Elements of Ocean Engineering

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Lecture - 28
Static Analysis of Mooring Cable

So, we will continue with the discuss analysis of static cable. So, we are doing the mooring analysis. Now there are the 2 basic equations that we have derived last class.

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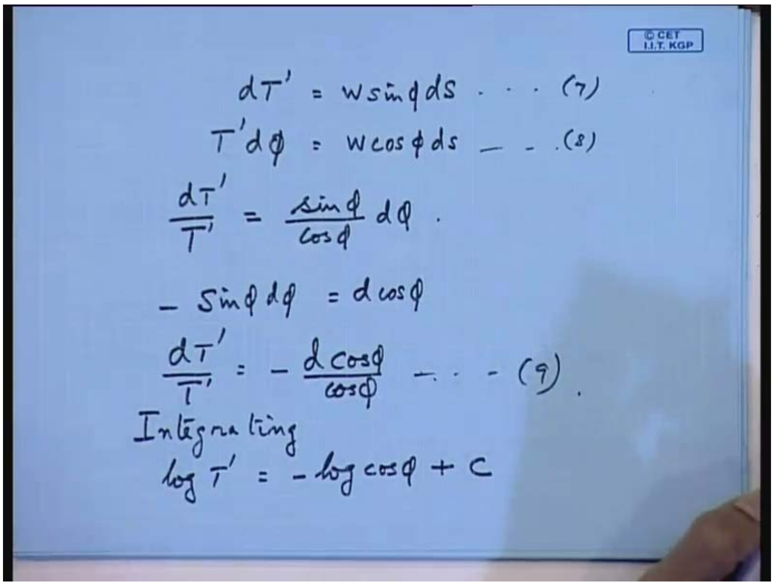

That is your d T prime, this is equal to W sin phi d s and the other one that we have derived was T prime d phi is equals to W cos phi d s. Now if you divide these two you will get d T prime by T prime you write in this form sin phi over cos phi. So, this we have derived. Now, in this equation this you integrate now sin phi d phi; you can write as this I think you can write this as d cos phi let us put a negative sign. So, this expression becomes. So, minus d cos phi over cos phi so now you can integrate this. So, this will be your expression if you integrate this will be log of T prime this will be minus log cos phi I have to find out the constant C plus C. So, this is your expression if you integrate this expression I think this was equation number 7; this was 8 probably you just check and this was 9. Now, starting from our, this chain cable.

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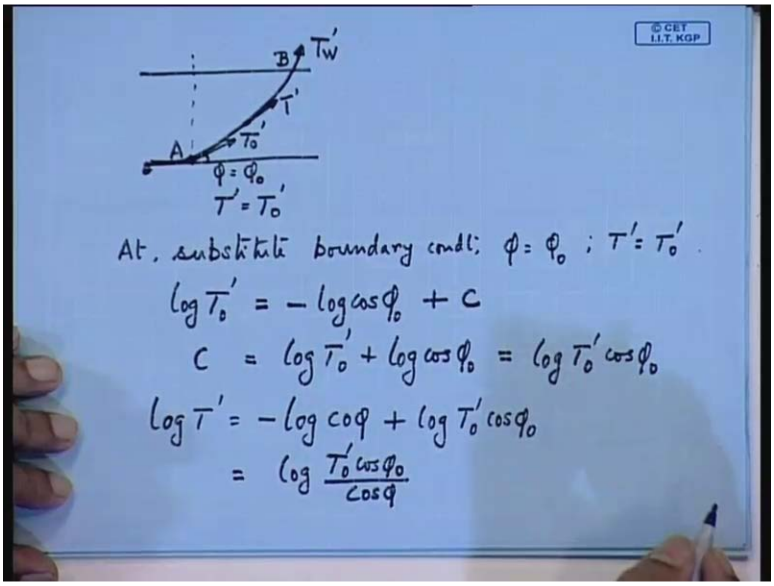

The diagram that we had drawn now, there are 2 points which are your boundary conditions. So, boundary conditions will be at the sea bed under the water line. So, this is your chain cable it is assuming A, it is a shape of a catenary and at this juncture it becomes horizontal with this sea bed. So, now, your, it is anchored at this point. So, this is your T, T prime there is a tension in the cable and out here at the water surface this becomes this is T W or T prime W just at the water plain it will become that. Now, here at the, this juncture it meets the sea bed. So, if you draw a tangent and the tension will be T prime subscript 0 and this angle will be phi equal to phi 0. So, these we have to these boundary conditions in our problem in order to get the expression.

Now here at this point this will be T prime will be equal to tour prime 0. So, now, let us find out. So, here are the 2 objectives we have, we have to find out in previous diagram with d x value. This is what we were aiming at first you find out the length of the chain cable then distance along the x axis and along the z axis. So, we have to find out the length of this chain cable and this horizontal distance is x. And we have to find out that the length of the chain cable from here the total length you subtract the portion that is lying in the seabed. So, this will be a 0. So, the total length is x minus X 0 you have to calculate this find out x h. Of course, you know and calculate the tension T. So, this is the flux of the problem what you have to solve.

Now, the equation that you get out here so you give boundary conditions and boundary conditions are the one that I have restricted and here you substitute. So, how much what is your equation? So, the equation that we have got is log T prime. So, you say this is at A and this is let us suppose this is A point B. So, you at A, you substitute boundary conditions. So, boundary conditions are phi equal to phi m and another is T prime is equals to T prime dot. So, then what happens to this equation.

So, log of what you will get say log T prime naught. So, we have to calculate the value of C. So, this is minus log cosec phi naught. So, now, you can get your C naught from this T if you have T naught and phi m so but how to calculate you know these values. So, this is see T T naught actually it will be slant at this point that is a horizontal force or tan curve and the inclination of this point. So, those I think you have to find out from instruments otherwise you cannot measure.

Now, here so from here value of C what is the value of c. So, this is log of so if want to analyze this chain cable see you have to find T prime naught and phi naught. Now, what is normally done in this you have to have certain instrument? So, you have to do some line instrumentation; you calculate the horizontal force of tan curve at this point and T prime you can put some this thing what is call stargaze out weal and calculate the friction in at others various points. So, normally in various things you know various structure or this they do this kind of instrumentation the similar thing you will find at the 2 end find out the stargaze in piddles. So, I think probably in your affected also you will talk about the severe piddling there also you have 2 instrument.

So, here you so now, we are getting C from this. So, this you can simplify then what is expression? So, this will be log of T prime naught whatever by cos phi naught A into B. So, now, you are getting the expression. So, your expression is coming like this log of T prime. So, this is equals to minus log cos phi another value of C. So, C will be log of T prime naught cos phi naught. So, now, here you can simplify. So, this will be log of T prime naught cos phi naught over cos phi. So, now, we have simplified this. So, from this equation you can calculate your since both sides are log.

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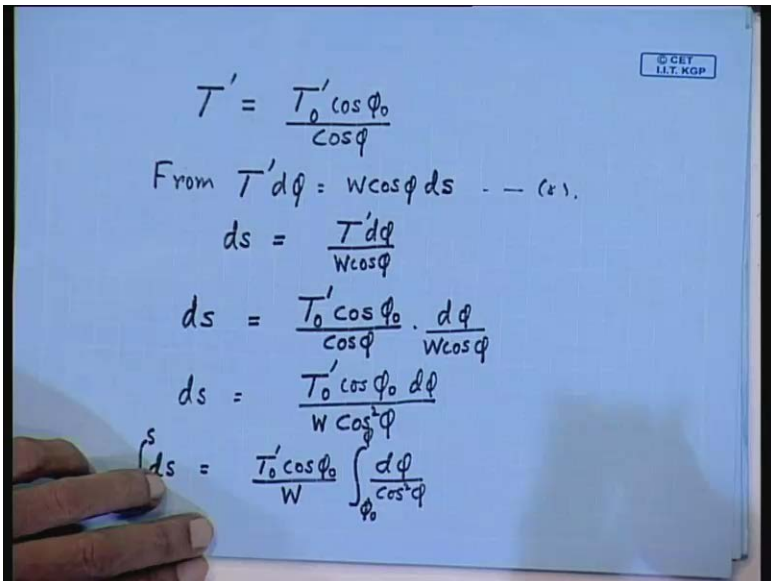

So, this T prime that is your tension becomes equals to this is T prime naught cos of phi naught over cos phi this is your expression for the tension at any point of the chain cable now this prime indicates what real of the first equation that we had done what was the significance of the prime actually we are connected for hydrostatic pressure. So, if you look at this equation. So, what I have written. So, what was the correction this expression. So, this expression was your T prime so; that means, that is just the correction for you hydrostatic pressure I think I would. So, this is your collection for T prime is the tension in the chain cable corrected for hydrostatic pressure. So, this is expression, and remember we are taking the weight of the chain cable that is taking into account to your bouncy the bouncy term. So, that is the net weight which is acting downwards.

So, now the other terms that are coming is we have the first equation that we started out the 2 most important ones are this. And these two expressions you should remember the other equation will follow from this now from 8 you write now, we have to calculate the length of the chain cable. So, you may use this equation. So, from 8 write the serial numbers you just check, because if you put down the correct serial number. Now, what you can find out from this is the length of the chain cable d s. So, this is equation 8. So, from here we can calculate the first you find out the expression for d s. So, d s is d prime d phi divided by W cos phi now, you integrate this expression; you put the boundary conditions and integrate now T prime what is a expression for T prime.

So, T prime you substitute this expression. So, d s will become. So, max here I will the max is not that difficult. So, you can easily follow the only thing we have certain spells od integration that is all. So, in differential equation are not comb. So, this a very simple analysis so this will be your d phi over W cos phi. So, now, your d s is coming as cos square phi. So, if you want to find out S you just integrate this, the right hand side W cos phi. So, 1 W comb will be there now it is. So, now, you integrate this expression. So, we are interested in calculating the length of the chain cable, but for which portion that is of the catenary portion.

So, you put the limits of integration as S and s naught. So, this will be you just integrate this. So, what are your variables? So, you bring this the T prime naught that is your initial condition at phi naught. So, this way you are supposed to know this from instrumentation or from physical examination of the chain cable by diverse going down. So, this you put this divided by W and this especially you integrate between phi naught and phi. So, that is d phi over cos square phi now if you integrate you check your integration terms for this will be…

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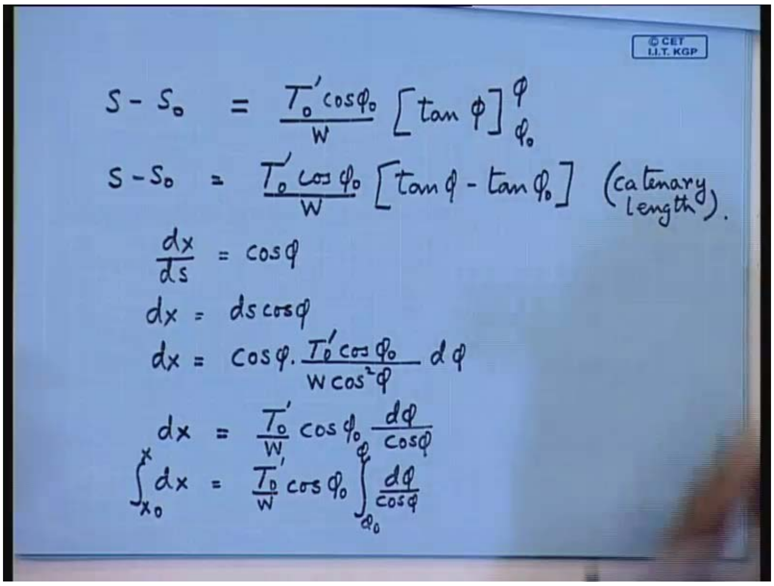

What is the expression for integration of cos square phi? So, this will be S minus S naught and at right hand side you will get tan phi. So, do the few you just check on any book on integration and differentiation you will get this. So, this will be from phi phi naught. So, ultimately your equation is coming like this S minus S naught this is equals to this will be cos phi naught over W. And this expression will be tan phi minus tan of phi naught for you are getting this expression. So, this is the catenary part you write in brackets catenary length now, please do not talk this is your catenary length. Now, you find out the horizontal component of x say; this is your expression for your x the horizontal total and again your X naught you subtract from the ankle length. So, this is your X naught. So, you find out this distance. So, this distance is X minus X naught. So, what is the expression say take a element of length about this line.

So, you call it d x above and this length is say d s now can you get the expression the angle is you write phi it is a phi angle. So, from this you can calculate d x. So, that is d x is d x cos phi good. So, the expression is d x over d x. So, that is equal to something cos phi. So, from this d x is equals to d x cos phi. So, now, you substitute the expression for d x. So, what was the expression d s that we had got? So, d s was this expression.

So, you write cos phi you bring it forward. So, multiply it by d s. So, d s is T prime naught cos phi naught W cos square phi naught. So, this again we multiplied by d phi now you simplify this how much we get and then you integrate. So, this becomes T prime naught over W is this the constant term the other term that you will get is the variable term is d phi over cos phi . So, now, you integrate if you integrate the expression for x. So, we can write in this form. So, that is X naught X d x. So, this will be T prime over W cos phi naught now you integrate this expression d phi over cos phi. So, this is your phi naught to phi. So, you integrate his you will get X minus X naught. Now, what is the integral of this 1 by cos phi?

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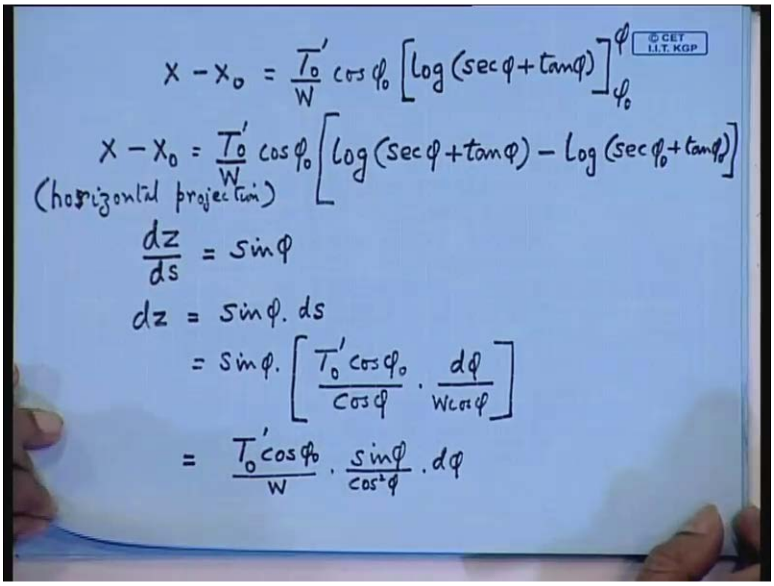

So, X minus X naught is equal to. So, integration of 1 by cos phi how much? You check your again your, you will get this expression. So, this is little bit large. So, this minus X naught is now similarly, you find out you calculate Z minus Z naught you calculate Z minus z naught. If you want to calculate Z minus naught then what is the expression that you should hey [fl] stop talking you find out Z minus Z naught I have shown you how to calculate X minus X naught and after that you calculate the tension t. So, what is the value of d z over d s? Now similarly, we have started in this expression that is d x by d s. So, it is just simply cos. So, this will be cos or sin. So, this will be sin phi.

So, now, from this expression you can calculate Z minus Z naught that is z particle projection of a cable line. So, you have calculated the horizontal projection that is x minus X naught. So, this is called the horizontal projection of the catenary this is the horizontal projection. So, this expression is simple you take d z over d s is equal to sine phi. So, from this expression if you integrate what you do substitute the value of this d s. So, your previous expression what we have substituted d s was this is sin phi multiply by what was the, you find out what is the expression for d s? Now remember in doing expression for d s we had substituted T prime is not it?

So, ultimately with T prime naught that is at the sea bed we are getting this now, you do this do the integration is it expression for d s is it right. So, now, you what it is called? So, you take out the, your constant terms outside. So, this is coming as dived by this is cos phi naught then the other term that you get is sin phi. So, we are getting sin phi out here. So, this will be cos square phi, cos square phi cos phi will not be cos phi squaring again d phi. So, if you integrate this you will get Z minus z naught. So, that is your vertical projection of the catenary, so that here if you integrate Z minus Z naught integration will be d Z.

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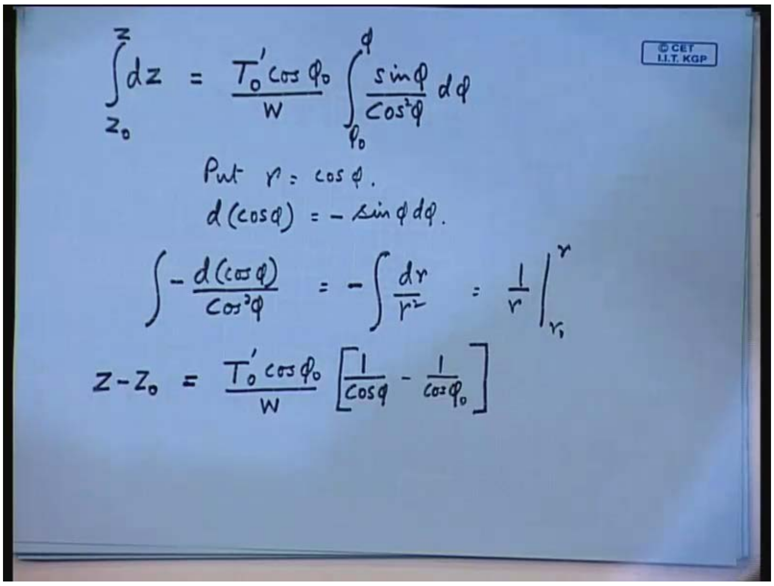

Z Z naught and what is your after integration; what you will get for this? So, this will be phi naught limits will be phi naught phi now you integrate this expression. So, integration is if you remember here you put simply put sin phi d phi equal to this expression put r equal to cos phi. Then you will get what is your d of cos phi it is minus sin phi d phi. So, this expression you can write, but you change your limits by phi and phi naught. So, this expression will be integral of how much say minus d of cos phi and this will be cos square phi.

So, now it is simple. So, this will be how much? So, now, can you do it? So, if you do this, this will be 1 by r. So, our expression is coming as Z minus Z minus z naught. So, this will be how much. So, this is the expression for Z minus z naught. So, has to be you have found out all the items, we have found out the length of the chain cable is 1 is S naught. Then we have found out x minus X naught your x minus X naught is not delta your x minus X naught you have found out. So, the most complicated expression is the horizontal length on the sea bed is not it? This one because you are having this log term the other is the Z minus Z naught now, you calculate your tension.

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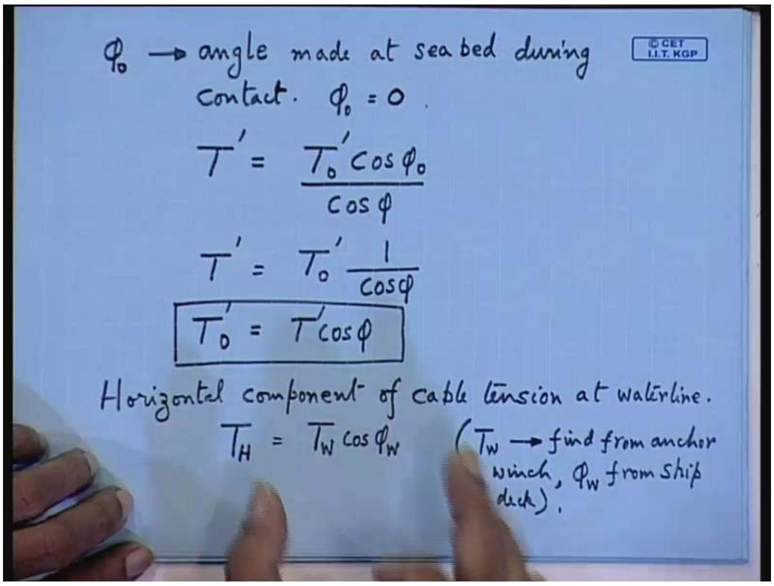

Now, phi naught phi naught is the angle made at sea bed you write during contact. Now if you want to simplify this equation; you can write phi naught equal to 0. Now, what was your expression for the tension corrected for your hydrostatic pressure cable line tension? What was the equation we had got remember that first we started out with? So, I have given you the expression for d T prime is not it? Then from that we have derived the expression for T prime. So, what was your expression for T prime? So, T prime you have got as this expression this is T prime naught cos phi naught over cos phi. So, this expression was coming from that 2 equations that d T and T prime equations which we have divided and we have found out.

So, now can you find out from this expression here you substitute this phi naught equal to 0 since we are signifying this you put. So, cos phi naught is going to be one. So, your expression for T prime coming as T prime naught over 1 by cos phi. So, this and this expression you remember now, next you calculate horizontal component horizontal component of tension at water line. So, what was the horizontal component at seabed? So, horizontal component with tension you write at water line that is your winch; winch tension was coming, how much winch tension you have written this as T prime W now how much is the angle? So, angle was given as phi W. So, here you find this is T H. So, we will come across a simple tension you know it will come from free body mechanics.

So, how much is the horizontal tension cable? It is quite simple you just take the component. So, at water line the winch tension is T W. So, how much is T H T H; obviously, will be T W multiplied by cos phi W. But you have to know both T W and phi W now this T W and phi W. You have to know T W is actually you can straight away find out from anchor winch, but phi W how are you going to find out. So, phi W also you have to find out from ship dick. So, this is fine now, what was your expression for T prime?

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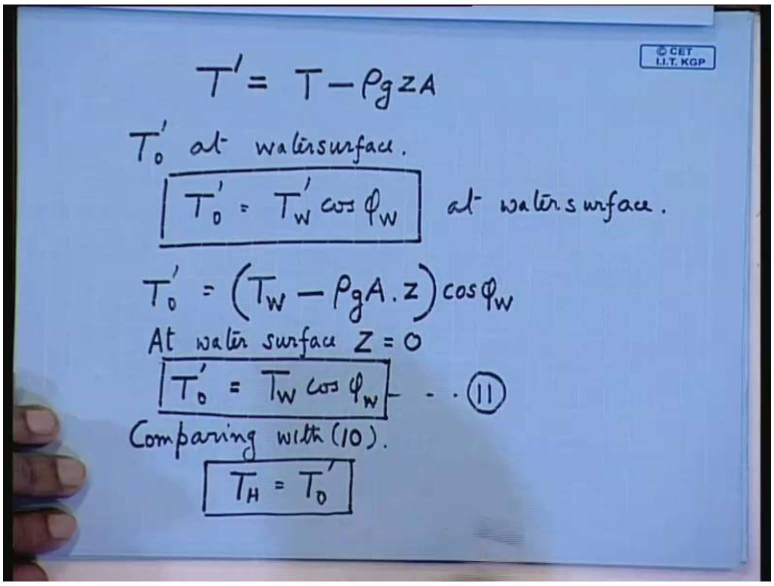

So, we will come back to this equation that is our T H later on, but we you tell me the expression for T prime. So, T prime was the corrected tension minus what the hydrostatic pressure. So, rho g Z A, A is your cross section area of chain cable now here what is the expression for T prime naught you tell me this T prime naught at water plain what are the expression for T prime naught. So, T prime naught is T prime multiplied by cos phi now water plain what is going to happen this expression of water plain if you write.

So, this T prime naught will be equals to T prime you just simple put subscript W. So, this will be cos phi W. So, this is your at water plain or at water surface you simply put T prime W at here and cos phi W that is all now you compare. Now I have to water surface this expression that is T prime naught now, you just see that you have put a prime out here; that means, you have corrected for hydrostatic pressure. So, this will be T W minus how much you write the expression and you then you put Z equal to 0. So, these expressions this is cos phi W.

So, I have simply written the expression for T prime naught now, at water surface at water surface what is the value of Z, Z at water surface is 0. So, then your expression for T prime naught becomes what? So, this is simply T W cos phi W now, you compare the expression the expression that we have obtained earlier. So, what we are getting now? So, previously we have obtained T H is equals to T W cos phi. So, this you put some number out here. So, this is 9 or 10 whatever we are forgetting. So, compare with 9 this is 10. So, this you write this as 10.

So, compare with 10 how much we are getting? So, you write this as 11 comparing with 10 we get T H equal to T prime naught. So; that means, what is our inference that is we are balancing the horizontal force at d follow so; that means, in your free body diagram. So, your if your T H is in this direction you are simple getting T naught in the opposite direction. The horizontal forces has to be balanced what about the vertical forces that T W sin phi is used to balance the weight now, what is the condition at seabed?

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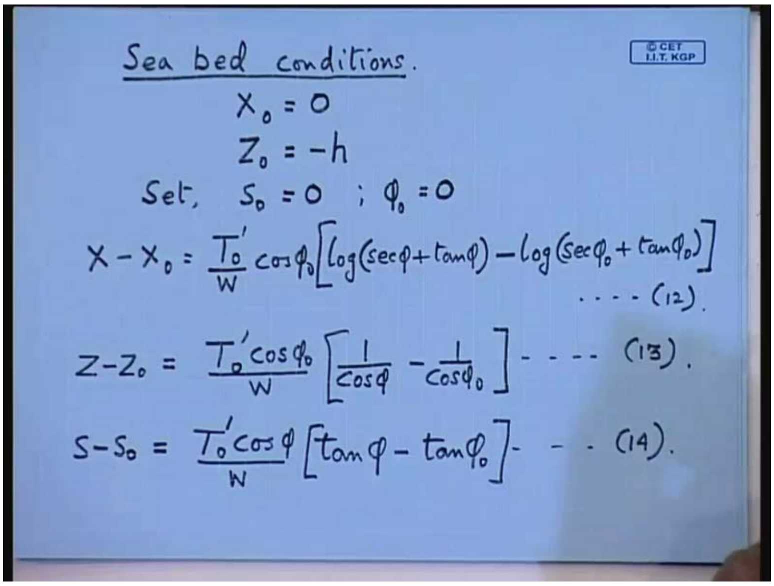

Now, you choose the conditions at sea bed your mathematics is not that difficult, but you have to work out. So, X 0 you put this; you started from X 0 equal to 0 now, what is the value of Z 0 sea bed minute? Sea bed is minus h or plus h minus h and you set S naught this you start at 0. In order to simplify the problem you are making all these assumptions phi naught will be equal to for chain lying horizontal on the sea bed. So, your phi naught equal to 0 now, you write down the expression for S minus naught. See what are the 3 equations that we have just now derived? The 3 equations X minus X naught. And your this S minus S naught Z minus Z naught now, your x minus X naught that we have derived is it is a quite a long expression. So, X minus X naught this is equal to T prime dot over W now these are fundamental equations which we have to use now.

So, this multiplied by log of derive you will get long expression. So, this is sec phi plus tan phi you put some number else otherwise you will become late to write this down. So, this is sec phi naught plus tan phi naught, now you tell me the expression the expression is Z minus z naught. So, this is one relationship which we should not forget. So, whereas, write this as 12. The other expression that we have derived is Z minus Z naught. So, what was you expression for Z minus Z naught that is the vertical this thing. So, multiplied by 1 minus cos phi naught now, you tell me S minus S naught. So, that is the expression we require now.

So, this is say equation number thirteen. So, S minus S naught how much we have derived this expression? We have just derived now I see this papers lose what was your expression? So, T prime over multiplied by cos phi naught over W now you in all these expression; you substitute this value equal to this sea bed conditions and see how much you get. So, they are the 3 important equation 12 13 and 14. So, these 3 equations actually give you the diameters of the chain cable the length of the cables. And there projections at the horizontal and vertical axis now, you simplify with this sea bed conditions. So, sea bed conditions if you in this expression for S minus S naught cos phi naught is how much? So, this will be cos phi naught I remember you are starting with S naught equal to 0. So, you simplify 14. So, put boundary conditions in 14.

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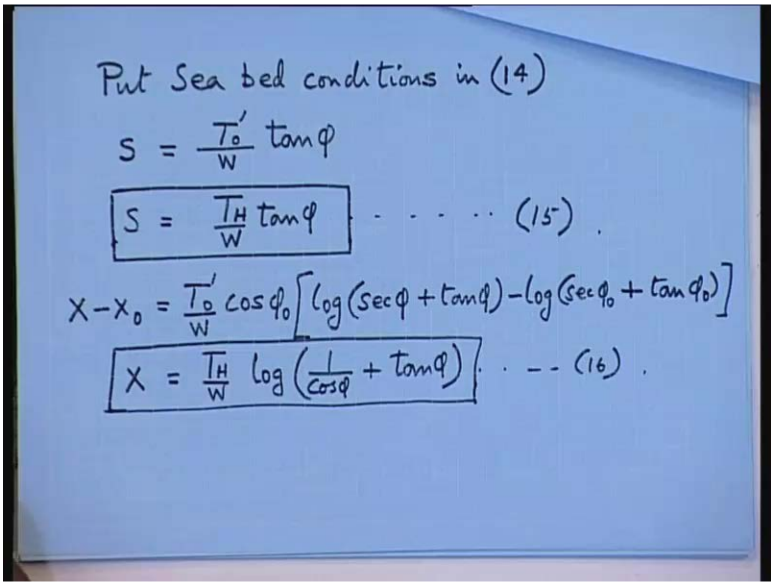

Or rather you write put sea bed conditions. So, this will be simply S, your S naught is 0 and there on the right hand side we get 0 cos phi naught you will get 1. So, this will become T prime naught over W and tan phi naught will be how much your phi naught is we have assumed phi naught to be 0. So, this expression simplifies to obey this expression tan phi. Now, we have found out T prime naught is equals to what was your expression for T prime naught that you have derived that is equal to T H somewhere I have written this. So, you substitute that. So, this equation let us write this as see how much and these sequence are not following actually see from 15. So, S becomes equals to T H over W multiplied by tan phi.

So, now, we have obtained a very simple equation for S able to see that these all simple sea bed conditions and mathematical algebra that is all. Now, you derive this for X minus X naught, what were the expression for X minus X naught? And you substitute this sea bed that is all. So, x so this we have you just tell me what is the equation what should I put equation number 15? I have already put inside let us put this as sixteen 15 then you do not put any number out there. So, your X expression for X minus X naught was T prime naught over W. Now, you signify that you put boundary conditions you will get the answer. So, this will be log of. So, remember this is only study kind of this is we have not started dynamic analysis the other one was cos static is not it.

So, here you substitute how much this becomes T prime naught is T H is not it. So, you just write T H, T H over W your cos phi naught becomes 1 and here this will be log of the other term will be how much. So, this will be simply log of 1 by cos phi plus tan phi. So, this is equation number how much is this 16 or what? Now, you tell me what is the expression for? So, this is your X X naught we have started as 0. So, your x is equals to this now you find out z what you substitute in this expression when you substitute you do not put Z naught equal to 0. Because your sea bed condition what were sea bed conditions sea bed condition; you put Z said Z naught equal to minus h do not put Z naught equal to 0 in the other equation. So, this you complete it in your home. So, will complete this and then we will find out the expression for Z plus h there is still a long way to go.