is $g_0 g_1$ vertical, this is the vertical moment of $g_0 g_1$.

So, because of the shift of this wedge from g_0 to g_1 , the center of gravity of the ship itself, will shift or the center of buoyancy of the ship itself, will shift, means, because of the transfer of volume from one side to another in the vertical direction, not from one side to another in a vertical direction. is also G is moving.

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vertical shift

$$B_1B_2 = \frac{V \times (G_0G_1)_v}{\Delta}$$

$$V \times (G_0G_1)_v = \int_0^L \frac{1}{2} \cdot \frac{B}{2} \cdot \frac{B}{2} \tan\phi \cdot dl \cdot \frac{1}{3} \cdot \frac{B}{2} \tan\phi \times 2$$

$$= \frac{\tan^2\phi}{2} \int_0^L \frac{B^3 dl}{12}$$

$$= \frac{\tan^2\phi}{2} I$$

$$B_1B_2 = \frac{1}{2} \cdot \frac{I}{\Delta} \tan^2\phi$$

$$= \frac{1}{2} \cdot B.M. \tan^2\phi$$

So, when the center of volume or the center of gravity shifts from this point to another height, this whole center of buoyancy of the ship itself, will shift from a lower to a higher point. So, that is what this is, so B_1B_2 is equal to V into G_0G_1 vertical divided by Δ , and this one vertical will be given by half base into altitude, this is the same thing into dl , note it is it will be either 1 by 3 from the bottom or 2 by 3 from the top, either way you are looking at it, 1 by 3 into B by 2 $\tan\phi$ into 2, because one is there at the top, one is there at the bottom, same thing a same distance. So, this becomes $\tan^2\phi$ by 2 integral 0 to L $B^3 dl$ by 12; then, this is equal to $\tan^2\phi$ by 2 into I , where $B^3 dl$ by 12 we know is the moment of inertia.

Therefore, that $B_2 Y$ is equal to $B_1 B_2 \sin \phi$. Let us look, I think it is correct, $B_1 B_2$, $B_1 B_2 \sin \phi$, you will look at this this triangle this small triangle here which have darkened in that triangle. You have B_1 , a triangle made with $B_1 B_2$ and Y . So, you have this one triangle, so in that you will get $B_2 Y$ is equal to $B_1 B_2 \sin \phi$; $B_2 Y$ is equal to that which is equal to half $B_0 M \tan^2 \phi$ into $\sin \phi$.

Now, again note that $B_2 Y$ is the perpendicular distance between this line, and this line, there are two parallel lines; it is same as XZ , XZ is also the perpendicular distance between the two parallel lines, it is the same thing; so, this is equal to XZ . Now, we have split GZ into GX plus XZ .

Now, XZ is given by this, given here, GX we derived from our horizontal moment somewhere there here; GX is equal to $GM \sin \phi$. So, two expressions we have got for GX and GZ , as the result of which GZ becomes $\sin \phi$ into GM plus half $B_0 M \tan^2 \phi$ square ϕ .

Now, note our earlier derivation said that GZ is equal to $GM \sin \phi$ which now says is equal to GZ is equal to $\sin \phi$ into a little more thing. Now, if you do the calculation, you will see that these things are very small; BM itself is a small quantity.

Now, you will see multiplied by $\tan^2 \phi$, we usually have ϕ very small; therefore ϕ itself is small, and $\tan^2 \phi$ is very small. Now, it is again multiplied by $\sin \phi$, which is also small; so, it is a second order, it is a very small order quantity, it does not really matter, but this is the real expression for the whole thing.

So, this is the final expression, you have to remember this for this course; that is what I am saying, but other things you have to remember forever like, GZ is equal to $GM \sin \phi$, something you have to remember forever; all your carrier you will remember that. And this for the timing you have to remember it.

Thank you.