

# Hydrostatics and Stability

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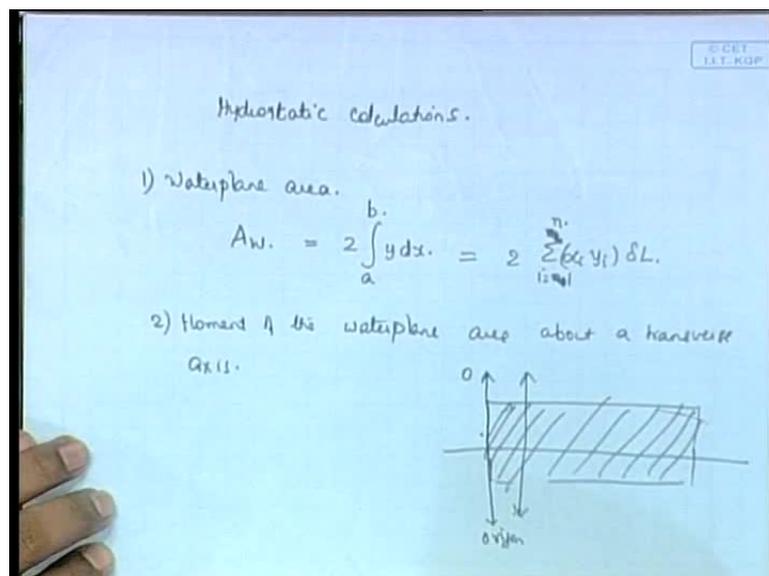
Module No. # 01

Lecture No. # 11

## Hydrostatic Curves - I

Let us start with what are known as Hydrostatic calculations. We already mentioned little bit about Hydrostatic Curves. Now, the first thing that we need to calculate is the hydrostatic calculation that you will need to perform is to find the water plane area.

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Now, in some ways while doing some of the problems I have done lot of this; but, mathematically I did not do it in detail last class in those problem. So we will do that in this chapter. The first thing you are calculating is the water plane area. Note, that is always written as  $A_w$ , it is the water plane area. As you know, it is the area at the draft where the waterline is cutting the ship, the area that is known as the water plane area.

It is twice  $y dx$ ,  $y$  is the half ordinate and integral  $y dx$  will give you the total area and twice that will give you the both the sides of the ship - port and star board. So this is the

integral equation for  $A_w$ ; now this we rewrite it as this. The same integral equation we are writing it in a discretized form (Refer Slide Time: 02:00).  $\Delta x$  the distance between two stations is  $\Delta L$  and this integral is always written as a Simpsons rule or a trapezoidal rule. This  $\alpha_i$  represents their multiplier, either the trapezoidal multiplier or the Simpsons multiplier.

So it is the same thing; it is this integral only. So the first one, you are actually integrating this between water line  $n_1$  and  $n_2$ , the station  $n_1$  and  $n_2$ . Therefore, let us say that it is probably not  $n_2$ , we can even call it  $n$  station equals  $1$   $i$  equals  $1$  with sum  $n$  to avoid confusion because what we are really doing here is this (Refer Slide Time: 02:53). It means  $x$  is going from  $x$  equal to  $a$ , which is most likely  $x$  equal to  $0$  and it is going up to the total length;  $L$  is the total length of the ship,  $\Delta x$   $x$  is going from  $0$  to  $L$  the total length of the ship.

So in this case, it is going from  $a$  to  $b$ , it goes from station  $1$  to the last station or the station at the aft to the station at the forward point. So that will give you the total water plane area. Then next one is now, you should know this; what is meant by transverse axis and what is the longitudinal axis? In fact this part, that is this naming of transverse we will see which is coming next, transverse moment of inertia transverse axis, longitudinal moment of inertia longitudinal axis.

It is a little confusing, especially first time when you are doing you will definitely be confused; so at least you should be clear at the time of exam, even if you are confused which is which now, at the time of exam be clear. This you should know at all times means if you have the ship like this (Refer Slide Time: 04:20), if this is the length of the ship, this axis is always a longitudinal axis and this is the transverse axis.

Therefore, what are we doing here? We are finding the moment of the water plane. This is the water plane area (Refer Slide Time: 04:30), let us say this is the water plane area; that means, this is the region where the draft is cutting the water line, waterline is cutting the air, its reaching the air. So, the moment about a transverse axis, this is a transverse axis. What are we doing? We are actually finding the moment about this point of the water plane area. This is the first problem; we can do one thing. We take the transverse axis at the origin; let us take this one, let us call this as station  $0$  which is the origin.

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$$M_x = 2 \int_a^b xy dx \approx 2 \left( \sum_{i=1}^n \alpha_i x_i y_i \right) \delta L.$$

$$x_i = j_i \delta L.$$

$$\approx 2 \left( \sum_{i=1}^n \alpha_i j_i y_i \right) (\delta L)^2.$$

3) x-coordinate of the centre of flotation

$$\underline{\underline{x_c}} = \frac{M_x}{A_w} = \frac{2 \left( \sum \alpha_i j_i y_i \right) (\delta L)^2}{2 \left( \sum \alpha_i y_i \right) \delta L}$$

The station 0 we will call as origin and therefore, your moment of this water plane area moment  $M$  becomes twice  $x y d x$ . It is clear now, why it is should be  $x$ ;  $x$  is the horizontal distance of each point and this  $y d x$  is giving you - if you do integral  $y d x$  you actually doing the water plane area  $x$  into  $y d x$  is giving you - that moment.

This again we will write it in the discretized form or the discrete form, so it becomes 2 into sigma; again let us call it from  $i$  equal to 1 to  $n$   $\alpha_i x_i y_i \delta L$  this can be written like this (Refer Slide Time: 06:00) and where  $\delta L$  is representing that  $d x$ . Now, how can you write  $x_i$ ? Let us assume in the simplest case  $x_i$  is actually  $x$  of each station you can call it like this  $x_i$  equal to  $j_i$  into  $\delta L$ .

$J$  is the number of station and  $\delta L$  is that distance between the station; so 1 into  $\delta L$  will give you  $x_1$ , 2 into  $\delta L$  gives you  $x_2$ , like that you are getting  $x_i$ . So, this equation itself we can write as 2 into sigma  $i$  equals 1 to  $n$   $\alpha_i j_i y_i$  into  $\delta L$  square. We have just taken that  $\delta L$  outside, now why it is written like this? It is written like this because this  $\delta L$  square is a constant it can be taken outside, so your idea is to find this summation.

We have already discussed how to find summations, how? We are going to draw the tables. How do you find the total area? That we will come to it next but, there is the goal of writing it like this. The sigma is actually representing the table summation in that

column, so there is the moment of area. How will you get the x coordinate of the center of floatation will see this, this is the third coordinate.

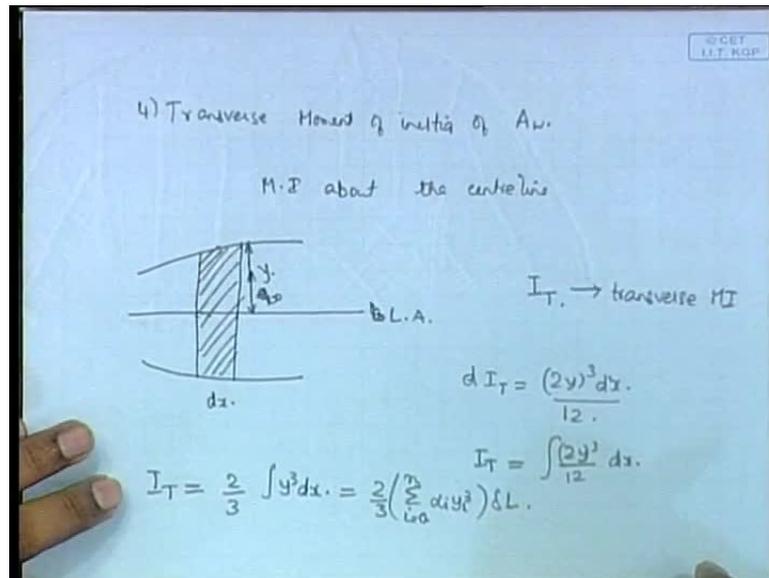
The meaning of the x coordinate of the center of floatation is I have already told you center of floatation means the centroid of the water plane area. So you have this area, you want the centroid of that water plane area. How will you find it? You will do moment of that water plane area divided by the water plane area. So that will give you the centroid of the floatation the x coordinates of the centroid of the floatation. Let us call it  $x_f$  is equal to  $M_x$ . What we have done here is,  $M_x$  is divided by  $A_w$ . Now these notations do not change; in this course at least do not change it.

Let  $A_w$  always represents the water plane area,  $M_x$  this moment because there are lot of  $M$ 's that comes  $M_x$  you will see, some  $M_t$ , different  $M$  will come; so each of them denoted by the same letter itself. So this represents  $\frac{2 \int x y \, dA}{\int y^2 \, dA}$  into  $\frac{2 \int x y \, dA}{\int y^2 \, dA}$  - what was  $A$ ?  $A$  was defined as  $\int y \, dL$ , so  $\frac{1}{2} \int y^2 \, dL$  will cancel,  $2$  will cancel. So, this gives you the x coordinate of the center of floatation.

Now, we will do examples that will explain it more clearly but, you can see **what you how you create the sigma**, do you remember? This is what you are calling as LCF- Longitudinal Center of Floatation; this  $x_f$  is what you are calling is LCF.

Now, this is one summation of one column in a table. I will draw the table then it will be clear. So, this is expression for  $x_f$  then, the notation is that  $x_f$  is always called as longitudinal center of floatation, so remember that.

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Now, the fourth one is a transverse moment of inertia of a water plane area. This is actually transverse moment of inertia, what is transverse moment of inertia? It is the moment of inertia about - this is the confusing part. Transverse moment of inertia means the moment of inertia about the longitudinal axis. I will explain this but, it sounds confusing; the meaning is that why is the transverse moment of inertia, why is about the longitudinal axis, because of this?

Transverse moment of inertia means moment of inertia in this direction; **there is obviously transverse**, it means this is the longitudinal direction; this is the transverse direction; this is the center line in the middle - that center line is called as longitudinal axis (Refer Slide Time: 11:08).

Now, you want the transverse moment of inertia means that moment of inertia in this direction. Obviously, from where do you calculate? You calculate it based on this line in the center, so transverse moment of inertia is the moment of inertia about the longitudinal axis. Is it clear or I should say this again. What I am saying is, what you need is transverse moment of inertia, means the moment of inertia in this direction. The ship is like this; this is the aft the ship and this is forward of the ship and this is one side and this is the star board side, so if this is the center line, this line is called as a longitudinal axis (Refer Slide Time: 11:43).

So, what we want? We want the transverse moment of inertia; transverse moment of inertia is a moment of inertia by multiplying distances in this direction it is the transverse moment of inertia means in this direction. From where do we take the distance, from the center line, so it is the moment of inertia about the center line - is it should I say I think is clear now - so this is the transverse moment of inertia, it is the moment of inertia about the center line.

These things do not that is why I said it you keep forgetting this line which is but, remember this is the longitudinal axis; there is no doubt about that. Longitudinal and transverse direction should be very clear, longitudinal axis is clear if transverse axis is clear.

Then you can just think transverse axis transverse moment of inertia has to be in this direction; moment of inertia in this direction, so you are actually needing the distance in this direction. So this distance can only be from some fixed line like this, which should be a longitudinal axis.

So that is the meaning of transverse moment of inertia would like this - I mean we can draw it here. Let us assume that the ship is this (Refer Slide Time: 13:00); this is your ship, this is the center eye of the ship and this is what we are calling as the longitudinal axis. So, what are we multiplying? We are actually finding the moment of inertia let us say a small area like this. You take the small area thickness  $d x$  of length  $d x$  you are actually finding the moment of inertia of this area about this point. You have to multiply a  $y$  here, you take the area and you multiplied with  $y$  that is actually going to give you the moment of inertia.

Again, this is always written like this,  $I_T$ ;  $I_T$  represents transverse moment of inertia about the longitudinal axis or transverse moment of inertia about the center line. This is the transverse moment of inertia. Now, look at this figure, let say that this length is  $y$  (Refer Slide Time: 14:05). What will be the moment of inertia of this? We have taken only the  $d x$ ; let us consider it to be a rectangle because its small length, so  $d I_T$  will be  $2 y$  cubed into  $d x$  divided by 12.

It is a rectangle and it is the moment of inertia about this point that you know already, if you are finding the moment of inertia about this, it is this length cubed divided by this length into that length, so this total length is  $2 y$ ;  $2 y$  cube into  $d x$  by 12 that gives you  $d$

I and integral of this; that is I T will be integral of this 2 y cube by 12 d x integral of this so I T will be this - which we will write in a discrete form - which is as I T is equal to - so 2 cube is 8 8 by 12 - so it is become 2 by 3; 2 by 3 into integral y cube d x, which is equal to 2 by 3 into sigma, let us start from i equal to 0 or you can start from the i equal to 1 means, you are calling the station 0, you are calling the station 1 it is into up to n into alpha i into y i cube into delta L.

You have inputs clear; I do not need to explain it again. Yes, 8 by 3 is 4 by 8 by 12 is 4 by 3 you know (( )) this is 2 y cubed this one right this is 2 y cube, so its 2 by 3 then this is what I was explaining last time, but did not write down the equation as such. I already told you that how do you find the moment of inertia about the center line of the ship? How did we do it last time? In case, I know that there is, when we are covering a lot of portion like this; we have a lot of interrelated things which kind of sometime creates confusion.

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The image shows a handwritten table and formula on a blue background. The table is enclosed in a hand-drawn box and has four columns: 'St d', 'SM', '(1/2 ord)<sup>2</sup>', and 'Func. of Moment'. The rows are labeled 'St 0', 'St 1', 'St 2', and then vertical dots. The values in the 'SM' column are 0, 1, 4, and 2. To the right of the table, there is a note: 'Func. of Moment. (0 = 0 x 0)'. Below the table, the text 'Σ Func. of Moment' is written, followed by 'Moment.' and a formula: 
$$I_T = \frac{\Sigma \text{Func. of Moment} \times \frac{2}{3} h.}{3}$$

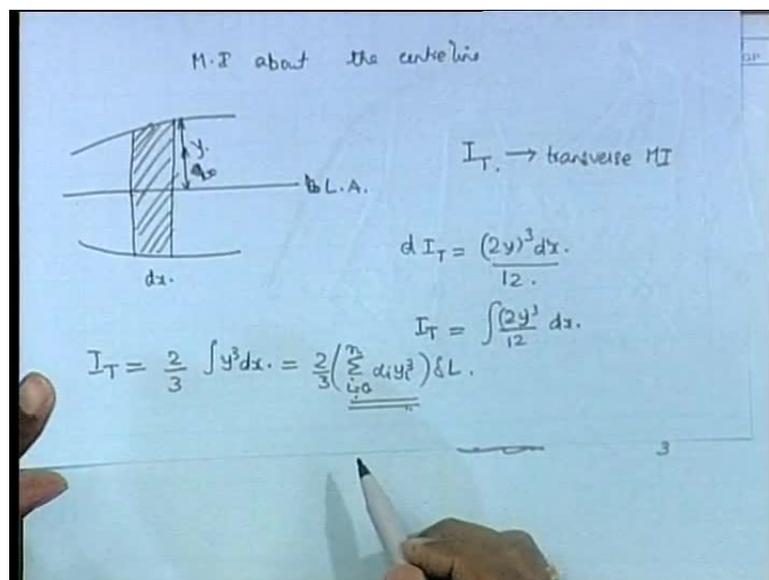
For an instance this one, I already told you, how you are going to calculate the moment of inertia about the centerline? That was like this that I mentioned there, we will do that but, you will write a table like this station 0, station 1 like that stations like this, you will write, then you will write Simpson's multiplier 1, 4, 2 4 are like that.

Then here you are writing the half ordinate cube; we were doing a half ordinate cube and then this is half ordinate cube and here we doing a, if you remember we called it as a

function of moment and this was actually given to be, lets name this column 1, this column 2, column 3, so this column 4 is equal to column 2 into column 3; I mentioned like this function of moment.

Now, you do this you get a summation here of different stations sigma function of moment. So, this I you are getting is I about the centerline which is I about the center longitudinal axis, which is known as the transverse moment of inertia, which is I T. This I T is equal to, so this sigma function of moment, I mean this is how I define it there multiplied by, it should be 2 3 into h. This you have to derivate it but, have to remember at least this final so, this function of moment is this (Refer Slide Time: 18:57).

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This is really your function of that moment alpha i into y i cube again, I will repeat alpha i represents the Simpson's multiplier or the trapezoidal multiplier and y i cube; y i is representing the half ordinate - half the distance from the centerline - half ordinate so half ordinate cube.

(Refer Slide Time: 19:21)

$dI_T = \frac{(2y)^3 dx}{12}$   
 $I_T = \int \frac{(2y)^3 dx}{12}$   
 $I_T = \frac{2}{3} \int y^3 dx = \frac{2}{3} \left( \sum_{i=0}^n \alpha_i y_i^3 \right) \delta L$

$I_T = \sum \text{function of Moment} \times \frac{2}{3} h$

So that is what we have done here, this function of moment; this function of moment is, what is it? 4 is equal to 2 into 3, 2 is the Simpson's multiplier into half ordinate cube and we are summing it up; we are summed it up and you are multiplying with delta L, which is h in this case into 2 by 3. So this is how you get your moment of inertia about the center line. This you will have to do for the exam, some problem will be there to calculate the moment of inertia about the center line.

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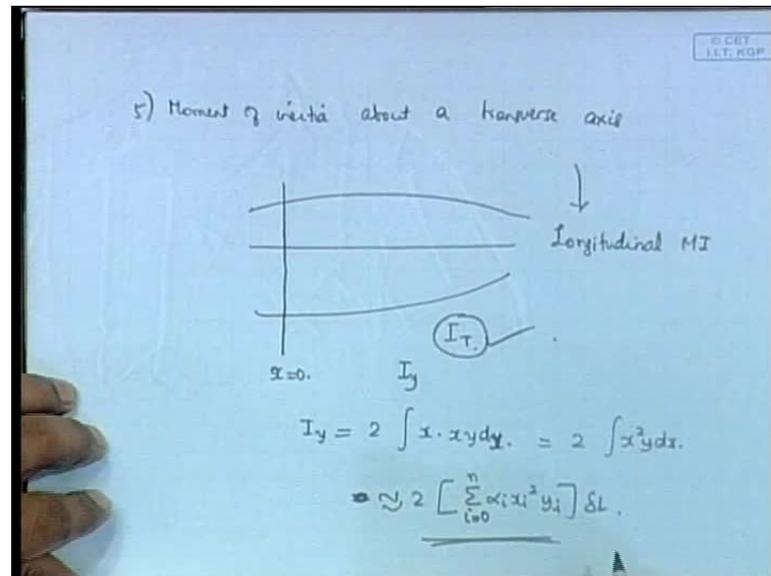
4) Transverse Moment of inertia of Area.  
 M.I about the centerline

$I_T \rightarrow \text{transverse MI}$

$dI_T = \frac{(2y)^3 dx}{12}$   
 $I_T = \int \frac{(2y)^3 dx}{12}$   
 $I_T = \frac{2}{3} \int y^3 dx = \frac{2}{3} \left( \sum_{i=0}^n \alpha_i y_i^3 \right) \delta L$

So, this formula should be firmly in your mind. Of course, it is very easy to derive. Just I guess it is better to just remember this method of derivation, making this rectangle and you find out I, about this line (Refer Slide Time: 20:02).

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Now we can have, as I said this is where the confusion starts. That is, I will write it 5, the moment of inertia about a transverse axis. This is another one, which is slightly different that is here we are doing the moment of inertia of the area about a transverse axis.

Which is the transverse axis? Suppose this is the ship; suppose this represents the ship (Refer Slide Time: 20:50). What we found in the previous section was the moment of inertia about the longitudinal axis, which is called the transverse moment of inertia. Now, we are finding the moment of inertia about a transverse axis; in this axis, which is known as a longitudinal moment of inertia. So this is known as a longitudinal moment of inertia.

Now, this is the transverse axis passing through the origin of coordinates; we say origin of coordinate. By origin of coordinate, we mean station 0 that is what we usually are calling it as  $x$  is equal to 0;  $x$  equal to 0 is it does not have to be but right now, I am calling  $x$  equal to 0, that means coordinate system is you have to choose, we can always choose where ever you want and we people do choose it differently.

For example that is the next one but, right now, let us assume that  $x$  equal to 0 represents the origin, so you are having the moment of inertia about a transverse axis passing through the station origin, which is the first station that is known as the longitudinal moment of inertia.

It is usually designate as  $I_y$  previously, the previous one is  $I_T$ , this one is  $I_y$ . Now, in your problems, all those problems that you have to done, I do not think we have ever done a longitudinal moment of inertia means, what we are doing now? We have never calculated the longitudinal moment of inertia. So, just as a aid to help your exam, so that you do not confuse where to take the moment of inertia. You are never going to use this one means you are never going to do this; you are only going to do previous moment of inertia what we have done so far.

Of course, you have to know it but, know that your moment of inertia always has to be about the center line because I have never done any problem dealing with trim; trim is where this moments becomes important, when you take the moment about this transverse axis, it is actually to do this motion, I have not discussed trim at all and we would not be doing most of trim.

You are only interested in heel, so you are only interested in this  $I$ , so note that you are going to find always  $I_T$  then not  $I_y$ , so for the exam find only  $I_T$ , then this is defined as  $I_y$  is equal to  $2 \int x^2 y \, dx$  - what it will be  $x$  into? It is  $x \, dy$ . It is the second moment of inertia, so it is  $2 \int x^2 y \, dx$ . So, this is equal to, in a discretized form it becomes  $2 \sum_{i=0}^n x_i^2 y_i \Delta L$ .

I mean there is nothing to get confused in making this thing discreet; discretizing it that is  $2 \int x^2 y \, dx$  is of course, the integral one new term came Simpson's multiplier. So that  $\alpha_i x_i^2 y_i$  becomes  $x_i^2 y_i$  and  $dx$  is  $\Delta L$   $\Delta L$  is the distance between the two stations, so this is the expression and to make it slightly simpler.

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$$I_y = 2 \left[ \sum_{i=0}^n \alpha_i j_i^2 y_i \right] (\delta L)^3$$

$$x_i^2 = j_i^2 (\delta L)^2$$

M.J. → center of floatation  
~~transverse axis~~  
 transverse axis through the C floatation  
 = barycentric axis.  $I_L$

$$I_L = I_y - A_w x_F^2$$

This can be written as two into sigma i equal to 0 to n alpha i j i squared y i into delta L cube, means x i square all i we have done here is, so x i is equal to j i into delta L means distance of the station two from the origin is twice the length between the two stations. So, j i into delta L, so x i square is equal to j i square into delta L square or j i delta L the whole square.

This is your I y, just know this expression anyway it is important but, you would not be solving it, so this is the longitudinal moment of inertia. Now you can also do the longitudinal moment of inertia. This is about the origin of coordinate, which is the station 0. Now, you can also find the moment of inertia about which point, the center of floatation. Center of floatation actually you know what is center of floatation? It is the centroid of the water plane area.

So, initially you had at the beginning of the water plane area that is at of most point. Now, you do not have to do it about this axis I but, you can do it about an axis I which is parallel to this but at the center of floatation that is also a longitudinal moment of inertia. In fact, when you just say longitudinal moment of inertia you usually take it about that axis; about the center of floatation and the axis through the center of floatation is called a barycentric axis. **It is the longitudinal axis, I mean it is a transverse axis;** it is a transverse axis through the center of floatation, it is known as the barycentric axis.

This moment of inertia about the barycentric axis, let us call it  $I_L$  therefore,  $I_L$  will be equal to  $I_y$  minus  $A w$  into  $x F$  squared. This is just coming from the basic definition of the moment of inertia which says that moment of inertia about any parallel axis is found out by using the area of that section into the distance squared using that - what is it called? It is a parallel axis theorem or something know - equation you get  $I$  about  $L$ , which is  $I$  about the centroid, so what is the  $x$  of that centroid? Its  $x F$ ; it is the  $x$  coordinate of the center of floatation which is called as the Longitudinal Center of Floatation LCF, so that is  $x F$ . Once you have that  $A w$  into  $x F$  square, so this formula will be useful to remember.

We have written a couple of formulas; we can just apply it in one problem and you will see how it is giving you the different moments of inertia and areas and all that.

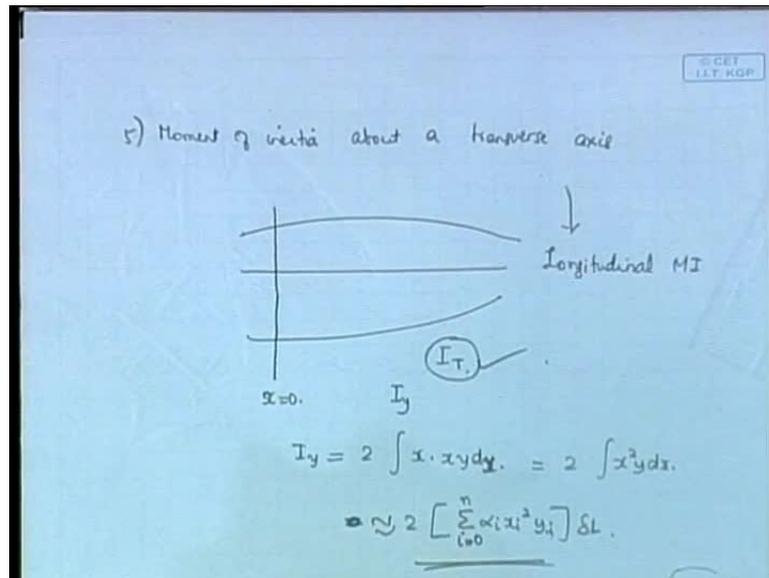
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St. No.	T.M.	Ordinate $y_i$	Level $J_i$	Function of Area of $y_i$	Function of Moment of $J_i y_i$	$I_x$ $(\frac{1}{2} y_i^3)$ of $y_i^3$
0	1/2	✓	0			
1	1	✓	1			✓
2	1	✓	2			
3			3			
4			4			

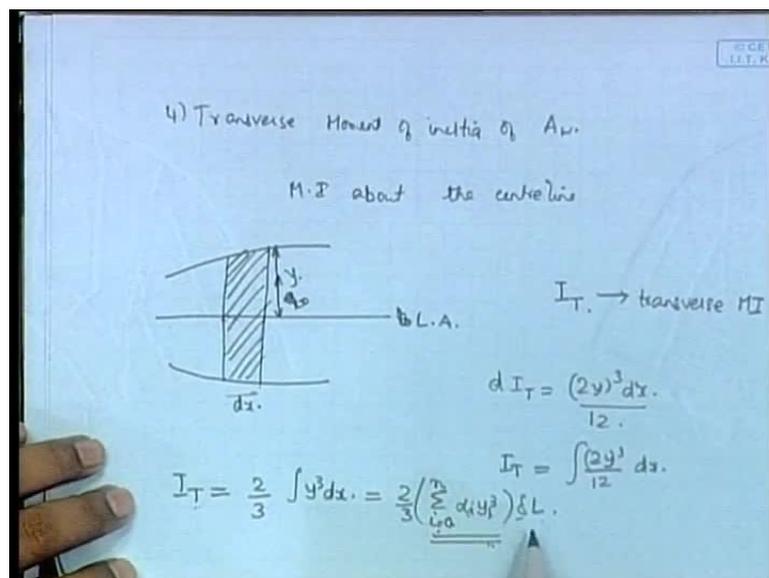
$I_x, I_y, LCF.$

Some problem is given such that you are told what is the station number and you are given with a table which gives you these details, the station numbers - like this you are given the different station numbers. You are given the half ordinates which means  $y$ , which is this distance; let us name them, this is column 1, this is column 2, this is column 3. In column 2, we write the trapezoidal multiplier; this time you are using the trapezoidal rule. We can do it as either as half 1 1 like that or 1 2 2; so this becomes half 1 1 like that, so you have the trapezoidal multipliers.

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Now, we look at this equation. Some of the thing that we derived here; this is one we need  $I_y$  and then we need  $I_T$ , which is this. So, our basic purpose in this problem is to find  $I_T$  and  $I_y$ . So,  $I_T$  which is given by this formula, which you have done and  $I_y$  which is given by this formula, so these two things we need. You are finding  $I_T$ ,  $I_y$  and LCF - three things.

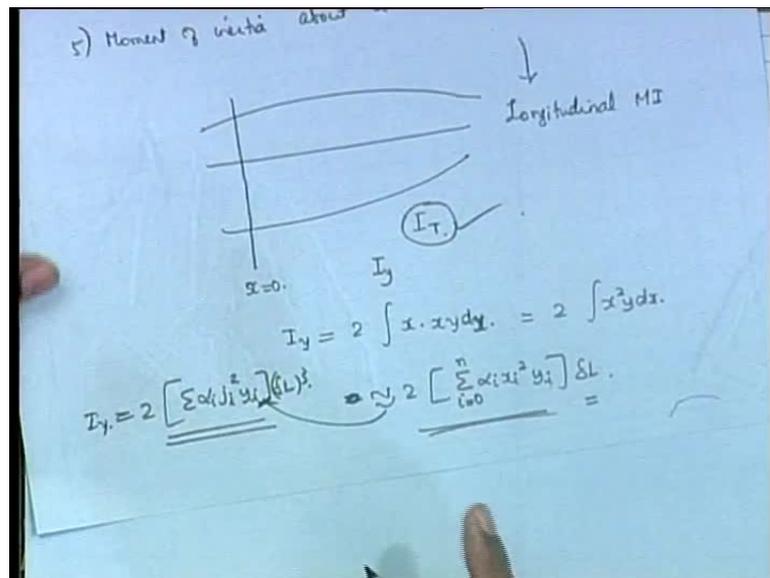
First, what are the things that you need to do this problem? This is what you are given; you are just given with the station number and half ordinate. First you make them

trapezoidal multiplier - that column - then you make a lever. The meaning of lever is actually  $j_i$  means, the position of the station. Lever is actually, what is the distance between the stations? It is  $\Delta L$ . So, the lever goes like this  $1 \Delta L, 2 \Delta L, 3 \Delta L, 4 \Delta L$ ; so this  $\Delta L$  is taken outside and it is multiplied later and lever is just 1, 2, 3.

What we first do is, the origin we call it as 0, 1, 2, 3, 4 and for different stations, you are going to have different levers. There is one column, this is called as column 4 then, function of area  $\alpha_i y_i$ . Now, this is to find the total water plane area, so you will have to find the water plane area. To find  $\alpha_i y_i$ ; this is the expression for water plane area  $2 \int \alpha_i y_i$ .

So, you see here you have to do  $\alpha_i y_i$  summation you have to get. So, you need  $\alpha_i y_i$  as one column. So, this will give you the water plane areas; this is column 5.

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Since you are asked to find  $I_y$ , you have to calculate  $\alpha_i x_i y_i \Delta L$  - I am not sure where I put the next half of it - this is actually can be written as **this** - I wrote that somewhere this - the expression for  $I_y$  (Refer Slide Time: 20:44). To get  $I_y$  you need this expression that means what you need? You need  $\alpha_i j_i^2$  into  $y_i$ .

That means, you need  $j_i^2$  into  $y_i$  therefore, this one. This is the function of moment, this is your column 6, so what are we doing? You have  $u^2 j_i$ ; you

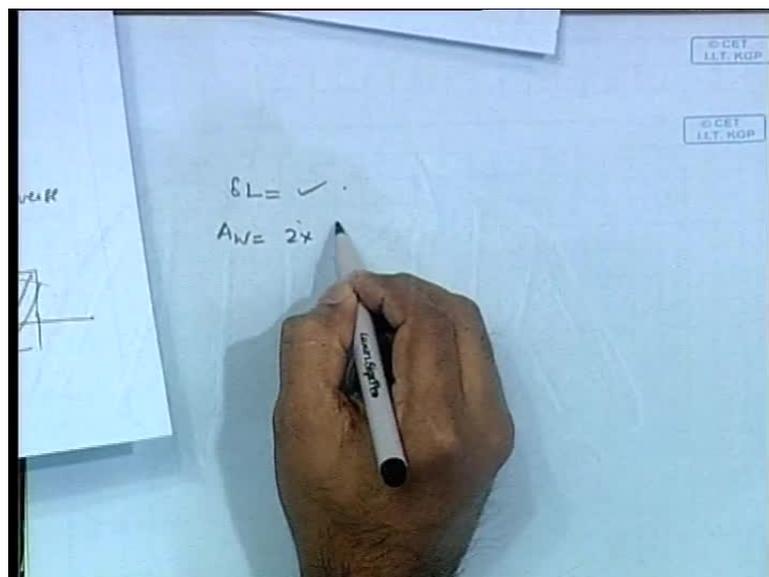
multiplied with half ordinate, so  $\alpha_i$  is this column,  $y_i$  is this lever column square into  $y_i$  is this half ordinate column. So, multiplying will give you another column the function of moments, this is to calculate  $I_y$ . So, in this you will be calculating  $I_y$  then, we need another column.

Actually the one you will be using for your exams that is to calculate your  $I_T$  - I have already told you,  $I_T$  is very important - though in this problem we are doing it; you will never do  $I_y$ , you will doing only  $I_T$ .

So to calculate  $I_T$ , I already told you need to do half ordinate cube into  $\alpha_i$ . So, this is to find  $I_T$  which is in fact the most important in this which will you have to do exam so this is  $I_T$  - half ordinate cube into  $\alpha_i$ . This is actually  $\alpha_i$  into  $y_i$  cubed, so this is another column  $\alpha_i$  comes from this which is your trapezoidal multiplier and  $y_i$  cube is your half ordinate cube, you have another column, so this is your table.

Of course, to do a problem like this it will take 2 or 3 hours, you cannot do it in the exam. Just one small section of it, if I am given probably I give you some 4 or 5 stations, 4 station probably. I ask you to find the moment of inertia, just know that how to do the method, so that is all I can ask; for example, this problem is very long we cannot do it.

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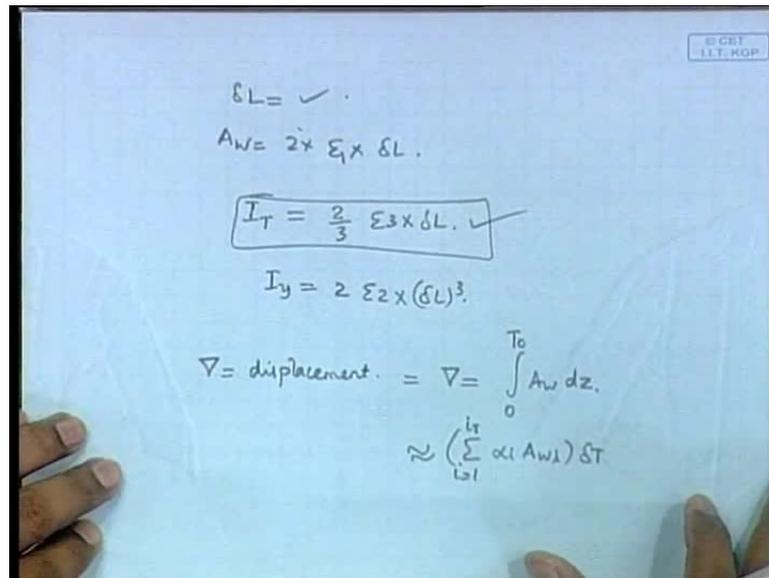
St No.	T.M.	Ordinate y	Level j <sub>i</sub>	Function of Area of y <sub>i</sub>	Function of Moments of j <sub>i</sub> <sup>2</sup>	$\sum$
0	1/2	✓	0			
1	1	✓	1			
2	1	✓	2			
3			3			
4			4			
				$\Sigma 1$	$\Sigma 2$	$\Sigma 3$

$I_T, I_y, LCF$

These are your columns and once you have this you have to do. You are given delta L - the distance between stations or whatever it is - then A w. Let us look at the formulas; the formula is given 2 this thing into delta L. So here we do 2 into, in this we have alpha i y i this summation sigma.

Let us call this as sigma 1, this as sigma 2 and this as sigma 3. This represents the summation of all the elements in this column. This is the summation of all this sigma1, sigma2, sigma3. So, 2 into sigma 1 into delta L; by just doing, this will give you the water plane area at that particular draft.

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If you are trying to find  $I_T$ , let us look at the formula for  $I_T$ ; this formula for  $I_T$ , it is equal to 2 by 3 into this; this we need. So, in this column we need this one  $\alpha_i y_i$  cube, so this is your  $\alpha_i y_i$  which is summed up to get sigma 3 means, you have different element; you have added them and you are getting sigma 3...

So, 2 by 3 into sigma 3 into - I think it is delta L - delta L; this will give you  $I_T$ ; the transverse moment of inertia which is the moment of inertia about the longitudinal axis that is the center line, that is very important. Of course, you have to be able to draw a table and sum it up; you have to remember this formula also 2 by 3; this 2 by 3 term and all has to be remembered, so 2 by 3 into sigma into delta L.

Best thing is not to by heart it, it will be confusing because there are so many such terms, best thing is to remember the derivation part. So, you just draw that center line and you draw that rectangle, you find the moment of inertia of the rectangle and so on, that is  $I_T$ . Your next question is to find  $I_y$ , so we have to use the formula for  $I_y$  which is 2 sigma  $\alpha_i y_i$  this (Refer Slide Time: 39:28). So, 2 into sigma  $\alpha_i y_i$  square  $y_i$  delta L cube, in this table  $\alpha_i y_i$  square into  $y_i$  this column, so 2 into sigma 2 into delta L cube.

If you do this you will be getting  $I_y$  then, what is this last? This is the different moments of inertia you can find. We have defined two types of moments of inertia mainly, the longitudinal and the transverse; one about the alternately longitudinal moment of inertia

about the transverse moment of inertia about the longitudinal axis and we have also of course, there is also something known as that about the barycentric axis.

Three moments of inertias that we have defined in which you have to remember this expression for I T and you have to know how to make the table. Now, next one is a little simpler that is, suppose we want to find out the del there is the displacement that is del is equal to, this is one expression for del that is the displacement.

So, A w is the water plane area, so you have the total water plane area and that into the this - at each point at each water plane you some demo you get the volume. So, this also can be written as a discretized form, it becomes sigma i equals may be 1 or 0, 1, 2 i maximum or i about T means I T represents the i value that is corresponding to the draft, we are summing up tile the draft point into alpha i into A w i into delta T, instead of the previous case where we had the delta L we have delta T here, which is the distance between the different water line not between the different station, but between different water lines.

I told you sometimes you can find the volume in two ways, using the sectional area you sum it up like this or you can find and sum it up like this; either way you are getting a volume. So, sectional area multiplied to the longitudinal distance or the water plane area multiplied with the drafts both gives you the volume, so this is how you find the displacement.

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base line = keel.

$$M_B = \int_0^{T_0} z A_w dz.$$

$$\approx \left( \sum_{i=1}^n \alpha_i z_i A_{w_i} \right) \delta T.$$

$$M_B \approx \left( \sum_{i=1}^n \alpha_i z_i A_{w_i} \right) (\delta T)^2$$

$z_i = J_i \delta L.$   
 $z_i = J_i \delta T.$

$$VCB = \text{vertical distance} = \frac{M_B}{V}$$

Now, just the extension of this same as before there is the moment of this water plane area about the keel; about the bottom of point you are finding the moment, we call that as the base line - the keel. You know keel means the bottom most point that line we call it as base line. So, when you are finding the moment about the base line, you are actually finding the moment like this; this distance, you are multiplying with this distance going up from the bottom - from the keel.

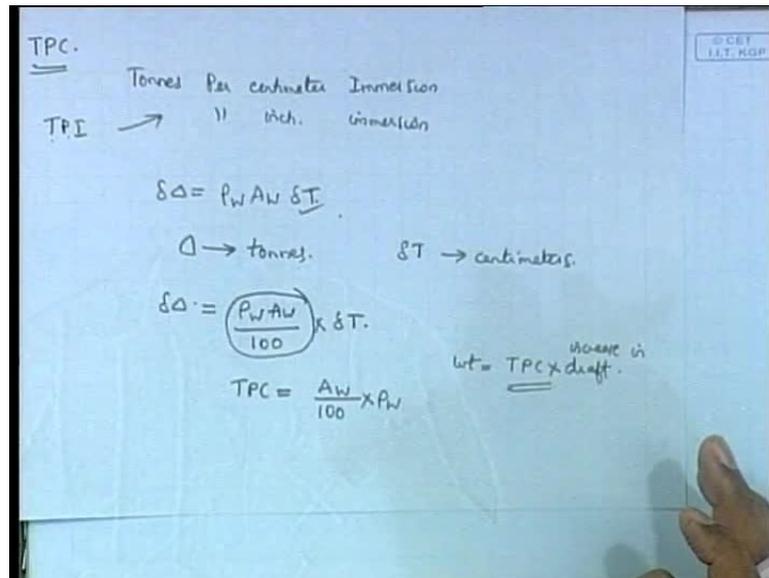
This is  $M_B$ ,  $M_B$  is equal to 0 to whatever is the draft  $\int T \text{ into } A \text{ w into } d z$  it is not  $z$  it should be  $Z$  actually,  $Z \text{ into integral of } Z \text{ into } A \text{ w into actually you should change in this it is not } T$ , it is actually  $Z \text{ into } A \text{ w into } d z$  then gives you the moment about the base line. Again you can discretize it, this summation is given by  $\sum_{i=1}^{\text{last value } i} T \alpha_i Z_i A w_i \text{ into } \Delta T$ . If you want you can do one more thing this  $Z_i$  can also be replaced the same way as we did last time...

For example, we said that  $x_i$  is equal to  $j_i \text{ into } \Delta L$ , just like that you can have  $Z_i$  let us call  $j_i$  itself but note that this is the vertical distance this time  $j_i \text{ into } \Delta T$ , we can write like this. So, this  $M_B$  can be written as  $\alpha_i j_i A w_i \text{ into } \Delta T^2$ , you can write it like this as well.

Last one, that is to find your vertical position of center of buoyancy. How will you find the vertical position of center of buoyancy which we called as VCB? It is the vertical distance, it is equal to  $M_B$  divided by  $\Delta$ .  $M_B$  is the moment above of that volume about the base line and this VCB will give you the distance from base line or from the keel. So,  $M_B$  divided by  $\Delta$ ;  $M_B$  is calculated here,  $\Delta$  is calculated here, we just calculated previously so by using these two you can get VCB.

This is mostly all about hydrostatic calculations. Now, some small things, so what we have done, the basic idea is to calculate the different types of moments and moment of inertias that we have calculated, which is again a second moment only, about origin, about the base line etcetera, either way you calculate the different moments; you calculate the areas, you calculate the moment of inertia and you can calculate correspondingly the LCF, VCF etcetera, as we are seen.

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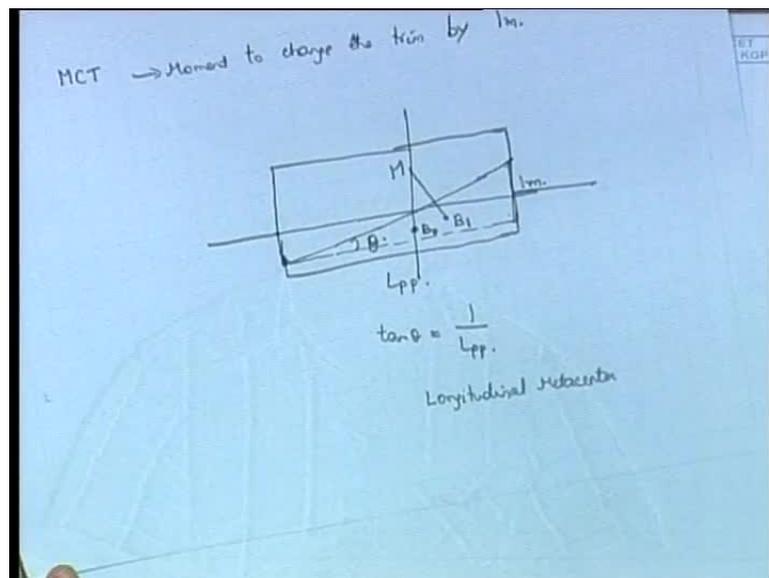
These are basically hydrostatic calculation; we have covered everything. Now one additional thing that comes another term that we use is, we usually call it as TPC, sometimes it is called as TPI it is actually the same thing Tonnes Per Inch immersion means TPI Tonnes Per Inch or Tonnes Per Centimeter same thing but, it has different dimensional systems.

Now, what is the meaning of it? It means that if you have ship or a body floating in water how much ton should you add to produce a increasing draft by 1 tonnes per centimeter means, to produce an immersion of 1 centimeter how much tonnes of weight should you have to put on a vessel, that is meaning of tonnes per inch or tonnes per centimeter; if you are talking about inch it is tonnes per inch.

That definition itself describes it, so that is weight required to put to increase the draft by 1 centimeter. Now, if you increase the draft by delta T, your displacement will increase by this, which is your increase in mass. Now, in this case we are going to measure delta having the mass in or weight **actually then it should be g should be there know tonnes no, sorry, tonnes is actually kilo gram into 10 or 1000 kilo grams is 1 ton, so delta is an tonnes and no this is correct and delta is an tonnes and delta T in centimeter there is the only thing to check in this.** We measure delta in tonnes, always the weight of the ship you never tell in kilo gram you always talk about in tonnes, because it is easier it is that is huge quantity...

Then delta T is in given here is centimeters, so once you have that when you convert it to your sg units that is, so you will get delta will be equal to - this is to do that tonnes and centimeter conversion gives everything in meter and Newton. This is your expression delta provided you have it in tonnes; delta is a tonnes and del T is an centimeters. So, this is called TPC, your weight is equal to TPC into increase in draft, so just know this formula TPC is tonnes per centimeter immersion.

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Now there is another term that comes along with this, that is known as MCT just like TPC, it is MCT is very similar it is known as moment to change the trim by 1m it is large but, it is 1 meter, so then I mean, moment will be large you have to have a large moment to produce the trim.

It is defined as MCT, note that this is not heel this is trim, so when you have the ship like this we are seeing what is the amount of moment that should be given to cause to trim by 1 meter - this distance is 1 meter (Refer Slide Time: 51:47).

So, if you draw that figure; let us draw a box easier to get the angles. So, let us assume that this is the ship that is floating like this, suppose that it trims let us draw like this so this is your water line. Now it trims, let say, there is moment causing and it trimmed like this and the total length of box or the ship in this case let us call it as  $L_{pp}$  the length between perpendiculars - I just finished this and stopped - suppose that you have caused

the ship to trim by 1 meter; let us call this angle theta, we are talking about the trims, so we call it theta.

So, it has trimmed as 1 meter - it has moved as 1 meter - therefore,  $\tan \theta$  is equal to 1 by  $L_{pp}$  this comes directly. Now, we defined heeling and we talked about the shift of center of buoyancy in the transverse direction like this perpendicular which gives you a meta center, we have defined thing like that.

The same concept you can apply trim also we which is very similar means, it trims like this or like this and because of it the center of buoyancy will have the longitudinal direction also means, it will be some were here or it will be somewhere in this point. So, if the ship heels or trims the B will move correspondingly; let us see trims like this. Therefore, B will move like this, the figure becomes exactly similar that of heeling except that of course, it is a longer distance other than there is no difference and these distances are longitudinal distances not the transverse distances.

So, everything becomes same like this. Initially, let us assume g at the center and B is here. Now, B will move in another direction, it is B 1; this is another metacenter, we called this metacenter as longitudinal metacenter; the other metacenter we are talking about heeling, trimming; we call it longitudinal metacenter. The mathematics is exactly the same; there is no difference; whatever you have done for the previous one you apply here. So, it is not called difference but note that the length is very small B, this is 1 meter, this is length of the ship probably 300 meters, so 1 by 300 is your  $\tan \theta$ , it is very small; we are talking of about very small values. May be I will stop here - this is your longitudinal metacenter - I will stop here; Thank you.