Elements of Ocean Engineering
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Lecture - 18
The Wave Spectra (Contd.)

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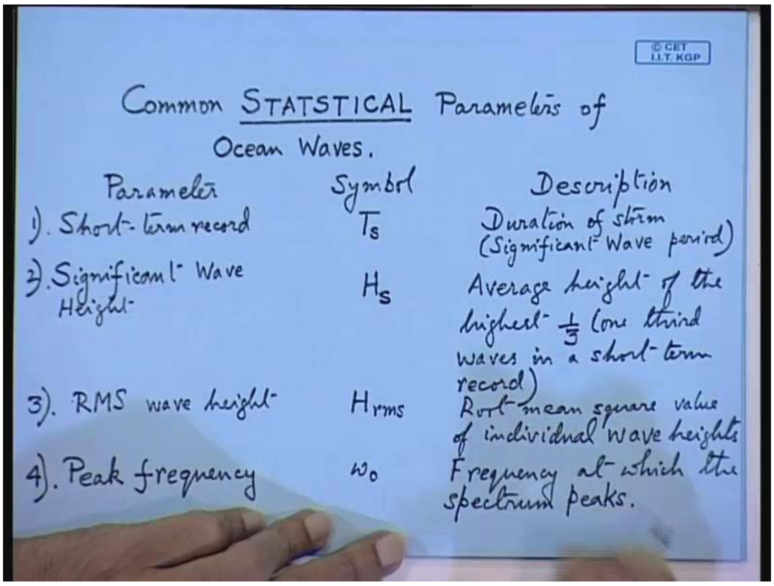

Today, we will continue with the Wave Spectra and tried to finish it. Now, before we going to the deeper discursion that has first have a look at what are the statistical parameters of the ocean waves? So, here you write parameter then symbol and description, because after we will come across notations for this parameter, especially you will find in these spectra definitions. So, you may not be knowing, so parameter number 1 you write short term record, say short term record, record means wave record for a certain duration of time.

Now, here we denote this by the time period T S, and this is called the duration of the storm. So, some of the terms are typical to the oceanographer you know duration of storm, and in mathematical terms you can always say that this is a significant time period, significant wave period. So, this is also important, so all these things you will learn in detail in your oceanographic class, but since in our case we have to come across in our spectra.

So, next to this is your significant wave height, so this is denoted by H S, so commonly you will find these coming in your oceanography class this H S and T S. So, this is the average height, now this is H S is very important from this point you will find this is called the average height of the highest 1 3 rd waves 1 3 rd rather you write 1 3 rd waves. Now, this is done in a short term record, so I if I give you a wave record of the wave height how you calculate H S.

So, in a descending order you take the wave heights, you make a chart in the descending order you plot all these wave heights, and take 1 3 rd of the highest you take 1 3rd of the waves from the highest 10 and mean of that there is a average this is mean you give you the H S. So, H S and T S this also called as a significant wave period corresponding to this value of H S.

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Short term record actually here it is normally you take wave records for say 15 minutes, long term record will go for say few days or few months. So, here this is the crucial point and the other one will come across is called the root means square R M S, so those of you who have done it will be statistics you will come across this. So, this is called R M S wave height, so in short they write H or R M S, so this is the root means square root of the mean square wave heights value of individual wave heights.

So, R M S we will see where this comes and the other one is called the peak frequency, now all of you must have a look at the spectra, you have a look you turn your pages and see this spectra, and there you will find the peak frequency. Now, this is denoted by omega naught, and this is called frequency at which the spectrum peaks there is the largest value of the spectrum, there is called the peak frequency.

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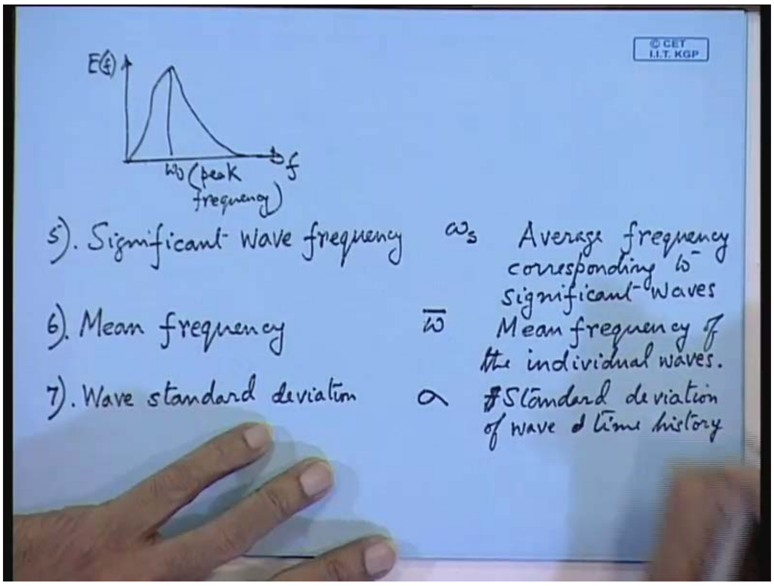

You will find in your wave record you have plotted the E f the spectrum is like this, so this is your omega naught. So, this is your E f verses what f, so this is your peak frequency, so this is another parameter which is important, now I have already given you spectra for JONSWAP and Pierson Moskowitz. So, today I have give you two more spectra, now coming let us finish this definition then significant wave frequency.

So, this is number 5 is significant actually all these in details we will study in another course that is called a ocean technology and ophtho technology you having in 4'th year. So, is very difficult to cover all the aspects in one course, so this called significant wave frequency, now this symbol is given as omega s now, so this is the average frequency corresponding to the significant waves.

So, all these are related your omega is H S and T S wherever you come across this s this stands for significant. So, that simply you remember going in to all these the other one is the mean frequency, so that is your omega bar, so this is simply the mean frequency of the individual waves. Now, all these pertains to short term require not long term, mean frequency of the individual I am not writing short time in any case, where it is understood mean frequency of the individual waves.

The last term is not that much used, but still this is called the wave standard deviation, so those of you who were not familiar with statistics they just you look of any statistical book. Now, in your in my previous class I have already talked about variance, now what is the physical meaning of this variance why it is called variance that a square or that E f term. The variance means bearing with what, variance means variance is the statistics what is the definition of a variance.

Variance means a particular the number which is deviated from the mean and you square of that, now in the elevation that we have done where you have talk about the mean elevation is taken as 0 that is where you are getting the square term. So, that is one of the interpretation why we say that, that is the delta bar is the variance, now here you have waves standard deviation. So, standard deviation is root over variance, so this is called the standard deviation. So, you just look up any statistic books you create this what is called the variance and standard deviation of wave time history, so these are some of the definitions.

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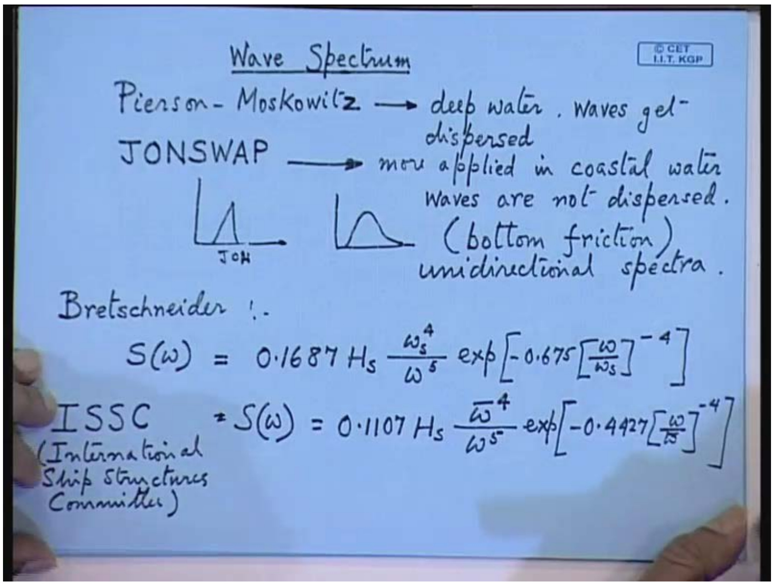

Now, let us go to the spectra, now spectra I have given you the two formulas of one I have given you what is called the Pierson Moskowitz this is I think this is the spelling Moskowitz. And the other one which is important is the JONSWAP this is your joint north sea wave project, now these two spectra’s are important, so this is your wave spectrum. Now, JONSWAP is actually a extension of your Pierson Moskowitz, you see in the Pierson Moskowitz I have added one more term, there you will get JONSWAP

But, where you are going to apply Pierson Moskowitz and where you are going to apply Jon swap, this is very important if you want to do structural analysis. Now, Pierson moskowitz is normally applied in deep water, so this a actually a deep water spectra, now in deep water actually you will find waves get dispersed. So; that means, if you come across your hydrodynamics or your oceanography, there is the various waves get dispersed in deep water.

Whereas, JONSWAP you apply is more applied in coastal waters, now in coastal water what happens? You will find the JONSWAP has a sharper peak. JONSWAP will come like this sharp peak, and Pierson Moskowitz will have a more or less a Rayleigh distribution, the peak is more having a curve like this. So, here actually this is more applied in coastal waters, dispersion deviation ship waves are not dispersed.

Now, waves are not dispersed why, waves are not dispersed because of waves in shallow water they do not get disperse to other frequencies because of bottom friction bottom friction. So; that means, in this case this is a unidirectional spectra, you apply unidirectional spectra, so I have given you two spectra, one is the theta term that is your directional spectra, and the other is the unidirectional spectra. So, unidirectional spectra more or less you apply JONSWAP model for coastal waters.

So, actually you have to be careful where you apply this spectra, now there are other spectra also this is called Bretschneider, but this is a little bit out dated now, but it was widely used at one time this is called Btschneider. So, here you will find this Bretschneider is given as S omega, omega is your circular frequency and this is a S omega is that E omega.

So, this is given in some other expression this is 1.687 H S is your significant wave height, similarly your this is omega S this is similar, wave frequency significant wave frequency divided by the wave frequency, and this is multiplied by exponential is E to the power minus 0.675 then here is a non dimensional term omega over omega S this is s to the power minus 4, so this is called Bretschneider.

Now, the other spectra that is of interest is the ISSC spectra, ISSC is stands for International Ship Structures Committee. So, if you want to doing structure analysis, you can take help of this spectra ISSC gives this formula, so sometimes in this you write International Ships Structures Committee. Now, sometimes you will find this expression in your ABS book American Beuro of Shipping those of you who are consulting ABS or mobile offshore giving units, they will come across this ICC spectra. So, this is given by international ship structures committee.

So, S omega is given as 0.1107 and this is H S, so we are more interested in our ocean engineering class, the behavior of the offshore structure with response to the waves. So, here actually the mean value is taken and the exponential term that is E raise to the power this you just change the factor this is 0.4427. And this is again omega by omega bar to power minus 4 same explanation with modified only, so these are the four spectra with which normally the oceanographers come across. But, normally other countries especially Japan the I do not have the take you make your own spectra that is for design.

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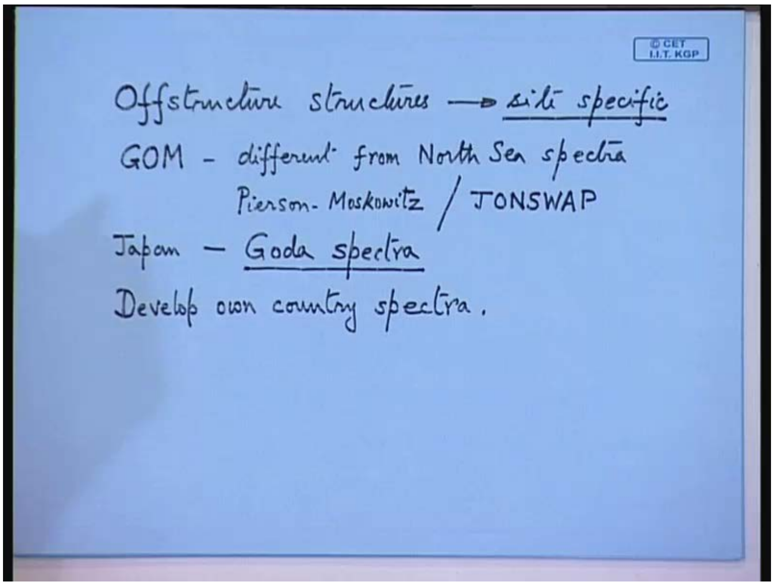

Now, offshore structures are you will find they are very much see offshore structures are site specific, offshore structures are essentially site specific especially your fixed platform or your floating platform they are fixed at a particular location. So, there are site specific in nature, now if you want to do say built your offshore structure in see gulf of Mexico, now this GOM in short it is called gulf of Mexico you have a different spectra then in north sea.

So, it different from north sea spectra then what do you going to do, so you have been given a specific task or designing a sea fixed structure in gulf of Mexico. Now, gulf of Mexico is having different from north sea spectra, now you say that your professor has given you JONSWAP Bretschneider s s c, and all these formulas. Now, here if you do not have any spectra you go for deep water or either you apply this Pierson Moskowitz or JONSWAP.

Now, many countries they had started developing their own spectra especially Japan, now Japan actually does not go for either Pierson Moskowitz or JONSWAP, Japan uses their own formula that is called the goda spectra. Now, you this I do not have in my book because this is not remember Japanese, so they use this goda spectra, so you will find they have done a lot of research they have taken a lot of data along the coast of Japan, and they have made their own spectra there is called the goda spectra. So, these actually requires lot of data analysis.

Now, which the other countries which are interested they can do, and where if you do not get this then you can blindly apply Pierson Moskowitz or JONSWAP. Now, JONSWAP actually has been developed from a project that is in the north sea, so it may not be applicable to other locations in the glob listen it. So, for to be more specific you better to develop own country spectra, but in all country I do not know whether your national institute of oceanography I done that or not, but now a days I think they are doing it. So, develop your own country spectra to be more specific, now this is used in the offshore structures. Now, so this is the discussion of the spectra, then next let us have some ideas since we are dealing with the sea what is the sea state.

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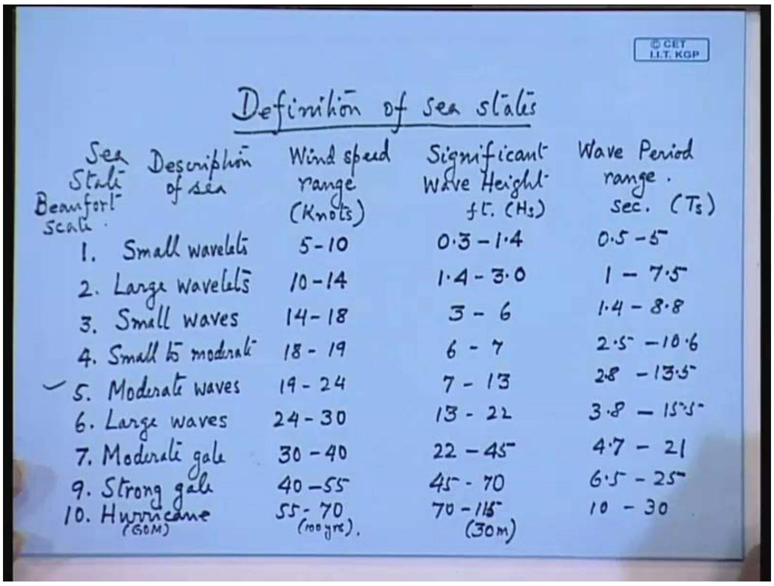

So, definition of sea states this is actually a layman’s knowledge, but in for more scientific definition, if you want then the oceanographer they always refer to the spectra. Where since the common man he is unable to interpret what is a meaning of spectra, he normally goes by a sea state definition. So, in normally you will find sea state there are various descriptions, so this is description of the sea.

Then it is describe with respect to a certain wind speed range why because most of the waves are created by wind. So, there is called a wind generated wave, so this is your significant wave height and this is wave period range, now in this table we have given a wave period range. So, normally this wind speed is given in knots, wave height here you can convert the feet in to meters and this is given in seconds.

So, sea state one this is the least of the waves, so here you will get small wavelets, wind speed is now 5 to 10 knots. Now, wave height that is small only 0.3 to 1.4, but these are all significant wave height, now the corresponding period that will give is 0.5 to 5 second wave period is your T, so these are your H S values H S you write T s. Now, next is sea state 2 this is the large wavelets, so this is 10 to 14 knots, this is 1.4 to 3, significant wave period is 1 to 7.5.

Number 3 you write small waves, so here it is still larger when speed `14 to 18, and this is you are getting 3 to 6 and here is a 1.4 to 8.8. Now, 4 you get moderate small to moderate is 18 to 19, 6 to 7 and this is 2.5 to 10.6 last is moderate waves, so 19 to 24, now actually what is a this wind speed you may not be able to remember, but this H S and T S values you simply have you take note of this right.

So, 19 to 24 is given 7 to 13, 7 to 13 is 2.8 sorry this is 2.8 to 13.5 and sometimes in your interview they will ask you what is the significant wave period and wave height. So, 6 large waves, so if anybody ask you give the value of this moderate waves difficult to, so 7 to 13 feet will be how many meters say around say 4 meters, 3 to 4 meters you just tell that the it is about 3 to 4 meters, and wave period you tells a 10 seconds.

So, just off the shore concept, so large waves you get 24 to 30, so here we are getting 13 to 22 and then this is 3.8 to 15.5. So, then is now actually from waves we are going to storm conditions, now this is actually more important for offshore structure design, so this is we are coming to moderate gale. So, when you design an offshore structure, you have to design for the storm condition, so moderate gale is now the seas actually becoming more and more rough. So, you are getting 30 to 40, so and this is 22 to 45, so this is 4.7 to 21 then strong gale.

Now, whenever you design an offshore structure you look for these three conditions, so 40 to 55 here coming. So, this is 45 to 70 and this is 6.5 to 25, so wave period has increase, and last one is hurricane, so this is sea state normally these are given in a scale it is call beau fort scale, you will come across this beau fort scale. So, hurricane is 55 to 70 and we are getting 70 to 115 feet 10 to 13 seconds is significant wave period. So, 115 feet is how many meters, 115 is actually this is 30 meters. So, hurricane conditions you will get at 30 meters, so these are frequent in gulf of Mexico.

So, gulf of Mexico offshore structure to be design for hurricane condition before 10 and but this statistic has been taken over I do not know this time period, they were this significant wave height for how many wave we have taken. But, normally data is not able you tried data for 25 years or 100 years data correct data for 100 years, so that is the maximum that we have got.

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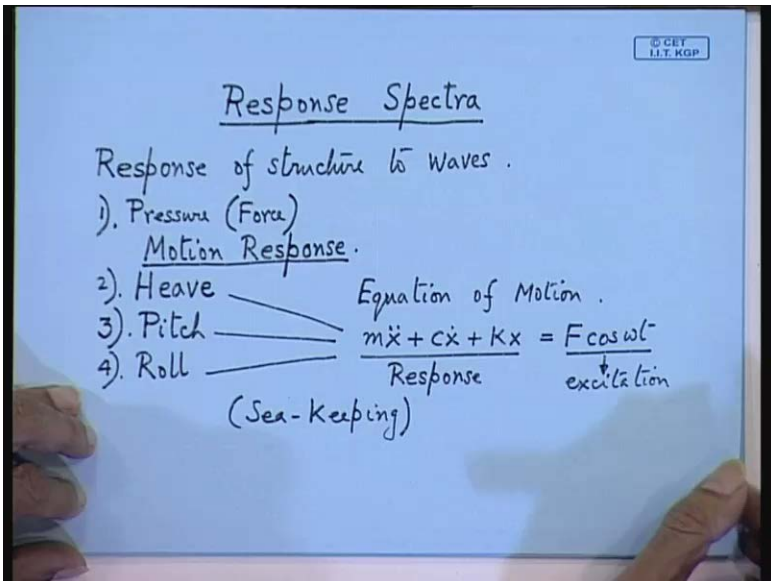

Now, after this let us come to an another important factor before we will wind up this spectra discursion is called a response spectra. So, we have more interested in studying the response, response of the structure to waves listen it, a ocean engineer he is more interested with response of structures to waves. What is this structure response, now numeral architects they are more interested in response of structures to wave.

So, what is this response now response can be number 1 pressure, pressure is a response pressure or pressure is force per event area or sometimes you write this as force. Now, what is the other response, others are basically motion response, now what is the motion response, we are interested mainly heave, heave is I told you in offshore engineering class is a very important response of offshore structures, because if you get too much of heave you have to stop grieving or taking oil from the wave.

Because, that will snapped your marine riser, now next important motion is pitch, so all is motions you study in your sea keeping class. Now, basically in your vibration you are just you have to formulate here equation of motion for all these 3 states 1, 2, 3, 4 the other one you will get is roll. So, you see keeping class you will be told how to formulate this equation of motion in heave pitch and roll.

So, I will just I will give you a drams of what is there is it is told for you write down or you have to find the displacement for this, this we have already discussed in your vibration class, and this will be how much this will be F, now in the easier ways the write this as cost omega t. So, this is your excitation force, this is your excitation and this is from this you this is basically your response, so there is a you have to straight in all these three conditions you have to formulate this equation.

So, this is another study by itself which of course, you come across this is called sea keeping studies. A sea keeping is nothing but a study of motion or motion response, now I will just give you some incurring of what is this since we are discussing spectra, now there are two things in that is important one is you come across what I have told you is excitation and response.

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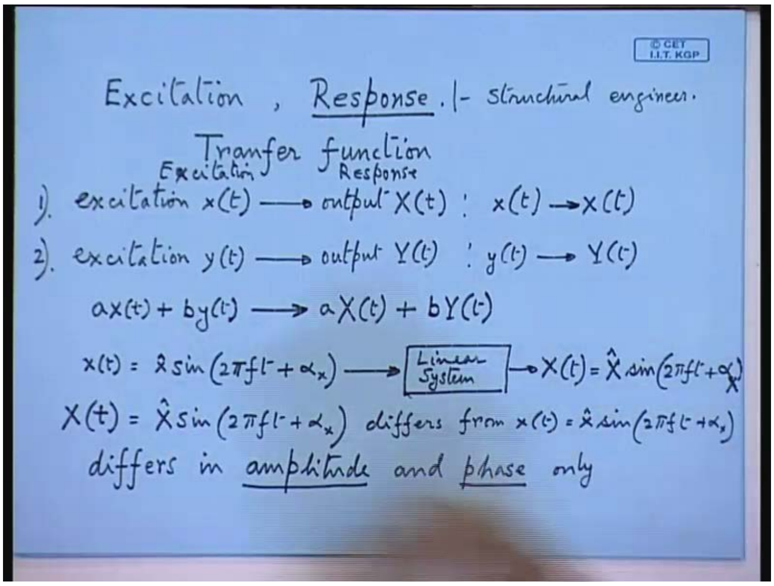

These are two things, which is from the equation of motion excitation and what is the response. Now, how we are going to study this, now this is best studied by what is called a transfer function, you have to formulate a transfer functions, where you are going to transfer the excitation to the response. So, in structural engineering we are more bother about this, so this is of your interest to your structural engineer.

Now, this transfer function you must have come across in your study on this is feedback and control, in electrical engineering you must have studied I do not know they should have taught you over these feedback and control. So, there also you will come across this transfer function, now in simplest term you write excitation suppose or excitation is nothing but wave excitation, now excitation you write in small x, now these gives an output.

So, all these terms you will find in the signal processing, signal processing I do not know because this is a feedback and control and all these things will come under this say output is your response X t. So, what we are getting is that there is a transformation from a excitation to the output, now this is small x and this is capital X and just I am writing you. So, this is let us say this is one wave, so number 1 excitation, now number 2 excitation you write from another wave you are getting this.

Now, I told you that the sea is represented by a summation of harmonic waves, so here you get another y term, say another wave is coming with by y t and this is giving another output and these out put you will write as capital Y t. So, here y t is giving this is giving X t and this is giving capital Y t, so now, when you add these two you have a x t plus b y t.

So, this is your input, your input is your excitation, now you just add linearly this is a linear addition between this is x t and y t the only thing that you have to do is multiplied by the amplitude of these wave, where x t is having a amplitude a and y t is having a amplitude b. Now, what is to be noted you just see the response, so here this is your excitation and this is your response, so this is excitation and this is your response.

Now, one thing you note is that these amplitudes will remain the same, so x t will give you capital X t, further plus you have the same amplitude of your response, but your formula in the harmonics will be different. Now, here say x t you write this as your sign function x t you write this as you put a small cap on x instead of writing the cost function, you can write this as. So, this is actually immaterial whether you write this as a cost or a sign function.

So, this is you write this as a alpha small x, now these equation that I have given you that we have formulated this is a linear system, there is no non-linear term. So, neither you have producing a square term or q b term out here, so this is a linear system, so this you write this as a linear system. Now, this linear system is giving out put as capital X t, now here you write capital X with the cap, and again since sign is out here, so the response also will be in a sign function. So, this is 2 pi f t, f is your frequency plus now there will be change in the phase angle, so you write capital X.

Now, you do not same thing x small x you do not put it out here, so you write this capital X t. So, we had seen what is happening, so a and b are the constant which are remaining the same, but x t you see that this x cap is your amplitude, so X t is this, so which I have already written. So, this is f t plus alpha x, so there is only difference in amplitude and phase, so differs from small x t, now small x t you have written this as small x with the cap multiplied by you are doing it for the same frequency that is 2 pi f t plus alpha x small x. So, you write this differs in amplitude and phase only, and this is very important, so differs in amplitude and phase only.

Now, this is to be noted, so this is given by this x cap capital X cap and small x cap, and here also that is the amplitude difference and phase difference is there. Now, you write your amplitude response function, so these actually in numeral architect we called this as response amplitude operator.

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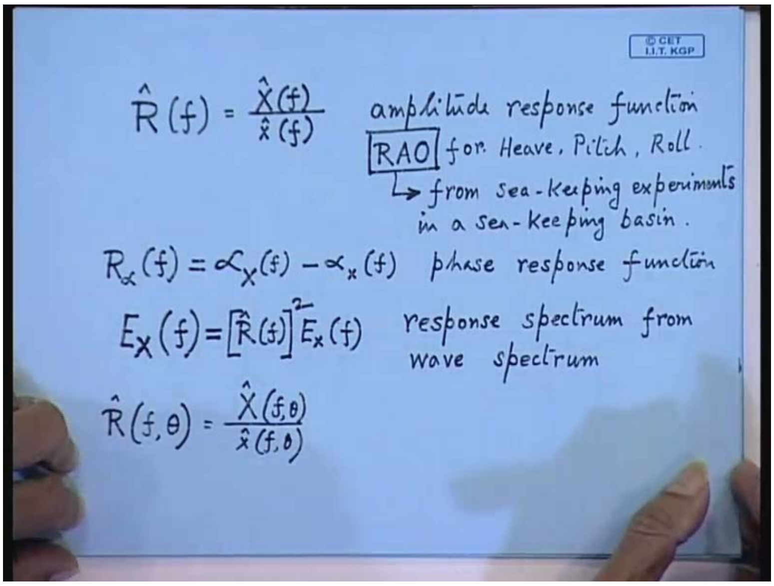

So, here this is written as R cap f, so this is your capital X with a cap f denotes that this is a these expressions are dependent on frequency, small x you write. So, this is just the ratio of the amplitudes of the response to the excitation, so this is called a amplitude response function. Now, this where we are interested because we are more interested in getting this value of x from a wave.

Now, the wave amplitude you know that is you are coming wave amplitude you get from the waves spectra right. Now, these how to get capital X f this is where your heave, heave response or pitch response we will be this x f, so you get this in developed we say we call this as RAO you will find this very common. So, in your numeral architecture book leave RAO you will find RAO you have to calculate this RAO for heave pitch and roll.

Now, these actually this is not easy to calculate, now you have to find these from sea keeping experiments. Now, sea keeping experiments you cannot do that in a twin tank, now what you have we have got is a twin tank, so this is done in a sea keeping basin or in a sea keeping tank. Now, these madras ideas got which we do not have, now this the specialty of these sea keeping basin is that, you have produce a random sea or you have to produce the sea surface elevation.

So, that is called a Gaussian surface, Random Gaussian surface wave have to be generated for your sea keeping experiment. Now, only after you do these sea keeping experiment, you can calculate this RAO for heave pitch and roll, so these has to this is where we are more interested, but unfortunately a experiments you cannot conduct in our wave tank. So, anyway, so that is this is called your amplitude response function or amplitude response spectra, now, so this is just the ratio of the amplitude of the response to the excitation, but what about the phase angle.

Now, the phase angle you will find this is given as this alpha and that we do not write this as the ratio of two phase angles, but rather we take the difference. So, this is actually the difference of the two alphas this capital X is your response, and the other one is your excitation. So, this is your phase or rather this is called a phase response function, now phase response function you will simply remember that this is the difference between the two phases, you do not take the ratios of the two phases or we have done for the amplitude.

Now, from these two how will you get the response spectra, actually in your measurement from your sea keeping tank, you do not measure directly these two ratios of the displacements because that will cause error. But, rather you have to take the energy spectra, so how do you get this E X f this is called the response spectrum, now your response spectra that is the spectra is always that we have analyzed from our previous class that it is always the square of the amplitude.

So; that means, you square this R f, so this thing you have to found out the these has to be squared because we are dealing with the amplitude, now this square you multiplied by the wave spectra. So, this is small x, so this is capital X now capital X stands for your response the small x is stands for the wave, so this is how you will obtained the response spectra from what from wave spectrum.

So, oceanic engineer they are more interested in this wave spectrum and your response spectrum. So, this is what we will obtained, now this is actually not having any direction, so this is not having any direction, now if you introduced direction in to this then this R f theta you just introduced another dimension that is all R the frequency and the direction of propagation of the wave that is the denoted by this theta. So, here of course, if you make this then this will be capital X this will be f theta, and this will be small x this is the only change that you will required, so introduced another parameter that is theta that is all, so E X f will be how much?

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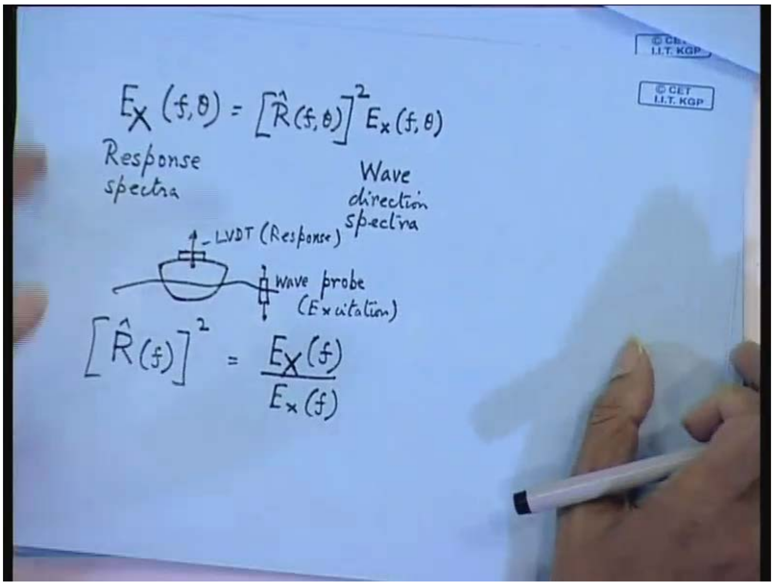

So, if you get R f theta, so; obviously, your response spectra will be, so this you write to be the terms. So, this f theta you just square this one, so this will be R f theta square you square it and if it is x e, so E x will be your wave, but then you have to do the f theta for your you introduced the direction spectra from the wave. So, this is your response, this is wave direction spectra, now in ocean engineering calculations this is either you get this there is a direction spectra or you get this X f.

Now, here there instead of directly calculating this R f theta, you calculate this R f square and you know you note this E X f this is what this is your wave spectra. So, wave spectra you can either obtain from JONSWAP Pierson Moskowitz, if you get you multiply by this R f square you get this. Now, what they do is normally if you want to find this R f square, you take measurements in your sea keeping tank or sea keeping basin, you have there are two things which we will have the ship you can generate this in sea keeping tank.

So, you measure the response of the ship you make a model, say this is your wave you put a say Uccelloro gram or motion measuring that LVDT you can now measure heave by an instrument called LVDT mounted on the ship you can measure that. So, this you can measure that is say example is the heave response, and you have a wave boyar or a wave probe say this is your LVDT and this what you require is wave probe.

Now, wave probe will give you the wave elevations or wave displacement wave amplitude. Now, from these two things you can measure this thing E X f you can plot your this is actually giving you are excitation listen it from the ship you are getting the excitation, and from the wave probe you are getting the sorry ship is giving the response. So, here you are getting the response from this LVDT, and this is giving you the excitation alright, so this you can get the displacement from these two instrument you get.

Now, what you do is you get this value, now actually you have to find out this motion RAO'S. So, this square is what, so this you can get it from the sea keeping tank, sea keeping basin normally you measure the ship response, and the wave probe will give you. So, these two spectra you can obtain, you divide this you will get the square of the RAO, now from this RAO you can get a square root of this, so that is how you can obtain the response. So, this brings us to be end of the discussion on the spectra, now next class we have give some description of this offshore structure before we going to the structure mechanics.