Elements of Ocean Engineering
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Lecture - 29
Static Analysis of Mooring Cable (Contd.)

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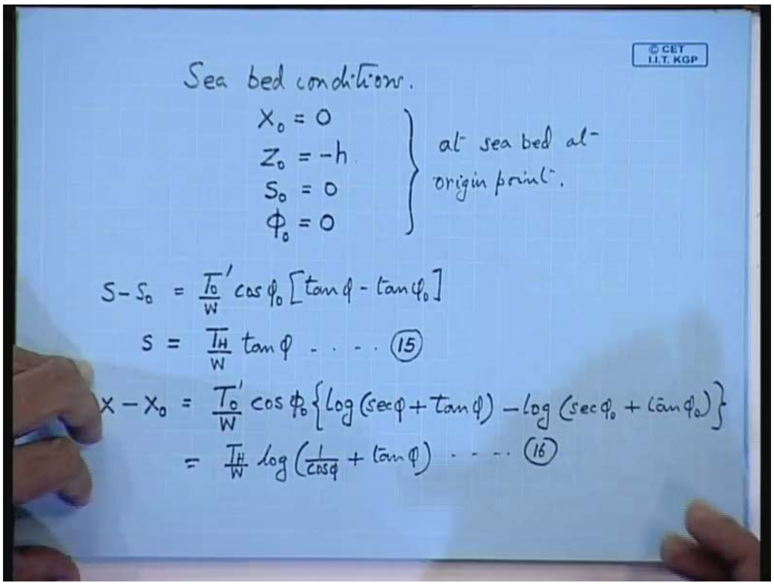

So, we continue our discussion with the mooring cable. So, last class, we I have derived all these equations. Now, sea bed conditions that we have said sea bed conditions. So, we have X 0 there is a horizontal distance that we are starting with X 0 equal to 0 and Z 0 is the depth at sea bed. So, this is equal to minus h one of the other conditions is S 0 that is the length of the chain cable at the origin. So, this is your the our origin point is here. So, this is your origin point so; obviously, S you bring it to 0, it will be 0 now S 0. We are starting this at equal to 0 and there is another option that is the angle that is phi. The angle of inclination of the tangent of the catenaries with the X axis is that is this is equal to 0 at the origin. So, this you can state these are at sea bed at origin at sea bed at origin point I think this I think it will make it clear actually.

So, from this we have derived the equations that are our length we have put the boundary conditions. And we have got that S minus S naught is equal to how much this is T prime 0 over W cos phi naught tan phi minus tan phi naught now if your phi naught is equal to 0. Then this; obviously, will be equal to 0, cos phi naught will be equal to 1. So, S naught is we are saying that S naught equal to 0. So, our expression simply becomes at sea bed. So, this will be and what about T naught. So, T naught we have derived in your earlier expression that T naught is equals to T H at sea bed. So, this T H is equal to T prime naught.

So, here we get. So, our equation becomes T H over T H over W multiplied by tan phi. So, this equation you can write this I do not know, but your serial number you check so in my 15. So, this is equation this should be equation number 15. So, this one equation we have got from the length of the chain cable. Now, from the horizontal distance you simplify you put the boundary conditions.

So, X minus X naught is equals to. So, this is your, I have derived actually in last class since most of you are coming in late. So, just repeating actually, but will start with this. So, this is sec phi naught so plus tan phi naught. So, this we have simplified into X naught is equal to 0. So, this comes downs to T H over W log of 1 by cos phi plus tan phi. So, this you can write this as 16 followed and the other equation that is the vertical projection we have derived this as Z minus Z naught.

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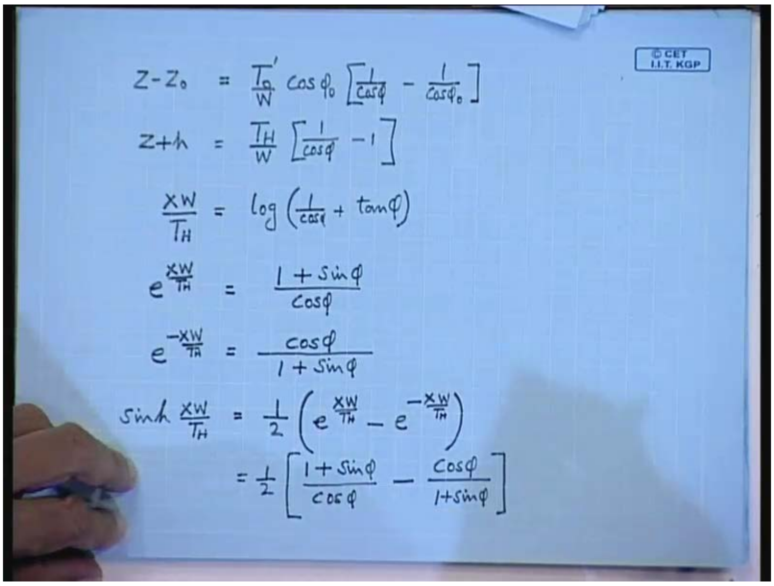

But Z naught is naught equal to 0. So, this you be careful when you are substituting Z naught equal to minus h. So, this is T prime naught over W cos phi naught into 1 by cos phi minus 1 over cos phi naught and here there is naught much of the mathematics. So, this becomes Z naught is minus h. So, this becomes Z plus h. So, this Z plus h is equals to T H over W multiplied by 1 over cos phi minus 1. So, after now, this is fine now you find out the expression. So, we have to simplify this tan phi term.

If you are our main goal is to obtain 2 things; one is the tension T and the other is the length of the chain cable. So, there are 2 things which you have to find out now, what is the formula for X W divided by T H X W? So, you use the second equation. So, this becomes X 0 is 0. So, this becomes simply X. So, from this equation you get X divided by k H is simply log of. So, log of 1 by cos phi the sec phi. So, this is what plus tan phi now from this you obtained 2 expressions.

So, actually if you simplify this, this will be e raised to the power X W divided by T h. So, this is equals to how much. So, this will be 1 plus sin phi over cos phi now similarly, you find out e raised to the power minus X W T H. So, this will be how much it is just the reciprocal. So, this will be. So, this is simple math. So, those of you who are coming late you just write this down. So, I am just using expressions from your previous expressions

Now, there is one important thing after this you find out expressions for sin hyperbolic X W over T H I have given you the expression for the exponential e term. So, from this you find out sin hyperbolic and cos hyperbolic these are 2 hyperbolic functions an then you simplify. You will simply write down the expression for sin hyperbolic X W by T H. So, those of you who do not know you simplify this. So, this is your half you substitute the values of e raised to the power.

So, what is the equation plus or minus e raised to power minus X W by T H. So, this is your expression for the sin hyperbolic term. So, now you substitute the values. So, this will be how much. So, this is half of 1 plus sin phi over cos phi and this will be minus cos phi over 1 plus sin phi you simply this and you get a very simple term.

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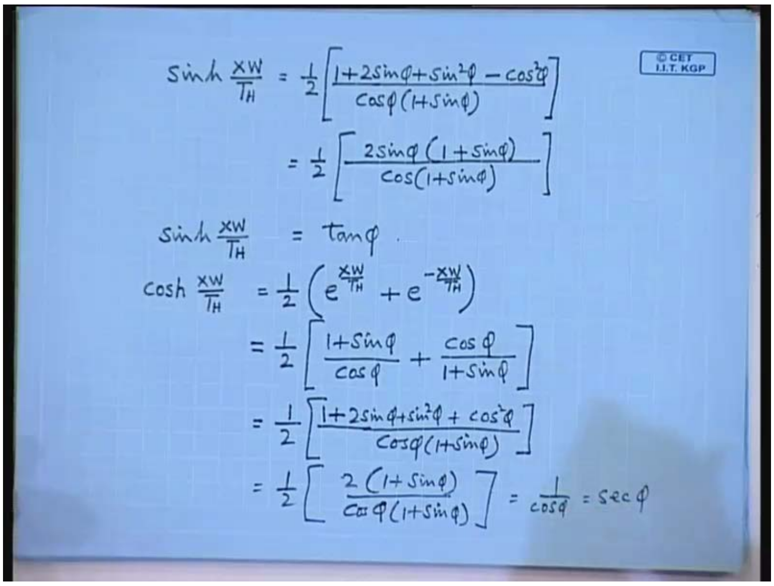

So, you just carry out the multiplications. So, this will be 1 plus sin phi whole square. So, this your denominator you will get this is cos phi multiplied by 1 plus sin phi. And on top you will get 1 plus sin phi whole square, this will be how much 1 plus 2 sin phi plus sin square phi and this 1 will be cos square phi. So, this we are getting half now numerator, what is the value of this? So, denominator is still cos phi into 1 plus sin phi. So, you will find that 1 plus sin phi, you can bring it out 1, why is sin square phi plus cos square phi cos square phi will go? So, sin square phi will be twice sin square phi is not it. So, you bring this out 2 sin phi out. So, this will be 1 plus sin phi. So, this is you are getting a very neat term.

So, here so half your, this will cancel; this will be simply so very neat equation. So, your sin hyperbolic now similarly, you find out the expression for cos hyperbolic that will also come up to a very simple term. Now, we will use these 2 expressions in our calculation. So, what is the expression for cos hyperbolic? So, this is cos hyperbolic X W over T H. Then your similar expression you use I have given you the expression for the e values, exponential values. So, this way instead of minus you write this as plus. So, there are plenty of assumptions we have to make here. So, the one that is of importance is the condition of the sea bed. So, you just simplify this and you tell me how much you get.You substitute the values that we have found out sec phi, you are right now this T H that we have drawn T H is balancing your T 0 component is not it anyway use that later on.

So, this if you break this down; this will be half of this is 1 plus sin phi over cos phi; this will be how much 2 sin phi plus sin square phi thus you are getting 1 cos square phi that is creating the problem. So, this will be half of 2 will come out. So, this is done now from this equation. So, what is the expression for S chain length?

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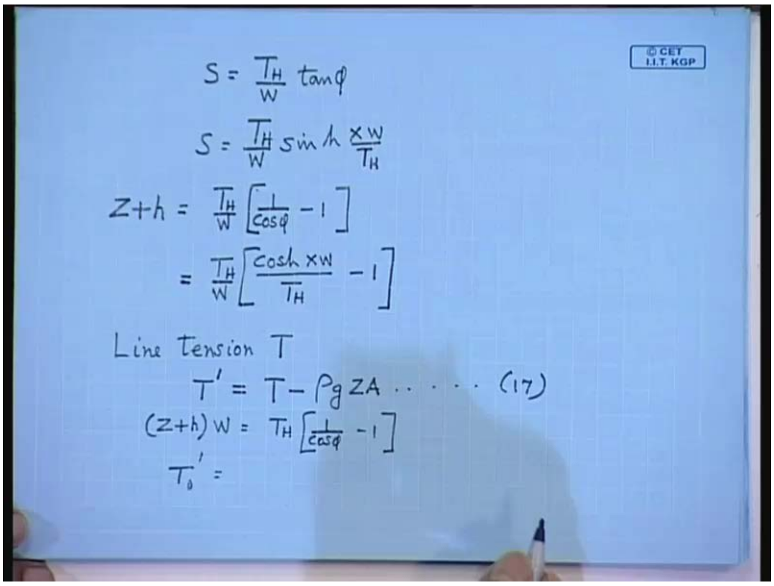

You look at your previous equations T H by W tan phi. So, S we have started out with after simplification this becomes this is that sea bed condition. So, this is T H now, what is the expression for tan phi? So, S equal to is this hyperbolic term that we will be getting out here. So, this is coming as sin hyperbolic X W by T h. So, we are getting rid of the, you are getting rid of the angle that is this phi, but in here you have to know. So, W is already known. So, that that you can weigh the chain cable per meter that will give you the W, W of the weight chain weight of chain cable per meter that is the most condition.

So, here, but this X is a variable now, T H; T H we know T H or not there is a horizontal tension. So, T H is how much T cos that we have found out from this expression T cos phi no if you know the angle you can find out that T H expression T W cos phi. So, only unknown is variable is X, if you know X you can find out length of chain cable. Now, this is one expression we can use and the other expression is; what is the expression for Z plus h? So, remember we started out with what was the expression. So, X minus X the other expression that we have got was Z plus h. So, that is again T H over w. So, 1 1 by cos phi, you replace. So, we are getting this as T H over W 1 by cos phi minus 1 now, you replace this 1 one over cos phi. So, 1 over cos phi is cos hyperbolic. So, you are just getting rid of the phi angle. So, 1 by cos phi is cos hyperbolic so but outside you write T H over W. So, this reduces the hyperbolic term. So, cos hyperbolic of X W over T H minus 1 so here what are the para unknown parameters? So, n T H W you know. So, X is the unknown term. So, H you already know.

So, from this you can find out Z. So, the now the other things that are remaining is line tension T which is our primary focus you calculate line tension T. So, I will just give you the expression your line tension T. You can find out from the corrected tension that is T prime is equals to how much T minus rho g Z a. Now, you find out from this expression and the other expression that you require. So, this is 15 16 and this say let us say this you put this as 17. The other expression you will be your Z plus h equation now from this Z, Z plus h equation you get Z plus h. This Z plus h multiplied by W is what T H you can write 1 by cos phi if you want to keep the angle out here.

So, if you want to dismiss the angle you put the hyperbolic term. So, this is 1 by cos phi minus 1 and the expression for T prime naught we have derived the expression for T prime naught. This is somewhere in last class, we have done that in now you tell me this in terms of T, T prime there are too many equations actually. So, in terms of T prime, what was the equation that we had calculated this is that water surface we are getting this as T h. But you have got this expression. So, this is your T prime naught is T prime multiplied by cos phi and what about T h? So, we have said T prime naught equals to T H is not it.

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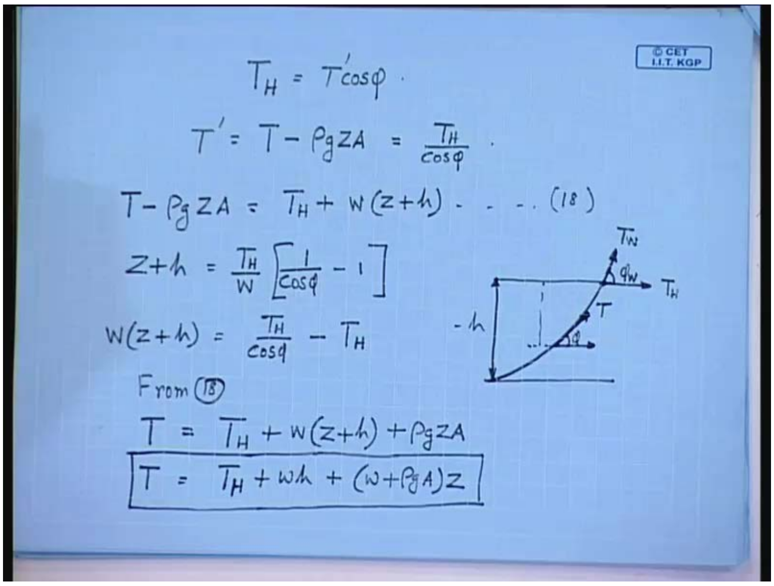

So, therefore, T H is also same expression T H is also T prime cos phi. Now, this expression for T prime is equals to T minus rho g Z A and this is equals to this T H over cos phi. So, now, we are getting this thing, but our still a problem is you are getting the phi term. So, you get rid of this phi term. So, how do you get rid of it? So, in 1 by cos phi is how much? So, we have dragged this on by cos phi that expression for that hyperbolic term will come cos hyperbolic. But then again this expression will come out T minus rho g Z a, but you do not write in the hyperbolic terms you write the hyperbolic terms this will be little bit awkward.

So, 1 by cos phi is Z plus h cos hyperbolic I want to get rid of this you can use this expression. So, this expression is say 15 16 17. So, from this we can use there is no hyperbolic term out here. So, then what happens? So, T minus this is equal to T prime T H by cos phi. So, what is this expression for T H by cos phi? So, this you find out from this. So, Z plus h is equals to T H over W 1 by cos phi minus 1, take it inside the bracket.

So, this will be W multiplied by Z plus h. So, this is equals to how much we are getting out here. So, this will be T H over cos phi minus T H this minus T H is coming. So, our expression becomes how much our expression becomes T H plus W multiplied by Z plus h. So, we have got rid of the hyperbolic term otherwise it will create problems. So, now, from this equation you can get T. So, what is the equation for T? So, ultimately so we are getting this is say equation number 18. So, from 18, we are getting the expression of T is coming as T H plus W multiplied b Z plus h plus rho g Z A.

So, we have got this expression now, we can simplify this further. So, this you try to bring this out. So, here Z is common. So, we can bring this out. So, this is T H plus W into H now Z is a common factor. So, this is plus W rho g A multiplied by Z. So, this you can calculate provided you know the parameters which should be known are W, W is the weight of chain in water rho g. Of course, the density and all that you know cross section f the cable you know. But you should know Z depth at which you are doing the calculation from there you will get the T.

So, T will come out somewhere here at a particular Z. So, this point so this is the Z point you have to find out cannot see now what I was talking about is at a particular. So, this is the expression that we are getting. So, if you want to calculate the tensions T at a particular Z value that is at a particular depth from the surface. Now, you can calculate that now, because all these A is the cross section area of the chain cable rho g. You know W; you know now, this H is what? H is the depth of water, the only thing that you have to find out is T H. So, T H is the W cos W phi bottom. So, this you can use.

So, now the other expression that we can find out after we have found out this the next point is. So, from this diagram this will make it amply clear. So, this distance is minus H is not it or you can write H whatever it is. So, now, your catenary is in this form now that T that we had found out at a particular Z value. So, you take a tangent at this point and your, this is your t. So, this is your angle phi followed and this is your T H now, what is this? So, this is your water plane. So, this is your T W and this is your phi W. So, what I am talking about is that you have to know this value depth. So, at this you can calculate the value of tension. Now, you find out the other point is T Z, T Z and then the parameter is length minimum length of the chain cable your T Z your Z direction is in the upward direction you find out the vertical component . So, how do you start?

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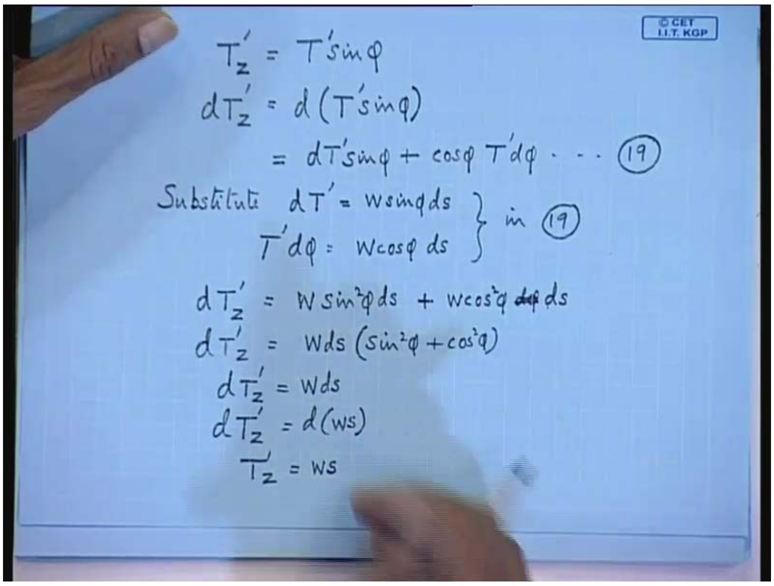

So, T prime Z that is the correction after the water pressure is T prime cos phi or sin phi T prime sin phi. So, this is your equation. So, now, you differentiate both sides. So, T of prime Z is how much you find the differential of this expression that is T prime sin phi now mind you both are changing. So, that is T prime is naught a constant. So, this will be how much? So, this will be d T prime differentiation by parts. So, the other term you keep it as constant sin phi plus what is the differentiation of sin phi? So, this is cos phi d phi will come and T prime will come now, you substitute this 2 expressions.

Now, at the beginning we started out with this d, what was the value of d T prime? That is W sin phi d s and the expression for T prime d phi was W cos phi d s and you have to use this 2 expressions and substitute. So, these your equation so the one that you had derived. So, that will be equation number how much? So, let us see this is our equation 18 and this is our equation 19. So, we substitute in 19 and see what happens. So, d T prime is this 1 and T prime d prime, you just simply substitute this.

So, substitute that in this 2 expressions in 19, but you will be getting only d T z. So, this because the first term becomes W sin square phi d s and second term so your T prime d prime is the again W. So, this will be again W cos square phi. So, this is W cos square phi d phi is not it, this should be d s. So, now, you can take out the common factor. So, d T prime z. So, you can take out W d s. So, you get some a beautiful equation. So, this is sin square phi plus cos square phi which is equal to 1. So, we are getting simply this as W multiplied by d s. So, d of T prime Z now you find out T prime Z from this equation.

How now, if you want to find this out you write this d T prime as d of can you write like this now although W is a constant you can take it inside so; that means, we are getting T prime Z is equals to W multiplied by S. So, in physical what is the meaning? Physical meaning is the horizontal component of the tension is balancing your weight. So, not the horizontal the vertical component of the cable tension and the horizontal component is balancing T H. So, W is a constant that we will leave out actually and you just simply integrate both sides.

So, here it will come. So, this is your differential actually. So, this expression you remember. So, if you integrate the differential you will get the original equation. So, this is equation number how much. So, this, whereas say this is equation number 20. So, 2 important relationships we have derived 1 is your tension T and the other is T z. Now, the other component that is remaining to be found out is the find minimum length of chain cable.

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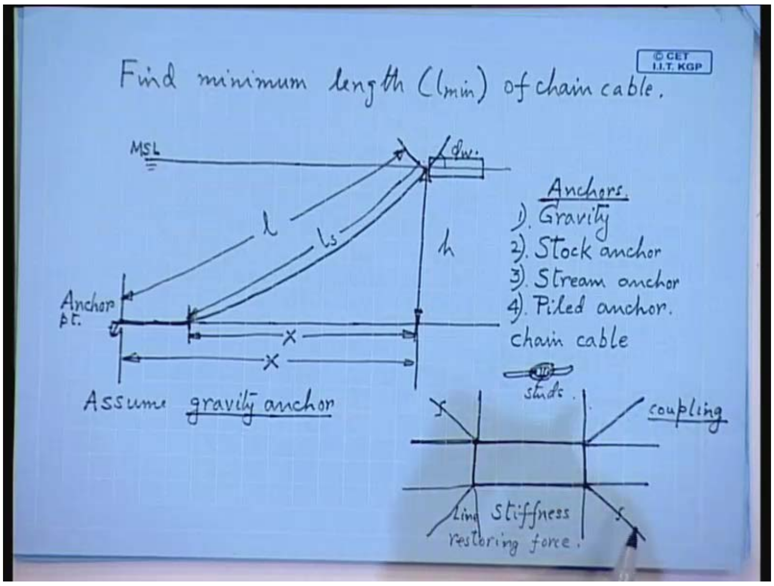

So, you just tell me when the minimum length will occur or when do you get the minimum length? When there is maximum tension in the chain you can just take your key chain and see you just pull your chain. And then the, if you increase the tension; obviously, the length of the catenary will shorten is not it. So, here also similarly, you will get this equation now let us try to derive this…Find minimum length. So, you have to calculate this l minimum now remember we have already found out the expression for S. So, you use that expression. So, this is minimum length l minimum of chain cable. Now the catenary say this is your mean sea level or the water line now your ship or barge whatever you have taken is floating at the mean sea level. So, like this.

So, let us say that this is your mean sea level now, the chain is hooked up at the anchor winch. So, it is assuming some kind of a form like this now at this point it will start lying horizontal on the sea bed so; that means, to some extent it will come horizontal and then you have anchor. So, this is your angle that is if you take the tangent out here. So, your angle is becoming phi W. So, l s is this length you start from here do not start from the anchor. So, this is your anchor point now l s is this 1 that is the length of the catenary, this you take from this boat. Now, what is your one if this be the length of l s? Now, what is your l? So, this distance is your l now, you take the horizontal and vertical distance. So, your depth of water is H you give the proper sign also.

So, here so this l is X actually in the derivation of the, of our formulas that is our expression for tension there are too many substitutions we have used is not. So, it is better you work this out in your hall otherwise in exams you will have trouble. So, this of course, is taking this also as X and this diagram we will have to use. Now, there is one assumption is you assume gravity anchor now, what is the specialty of this gravity anchor? Now, there are number of types of anchors you will find in your the main engineering terms if I have 1 is gravity anchor. The other anchor is called anchors can be of different types anchors you have first is gravity.

But normally this gravity anchors are use only for an anchoring say offshore huge large structure, say offshore platform T l c s. And all these thing number 2 you have the, what is called as stock anchor. So, sometime in the, this class, you just remind if I get the diagrams of this type of anchors and I will show you. Then you have stream anchors these are the smaller varieties of anchors this is only to be employed in rivers.

So, these are called stream anchors well last 1 you will find 1 2 3 they are called piled anchors. The anchor is actually a pretty common term in civil engineering. Although you have in naval architecture we mean that anchor means you are dropping a weight on to the sea bed which is actually clawing the sea bed or going inside the sea bed and giving you horizontal force, but anyway. So, this, these are the various types of anchors you can use now anchors when you use these anchors. They are to be used with a certain what is called chain cable I do not know whether you have this subject on this thing mooring systems in your ship design. You will find chain cables are really the not your this key chains, it will look like this I will give you the diagram. So, here you have this sort of ring is going to come. So, these are called studs.

Now, mooring systems, if you want to analyze then it is naught just one simple catenary chain normally ships offshore the platforms. They are not moored by 1 chain cables you have say I have a chain cable out here or you may have as many as in 1 corner of the say boat or 1 corner of the semi submersible. You may have as many as 3 chain cables; 1 in this direction, other in this direction. And this direction you can never moor a ship with 1 chain cable; that means, the offshore platforms are literally held in position by this sort of this is called a spread mooring system. Now, you can just understand the mechanics the trouble we had with only 1 chain cable.

Now, if I give you a configuration like this you have to calculate the total forces that are coming on to the structure the mooring forces. Or the restrain forces coming from as many as how many as many as 12 chain cables. And not only that nowadays actually the you all the naval architect you will find they ask you that you find out whether the ship will be able to be in position with say 1 chain cable snapped say I cut 1 of the chain cables or I cut this 2 chain cable. So, will my ship still be holding in this position or it will start drifting away?

So, that is another part of mooring analysis which is done. So, in mooring analysis the picture is not this simple just the equations that I have worked. But first of there are 3 types of analysis that I have told you the, this is the first one that you do is the static analysis then quasi static and then dynamic, but here there are actually. So, many chain cables and here you have to find out the, what is called the coupling of all these forces coupling terms will come.

So, this we will try to I will try to analyze this say in later classes. So, you the whole mooring system if you want to analyze you have to analyze all the chain cables and all the forces coming from all these chain cables to the ship and whether you are able to restrict the response of the ship sufficiently So, this all this chain cables will very exert what is called a springing force you come to this we have to calculate stiffness.

So, this called line stiffness; you have to calculate from this you calculate the spring force or the restoring force. So, you can see the mooring analysis is quite complicated, you just in your class the example we just dealt with only 1 line. But ultimately you will find you have to make a dynamic analysis with all this chain cables. So, normally if you start doing this expression you land up in a sort of a finite event ideologies or matrix form of equations. So, ultimately you will come into a FEM analysis.

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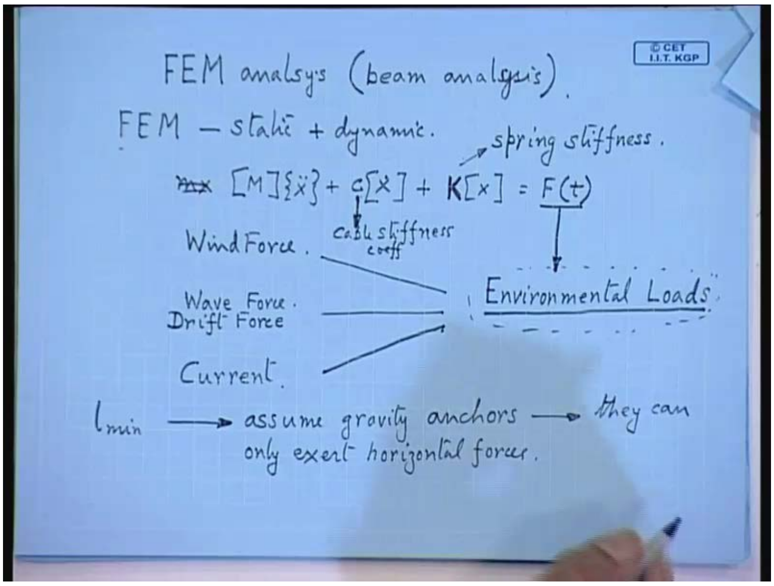

Now, these FEM analysis is also not very simple. Because in the, this in this we have assumed that there are no hydro dynamic forces acting on the cable line. That means, we have vanished the F and d term you remember in the beginning when you started simplifying. So, that is also a very dangerous assumption, but since for this is simplifying the case. Then we think that that is equal to 0, but all the hydro dynamic terms will also come and then you do what is a normally done a FEM analysis. So, this is called a FEM beam analysis I will try to give you some exposure if you have some time.

So, analysis this is your term. So, FEM beam analysis has to be done for all these chain cable and not only that there will be coupling of all these terms. So, nowadays the present method is you do straight away you start doing FEM. So, FEM is you do this static as well as dynamic analysis; you are with lot of your restoring forces. So, in our equations you have to start up if you look at your dynamic analysis. So, the equation is now all these you get in matrix form.

So, this is your K X now, the right hand side is what now this actually we are in our calculation we have not calculated this have you thought about it here. So, we assuming a winch tension out here is not. So, with that we had calculated, but out from where this winch tension is coming. So, this mooring analysis is usually done from F t calculation that is your environmental loads. So, this is a. So, you have to find out this first environmental load calculation. So, unless you do this all these equations that you have done is useless. So, we are assuming a certain winch tension of course,, that is this is related to what T W and T H this is maximum.

Obviously, this is maximum winch capacity is not it that we are assuming, but a winch capacity calculation you have done from what this equation. So, this is your starting point. Now, environmental loads you will find. So, there are the number of environmental load. So, here I do not know in your hydro dynamics class you will come across. So, first one is what is your M of the ship is your wind force now ships are normally floating in water at the water line. You will have wave force that I do not know whether you have come across this another force which is called a drift force. So, these are you environmental forces and down below you have current.

So, these are your environmental force calculations. So, this F t term you have to find out from this forces. So, that is your starting point. And this c c I will talk about later on your c is yours is called a cable stiffness coefficient and how you find out K, we will come to this k is called your this is called spring stiffness. So, you are the whole mooring system has this some kind of a springing action. So, this called a spring stiffness. So, all this stiffness is you have to calculate it.

So, anyways so you start from here then you find out this K c and M. Of course, you know from the disposition of the cable, but your file name is FEM static and dynamic analysis. So, anyways so that will be quite complicated, but we have to stop our discussion with this. Now, the minimum length of and minimum of chain there is a the last item we have before we complete it this has to be calculated. Now, the assumption that we are making is gravity anchors. Now, gravity anchors the one important, the main item point to be remembered about gravity anchors is that they can only exert horizontal forces. So, this we will use this in our calculation.

So, those of you who are interested they can look any ocean engineering journal for mooring and anchoring systems mooring. And anchoring systems are very important for offshore structures. The other important point that you want to find out is is piping I do not know whether you know anything about piping or not. But piping is actually the realm of your mechanical engineers, but some portion you should know. So, unless you know that you will not be able to design your risers or your conductor pipes piping layout it is quite complicated. So, we will break for some few minutes and then we will again do this calculation.