Hydrostatics and Stability

Prof. Dr. Hari V Warrior

Department of Ocean Engineering and Naval Architecture

Indian Institute of Technology, Kharagpur

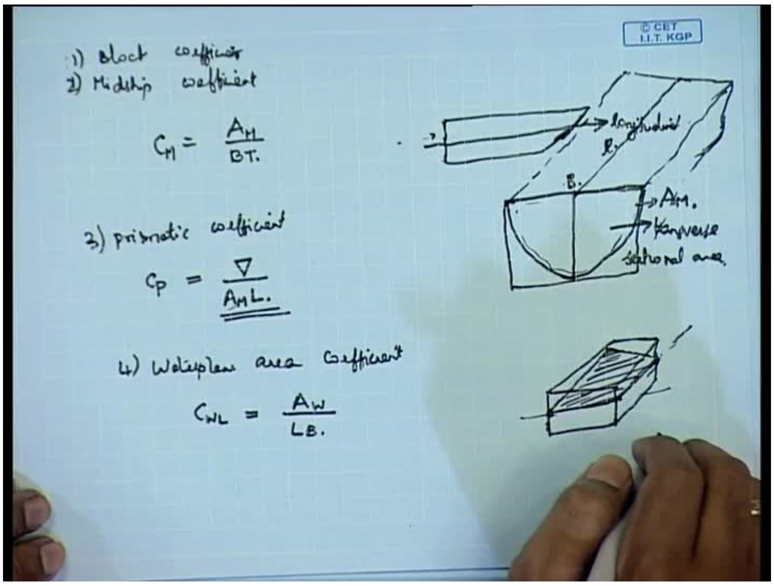
Module No. # 01

Lecture No. # 02

Archimedes Principle

Welcome to the 2nd lecture on hydrostatics and stability. In the last class, we discussed about the block coefficient and we left off at block coefficient.

(Refer Slide Time: 00:47)

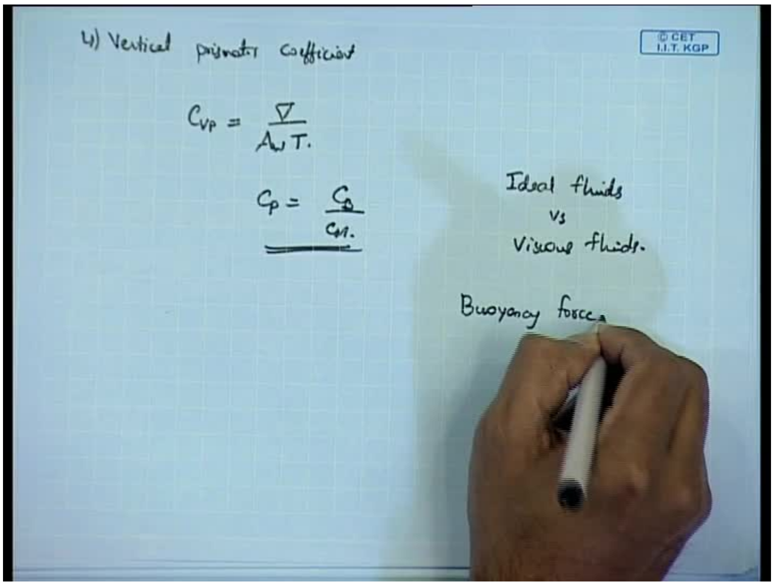


Now, there are a couple of other coefficients, which are also important, though the main one is a block coefficient. One of them is the midship coefficient. It is defined as the ratio of the sectional area under water to the breadth and drafts. It is defined to be C M. midship sectional area and C M is equal to A M by B into T. As I told you before, we divide the ship in two ways. If you define this to be the ship, this direction is known as the longitudinal direction. When you see from here, what you see in that area? It will look like this and the area is known as the sectional area. The direction is known as the traverse direction and it is known as the sectional area. Now, this area is what we are talking about here, so it is this area divided by B into this draft. The midship sectional area will look something like this; this area sectional area, which is written here as A M divided by B into T. So, you will have the sectional area at the midship section. It is known as the midship coefficient and it is designated as C M. So, this is the second coefficient. I already told you the first one is the block coefficient and then you have the midship sectional area coefficient.

We call something as the prismatic coefficient; it is defined as C P is the ratio of the molded displacement volume delta. Now, it is little more difficult to draw this figure, so I leave it. Prismatic coefficient is defined as the ratio of the displacement of the ship delta; it is the volume displacement of the ship. Delta is always the volume displacement and we call it as rho into delta. It is known as the displacement weight of the ship. So, you have C P, the prismatic coefficient as delta divided by A M into L, where A M is the midship coefficient. If you consider this to be the length of the ship, then midship sectional area into L will give you… This is length L of the ship, midship sectional area into L is this and it is the denominator. Delta operator takes up the numerator. So, this is the prismatic coefficient.

There is a water plane area coefficient. It is denoted as C WL, it is equal to A W divided by L into B. If you have a shape like this, A W is the water plane area and let us say that this (Refer Slide Time: 05:08) is the water line, this is the ship, this is the water line and then A W is this area. This water plane area divided by L into B, where L is this distance and B is this breadth. This is known as the water plane area.

(Refer Slide Time: 05:48)

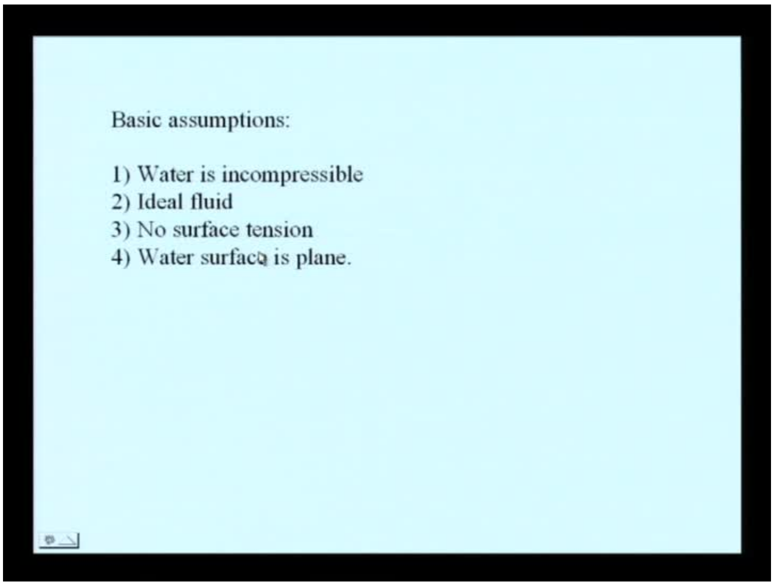


There is a vertical prismatic coefficient. Vertical prismatic coefficient is defined as C VP. It is equal to delta divided by A W into T. So, I leave it, as the definition itself says what it is. C VP is equal to delta divided by A W into T, where T is the draft; A W is the water plane area. You should know, what is the water plane area, for sure. Water plane area means, if you have the ship like this, there is a water line. At the water line, there is a horizontal area of ship and that area is known as the water plane area told as A W into T.

So, we defined a couple of things in this chapter. We defined a block coefficient and we defined what is known as the displacement, displacement volume, displacement mass or weight, midship coefficient, then midship sectional area, prismatic coefficient, vertical prismatic coefficient, water plane area. Among these, I think water plane area is very important. You have to know what is a water plane area, then the water plane area coefficient. These are some of the terms we learnt in this chapter.

This C P becomes C B by C M. The prismatic coefficient is equal to the block coefficient divided by the midship sectional area coefficient. So, this is where we leave chapter 1. Next is the second part, which deals with Archimedes principle. This is first principle of this topic. The most basic principle with which, all rest of the study proceeds. As you know the famous story - Archimedes is supposed to have discovered this. When he discovered it, he was so excited that he ran naked on the streets shouting ‘eureka’. That is the famous story of the Archimedes principle.

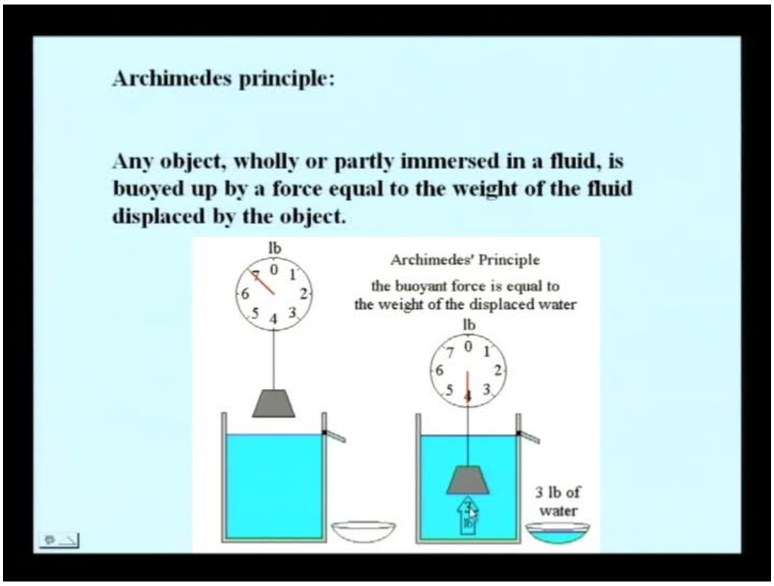
(Refer Slide Time: 08:27)



Let us take a look at what is Archimedes principle. Before we do it, we make some assumptions: water is incompressible; incompressible is a word used in fluid mechanics, which implies that there are no density changes in the medium. So, the water has a constant density. It does not have any variabilities in density. Water is incompressible and then water is an ideal fluid. It is an ideal fluid; it is again an important term in fluid mechanics.

You have two types of fluids: ideal fluids and viscous fluids. Now, all the liquids that we consider, including water are all viscous fluids. In many cases, we can assume that the fluid is ideal without any viscosity and that assumption is made in this case. Then, we say that there is no surface tension and that is an assumption made here because it makes changes to the water line. It would not be a straight line and that is last assumption that is water surface is plane, so the surface of the water is plane. So, these are some assumptions we make.

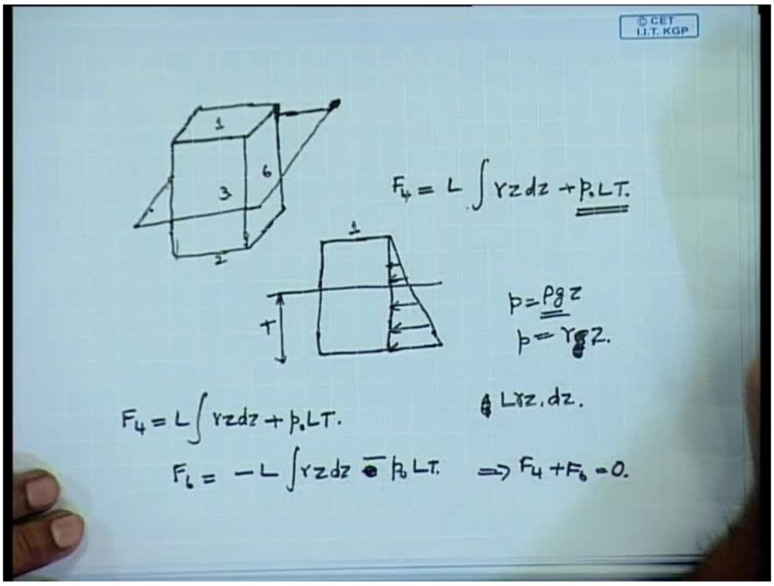
(Refer Slide Time: 10:03)



We now go into what is known as the Archimedes principle. The principle itself states that if a body is immersed in a liquid, it will experience an upward force and the magnitude of that force is equal to the weight of the liquid displaced. When the body is immersed, it displaces some liquid. The volume of the liquid displaced will be equal to the volume of the solid immersed. The part of the body immersed has a particular volume and that volume will be equal to the volume of the liquid displaced. In addition, if rho is the density of that medium; rho into that volume will give you the weight of the liquid displaced and that is the amount of weight that is displaced.

This figure tells that any object wholly or partly immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the object. So, the object displaces weight of the fluid. Here, you have seen this weight is put into the water and it has displaced the 3 pounds of water. So, this is experiencing an upward force of 3 pounds of water. This upward force is known as the buoyancy force and this buoyancy force is equal to the weight of the liquid displaced. As this figure clearly states, there are many derivations for the Archimedes principle. We will take a look at some of them.

(Refer Slide Time: 11:59)



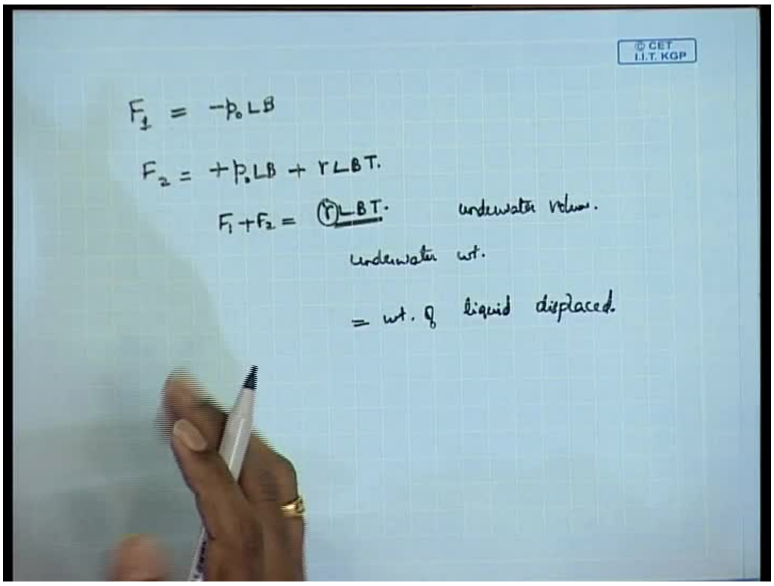
Suppose there is a solid, which looks like this; face 1 is this, face 2 is below; it is not this one, it is the face below, this is 3, this is 4, this is 5, this is 6. So, these are the six faces of this solid. Let us assume that the water plane is here. So, this is the water plane and it is touching the surface 6 and surface 3 at this point. Now, you know that as per the physics, all these faces will experience a pressure due to the liquid, but they will also experience a force.

Now, for example, this is the fourth side; this one, the curve. I mean the face opposite to 6 is 4. So, if you are familiar little bit with the hydrostatics of fluids, you will see that if this is the body here, you have the face 1 and if this is T, then you will have the pressure distribution. Pressure distribution is given by rho gz; this is the pressure at any depth. It is given by rho gz, where z is the distance from the free surface. Now, in this book, rho g is always written as gamma. So, we have rho g as gamma and gamma z will give you pressure at any point.

Now, the pressure gamma z into dz will give you the force per unit length of the side. If you multiplied it with L, you will get the force on the side 4. The pressure distribution will look like this and it will look like a triangle because it is rho gz. As z increases; the values keeps increasing, so the whole picture will look like a triangle between pressure and depth. Here, p 0 L into T, where p 0 is the atmospheric pressure. The side 4 will experience it; it is experienced on the whole area. It is the atmospheric pressure plus the pressure due to the liquid. This is due to the atmospheric pressure and this is due to the liquid. So, F 4 is known and then you need to find out F 6.

If you see, F 6 will be exactly opposite. Now, pressure into area p dA will give you dF. The direction of dA defines the direction of the force. Pressure has no direction and force has this direction. In this case, since the area vectors diverge in the opposite directions, one goes like this. It will be of a different sign from the previous one on F 6 that is minus L gamma z dz plus minus p 0 L T and both will be negative. When you add these two together, it implies F 4 plus F 6 is equal to 0. Both of them are opposite of each other and so when you do F 4 plus F 6, you get 0. It shows that when a body is put into the water, it does not experience any horizontal forces. This is known or otherwise, it would be a kind of free propulsion. Without applying any power, you get free propulsion and it does not happen.

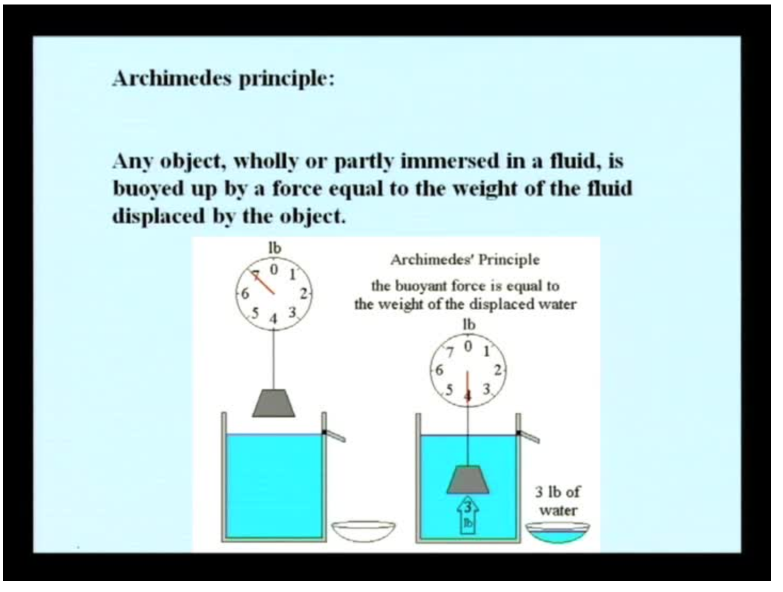
(Refer Slide Time: 17:41)



Now, let us look at the vertical direction. This one is F 1 and as you can see, F 1 will be equal to minus p 0 into L into B L is this and B is this. So, L into B gives the area p 0 L into B and this gives you the force on the side 1, force on the side 2, which is at the bottom. It will be plus p 0 into LB. If you take this dA as negative, then this dA you have to take it as positive.

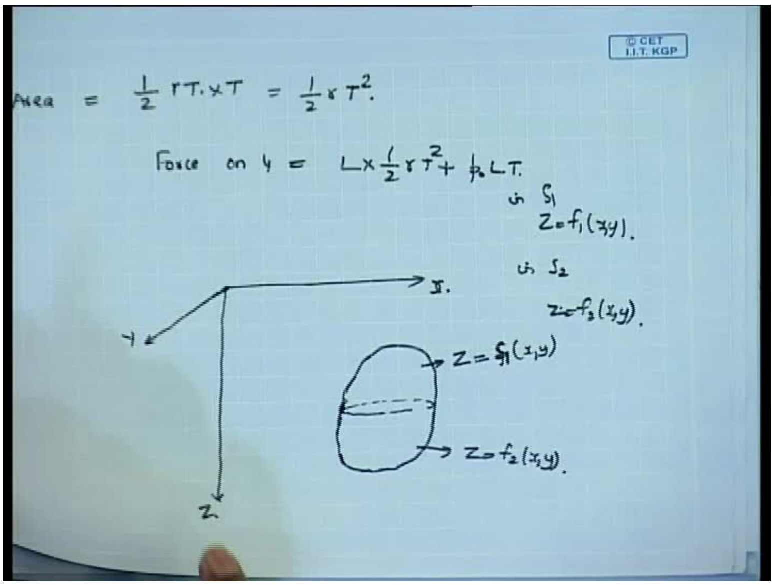
So, it is plus p 0 LB plus gamma into LBT LBT is our volume under water, this is T. Gamma into T will give you the pressure at that depth. It is just gamma z and so gamma into T gives you the pressure at this depth of 2. What is the pressure? It is given by this gamma into T. L into B will give is a multiplication with area. Therefore, you will get the total force. So, F 1 plus F 2 is therefore equal to gamma into LBT. LBT is the underwater volume and multiplied with gamma, which is the density of water. It gives you the underwater weight and what it really means is that it is the weight of the liquid displaced. So, you get that F 1 plus F 2; the net force on the body, which is a net upward force.

(Refer Slide Time: 19:25)



It is directed in the upward direction, pushing the body up. As you can see, it is given by 3 pounds. As you see here, 3 pound weight of the water is directed upwards. It lifts the body up and this is the weight of the liquid displaced. It is equal to the force of the body and this is the Archimedes principle. It can be done in some other ways. In the same figure, for instance that the area under a pressure length curve would give you the force on a body.

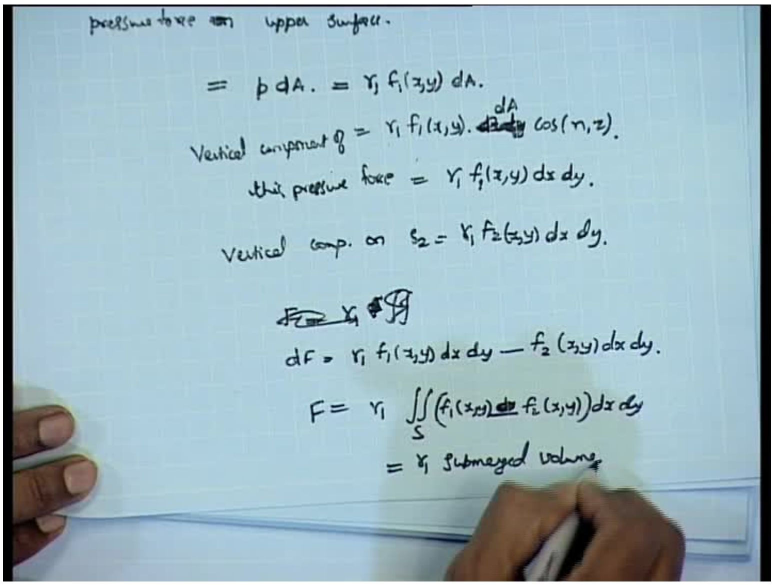
(Refer Slide Time: 20:38)



If you take that half base into altitude and if you consider one side as pressure, one side as the breadth, one side as the length, then you put it on a pressure length curve. The area under the curve will be half base. Base is equal to gamma T, the pressure at the base is gamma T and half base into altitude. Altitude is T and so you will get half gamma T square. This will be the area of the triangle and the force on fourth side. This will be equal to L into half gamma T square plus p 0 LT. It comes from the atmospheric pressure and similarly, it is the same as what we got before. So, this is another way in which you can do it. If you do not like integrals, you can just take it as the area under a curve and do it.

We can look at the general case. Let us assume we have a coordinate system like this – y, z, x. Suppose, you have a body that is immersed in the liquid at the center and let us say that the surface is given by z equal to f 1 of x, y. The bottom is given by z equal to f 2 of x, y and the surface is defined in this fashion. The top half of the surface or top part of the surface is defined as z equal to the upper part of the surface. It is defined as z equal to f 1 of x and y. The bottom half is defined by z equal to f 2 of x into y. Therefore, z equal to f 1 of x into y, which is the top surface in S 1. In S 2, z equal to f 2 of x y, this is the basic definition of the surface.

(Refer Slide Time: 23:32)

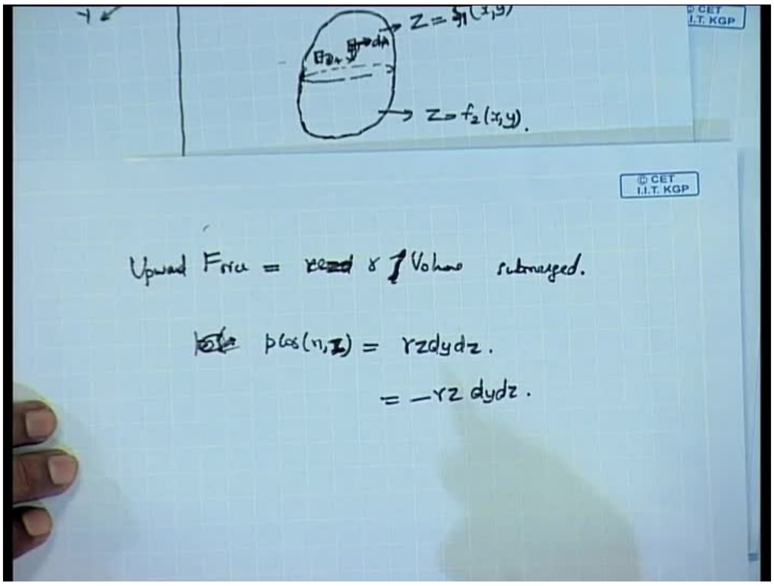


We can see that the pressure force is given on the surface. Let us take the pressure force on the upper surface. Here, p dA equals to gamma f 1 of x y into dA. Now, the vertical component of this force - if you look at this figure, we now consider a small element d, like this. This will be in direction normal to the surface; it is dA and it is directed normal to the surface dA. It will have an angle with the vertical, whatever be that angle. Gamma f 1 of the vertical component of this pressure force is given by gamma f 1 of x y into dx dy into cos of the angle between n and z and so it is a cos of angle between n and z.

This will give you the vertical component of the force. All I have done here is, if n is a normal like this, then the vertical component of the force that is the component of this force in this direction is this force into cos of the angle between them. It will give you the component of the force in the vertical direction and that is what is given here. Vertical component of the force is equal to gamma 1 into dA into cos n z. If you put that, it will be an element dy dx. This is basically the projection of dA cos n z. It is actually the projection of dA on to this plane. It is given by gamma 1 f of this. It becomes dx dy and similarly, the same reasoning we will get from the bottom side of the vertical component of the force.

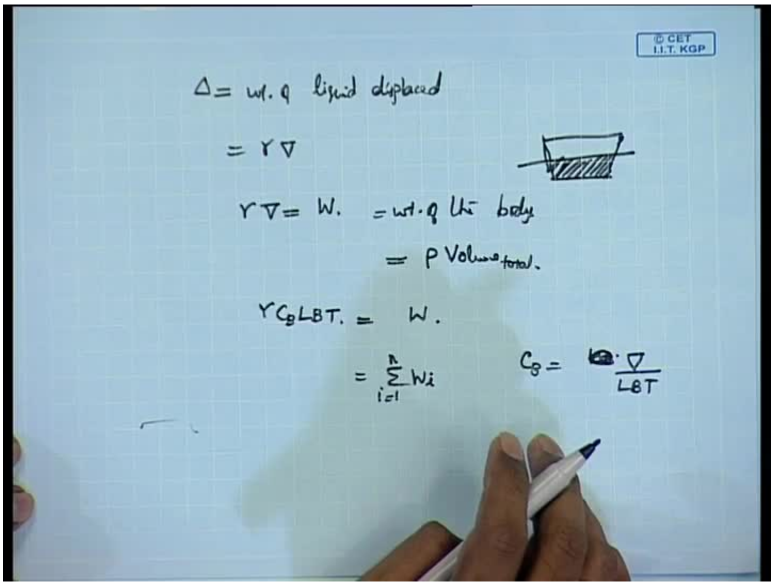
The vertical component on side S 2 will be gamma 1 f 2 and this is f 1 f 2 of x y into dx dy. Therefore, you get the total force F equal to the sum of these two gamma 1 into a double integral or I will write as dF equal to gamma 1 into f 1 of x y dx dy and the top surface. If that direction is taken as positive, the direction of the bottom surface has to be taken in negative as minus f 2 of x y dx dy. So, dF is equal to gamma 1 into double integral F is equal to surface area f 1 of x y minus f 2 of x y dx dy. Now, what is this? This is basically z dx dy. It is equal to the volume of the body and this volume is the submerged volume. So, gamma 1 into submerged volume.

(Refer Slide Time: 27:56)



The force on the body, the upward force or the vertical force is equal to gamma into volume submerged that is double integral over S. So, it is equal to the volume submerged. This gives you the upward force on the body and similarly, if you try to look at the horizontal component, it will be like this. In this figure, you will have to get the horizontal force. You have to take the projection on to this side, y z. It will give you p into cos of the angle between n and x. It will give you gamma into z dy dz and this will give you the horizontal force on one side. On the other side, the horizontal force will be minus gamma z dy dz, which will imply that when you add the two forces together, it will sum up to 0. Therefore, the body does not experience any horizontal movement. It does not experience any horizontal force as it is submerged in the liquid. So, there is no horizontal force, but the vertical force is given is equal to the weight of the liquid displaced.

(Refer Slide Time: 30:30)

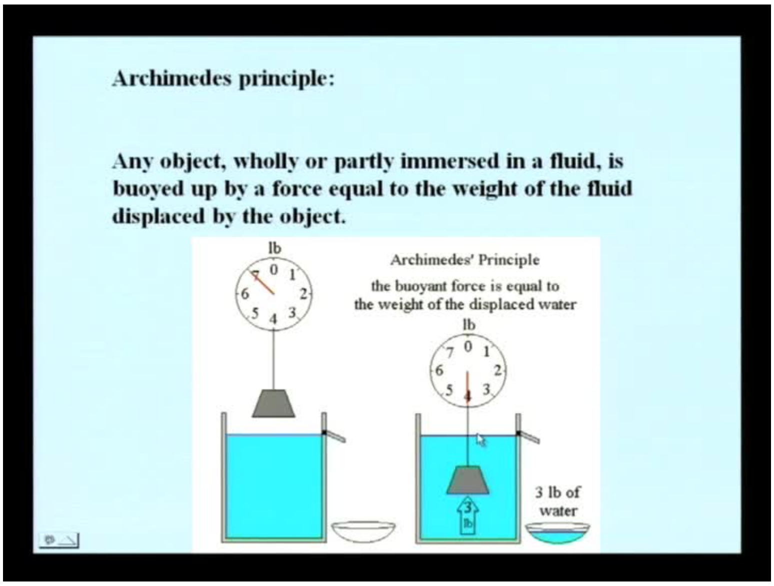


Now, for a body that is freely submerged in water that is for a body that floats on water remains at equilibrium. It is in a stable state without any support, it remains floating on the surface of water. You can give this formula delta, which is equal to the weight of liquid displaced. It is given by gamma into delta, where delta is the underwater volume.

If you have a body like this, it is very important to note that we are talking about the underwater volume and we are not talking about the total volume of the ship. This is the total volume of ship and we are talking about underwater volume of the ship. It is delta into gamma, where gamma is the density of water and not the density of the material of ship or anything. These are the two confusions that come up in many places that is gamma is density. This is actual the weight of the liquid and the displaced volume of liquid is equal to the volume submerged because, what is gone in has to go out as it has no other way.

If x amount or V amount of volume goes into the water, then V amount of liquid will be displaced that is delta. It is multiplied by the density of water, you will get the weight of the liquid displaced. If the body is floating freely, then this gamma into delta is equal to W and that is the weight of the body itself. If density of the material of the body is rho, it is rho into V, which is the total volume of the body. So, rho is the density of the material, it means rho into total volume of the body and it will give you the weight of the body.

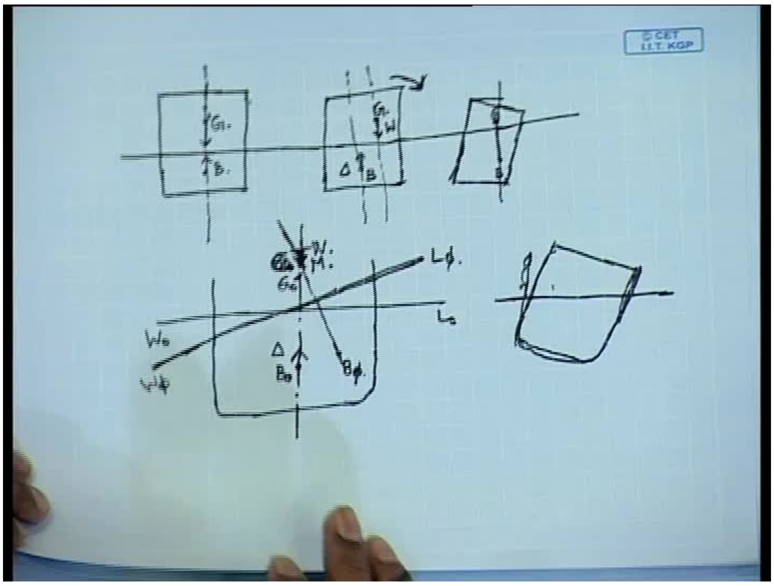
(Refer Slide Time: 32:33)



In this figure, we could say that the weight of the body if In this case, there is a tension acting upwards, there is a weight acting downwards and there is a 3 pound force acting downwards. In this case, I would say that t, the tension plus 3 is equal to w, the weight of the body. So, t is the tension in the rho plus 3, which is the upward force equal to w, which is the weight of the body. In all other cases, you will have delta equal to the weight of the liquid displaced. We can write this in many ways, for example, gamma into C B into LBT, where C B is a block coefficient. I have already told you that C B is equal to delta divided by LBT, so C B into LBT is equal to delta.

Gamma into delta is given by gamma into C B T. This is equal to the total weight on the ship and this is equal to the sum of all the weights on to n W i. It is equal to sum of all the weights in the ship, where W i is the weight of the i th item of ship. There are different items on the ship and that is given here. This is something about the forces. In this subject, we will deal with two things. We will deal with forces and we will deal with moments that occur in ship itself.

((Refer Slide Time: 34:23)



For example, a body is floating like this and the weight of the body acts down at G, which is at the center of gravity of the body. A weight acts down at the center of buoyancy and a weight or a force acts upward. Now, the center of buoyancy is defined as the centroid of the underwater volume. If a body is floating in water, a part of it will be in underwater. Centroid of that underwater part is known as the center of buoyancy. It is a centroid of the volume; the weight is actually the centroid of the underwater volume of the ship. We call that as the center of buoyancy as B and G is the center of gravity of the ship.

Now, the center of gravity of a ship depends upon the weights on the body. It does not have to do anything with water or the upward force from water. It has to do with the weights that are placed on the body. When you place weights at different places, you get the center of gravity. For instance, in this case, you see center of gravity and center of buoyancy in the same straight line. If they are in the same straight line, you can see that they do not produce any moment. So, this system does not feel any moment, but let us say that I removed some weight from here and pushed it here. Some of the weights on the ship on this body is taken from here and put on this side. So, there is a shift in the center of gravity.

You will have the system like this. Now, center of gravity is here and the center of buoyancy has not changed. Center of buoyancy is still at the same place. At this point, a weight delta acts at this point and the weight W acts at that point. The weight W acts downwards and at this point, delta acts upwards. We have the center of gravity and center of buoyancy at different points. They are not on the same straight line. In this line, it acts here and this acts in this line. We have two forces acting in the opposite direction, separated from each other. As you know, they will produce a moment. In this case, there is a tendency to trim like this. It has a tendency to heal; the word is not trim, sorry it is heal. It has a tendency to heal like this and then the body will become like this.

In this case, what will happen is that the body will tilt, such that G and B again comes in a same straight line. The moment it comes in a same straight line, the moment stops and the body becomes stable. In this case, the body remains in inclined position. So, this is what we mean by studying about the moments. There comes a very important concept that is many cases are not studied properly. This should be looked about the water plane of the ship.

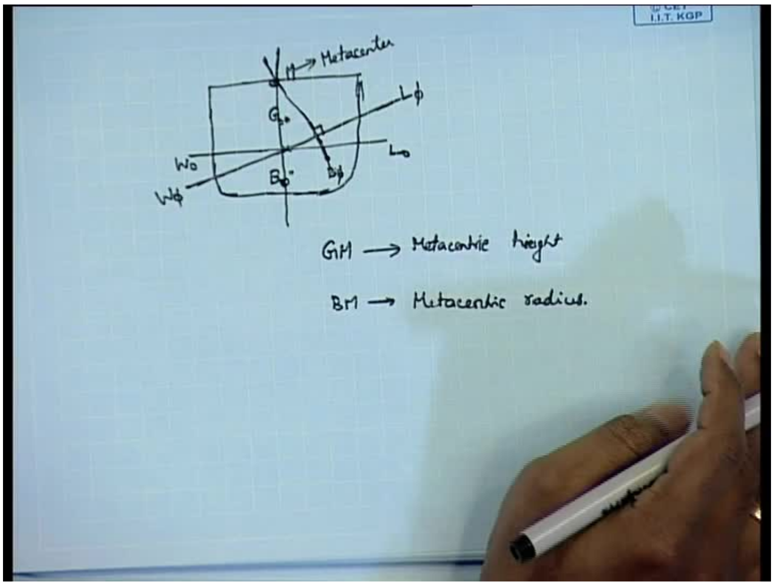
Now, this is the initial water plane of the ship, W 0 L 0. Suppose, the body tilts like this and it is very important that you understand this figure properly. What has happened here? This body was initially in horizontal state. The body has tilted in this direction and therefore, some part of the body has gone at this site and another part of the body has come out at this side and this is still the water line. Instead of trying to draw the ship like this (Refer Slide Time: 39:59) and then tilting it. It is easier to tilt the water line. Initially, the water line is like this, if the body has titled in this fashion and something has gone under here and came out here.

If you draw the water line in this fashion, this is what it is. What this figure shows that the body, which was initially at W 0 L 0, when it was in a horizontal position is tilted in this direction to become w phi l phi like this. We write the water line as tilting and this is done. This is called as W phi L phi and this is the new water line. This figure indicates that this body is not in a straight position, but it is in an inclined state. There are some other formulas that need to be satisfied. In case of floating bodies that is for the body float upright. It is just not enough that the body just floats, it is also important that the body or the ship float upright. It does not heal or trim too much, it floats upright.

There are some other formulas that needs to be satisfied and that is what we are going to do next. Initially, we have G here and this is known as G 0. At this point, you have B 0.

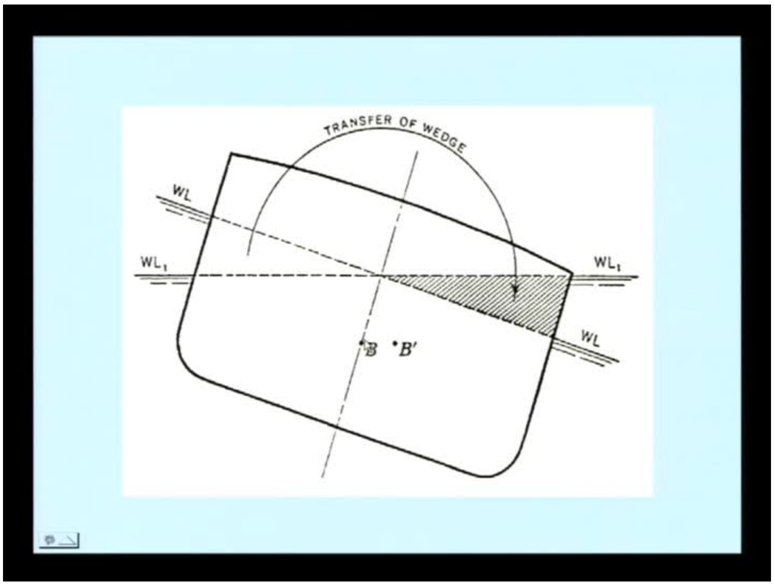
Here, you have W - the weight of the ship acting. You have delta - the ship the buoyancy force acting upwards. The body is tilted and B 0 shifted to this point B phi. It is the new point, this line gives the vertical at that point and here, it hits the vertical. Let us say that G 0 is here and B phi goes up and hits the vertical. Here, this point of intersection is known as M. Since the figure is a little confusing, let me do it again.

(Refer Slide Time: 42:47)



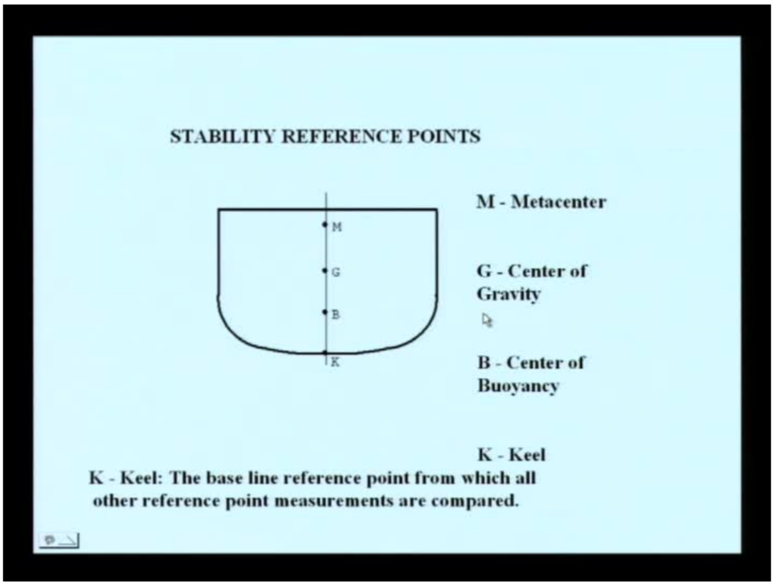
This is the body and the initial water line is here as W 0 L 0. Here, you had G 0 and here you had B 0. Something shifted and as a result, the body had to tilt this much. This is the new water line W phi L phi and it has shifted here as B phi because of this, some wedge has gone out here and some wedge is coming here. Therefore, B will move in the direction of increased volume. So, B 0 to B phi and here, if you draw a vertical, this hits the original vertical. Remember, this line is the vertical, because this water line is always vertical W phi L phi. It is the horizontal line and not vertical. W phi L phi is a horizontal line, so perpendicular to that is always the vertical line. So, B phi into M is a line and this M is known as the metacenter. A very important concept in naval architecture is known as metacenter point M. There are a couple of… before I go into that we should…

(Refer Slide Time: 44:34)



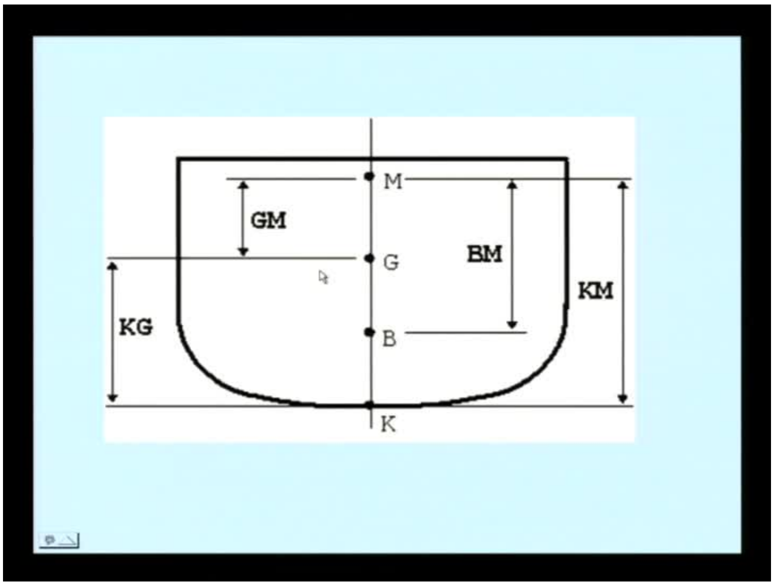
This is how it will look. In this case, the ship has tilted in this direction. Some part has gone in here. In this case, it is different; the initial is WL 1. When the ship was stationary, the ship was horizontal. When it tilted, the water line came to WL, like this and B shifted from here to here. Initially, you have B and this is B prime. So, there is a transfer of wedge to one side and this is the figure.

(Refer Slide Time: 45:13)



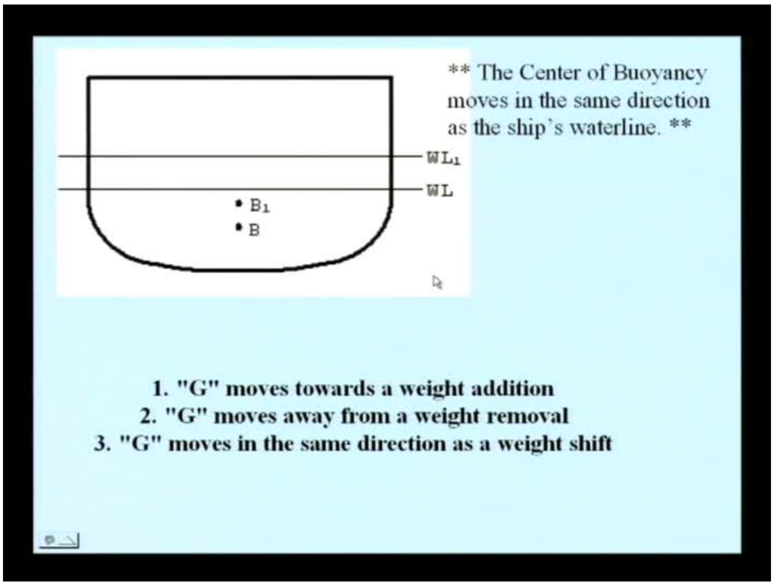
When you look at different points, these are the different points that you need to know. In this case, for example, M is a metacenter, it will be at the top mostly. G is the center of gravity coming below it. B is the center of buoyancy coming here, then K is the keel. Keel is the bottom most hull part of the ship. It is known as the keel and k is the basic reference point, from which all other points are measured. So, K is defined to be z equal 0 and all these have particular values. You have KB, KG, KM, GM and BM. We have different things and we call differently. For example (Refer Slide Time: 46:07), there are two things: one is called as GM - this is known as metacentric height and then BM, which is known as metacentric radius.

(Refer Slide Time: 46:41)



Here, you see that BM is known as a metacentric radius and GM is the metacentric height. You have KM that is another parameter.

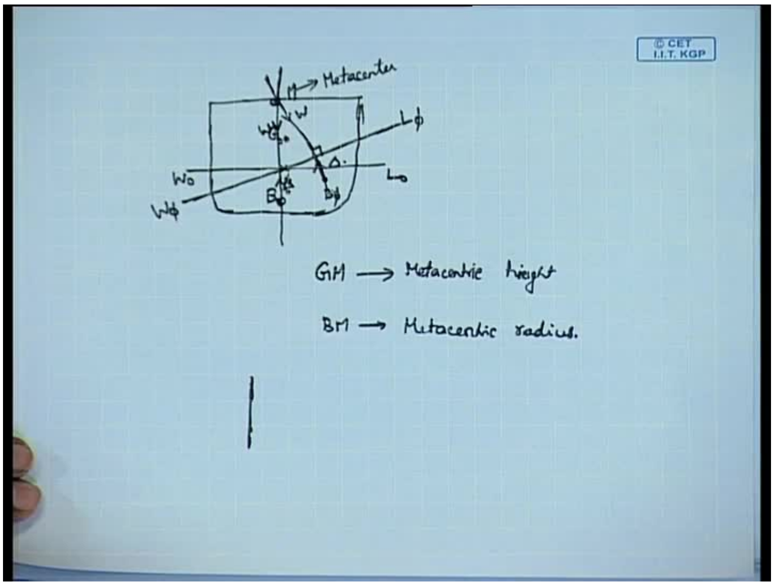
(Refer Slide Time: 46:57)



Now, couple of small points here, first of all, initially the center of buoyance water line was WL. Suppose, WL was the initial water line, B is the centroid of the underwater volume. I have already told you, it is the centroid of the underwater volume. So, you have B. Now, due to some reason, the ship sunk a bit more, such that the water line has raised above and it has now gone to WL 1. If that happens, then the center of buoyancy B 1 as the water line goes up. So that is how the ships center of buoyancy behaves.

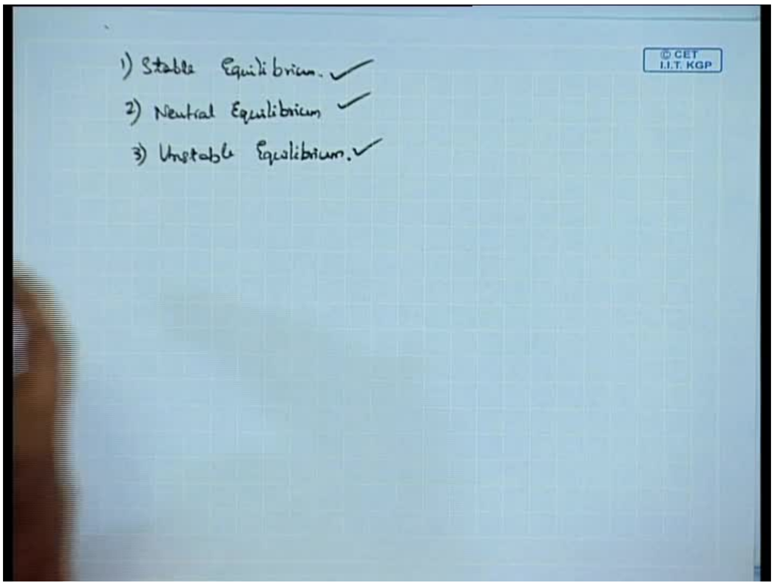
The next one is about G. In case you add weights, G shifts towards that point. So, whichever point, you add some weight, G shifts towards that point. If you remove weight from some place, G moves away from that point. So, G moves away from weight removal, G moves towards weight addition and G moves in the same direction as the weight shift. So, if there is a shift of weight, body will move in that direction.

(Refer Slide Time: 48:19)



In this figure, we have seen this M is above G and that is the condition possible. Now, another condition is also possible. As you can see, when it reaches this condition; here, you have a delta weight – W. You have this weight delta and displacement delta and you can see that. In this case, there is a tendency for the body to shift back to its original position of equilibrium. Before we do that we have to define what equilibrium is.

(Refer Slide Time: 49:06)



In general, we say that there are three types of equilibrium. We call them as stable equilibrium, neutral equilibrium and unstable equilibrium. The meaning of these terms are: a body is said to be in stable equilibrium, if it is in some position and some force acted on it or a moment acted on it to change its position or its orientation, the moment the force is removed, the body returns back to its original position. That kind of equilibrium is known as a stable equilibrium.

Now, on the other hand, if a body is in some position and suppose a force acts on. It continues as long as and it tilts, when the force is removed, the body does not come back to its original position, but remains as such without changing its position in a tilted position or in some other form. It is mainly in the tilted form with the angle of heal. If that happens, then it is said to be in neutral equilibrium.

The third one is unstable equilibrium. If the body is in a particular position and if a force acts on it or a moment acts on it and it tilts. If the body does not return to its original position, but it continues in its original tilting and increases the tilting, such that it capsizes. Such a situation is known as an unstable equilibrium. These are three forms of equilibrium: stable, neutral and unstable equilibrium. Unstable equilibrium is - once the force is removed also, it capsizes. In stable equilibrium - the body returns to its original position, after the removal of the force or moment. In neutral, it remains at the same position. So, these are some forms of equilibrium.

Now, we need to figure out, how this applies to the case of ships. I mean, how this stability parameterization that is the next step. We will continue in the next lecture. So, today, we will stop here. Thank you.