Elements of Ocean Engineering
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Lecture - 32
Mooring Systems (Contd.)

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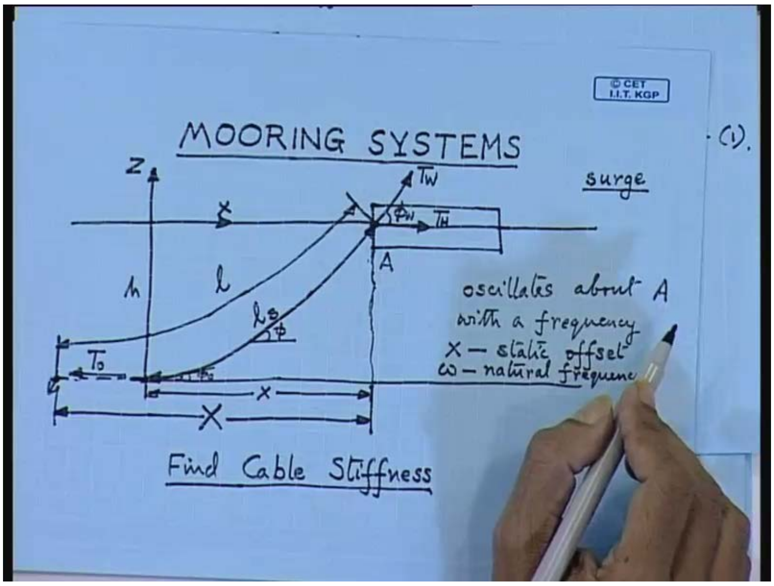

So, we continue with the discussion on mooring lines. So, we are simply analyzing one static table and today's class, let us try to find out the mooring stiffness. Now, which is very crucial to our calculations, the other part that you will require to know is the equations of motion. So, equations of motion are the vibration part. So, we will come gradually to this. So, this is our barge or our offshore structure. So, we are analyzing one simple line equation.

So, your anchor line, let us say that, this is making it is a catenary it is coming like this and it is lying on the sea bed. So, this is your point of anchor. So, this is your edge. So, this is how much? This is T H and this one is T W and this is phi W and out here, this will be say at the sea bed. This will be T 0, this is what we have so, T 0 has to balance here T H then this is our water depth. Now, in this equation we have calculated the length of the line, if you remember. So, there are two values see, if you take right up to the anchor point. So, this is our anchor point the whole length is. So, this is your point of suspension of the chain cable to your anchor point say that this we call, this is l and right to the point at which the line becomes horizontal you call this as l s. So, this is your distance up to the point of suspension of the vessel. So, this is small x you know capital X is the distance right to the end of the line. So, this one you mention this as capital X.

Now, today let us find out, find the problem is find cable stiffness. So, we have already solved the static line equation and we have derived the expression for T H l s length of the line and small x.

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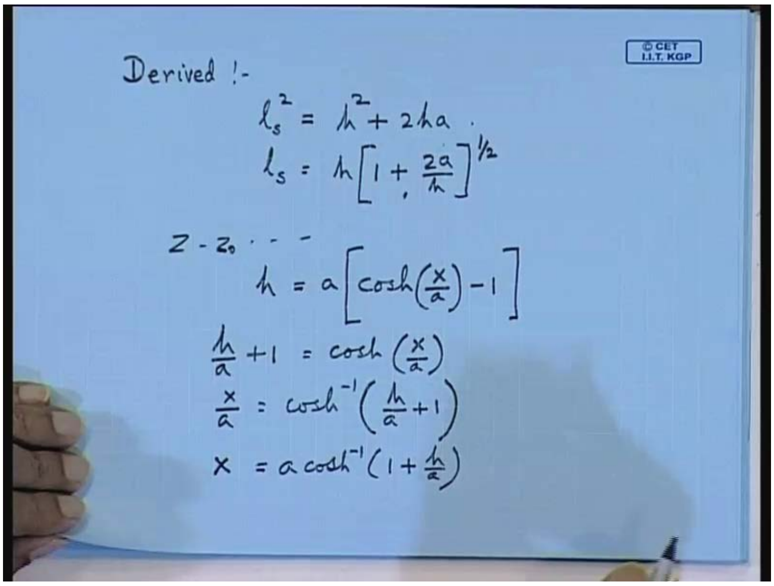

So, these formulas have already been derived. If you look into your previous class notes. You have derived this expression that is l s square, l s find is the total is a length up to be where it becomes horizontal the line. So, this is h square plus twice into h a. No please you do not talk. So, from this expression you what is the value of l s? Say l s you can write as so, this is you take h square out. So, this is one plus so, this will be how much? This is 2 a over h to the power half. So, this is the expression for l s.

Or so Now, what are the other equations that are required and you have already found out the expression for the depth of water. You have the expression that we have derived from z minus z naught, you look into your previous notes from this. We have found out you tell me the expression for h, there is a h we have put the z equal to zero at the water surface. So, h is we look back into your notes you will find this is a multiplied by cos hyperbolic of x by a minus 1. So, this is the expression we have derived from z minus z naught and put z equal to zero. So, from this you can find out what is now this x is the smaller x, that is this part of the length of the line.

So, from here what expression we are getting. So, h by a, you can take it outside. So, this is h by a plus 1. So, this is the expression for cos hyperbolic of x by a. So, from this we can get x by a so, that is coming as cos hyperbolic inverse h by a plus 1 or 1 plus h by a, so, this expression you can derive. So, now, you can derive write down the full expression for the length of the line. So, this is x is equal to a cos hyperbolic inverse 1 plus h over a.

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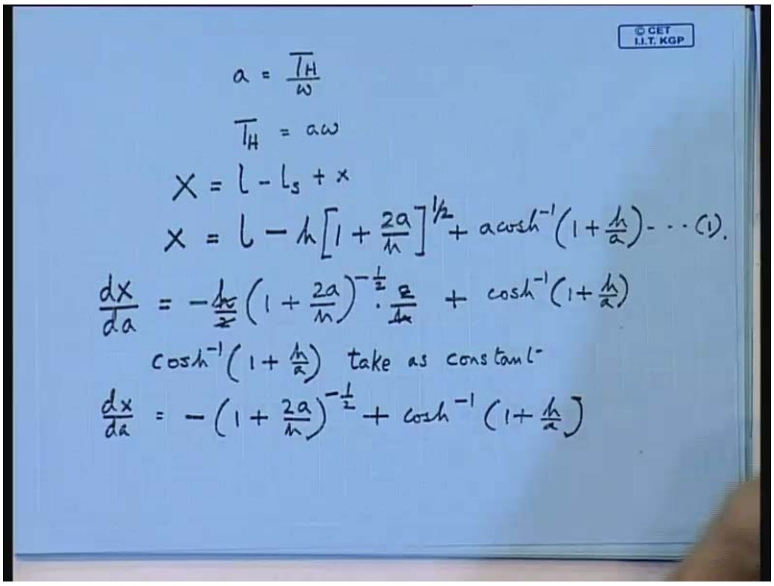

Now, one thing has to remember that the value of a that we have assumed, what is the value of a? a is nothing, but the horizontal tension divided by rate of change, that this w is not the total weight of the chain, but chain per unit length listen it. Or I have given the corrected for buoyancy. So, from here what is the value of T H, we are getting so, T H is a multiplied by w.

So, what is our expression for capital X? The small x we have already derived as a cos hyperbolic 1 plus h a. So, capital X, if you look at the diagram. So, what you will get? So, in this capital X is this distance plus small x. So, how much is this distance from the horizontal portion. So, here actually, if you take a tangent this will be phi naught.

That is your angle phi. So now, what is the equation for capital X. So, X is how much is this distance from anchor point to the horizontal porter. So, this is capital L minus l s plus small x so, capital X is L minus l s plus small x. So, now, you write down the expression for capital X. So, capital L is the total length of the line listen it so, that is even in constant. Now, l s is we have already found out, the expression for l s so, that is h multiplied by 1 plus 2 a divided by h to the power half and write down the expression for small x. Now expression for small x is this one, so, this is a cos hyperbolic 1 plus h a. So, this is the total expression for X.

Now, from this equation say, let us you put this as 1, you find out this d X over d a. So, what is the value, now L is of course, constant so, this will be zero. What about this expression so, this expression will be you this h is the depth of water. So, that is also constant, you can keep it out, but a is a variable so, this will be half. So, this will be divided by 2 multiplied by 1 plus 2 a h to the power how much, minus half. Any other term multiplied by 2 by h and what about the next expression. So, next expression actually you have to do some simplification or use the, if you want to differentiate the cos hyperbolic term it will be too much of headache.

So, this is product of two functions. So, how much will be you keep, first you would find out d a by over a. So, that is one and keeps the other part as constant. So, this will be plus cos hyperbolic inverse 1 plus h a. So, you have differentiated only you with a or multiplied by this and you keep a and differentiate cos hyperbolic inverse with respect to a. So, what is the function?

So, here what has been done, some simplification has been done. Now, simplification is you this cos hyperbolic thing has been taken as constant. So, in your maths book you just try to find out the differentiate of cos hyperbolic x. So, next class you just tell me of what is the expression for that d d x of cos hyperbolic inverse x. You know that it will be come up as some exponential function. So, it will differentiate, if you have to differentiate it, e series from that you will get. Now, in this simplified version this has been taken as constant.

So, that actually signifies lot of trouble. So, here how much we are getting. So, this 2 cancels out also h. So, d x by d a comes as so, this is our expression. Now, we have to find out two things, you know this. What is from this equation? The first one I have giving some number, no the other equation oh this one I have given a number.

So, from this equation, what is the value of a? Now, you find one thing you should remember that this after structure barge whatever you have taken, this is actually oscillates about a mean position. So, this you remember the moored barge oscillates about mean position as this is always the case. So; that means, what is this barge is doing you will find out, because of the variation in the external forces, we have assumed the external forces to be semi circle. So, it will go in this direction and it will come back, it will again drift in the reverse direction. So, like this it is going to oscillate about this position say, lets us position oscillates about a. So, this is similar to your vibration studies, oscillates about a, with a certain frequency with a frequency. Then what is this X? X is called static offset, X is known as static offset.

Now in our discussion, you have to find out the small x static offset and what is the natural frequency of vibration. Here we calculate this from your equation, natural frequency. So, the motion that out, here we are considering only one single motion say this is we are analyzing the, let us say surge. We find natural frequency of surge, here you want to find out natural frequency then you have to formulate your equation of motion. So, this is called your static offset vibrating about mean position a. So, this is how the barge is behaving naturally so; that means, if you take a your normally in your spread mooring system.

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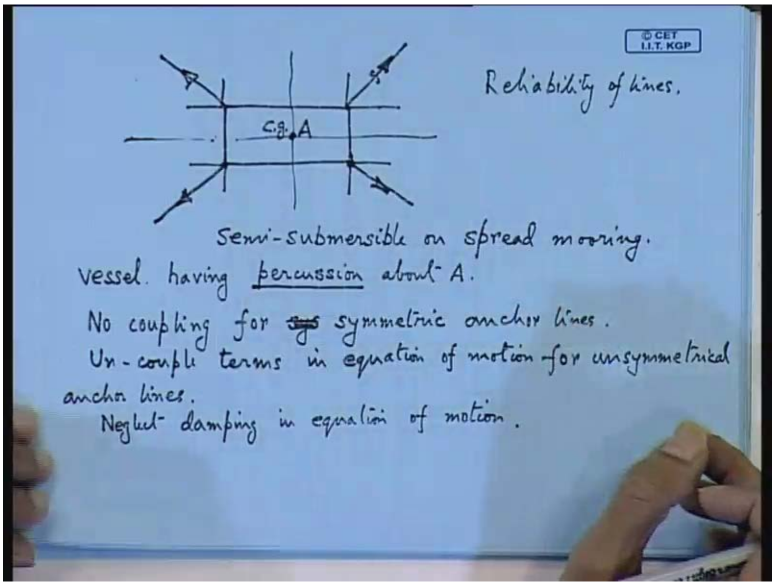

So, this is say, this is your offshore platform say (( )) t l p for that matter it will look like this say let us take a semi submersible t l p does not have a spread mooring system. So, spread mooring system you will find the chain cables for anchor lines being attached at the four corners say like this. So, this is the diagram of a semi submersible on spread moorings. This is your equation. So, one this now what you do you just makes an axis at the center and you also draw another axis at the anchor point. So, this is your local axis.

So that means, your cable tension is coming in this direction that is you come back to this diagram later on, but what I want to say is coming back to your the behavior of the vessel. So that means, what this is happening suppose your anchored like the semi submersible in this form. So, if you take the c g of this semi submersible say at the center say it will be located somewhere here. So, this c g it is once it is going in this direction then it is going like this so; that means, the vessel is actually going in this form round like this about this mean position A.

So, that is called vessel having percussion about A. Vessel is having a percussive motion about A this is going like this. The heave one I have not analyzed, but simply you have percussive motions. Now, the point that I want to tell you is that. So, this is how the vessel is anchored at the four corners. Now suppose, a storm comes though here actually, you will find that the equations are not coupled, whatever the equations that we find out from the equation of motions you will not get it coupled motion there is uncoupled motion. So, it is easy to solve. Suppose I snap one of these lines, anchor line snap a tsunami has come. So, in the it has actually happened in the navy floating boat, this was a floating boat and one of the anchor lines had snapped.

So, now you have to find out the forces say without this on the other lines calculate the maximum forces on the other lines. Now there you will have lot of coupling terms will come, because the anchor lines are not symmetric. So, you will find no coupling for symmetric anchor lines. Sorry, now the job actually becomes more difficult for formulating your equation of motion no coupling for symmetric anchor lines.

Now, we have to uncoupled terms, uncouple terms in equation of motion and this is actually more complicated. In equation of motion for unsymmetrical anchor lines vibration problem this is one aspect of the case. Another aspect you will find that, whenever you are finding out the natural frequency of this salt sewer or whatever it is, better not to take damping terms. So, neglect damping I will show you, I will give you the equation of motion for a damped system neglect damping in equation of motion. Now some of these problems, since this is not a sea keeping class many of you are interested in solving this kind of equations of motions in surge there is a chapter on sea keeping in there are some problems you can work out the problems. So, you neglect if you want to simplify your equation of motion this is the easiest job ok.

Now, the problem if you have, you will be more complicated. If you have a anchor line snap, you know anchor line snapped actually this is your clients they made require you to calculate with anchor line snap, because they want to check the reliability of the anchors, reliability of the anchoring. So, how many anchor line snaps the vessel can sustain we will find once, because your environmental forces will be there. So, you have redistributed the environmental forces on the remaining lines and you find out the maximum tension ok.

So, that is the how the procedure that is normally followed with the anchor line snapped so, reliability of lines. So, either they will tell you to calculate with one anchor line snap or two anchor line snaps, etcetera. Well of course, there will be a limit to that; you cannot survive with simply one anchor line. Otherwise then why you have given you so many anchors it is quite expensive, it not only that if you are going for this kind of catenary anchor. So, this distance can be as far as larger as say 2, 3 or 4 kilometers for semi submersible moored semi submersible. Just one example and here, what I was showing you only 4 lines, but actual practice you will find, you have 3 in one corner. So, this represent you will find out that, you have to do this for normally when you calculate the anchor strength of the anchor lines. You calculate on the basis of say semi submersible which moored at a particular location listen it.

So, extreme load conditions or other you write extreme 100 year storm condition. So, normally we are desirable to another, you have to do see the mooring calculations are not as simple. Because in the class you cannot go to, too much deep into the, because then we need to take the whole semester, but in normally if you go into your, this your offshore companies etcetera. There do I think orcaplex, you see this option is there with one snap line, you can study this with one line snapped, how much forces are coming etcetera.

So, extreme 100 years storm condition and normally the client will require, you to make a matrix. So, matrix you have to give him with say 100 years storm condition the maximum storm condition that is occurring this you have to get from sea spectra again the previous class. So, 100 year maximum wave, next we you have the 25 year storm criteria 50 years storm like this. You have to generate a matrix with say one anchor line snapped say two anchor line snaps, you do the calculations say in orcaplex with say four anchor lines, six anchor line, eight anchor lines and all this. So, you have to generate the designed data all right. Designed matrix has to be generated with number of options whenever you are doing some mooring system design you never give your clients say with say four anchor lines.

So, he will ask you that, you tell me what is the various configurations with various loading conditions. So, your extreme strong conditions now here actually what will come you find all this external forces are coming because of waves wind and current these are the normal forces are coming of course, on floating platforms. So, for all these spectra you have to calculate the maximum wave wind and current forces and these are actually submitted for at all this point it this is your point B. You find out all the forces that are coming at B C D and E. So, the job is not all that easy and we have to you know find out our matrix first with a number of in the designed situation design actually you find again.

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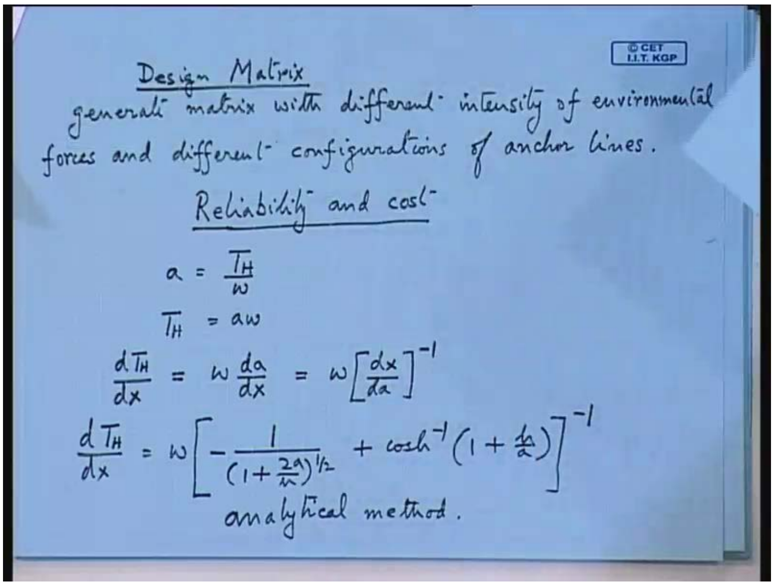

You have to coming again to your design spiral the designed situation you will find you generate matrix with different intensity of environmental forces. So, that is why you require the help of a software intensity of environmental forces and different configurations of anchor lines you this client is going to ask you about this. The client is not going to be satisfied because just giving one off answer. So, normally in your ship design or off shore structure whether it is be four anchor lines or whether you go for pipeline or whatever it is this he is going to ask for a setoff design options from that he has normally he selects. So, ultimate criteria is your in any offshore or should be is the cost criteria.

So, there is a tradeoff you will find there is always a tradeoff between reliability and cost. There is always a trade off now remember in the off shore business cost is very very important criteria, because there is only capital intensive venture a thousands of crows are actually pumped into your sea bed or your off shore platform. So, who is going to foot the bill so; obviously, there is always a tradeoff between this reliability and cost. Here you want the system to be more reliable then you have to pay for it and if you want to be less reliability then it is in lesser amount of cost.

So, this is always the situation. Anyways so, that is why the reason is they ask for a designed matrix the client. So, now, it is you have the softwares are all there. So, it is easy to design options. So, what I was talking about here we have found out the d X by d a term of course, we have made a simplification. And this is just one equation say, we are trying to calculate this in surge, but similar equations, we have to do for surge sway heave pitch and not only that. All this motions are going to be come together the ship will be having a surge and also it will be having a say pitch and at the same time it will be heaving so; that means, there will be number of forces will be acting on the anchor line. Which super impose one another are added and then you find out the total force. So, we will come to that.

But this the situation that is going to happen. So, d x by d a we have found out, now the point that we have to calculate is the cable stiffness. Now we have assumed that this a is equals to T H by w. So, now, you try to find out from this expression, what is the value of d T H over d X. So this is what? Now your w is constant. So, this is simple w multiplied by d a over d x , now this you can write as w multiplied by d x over d a to the power minus 1. Now your equation for x that we have got is, we have already found out d x by d a so, our problem is solved. So, now, you find out this d T H over d x. So, that is equals to w multiplied by what, this expression. So, minus 1 over 1 plus 2 a h just to the power half plus cos hyperbolic just 1 plus h a. So, this whole things is to the power minus 1.

So, this is the analytical expression we have got for d T H by d x. Now we are going to use this expression. So, this is called analytical method.

Now, where to use this expression, now you know what was the value of T H. So, T H you have to find out, there will be two terms for T H, one is a mean value, next you have to formulate the equation for T H.

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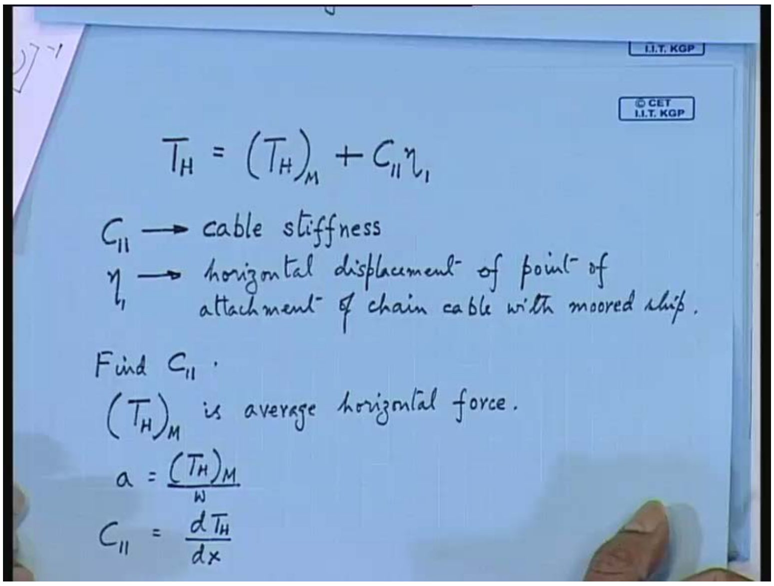

T H there is in one mean value, you write this as n and the other one is a variable term. So, this you write as C 1 1, eta 1. So, let us say this 1 1 stands for the research. Now this term C 1 1 is called cable stiffness. Then what is eta 1? Eta 1 is called, there is a term for this is the horizontal offset, but this is actually at the c g, no sorry, horizontal displacement you write. Horizontal displacement of point of attachment of chain cable with moored ship all right.

So, find C 1 1 and if you want to find out C 1 1, this T H M is called the average horizontal force, you have to make an assumption. So, T H M is average horizontal force. Now this you write this as a equals to T H M divided by w. So, you will find in this expression there is a second term, actually we have made this as constant listen it, this is the reason. This a, we have found out as average horizontal force divided by w. This is why d x look at this so, we have found out the C 1 1, what is the value? C 1 1 we have already found out, now C 1 1 cable stiffness you write this as d T H over d x. So, from this expression what is the value of d T H?

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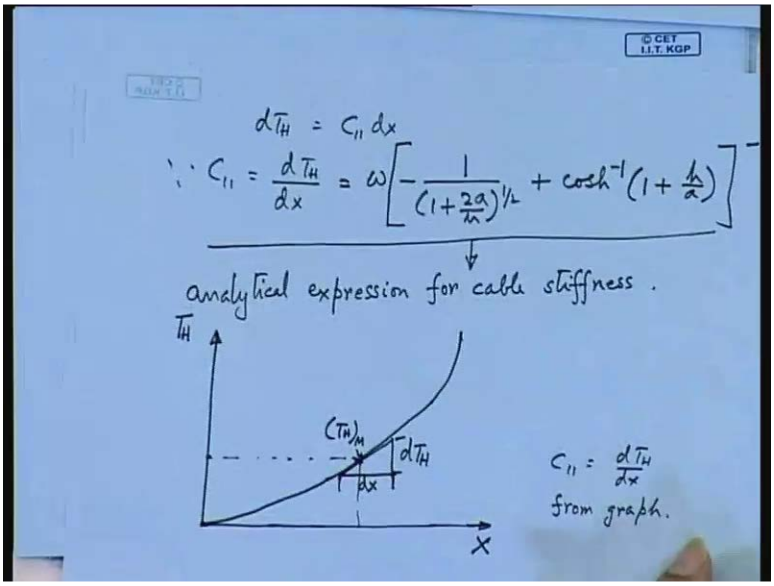

So, d T H is equals to C 1 1 multiplied by d x, now that is what you have written in this expression. You see here eta 1 is the horizontal displacement and this is the mean value, this expression. So, this is our expression for C 1 1 then. So, since C 1 1 is equals to d T H over d x. So, now, you have found out cable stiffness listen it. So, what is that expression? You have already found this out. So, w multiplied by minus 1 over 1 plus 2 a over h to the power half. So, this is called the analytical expression for cable stiffness. And do you of you who want to create graphically there is always a method so, this is called the analytical expression for cable stiffness.

Now, other one is the graphical method. Now if you want to do the graphical method you have to plot this values T H and x, now T H, we have the found out the T H expression, we will see look into your expression of T H for the previous notes. You will find then, you have to plot a graph like this and x you can find out from, can you find out x and T H what was the expression for T H. So, this you plot, vertical axis you plot T H and the horizontal axis you plot x and you will get a graph like this. And this will be some kind of an exponential curve.

So, T h expression, what was the T H expression? You have already derived one T H expression in last class; I told you expression for T H. That expression t, we have found out here, now T H you find out from T max, T max minus w h that is the easiest expression. So, from this you can find out T H and x, how you can find out x? x you use that expression l minus l s. So, this they are the two expressions you better use.

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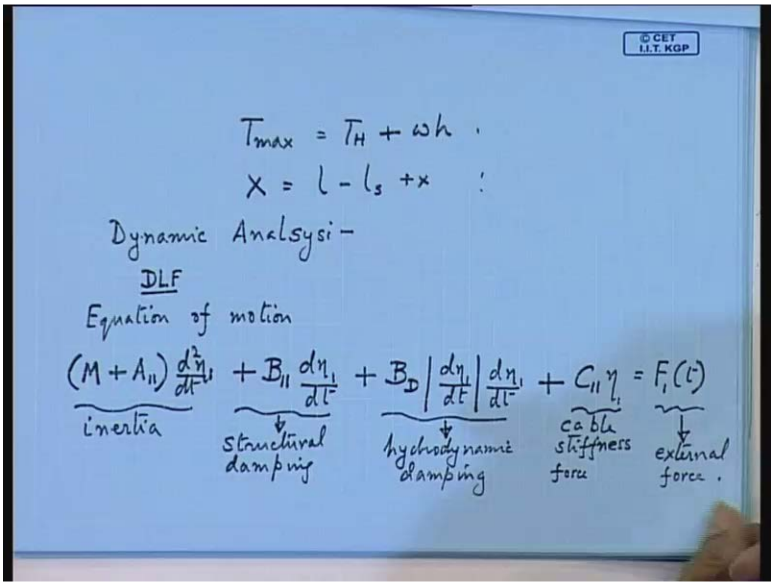

So, one is T H we can get from T max and T max we have already derived as T H minus sorry, this is plus and the h is negative. So, this expression we have already derived and x you can find out from l minus l s, what is the other expression small x, but you have already derived small x expression. And l s expression you have already derived c x minus x naught from that you can find out the expression for small x. Now you use these two equations and you plot your graph. Your graph will come something like this. Now you have to find out this value and this value you find out at a particular location of T H. So, that is why you find that a value? We have used T H M divided by w. So, this is always, a is always found with respect to a particular value of T H. So, you find out from here at one point.

You calculate the tangent, you draw a tangent at this line and then you find out this value of d T H and how much is this is your d x followed. So, you find out these two values from the graph at a particular point of T H. So, T H you can find out at say, in our case you can find this at T H M say, this is your T H M. So, from this you can find out C 1 1 is equals to d T H over d x so, this from graph.

So, the analytically expression, you can find out the correctness of this because, we have signified the other term. So, this you check by drawing a graph from the equations from the cable line equations we have got. So, this now what we are going to do with this C 1 1. So, we have found out cable stiffness from what now, there are three methods of calculations. One is called, first is the static energy then it is the quasi static and the other one is the dynamic analysis. Now in this book in the Ferguson book, the quasi static he has not done. So, first thing he has done this static analysis. Now from static analysis we are getting this values C 1 1.

Now, this you have to put as put your dynamic analysis program. So, the dynamic analysis is the more important and crucial. So, this you get an expression like your equation of motion. So, this is your dynamic analysis. Now, dynamic analysis in your vibration class what you calculate. Dynamic analysis dynamic analysis you come across a factor which is called D L F.

So, this you calculate D L F anyway the expression that, I am going to give you now is the very complicated one. So, say let us you find out equation of motion in surge. See equation of motion will come out like this any equation of motion. Now there will be one mass term that is your displacement of the ship plus we have to find out another term which is called added mass.

So, your surge motion you write this as eta 1. So, this your inertia term there will be other damping terms that will be coming out. So, B 1 1 is your damping, now damping actually there are two terms. One is the linear and the other is the nonlinear term. So, the linear damping is because of what the linear damping is coming because of your chain cables damping because of the structural damping that has come other term that is going to come and going to be make the equation much more complicated it is the hydro dynamic damping. So, that is called B D and hydro dynamic is the square of the velocity. So, d eta 1 is your velocity. So, this is d t multiplied by d eta 1. So, this is similar to the drag term this is called the hydro dynamic drag.

And the other one is your cable stiffness that is your stiffness force. So, this is C 1 1 eta 1. So, this is equals to F 1 t. So, this is your expression for one equation of motion. So, you take only the surge equation. But this is your, these are the terms. So, this is inertia this you can write this as structural damping, this is hydro dynamic damping. Now what is this C 1 1 eta this is cable stiffness or spring stiffness.

So, this is why you find that the anchor line is having a spring stiffness sometimes this is also called spring stiffness. And F 1 t is your external force. So, you can see the equation of motion. You also studied vibration now you apply this. Now you solve this equation by solving this equation what you will find, equation of motion once you solve what you get? Here you have to find out the dynamic displacement.

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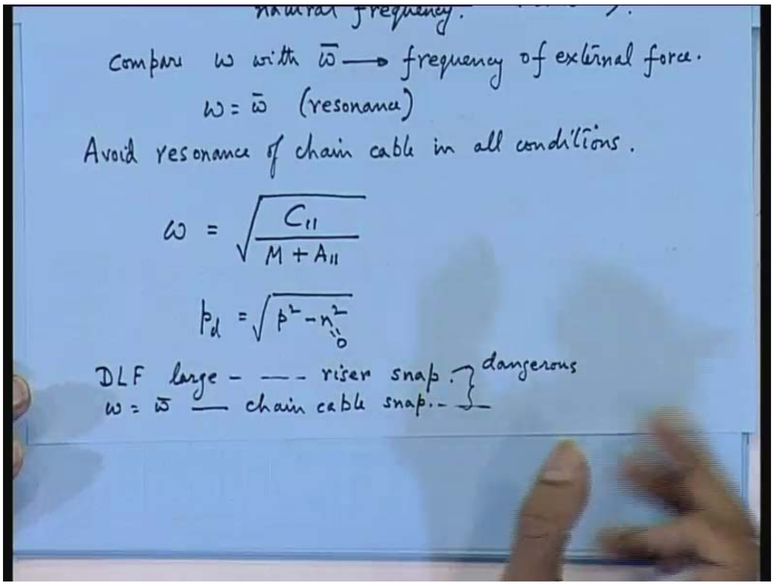

There are two terms. Find dynamic displacement or what else natural frequency this is very important. You find displacement and frequency, displacement is nothing but amplitude now why you are finding out the frequency. Compare omega with omega prime. So, this is coming from frequency of external force. Now when this omega is equals to become equal to omega prime then resonance will occur. So now, you have to avoid resonance of line. So, avoid resonance of chain cable in all conditions remember this, your system should not have resonance with external force and the natural frequency what is the natural frequency from this equation. So, what was the long equation I have given can you find out natural frequency from this vibration you have already done.

Now, for your simplification you just put B 1 1 and B D equal to 0. Then what is the natural frequency? So, natural frequency is say omega that you have calculated is root over C 1 1 over the added mass M plus A 1 1 any of course, here you have not taken the damping term. So, remember we worked out what was the p d damped frequency, root over p square minus n square listen it is. So, this is what we have not found, we have rather simplified our equations.

So, now you can find out his omega once you have calculated omega you can compare with omega. We will avoid resonance now, how we want to avoid resonance cable stiffness. So, you avoid resonance either by changing C 1 1 or M plus A 1 1, now which one is the easiest. Easiest is the variation if you can ready this C 1 1, because mass and added mass is very difficult to change. So, stiffness you change by you changes your cable tension. So, normally for this is what we have come across in take T M P also the cable tension they change in order to avoid the resonance both you change resonance the frequency has also your percussion excursion that amplitude of vibration. So, both are equally important why the reason is this actually for our calculation you have taken this as 0.

Now, the reason is, if you have this D L F large, D L F large will be a problem, because this will be to risers snap. Now omega equals to omega bar will lead to chain cable snap. So, remember these are the two very dangerous situations. The first one is pretty dangerous, the other one is now the riser actually we have only one pipe there are no distinct other risers took take care of the load, but chain cable we have buffered listen it; that means, the other chain cable can take the load risers snap is very dangerous the then you have position fire and all this will come. So, anyway with this we will end the discussion and after this I will just give you the some of the equipments that are used.