Hydrostatics and Stability

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Lecture No. # 19
Dynamical Stability – III

As I told you, this section is not included for the exam - all these sections. So, we will go into another type of heeling moment now.

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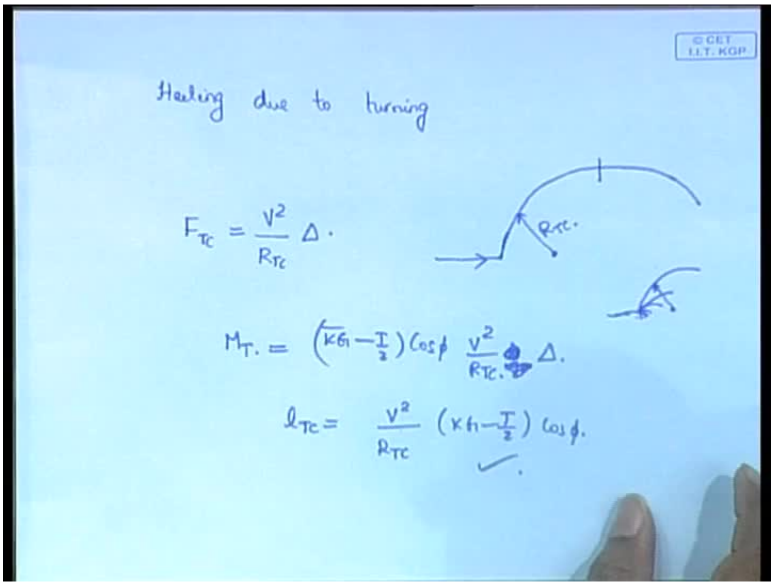

We were talking about different types of heeling. First, we talked about heeling due to grain. Of course, there is a lot of heeling. First, we talked about heeling due to the shift - horizontal shift - of a weight, then we talked about heeling due to a free-surface effect, and then heeling due to hanging load, and then we came into heeling due to a grain, and then heeling due to wind. We have done so much. Now, there is one more thing left, that is, heeling due to turning.

Now, what this suggests is that, we have a ship, initially, that is moving in a straight line. Now, if the ship is made to turn like this, it is asked to turn like this - what will happen to the ship, means, in terms of heeling? Of course, the ship will turn. Now, we have to see if there will be any effect of the turning on the heeling. So, all that is happening is the ship is moving straight, then, let us assume, suppose, that the ship has to go there, that means, the ship has to turn; so, it has to go in an arc. So, once that happens, will the ship heel? That is the question and in which direction? That also we will see.

Now, let us assume that the ship is heeling, or the ship is turning through a radius - this radius is R; TC stands for turning circle - radius of the turning circle; so, this is the turning circle. So, the ship is turning like this, up to this; now, this is R TC. Now, as you can imagine, when the ship turns or when the ship starts going in a radius, obviously, on it, a centrifugal force will start to act; that centrifugal force is given by F - we will call it TC. It is a centrifugal force caused due to the turning, is given by: first of all the acceleration - acceleration is V square by R. Let us assume that the ship is turning with the velocity V; therefore, V square by R TC will give you the acceleration, into delta of the ship, will give you the force due to the centrifugal force, which is the force that is acting away from. So, if the ship is turning like this, at the center, we assume that the centrifugal force acts at the center of gravity of the ship. At the center of gravity of the ship, there is a force that acts, towards, away from the circle.

If the circle is like this, the ship is turning like this, let us say, this is the center of the circle; so, away from the center, like this, a force acts - a centrifugal force, like this, it acts; so, V square by R TC act, into - this is the force acting.

Now, similarly, you can imagine, this force will try to push the ship away, means, it will try to displace the ship, but water resists it, means, water does not allow the ship to move means. Of course, ship moves like this, but like this. This force - the centrifugal force is trying to push it like this, but water prevents it; water stops it from moving in that direction. So, there is a reaction force from the water. That force, most likely, becomes equal to the centrifugal force, such that, the force is balanced; so, the ship does not move like this; laterally, it does not move or in transverse direction it does not move.

Actually, there are names for each of these motions. This is called heave, when a ship moves like this, you call it heaving; this is called surge. I believe this is sway, this is called sway; so, the ship tries to sway, but it prevents it from swaying - the water prevents it from swaying and as a result, that force is the same as this force, because force is balanced, such that the ship is not moving in this direction. Ship is moving longitudinally, of course, in the straight direction or in the radius it is moving, but it is not pushed like this, this movement is prevented. And, this force from the water will act at a distance of, means, where will the force, it will be uniformly distributed from the bottom of the ship to the point where the water ends or the water starts rather. So, between the air-water interface, from that point to the bottom of the ship, that is the distance over which the water force will be acting.

So, this water force can be assumed to act at that centroid, that is, at the middle of the draft; it is at T by 2. Therefore, there is a force, there are two forces now - one force acting at KG, that is, at the center of gravity, which is the centrifugal force, it is like this. And there is a force acting from the water at T by 2, like this. Therefore, there is a force like this and a force like this separated by a distance of KG minus T by 2; KG is this distance, this is T by 2. KG minus T by 2 is the distance between these two forces. So, this force is actually causing it to turn, tilt like this.

So, it might be slightly against your intuition. When the ship tries to turn like this, it will actually tilt like this; it will not tilt like this, means, in a bicycle and all, we think it will tilt like this; it is not like that for a ship. For the other thing, it is slightly different the phenomenon is slightly different, we will not go into that.

For the ship, one force acts like this here and one force acts like this here; so, it will cause it to tilt like this. So, when a ship turns, it will actually be found to tilt like this, heel like this; tilt means heel. So, the ship will go around heeling like this; this is the phenomenon of turning. Now, these things we can easily calculate, that is… So, first of all, this is the force, and what is the moment is the, the distance between the moment will be the force into the distance between the two forces - this force and that force; that is, KG minus T by 2. And, if it heels, the distance between them becomes KG minus T by 2 cos phi. I am not going to draw the figure. It is exactly the same as the wind heeling. If you remember that, it became h v plus T by 2, last time h v plus T by 2 cos phi - I derived one. Just like that, in this case, it is KG minus T by 2 cos phi.

So, the moment becomes KG minus T by 2 cos phi into V squared by R TC, this gives you the, into delta, this gives you the turning moment. And, when you divide by delta means, moment divided by delta, it will give you the turning arm. This is the heeling arm; so this will be with delta. Then, l turning, what is this? Let us call it TC itself, like what they have done, will be V square by R TC into KG minus T by 2 cos phi.

KG minus T by 2 is negative; means, let us see. G is, of course, we can always put KG very low, because you can shift all - yeah - it can be negative; so that will mean in… that is the very particular case, it is possible, KG, its possible, it can be negative. In general, it is not, but it can be negative, yes. When you make it negative, then KG is below, the directions reverse and the ship will tilt inwards. But, note that in general KG is greater than T by 2. KG is for the whole ship, in fact and in fact, most of the weight of the ship is up, in the top region, and that is where you are putting the cargo and all that. So, in general, 95 percent of the cases, KG will be… I do not know if there are some cases where KG can becomes less than T by 2; some particular type of small boats, probably very small boats, may be it can happen; but in general, it is like this.

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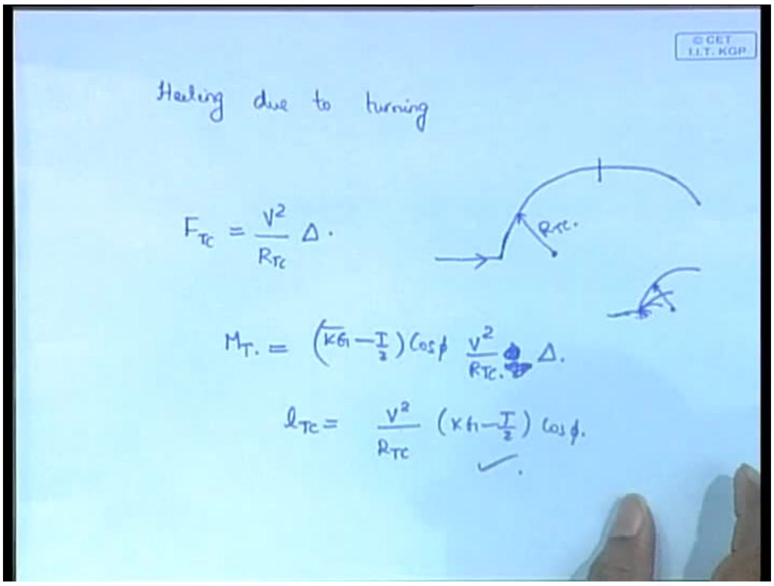

This is an important point. This is another question they generally ask, that is, when a ship turns or takes a circle, will it heel inwards or outwards? You will generally think, like a bicycle it should turn inwards. But, it does not heel inwards, it heels outwards. It is because of this, one moment like this, one moment like this, it causes it to turn like this; unless this particular case occurs, what you said.

If the ship be… stable? It is again like this. The stability again we have to… Stable means - are you asking will it capsize? That depends on the moment, heeling moment. It depends on how much it is heeling and how much it heels depends on what is the turning circle, KG minus T by 2 cos phi. As you can see, there is V square by R TC, it mostly depends on R TC. For example, suppose you make R very small. Let me see, how can you make R very small? This is a large R. If I make R smaller, it means like this. It is like this, means, if a ship is like this and suppose it tries to turn like this, that is the meaning of very small R; then, it will be unstable. But if it is a large circle, it does not matter; that is why you cannot turn a ship like this, it will capsize.

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There should be a… you are thinking something like a critical radius where it will be… Yeah, there should be. For any and every ship, there will be a critical radius below which the ship will capsize. If you try to turn beyond that, it will topple, that you can imagine. Even if it is a simple thing as a car, if you suddenly turn it like this, there is a very good chance that it will topple. Just like that it is there for ship also, yes; it depends upon R and V also. It depends on V also. If you increase the V, that moment will increase, it means, that a very large speed and a small radius if you make, it will topple. So, the best thing is smaller velocity and higher radius - it will be stable.

Now, there are some relations… This just you have to know, but of course, for the end semester. ((No audio)) No, not like a car, because first of all the medium is water. Water, it does not allow you to turn as fast as air. Air, the resistance is very less, so you can turn fast. Water, the resistance is much higher, 1000 times, in fact. So, it cannot turn like that, but if you try to turn very rapidly and in a very small radius, that ship will topple. You can do things like, you keep accelerating the ship, you know, you take it to its maximum speed and then you try to turn in a small circle, with a small radius, then it will topple, and that is what we are saying.

Then, next thing is we have to… We are just touching on dynamic stability from some slightly different point of view; it is not different, but let us look at this. We have already seen that by dynamical stability, we mean the work done under the GZ curve. It is the work done by, it is the work done by, the ship in opposing the heeling moment, that is the dynamical stability. So, if some heeling moment acts and it is trying to turn it, the ship tries to resist it, it does work there; that is known as the dynamic stability, we have seen that.

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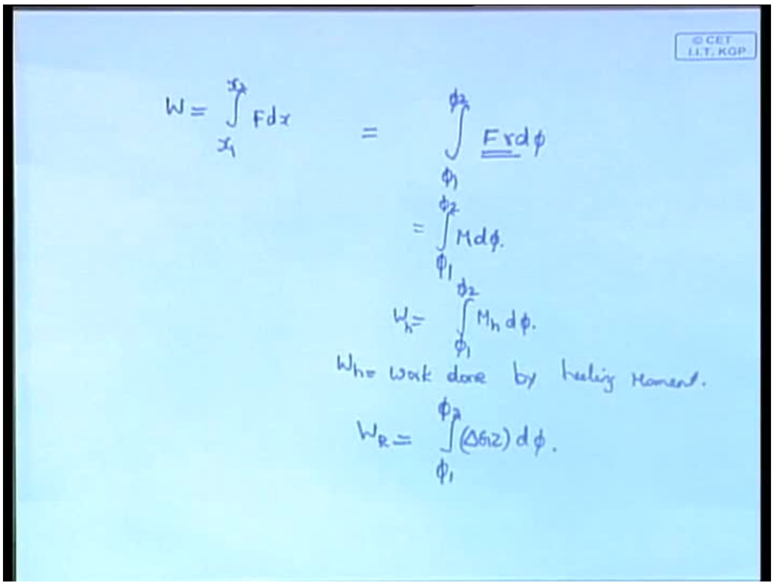

Now, we can derive it simply. First of all, you know that work done is always given by: when some force acts between two distances x 1 and x 2, the work done is usually given by x 1 to x 2 F dx - this is straight forward. Now, suppose that this dx is an arc, that means, this can be written as… So, it means, instead of moving in a straight distance, it is in an arc; so, this will be… if, I write it as F into r d phi and it goes between phi 1 and phi 2; all I have done is… It is going in; the arc distance is always given by r d phi. So, this is what we are having here, means, the ship, in this case, we are not talking about the, in the case of dynamical stability, we are not talking about the work done in moving like this; we are talking about the work done in turning, means, turning and going in an arc.

So, it is moving in phi; so, if some distance, most likely the distance between the G and the that arc is known, r d phi will give you the distance that it is moving. It is not a horizontal distance; it is a it is a it is a curve distance - that distance, it moves. So, that is what this shows; from phi 1 to phi 2 if it rotates, it moves a distance of F, is some force, into r d phi. Now, therefore, we write F like this, I mean, W is given like this; this F into r, we combine together to write it as M; therefore, this can be written as M d phi.

Now, so, now, What are we saying? Suppose, we give a heeling moment to the ship; therefore, the work done by the heeling moment can be given as moment due to the heeling or the heeling moment into d phi. This will give you M h into d phi; this will give you the work done by the heeling moment. Now, similarly, the work done by the righting moment will be. This is the work done by the heeling moment - W h, is the work done by the heeling moment.

Now, we have one more thing, we have the work done by the righting moment W R, it is given by… Now, the righting moment, you know that it is always given by displacement into righting arm; righting arm is GZ; displacement is delta; so, delta GZ gives you the righting moment. So, right, that into d phi gives you the righting work done by the righting arm or the righting moment.

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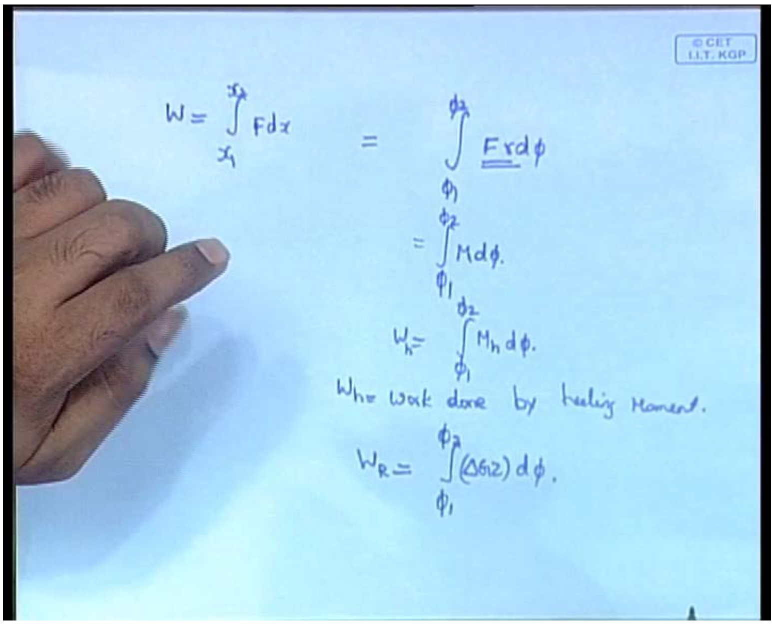

Now, suppose that ship is heeling and this righting moment is act; now, this ship will obviously come to rest at a state, when the work done by the heeling moment and the work done by the righting moments are same - are equal, identical. So, what happens is, at that stage, M h into d phi equals phi 1 to phi 2 delta GZ into d phi.

Now, force, actually, one thing I missed here - delta is actually mass; so, mass into distance would not give you the moment. Mass into G has to be multiplied, so there should be a G here; so, G is to be multiplied here so, it is delta. Delta is, remember, the mass of the ship, it is not the weight of the ship as such, it is the mass of the ship; so, delta into G will give you, means, delta is usually given in tons; as you know, tons is 1000 kilograms; so, kilogram is mass; so, delta into G; so, that G should be there. So, there is a delta into G here; therefore, phi 1 to phi 2 M h by G delta, I am just, because delta is a constant, G is a constant, means, they are not variables; delta is the total displacement of the ship that does not change due to heeling; phi 1 to phi 2, here, you have GZ into, or, GZ into d phi .

Now, this I have said, still I will repeat this. It is like saying this, what we have done so far is, suppose, this your GZ curve and here you have your l v. Let us say, this is your heeling moment curve, heeling arm; this is your GZ. Now, the first one, M h d phi represents up to this, let us take, let us take this. Now, between these two angles of phi 1 to phi 2, some phi 1 some phi 2; in this case, we assume phi 1 to be 0, and in this case, the first one, the first integral represents the area under the…Let us call this A 1, A 2 and A 3.

So, what is the first integral? It means, the integral on the left side, integral phi 1 to phi 2 M h d phi represents A 1 plus A 3, means, it is the area under the l v curve or under the moment curve; heeling moment is this curve; this curve represents the heeling moment or the heeling arm; so, area under that curve is A 1 plus A 3 and that, is represented by this integral. And, the area under the GZ curve is the area under this curve, it is A 2 plus A 3; therefore, what we have done is literally A 1 plus A 3 is equal to A 2 plus A 3, or rather A 1 is equal to A 2; this is the condition that we are doing; this area is equal to this area.

We have done this in the derivation of the wind heeling arm also, that is what we did. Therefore, at this angle, is usually called as phi dynamic, that is, the angle… Remember, we did this in the wind heeling arm also, that is, when the what is happening? What this figure shows or explains is this - initially, the ship is in an upright condition phi 1 is equal to 0, wind, some heeling moment; in this case, it can be wind or it can be turning, whatever it is, that heeling moment starts acting. Now, in the initial stages, the heeling moment is greater than the righting moment; the moment it starts heeling, the righting moment starts acting; because the righting moment always comes the moment it starts heeling; it has a tendency to come back; it is like the resistance to that heeling moment. So, heeling moment is greater than the righting moment and it starts, or it keeps going and it reaches a state, when finally, the heeling moment equals the righting moment - that is this stage.

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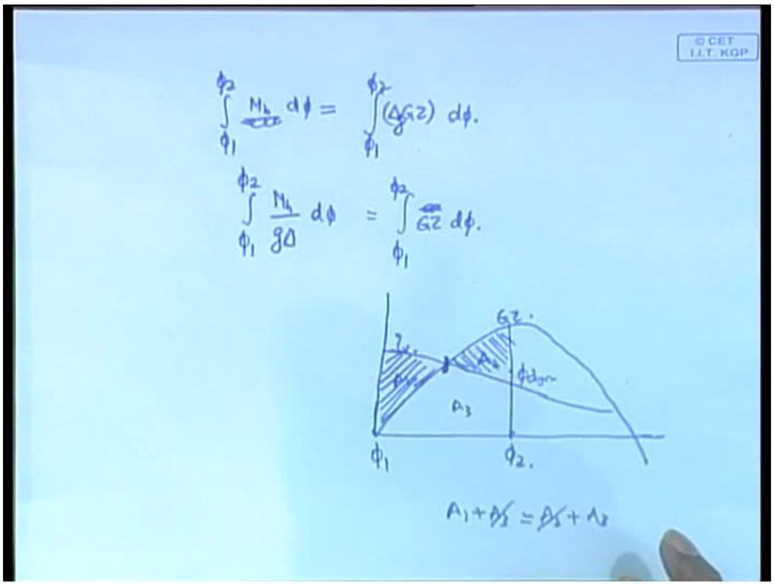

It is some kind of equilibrium where the heeling moment equals righting moment, but you have to notice one thing - in this process, from the upright condition to this moment, the wind or the turning has imparted some energy from into the ship, that is because there is a difference in area between the…so, this area. First of all, some work is done by the heeling moment on the ship, that is, the area under the heeling arm curve; then, work is done in resisting it; that excess work done by the heeling on the ship, is given to the ship; it is like an energy given to the ship. So, even though it has come to this dynamic equilibrium, in this static equilibrium case, it has an excess amount of energy; this ship now, has an excess amount of energy; that energy causes it to heel further. Even though, that is why, even though the forces are balanced, the energy is still more for the ship; ship has some excess energy therefore that ship continues to heel further. And, the moment it crosses this static equilibrium case, the resistance is more than the… still, that wind moment is acting, but the resistance is now more than the wind heeling moment, but energy is there causing it to heel further. And, till that energy dissipates, till that excess energy dissipates, it will continue to heel.

Therefore, though this is some kind of equilibrium, it will continue to heel till it reaches phi dynamic. So, we can say that phi dynamic is like a possible state when it will definitely stop; after that, the ship will not heel further definitely. And, therefore, this is the condition, this is the concept of dynamic stability.

Now, here, actually, they have done something which I described below, because someone asked me at that time - what is the really a worst condition? They are talking about this condition here, that is, they are showing here, though it is not really explained properly. It is like this, that is, the concept is this, that is, you have a pendulum hanging like this; so, there are three possibilities: pendulum hanging like this, a push is given to it, that means, that heeling moment is given to it or I am giving some energy to it, it moves like this; then it is in a condition like this, it has some, then I am giving a push to it - that is one possibility; then it is like this and I am giving a push like this - that is the third possibility. In which of these conditions will it go to a maximum? That is the question.

Though you might think that, like this, it is already in some heeled condition, would it not go any further? It is not correct; that is not where it will heel further. If it is like this, what will happen is that - if it is like this - when some energy is given, it already has a potential energy, because it is not… In this case, let us take this case - pendulum is exactly vertical, it is upright; ship is in the upright condition. In this case, it has literally zero potential energy, so whatever energy you are giving, causes it to move like this. Then, what they are saying is that, if it is like this, that potential energy added will cause it to move further; that additional potential energy adds to the energy already available to move further; whereas, in this case, it has less potential energy which causes that, it has like negative potential energy which removes from the energy added. So what the… Though it is not exactly explained, the meaning, the thing is - a ship, let us say, you can think, that is, when you have a pendulum in these three conditions: when you take it like this and you give it a push, it will go further - than - if it is like this and you give the same push - than - if it is like this and if you give the same push. That is the law and that is what will happen. Actually, think of it in terms of energy; you will figure it out.

It is the same thing for a ship. If the ship is like this and you give a push, it will, the inertia or that force due its motion itself, will cause it to move further, than it would if it is in the condition - in the upright condition; it is like this, some kind of additional force comes to it; some energy is given by the wind itself, but when it is turning, an additional force comes to it which causes it to move further; so, that is the condition though it is not given or explained any further than this.

Okay, I do not think anything more is explained in it than this - it says that the energy transfer from the push which is the energy given by the moment is added to the potential energy accumulated by the swing, that means, while coming from here, some potential energy is accumulated by the swing while it comes like this because of that, that will add to the energy and it will cause it to go.

When it is like this, it is like instead of potential energy accumulated, it is like potential energy lost from the swing; that much energy will be lost from the energy given by the push itself; so, this actually goes completely against your intuition, but that is what happens by physics.

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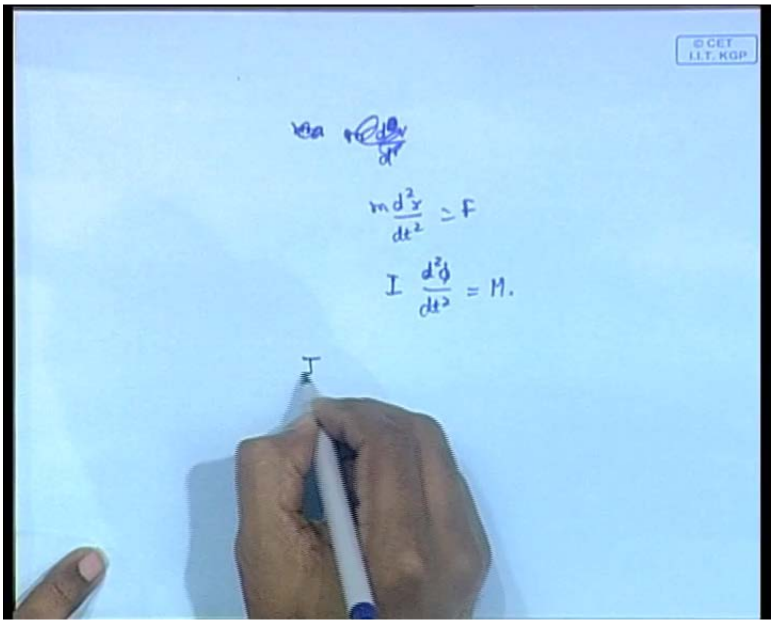

Alright, then we will go into a little bit about equations of motion. So, it is like this, that is, first of all, we write…You know the equation of motion is, first is, ma is equal to or m into d square, no m into dv by dt. I will write this: m into d square x by dt square is equal to net force; this is the first Newton's law of motion. Similarly, if you consider the rotational case, it will become I into I omega square, that is, half I omega square is known as the rotational energy. So, it is like this, the equation becomes I into d square phi by dt square which will become the moment; this is the Newton's equation in the rotational case. So, I into d square phi by dt square, d phi by dt is omega so, d square phi dt square is your omega dot; so, I omega dot is equal to the moment or the external torque acting. Instead of the force you have the torque, instead of mass you have I and instead of d square x by dt square, you have the phi. This is your rotational condition and that same equation we apply here.

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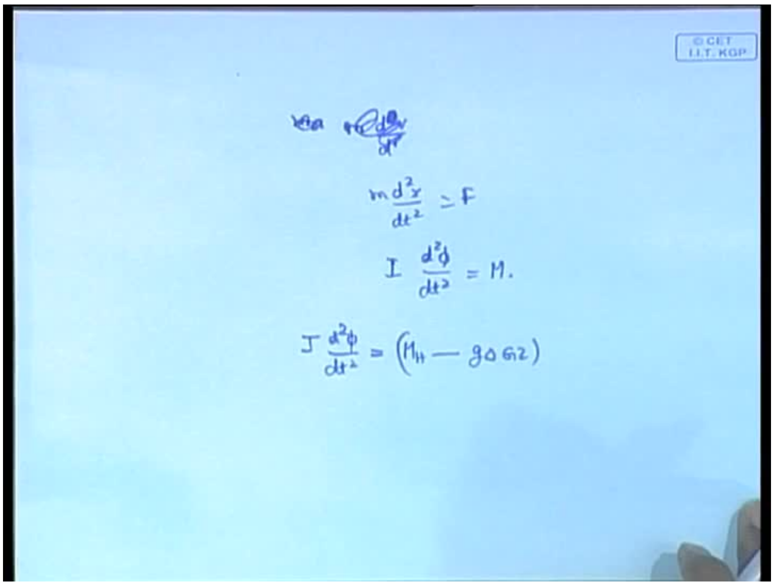

Now, the right hand side indicates the net moment which is the heeling moment minus the resisting moment. The resisting moment is the righting moment which is G delta into GZ; that is the resisting moment. The other one is the heeling moment, so, the heeling moment minus that will give you the net... This is the case when the first part of the problem means, when the heeling moment, in this case, we represented this region, where the heeling moment is greater than the righting moment - this is bigger than this, that is what this figure is. From here, this becomes bigger than this, that is the different part, that we are not bothered about right now.

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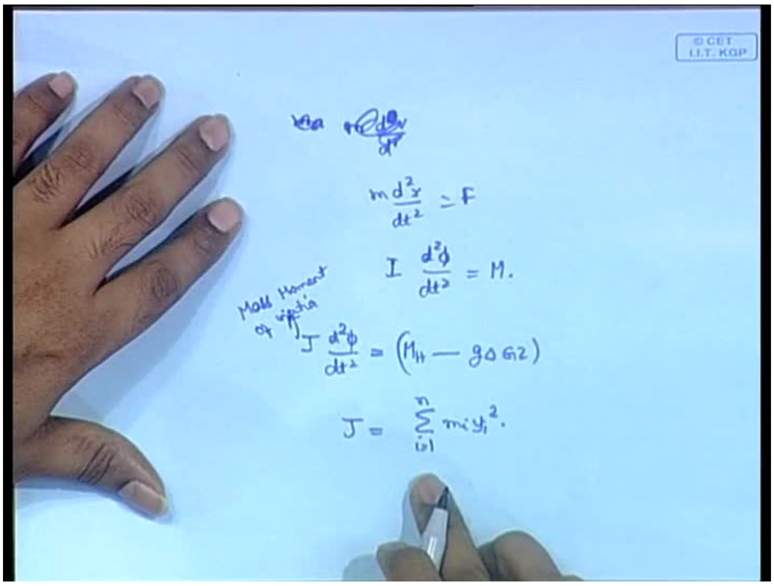

We are talking about this case where the heeling moment is greater, so the net moment is this minus this; that will give you the net moment. So, the heeling moment minus the righting moment will give you, J is still the same thing, our I… But slight difference is that, till now, we have talked about area moment of inertia in our earlier things; this is the mass moment of inertia. Like ma is the act force, like that, I related with M is the mass moment of inertia. Therefore, this becomes… J is actually defined as m y square; this is known as the mass moment of inertia and so, this is J. Now, once, you have that so, this equation, this we rewrite it as J d square phi by dt square plus g delta GZ is equal to M H; this is the equation.

We will do one thing; we will add a d phi to all the terms. Now, this J factor, let us take this component that comes with J, d square phi by dt square d phi , this term, we write it as… Now, d square phi by dt square can be written as d phi dot by dt, where phi dot is d phi by dt. So, d phi by dt, d by dt of that is known as d square phi by dt square; so, d phi dot by dt into d phi, which is equal to, which I still write it as d phi dot by d phi into d phi by dt into d phi. Now, I remove these two d phi; this becomes d phi dot into phi dot. If you have some doubt with this, you can ask me - any of these, one of one, going to the other, if there is any doubt in that.

You have d square phi by dt square d phi; now, this is d by dt of d phi by dt. Now, d phi by dt I write it as phi dot which is again d phi by dt only; so, d phi dot by dt into d phi.

Now, here, I just divide by d phi and multiply by d phi, so this becomes d phi dot by d phi into d phi into d phi by dt into d phi. Now, d phi d phi is cancelled; therefore, d phi dot into d phi by dt is phi dot - that is the definition of phi dot; you are defining it like that; so phi dot into d phi dot - this we are slightly rewriting it as this. So, the equation becomes J into phi dot d phi dot plus g delta into GZ d phi is equal to M H d phi.

Now, we will integrate this expression between two values of phi 1 and phi 2, or phi 0 and phi final; phi 0 means of phi 1, phi final is... So, J into integral of phi 0 to phi f phi dot d phi dot plus, g delta GZ, G delta into integral phi dot to phi f GZ d phi equals integral of M H d phi between phi 0 and phi f.

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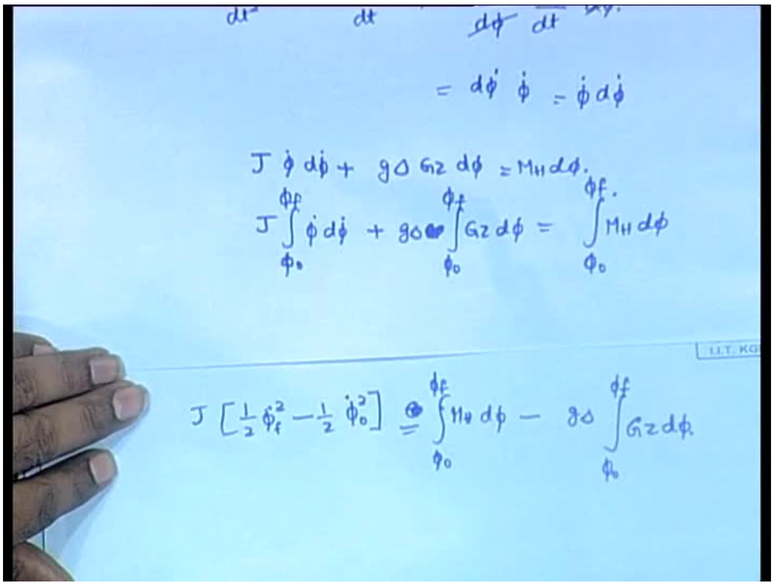

Now, I will just take this. Now, this I will become, this itself - what does it become? J into half phi dot square final minus half phi 0 dot square plus or that is equal to M H d phi phi 0 to phi f minus g delta phi 0 to phi f GZ d phi. Is there any problem with any of the steps? You see how we have got this know, means, integral of something, integral of x dx is x square by 2. Integral of x dx is x square by 2 so, in this case, x is phi dot, so phi dot squared by 2, so phi dot square final minus phi dot square initial by 2.

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No, phi dot is not initial, but it is phi dot, I should not have written like this, that is correct; that is a good point - left side, I will write again.

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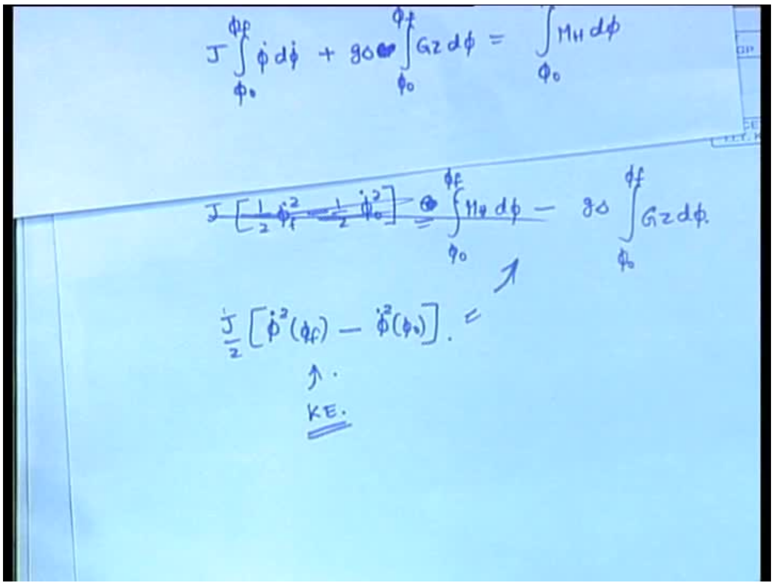

J into J by 2 into phi dot squared at phi f. Maybe, I will write like this, minus phi dot squared at phi 0. This is the correct way, so this is equal to this right hand side. Now, what is this left hand side? Half, it is actually, J, remember is I - your moment of inertia, so, half J, what is phi dot? Phi dot is omega so, half J omega square is actually the rotational kinetic energy of the system. So, you have two components here - there is a rotational kinetic energy and there is a potential energy. So, left side represents your kinetic energy, of course it is rotational, it is the kinetic energy of the system - this thing.

Now, in the case of, So, there is a total energy of the system which will be a sum total of its kinetic energy, rotational kinetic energy, there is no translational kinetic energy here. Of course, ship is moving, that is a different thing; we are not bothered about that. Let us just consider the sum total of this rotational kinetic energy and because of its tilting, there is change in its potential energy; because g will, because of movement, there will be a change in, let us see - why exactly should the potential energy shift? Potential energy changes; yeah, g is shifting vertically because it is heeling. Because it is healing, some masses are coming down; vertically it is moving. There is a movement vertically of the whole mass; therefore, g is shifting and therefore, there is a change in potential energy because the g is moving know.

Total mass is the same but g is moving, as the result of which there is a change in potential energy. So, the potential energy plus rotational kinetic energy represents the total energy of the system. Now, in a stable condition, your potential energy will be minimum, that is the basic law; your potential energy is minimum means, you have a stable condition. For example, you have a pendulum, the pendulum is stable when z is equal to 0 at the lower most point; at this point, pendulum is not stable, potential energy is not smallest, it is high; there it is not stable, potential energy is high. So, potential energy is low means, the system is stable. Therefore, when you have the minimum potential energy means that you should have the maximum kinetic energy, I am talking about the stable condition, in case of heeling; so, your rotational kinetic energy should be maximum; that seems to be the condition for stability, that will be when your potential energy is minimum.

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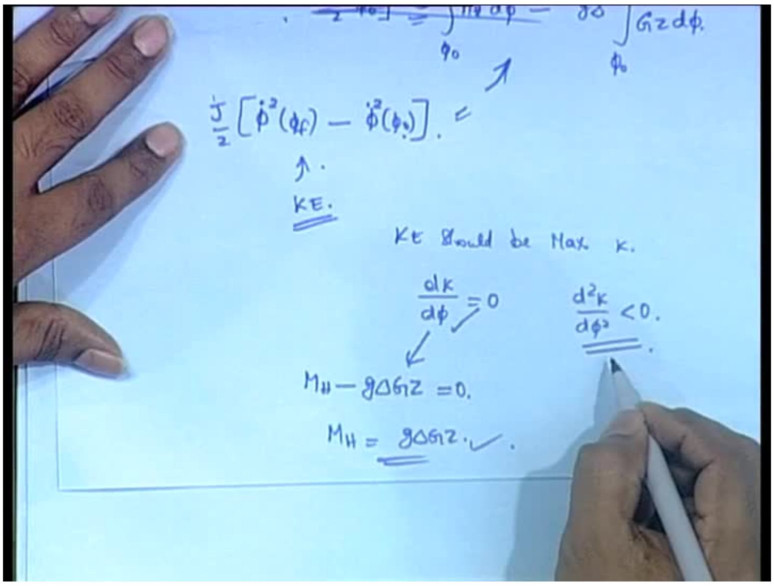

So, your kinetic energy should be maximum. Your kinetic energy should be maximum that means, the left, if K is the kinetic energy, then that means dk by d phi should be equal to 0 when kinetic energy is maximum, I am saying; dk by d phi should be 0 and d square k by d phi square should be less than 0. This is the condition for maximum kinetic energy that means, at what heel, at what phi is K maximum? That is what we are trying to find; at what phi is K maximum? Therefore, dk by d phi should be 0 and d square k by d phi square should be less than 0, that means, this represents K.

As we have seen, this equation states that this value is equal to this value, so d by d phi of this is equal to d by d phi of this right side, which means M H minus g delta GZ should be equal to 0; this is one condition, the first condition. This becomes like this which means that M H should be equal to g delta GZ - what does it mean? It just simply means that your heeling moment and your righting moment should be the same; that is the condition of stability and that straight away we know. But that is not enough. This also has to be satisfied then, only you will have the minimum potential energy or maximum kinetic energy.

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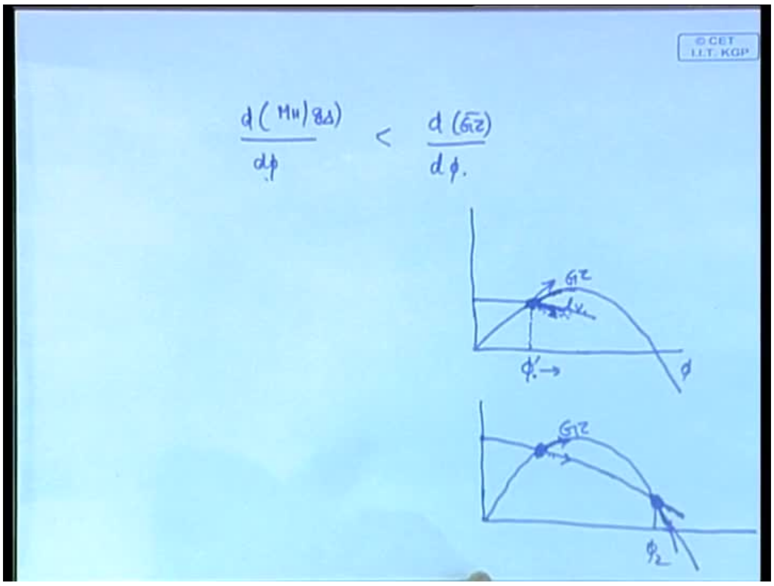

Now, that condition means the right hand, so d by, you have to do d by d phi again; therefore, this becomes d of MH by g delta by d phi, will become less than d of GZ by d phi. Now, this is also very obvious - what does this indicate? I will tell you what it indicates, that is, suppose you have this…Now, what it really - First condition says, what it says is that, the heeling moment and the righting moment equals like this point, this is the condition of equilibrium - stability - lower potential energy, so it is a condition of stability. Then, our next condition - what does this mean? It means that the slope, this is the phi and this is the moment; so, the slope of this should be less than the slope of this. The meaning of that is, if the slope of this is smaller than the slope of this - the meaning of that is - this goes up and this is coming down means, the moment you cross this point, this curve will be above this curve, and that is the meaning of it.

That means, from that point up, your righting moment will be greater than your heeling moment. When you do that, your resistance is more, and there is a tendency to come back; there is no tendency to go further; so, it is a position of stable equilibrium. Stable equilibrium means, it might go but the heeling moment might cause it to move; if it tends to move, then the righting moment becomes more than the heeling moment and it comes back to the original position; so, it stays there; that is the meaning of this mathematical inequality. This less than expression, the meaning of that is d by d phi of this thing is less than this, the meaning is that, the slope of this curve should be less than the slope of this curve.

The meaning of that again is, if the slope of this curve is less than the slope of this curve; when you cross this phi, this value of phi, let us call it phi dash - the moment you cross that you will have a higher GZ than an l v or, the righting moment will be greater than the heeling moment. This value will be higher than this value, if this slope is in this fashion; if this is going down and this is going up, this value will be higher than this value the moment you cross phi dash, therefore, your heeling moment will become less than your righting moment. Righting moment, remember, is always the resistance to motion. If the resistance is greater than the heeling, then there is no motion. That is all we are saying, so it is in equilibrium, stable equilibrium - stable position.

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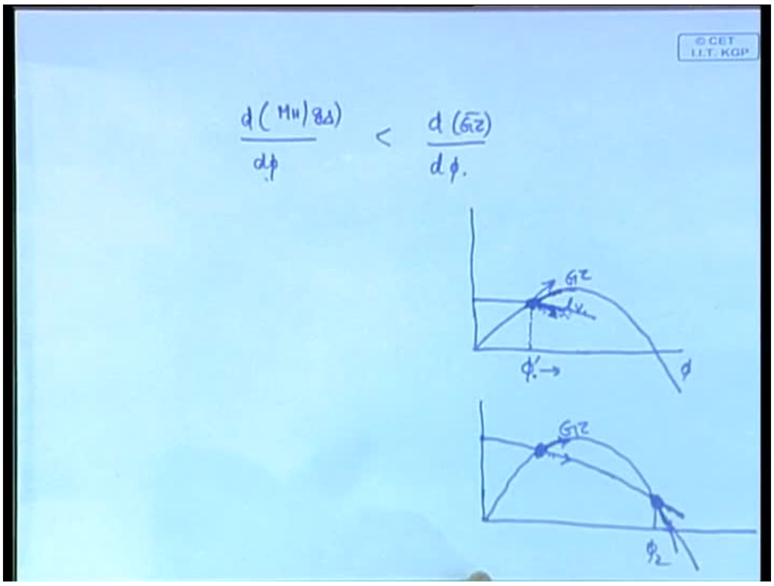

What happens if it is the other way around? It is like this, let us take this case, this position - what happens here? Here it is the other way around, the moment it crosses here, GZ is below the, this part along we will draw, this is GZ. GZ is below the heeling moment curve; so, righting moment is less than the heeling moment, the moment it crosses this. So, what does it mean? Heeling moment is always greater or the righting moment is always less. The resistance is always less than the force acting that means there is always motion, it will continue to move. Therefore, whatever it is, the moment it crosses this phi 2, it will continue to move only because the resistance is less than the force acting. The force causes it to keep moving and it will capsize; there is nothing to stop it from capsizing, it will continue to happen.

But it is different here; at this point, if it moves further, the resistance is more as a result of which, it comes back to its original position. The only reason is the relative position of the GZ and the heeling moment curves. This is the GZ curve and this is the heeling moment curve, the relative position is what determines whether it is going to capsize or not. So, from this one curve, we can say these two things - whether it will capsize or not, just looking at the relative position of the GZ and the heeling arm. There are two positions of equilibrium here, you can see. In both cases, you have the righting arm equal to the heeling arm; but at this point, it is stable, whereas at this point, it is unstable.

Now, there is something known as roll-period, let us write that equation. Let us consider first, there is no heeling moment means, nothing is there to cause a ship to heel as such - there is no wind, there is no turning - nothing. It is just moving in a straight line, but at that time also heeling can happen, it will keep moving that is called rolling in fact. It means, the ship, even though, there is no such external force, due to some reason, it will always, because of its motion itself, move a little bit like this; that kind of rolling will happen.

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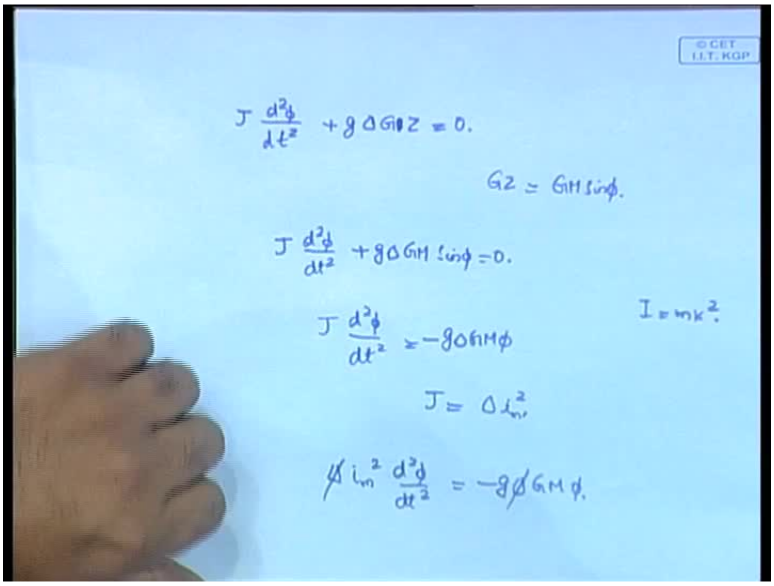

Now, the equation, previous Newton's equation becomes like this; it is g delta GZ is equal to 0, that means, I have just set the heeling moment to 0; so M H is set to 0. Therefore, J into d square phi by dt square plus g delta GZ is equal to 0. Now, remember the formula - GZ is equal to GM sin phi. There is such a formula which you have to remember. So, J into d square phi by dt square plus g delta GM sin phi equal to 0. Now, let us consider small angles of heel, that is, phi in the range of 0 to 15 degrees probably. Therefore, sin phi is equal to phi and therefore, J into d square phi by dt square is equal to g delta GM phi minus. So, what is this? This is the equation of simple harmonic motion, where you have your acceleration proportional to displacement or minus displacement, that is known as the simple harmonic motion.

In this particular case, it is just happening. Because of this condition, the ship is just going in a simple harmonic motion like this. Even though there is no heeling moment, it is still going in a simple harmonic motion, because of its motion, it is causing it to heel like this. And, from this, you can directly get the expression for the time period of the simple harmonic motion. It will become… Now that is one simplification we can do; this moment of inertia can be written like this that is or, in general words, you write it like this; that is, where k is known as the radius of gyration or gyration or something. So k is the radius of gyration - what do you call that? Gyration - right? Radius of gyration.

So, this J, if you write like this, delta is the mass of the ship in tons probably, so, i m is the radius of gyration and that squared; so, this is J; so we write it like this - i m square into delta into d square phi by dt square equals minus g delta GM phi; so, delta gets cancelled out; so from this, we can get an expression for the frequency of the rolling.

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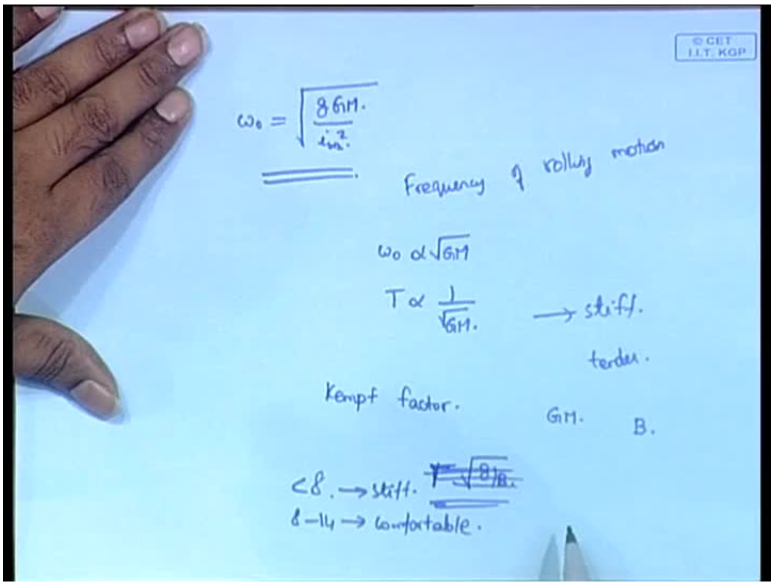

So, that is given by omega 0 is equal to root of g GM by i m squared. This is omega 0 is equal to root g GM by i m squared; this is the expression for the frequency of rolling. From this, you can get the time period - 2 phi by omega will give you the time period. This gives you the frequency of the rolling motion. Now, this tells you … In fact, we have derived this expression in a slightly different form sometime before. What this shows is that, again, GM is dependent, this frequency of rolling - I mean, that derivation was done from a slightly different direction, but still there, the expression was the same. What you are seeing is that, the frequency of rolling is dependent upon GM, or rather the frequency is directly propositional to root GM; so, time period will be directly propositional to 1 by root GM.

If you keep increasing the GM - same concept - if you keep increasing the GM, we know that it is good, from our stability point of view. If you increase GM or when GM becomes greater than 0, it is stable; like that you keep increasing GM, the stability will keep increasing, but when you keep increasing GM, the time period will keep decreasing, so it will become like this. When the time period is less, it will become like this; lesser and lesser, it becomes very fast like this. If the time period is higher, it will become - slowly like this and it will start moving. This is very comfortable actually, when it is very slow, we would not even feel the movement, otherwise, you will be moving like this.

That is a reason for not increasing GM too much, because the ship will become highly, what is known as stiff, the word is 'stiff'; which means, the ship is like this; this is called stiff. The other opposite of it is called tender, it is very good; it is good for us; but of course, that means, you keep decreasing the GM, which is also not very good, because from this stability point of view that might affect you. If you keep decreasing GM to a very low value… So, somewhere you have to do an optimization, that is, some intermediate value you choose where you have a value of GM, such that, both are ok; it is not too stiff, nor it is unstable.

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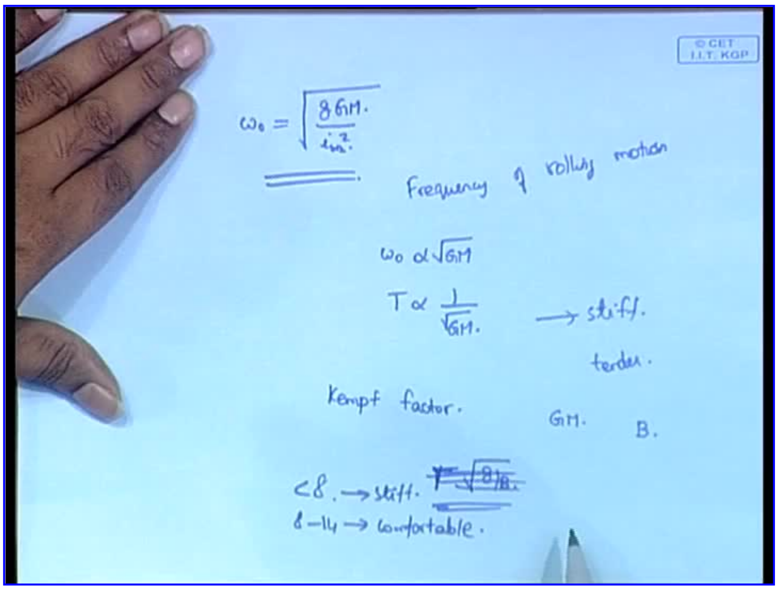

Well, this is not that important, but still there is something known as… I mean, this was proposed by Kempf and there is something known as a Kempf factor, which is defined as… Now, in general, it is known that… We have not done it, so you would not know. There is a relation between GM, not a relation, actually, it is like maximum limit between GM and the breadth of a ship. There is some relation; there is some form of relation between the, for stable condition - what will be the relation between the GM and breadth of a ship?

So, he brought in T into root g by b; this is actually not important; just know that there is something known as Kempf factor; this is the exact expression for it. It is like T is the draft, T into root of g by breadth. Now, when the Kempf factor is under 8, you say that, or if it is less than 8, then you say that the ship is stiff; if it is between 8 and 14, we say that it is comfortable. I guess you can just know that there is a Kempf factor because these things are probably not that important, but know that this Kempf factor is defined, such that, if it is less than 8, you have a stiff ship.

This factor becomes… So, this Kempf factor is used in designing a ship. When you are designing the GM, you do not always go to a maximum; there are definitely very strict requirements that say the GM should be a minimum of so much. So, you come to that minimum, you design the ship with that minimum; then everybody tries to increase the GM, because then it will become more stable. While going in the other direction, you keep increasing the GM, you check this and make sure that Kempf factor is in 8 to 14 range. So, once it is outside this range, it becomes stiff; at least, it should be 8; you try to put the Kempf factor 8, such that your stability is maximum and then its okay, the ship is okay.

Alright, I will stop here. Thank you.