Elements of Ocean Engineering
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Lecture - 17
The Wave Spectra (Contd.)

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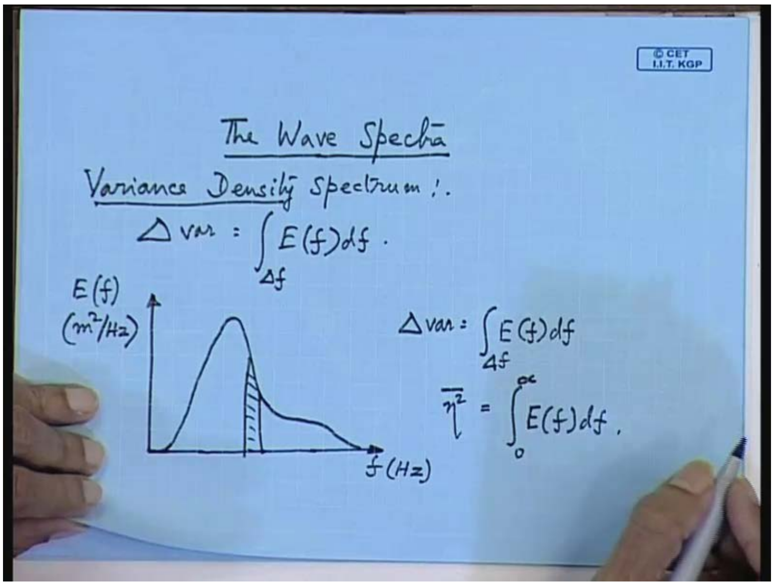

So, today let us finish this Wave Spectra. Now, here actually we have how to obtained the spectra I already told you, so this called of various density spectrum, so this we have obtained last class that is the density, that is you divided by the frequency. So, that is given by delta variance is equals to you sum it over delta f and this is your E f d f, now graphically what is the significance of this.

Now, if you draw the graph and last class we have discussed the amplitude spectrum also is it not, from the amplitude spectrum you have getting the variance spectrum, and if you divided by the delta f you get the variance this is called the variance density spectrum.

So, if you plot this graph, so you get now you just remember the units, so this in meter square per hat and this half obviously, will be your frequencies, now if you plot you get a graph like this. So, this is a continuous spectrum, so last class I told you how to obtained the continuous spectrum that reduce you the decrease the frequency interval, so you get this kind of curve.

So, now here this area under the curve is here what, this is you are delta variance, this is a shaded portion, now so far so good, so this actually we nothing but if you this square of the surface elevation. The mean of this square of this surface elevation, this is given by you integrate this from 0 to infinity, you will get E f d f. Now, you remember this, this eta square that is the mean of this square surface elevation is equals to half of a square where a is the amplitude of your harmonic wave.

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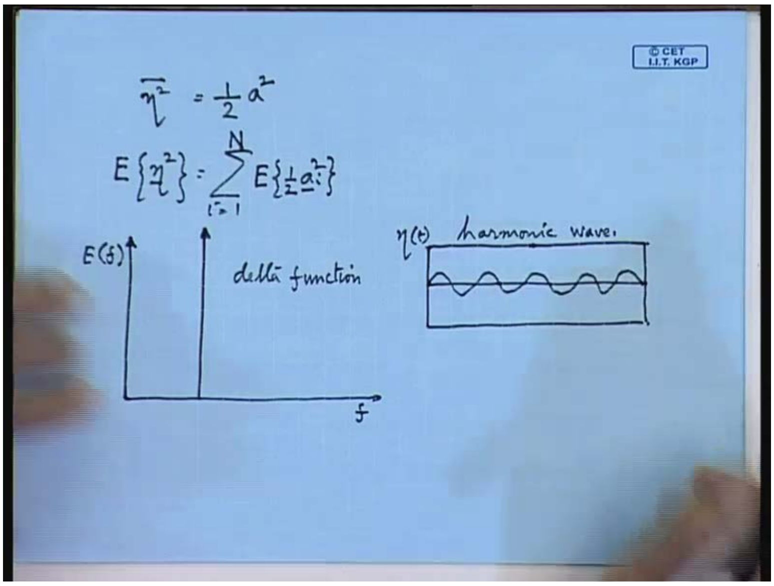

So, we are considering a number of harmonic waves, so this is the mean of the square of the surface elevation is required to half off a square followed, now so expectation nothing but your mean. So, if you do the expectation, so then you will get now underscore below the random, so obviously this will be equal to sigma 1 to n, so n is the number of frequencies you have this, expectation half of a i square, so this is to be remember. Now, coming to the nature of this spectrum you will find, if you are waves are fully harmonic you will get a spike.

If the surface elevations are harmonic, you will get what is called is spike that is means, at a certain value of f you will get a infinite value of f, so this is called a delta function. So, if your surface elevation are like this, so that this close to harmonic wave, so then you are getting a delta function, so this is you are surface elevation with respect to the time, this is your harmonic wave. But, a surface elevations are really harmonic, you will find the sea surface is, now if you gradually go from the harmonic to the irregular waves, you will find that your E f is having a frequency band.

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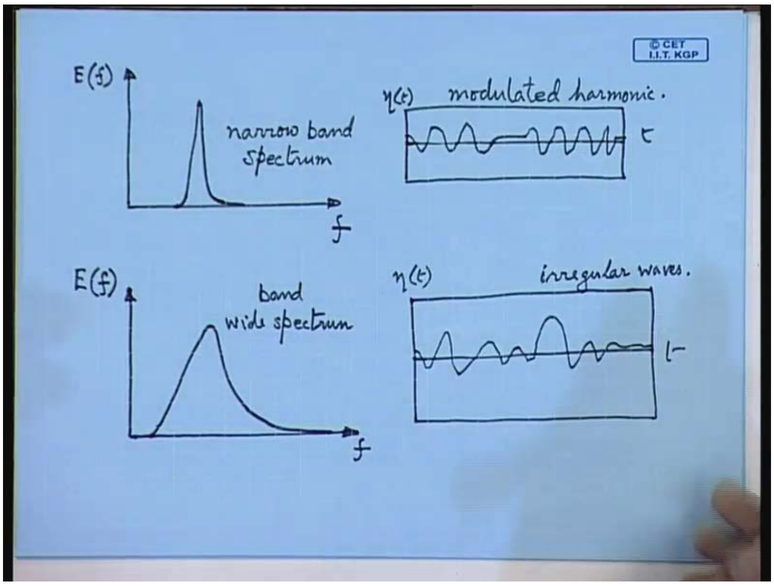

So, you are getting a spike of a delta function out here, but you get the frequency band like this, so this is again E f and this is f you get something like this certain peak. Now, actually oceanographers they are formulated a, their formula for this you will later on you come to see, what is the importance of this spikes, so this is called a narrow band spectrum. So, in vibration also you come across this, the narrow bank and wide band spectrum, so this is called a, now you are surface elevation you look like this, not exactly harmonic and either it is a replica of your sea surface.

But, it will follow closely the harmonic at certain time intervals, then you will get this, then again is something like this, so this is called a modulated harmonic, and this is your surface elevation. And there real situation is quiet little bit different from this, the sea surface, so that is called a wide band spectrum. So, has you degenerate form a harmonic wave to the sea waves you will find your spectra is also changing it is shape, so this you will find this is called a wide band spectrum.

Now, oceanographers and ocean engineers they are actually more interested with the nature of this graph, later on we will see what type of spectra you will have to use, so this is called a wide band spectrum. Now, in this case the sea surface is absolutely random that is confused in nature, so you are wide band this thing will be like this, this is called irregular waves; the surface elevation in eta t and this thing the horizontal the time axis followed.

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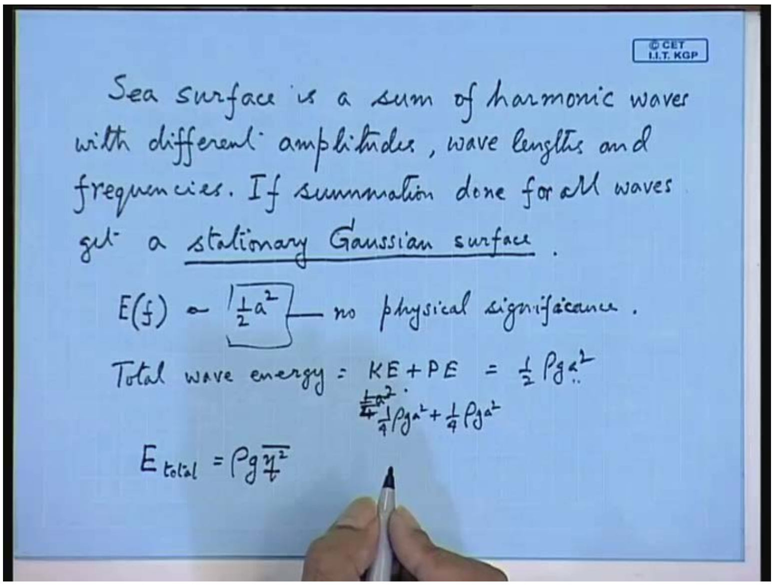

Now, the other thing, your previous class I told you that sea surface is an assemblage of a number of harmonic waves, your sea surface is a sum of harmonic waves with different amplitudes, wavelengths and frequencies. And if you plot this together you get, so I will not able to draw the diagram, you can see in one of these books, if summation is done for all these waves and for all waves you get a, get what is called a stationary Gaussian surface, so this is to be remember. If stationary Gaussian surface is obtained, if you some all these harmonic waves to (( )), now this is what we will done.

Now, we have obtain what is this E f is it not, now E f is obtain from this half amplitude square, but as self this half a square does not have any no physical significance. Now, you hydrodynamics teacher who will have told you how to calculate total wave energy that is equals to, I thing this are the sum of kinetic energy the potential energy. So, this will give you I think 1 4th of a square I thing, rho a g square something like this, so this is 1 4th rho g a square plus 1 4th rho g a square, so summation will be half.

So, if you get this a square, so now you can derived what is called energy is spectra, so this is your total energy spectra. So, this is given as you simply multiplied this rho g by this surface elevation square, the mean of this surface square. So, that means, your energy spectra we can obtain from the square of the surface elevation is it not.

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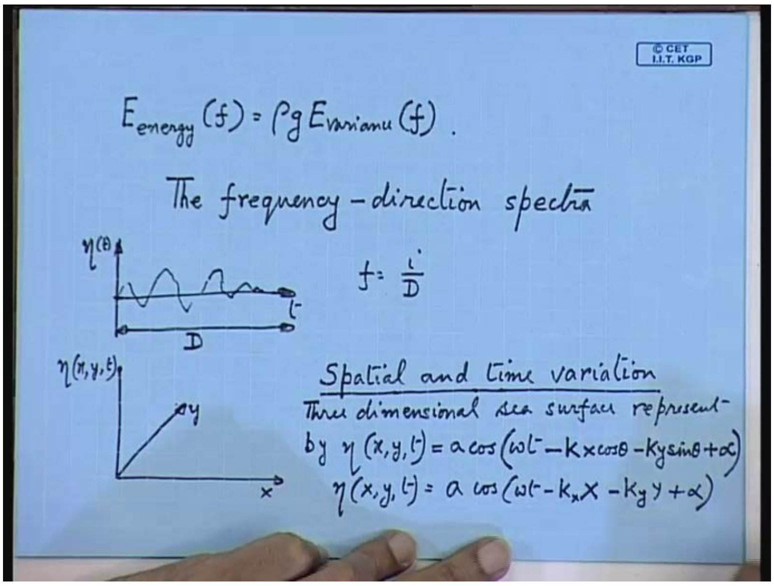

So, you write your energy spectra are E energy is nothing but you E variance multiplied by rho g, so this is to be remember. So, now let us go to another type of spectra, this is called the frequency direction spectra, now this spectra that we are analyze is respect to how you work in that spectra that is, you are measured this surface elevation in variation how you are worked in your spectra, that is your measured surface elevation with respect to time at a particular place is it not.

Spectra you have obtained from surface elevation with the respect to time, you have started the class with the time duration D, if you remember. So, if the number of frequencies you can obtained this i by D this we are talked about now, now what is actually gives, that is the variation of the sea surface elevation with respect to time at a particular place, is it not. But, actually in the actual scenario you have, if you want to take the wave height, so that is going to vary with not only with time, but with space. So, this is you are, say here you have mentioned this one say x and y axis, so you will get eta surface elevation x, y and t.

So, there are three parameters, that is there is a spatial and time variation both are there, so you have to accommodate, if you want to measure in the sea surface in three dimensions. So, three dimensional sea surface, you have to incorporate both the space and the time, so that is a three dimensional, three dimensional sea surface represented by eta x, y, t. So, you are a is again the amplitude of the harmonic, but here actually you will find, you have to incorporate two more terms having x and y, so this is omega is your 2 pi f, this we have, I have told you your previous class.

Now, you introduced two more terms, one needs your K x cos theta and the other is minus K y sin theta and your face angle will remain as it is, so you plus alpha you make the changes, if you want to have a three dimensional sea surface representation. Now, this you can write in the, this K multiplied K cos theta you can write short form or you have condense form. So, this is x, y, t you write the amplitude a cost of omega t minus K x that is a component in the x direction, you simply multiplied these by x minus K y this is also the multiplied by y plus alpha; so this is the surface elevation in three dimensional.

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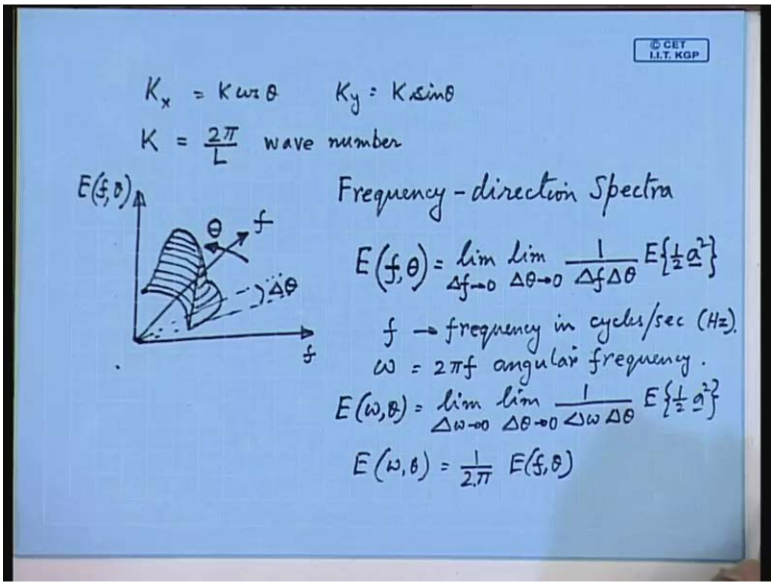

Now, here actually if you want to study this in three dimensional wave, this K x you write this as K cos theta and this K y is equal to K sin theta, and what is K, K is your wave number, K is 2 pi over l, this is your wave number. Now, why you are suddenly introduce this term K, there is no need is it not, simply make ((Refer Time: 19:22)) this as x cost theta y sin theta, but K you see this omega also have the 2 pi term omega is nothing but 2 pi f. So, obviously the other terms will be in certain angle, so you have to introduced 2 pi and alpha is already angle.

So, you have to from this spatial parameters we have to data angle parameters, so that is why if you want to add all these terms to made similar scaling parameters, so that is why you introduced the wave number into the equation. Now, here actually you will find that your diagram will look, the more complicated staff anyway, so if you do this you plot frequency axis in x and y coordinates, and this your E f theta. Now, you are get a three dimensional of plot or you will get something like a V, so it will look like this.

Then I am not that doing drawing, so there the various contour lines along the surface of this yield, so these are called your E contours, so this is you f and this direction also will be a f, but you have to plot this with respect to certain angle. So, this is your delta theta and your theta direction positive, theta direction is like this ((Refer Time: 21:37)), so actually this is similar to your r theta plot in polar coordinates, and all this thing that you have expansion is related to a r theta plot.

Actually if you plot this, this actually these are the, if you sum it you take the minus K sin outside, ((Refer Time: 22:06)) so this is x cost theta plus y sin theta is giving radial plot. So, if you want to take a wave surface at a particular point of time, we have to find out that distance among the r axis and you r and theta that is all, and your E f theta of course, is the vertical height. So, this is called a frequency direction spectra, so these actually as tell you about the wave surface, and the equation for the elevation I already given you.

Now, what is this E f theta, this E seems here having two parameters, one is f and your having a direction parameters E f theta, so you can write this, now E f is what is called be the variance density spectra, as I have told you in the previous class. So, limit you take delta f tends to 0 and in the limit this angular delta theta also be equal to 0, what will be your density, density will be you divide by delta f delta theta. And this E is your mean of half a square, now in underscores signifies that a is a random, for all these harmonics.

Now, if you want to plot the f is frequencies in what, this is frequency cycles per second, now this is actually called at Hertz H z, now if you want to plot in angular terms omega is equal to 2 pi f, so this is called angular frequency or radial frequency. Now, there is a relation between these two, so you omega theta will be limit you take, instead of delta x you take delta omega tending to 0, and what else and also your angular also tend to 0. So, 1 over delta omega, delta theta that elevation will remain the same half of a square, so what we get E omega theta is equal to 1 over 2 pi of E f theta.

So, from this is the, is the only factor that will come out to 2 pi, omega is equal to 2 pi from this 1 by from here is 2 pi, so this is your spectra and then the surface elevation that we will get out here.

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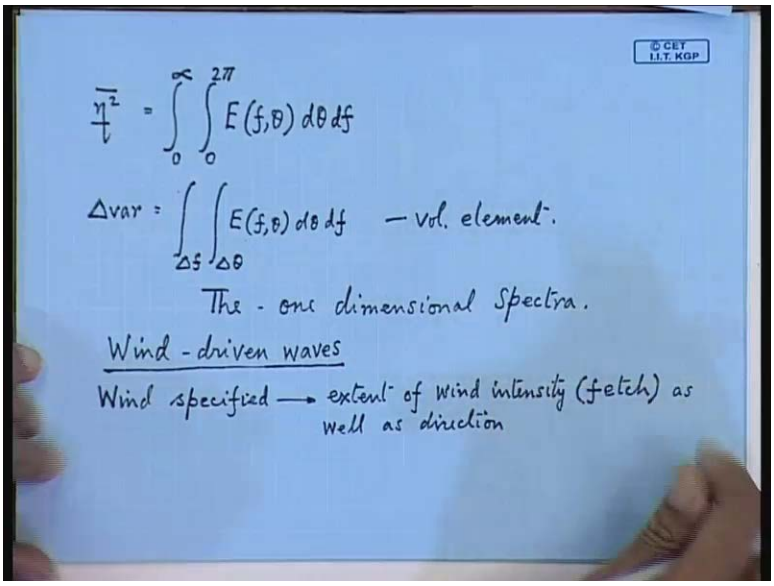

So, the mean of this surface elevation is equal to, you simply integrate on 0 to infinity and the angel will be 0 to 2 pi, so this eta square I told you, eta square is half a square for large value of your waves. Now, if you come back to ((Refer Time: 27:10)) this diagram what does this mean, so delta variance is, variance is that half a square density, so this is over delta f then you will get another integral, so that is delta theta, so E f theta d theta d f. Now, previously I told you that we started with the wave spectra, you can feel the delta variance in, this was a two dimensional spectra.

That is E f versus with f frequency we were getting area only, this E f d f, E f d f was your area delta variance, now in the three dimensional case what you will get, ((Refer Time: 28:09)) three dimensional case you get a volume, so you get this volume. For volume between this delta f and delta theta, so this is your volume element, so you can write here this delta variance, this is given by delta variance is equal to integral over delta f integral over delta theta E, this is a f theta d theta d f. So, this is called a frequency directional spectra.

Now, so is more complicated, but in the three dimensional case, we have to go like this, now coming this, the other point that I want to tell you, in normally the see here will come across the two terms. But, they are two dimensional, so that is called the one dimensional wave spectra, now this kind of spectra where you will find, there is why the direction is important. That is here by constantly varying what wind direction, so these are actually have, this is spectra that is we have analyze coming from what is called mean wind driven waves; these are all wind driven waves.

Now, in wind driven waves when you are making any ocean engineering analysis, suppose the direction of the wind, so wind you will find we are not gone into give us this wind. Wind it is specified by, is always specified by is two things, the extends of wind or extent of wind rather you will say, extend of wind intensity, now oceanography terms this is called a fetch, f e t c h. Now, in actual sea you find that wind also varies with the intensity as well as the direction, so this actually because of these wind actually r spectra, because more and more complicated.

So, then if the wind is varying, then we have to take recourse to a, there is a frequency directions spectra, ((Refer Time: 31:58)) this then no more your one dimensional spectra will be considerable. But, then you have to go for a frequency direction spectra, otherwise for a certain period of time wind is mini direction, then we are this spectra is ok, that is called a one dimensional spectra.

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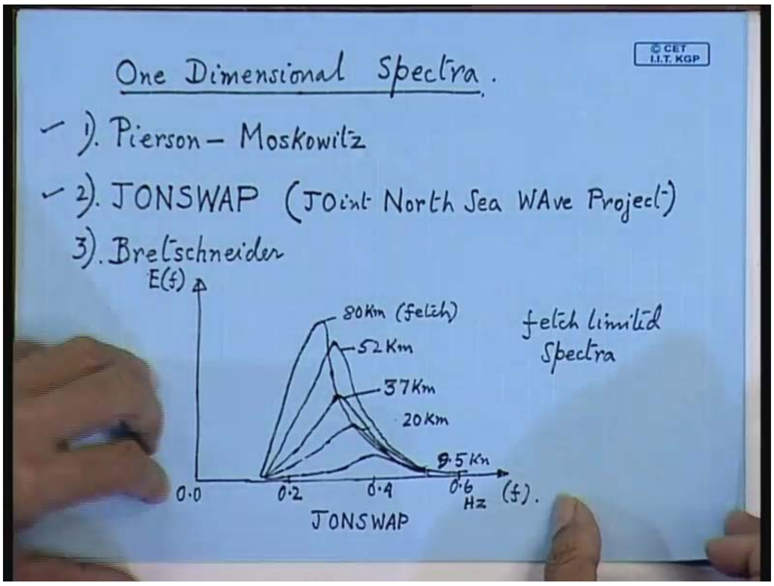

Now, there are two spectra very common in ocean engineering applications, first one is called this is both are one dimensional spectra, another two most important spectra in ocean engineering analysis you come to; number 1 these (()) is little bit tricky. Number 1, is your Pierson-Moskowitz I am just telling you the one is called Pierson-Moskowitz, but these are all one dimensional spectra. So, I will give you the nature of this is Moskowitz, in the other is actually variation of Pierson-Moskowitz that is one more term which is coming to the Pierson-Moskowitz, that is called Jonswap spectra.

Now, Jonswap was a project under the Norwegian countries, that is called joint North sea wave project, there is no from the capital letter you can see the Jonswap. So, here they made lot of studies, but Jonswap actually we also apply in the Indian ocean, Indian ocean I think you applied this Jonswap, now we have to select between these two, either Pierson-Moskowitz or Jonswap. The another spectra I think process of Baskaran will tell you, which is I do not have the formula is called Bretschneider.

The Bretschneider formula not having right now with me, but these two are quite common, where the Jonswap is particularly applicable to North sea. And if you find the Jonswap is not favorable in all location, then you can go to either Pierson-Moskowitz or Bretschneider. The Bretschneider I thing I am not having the formula, but you can consist I will tell you how from where Bretschneider, so here these are the three common spectra used in ocean engineering applications, but they are all one dimensional spectra.

Now, coming to this formula before we go into the actual physics of the problem, you will find that the Jonswap has been derived from what is called development of Jonswap with the frequency. So, here actually this is one dimensional spectra, that is your E f versus wave, and you plot you will find as the frequency is decrease your spectra is over is also increasing. So, this is a plot of E f versus f, (()) now this is Jonswap, so those of you go for further studies in the Norwegian technical university, you find the constantly use this spectra.

So, 0.2 is 0.4 and 0.6, so this is in Hertz, H z now you can see this spectra is getting developed, as you for lower frequencies, now higher frequencies you would not get the peak, but why it is like this, and the tail you find it is more or less same, so Jonswap is developing like this. So, last class I told you what is the meaning of it, fully developed sea, but Jonswap is developing that is that tail of this is converging, but you get the peak is increasing with decreasing frequencies. So, you get the various curves in, now this curves are first one is fetch you write this is not 0.5, this is 9.5, the next one this is 20 kilometers, now fetch is the application of the wind.

So, this is 37 K m, this is 52 the last one is 80 kilometers, so this is your fetch, f e t c h this is called fetch limited spectra, so fetch is an important parameters in designing the shape of your Jonswap spectra. Now, you find as you come from 0.6 to 0.2 cycles your spectrum is developing, spectrum is developing from high frequencies to the low frequencies side. And also it is developing if you increase the fetch from 9.5 you have 80 kilometers fetch, that is the sea is now building up.

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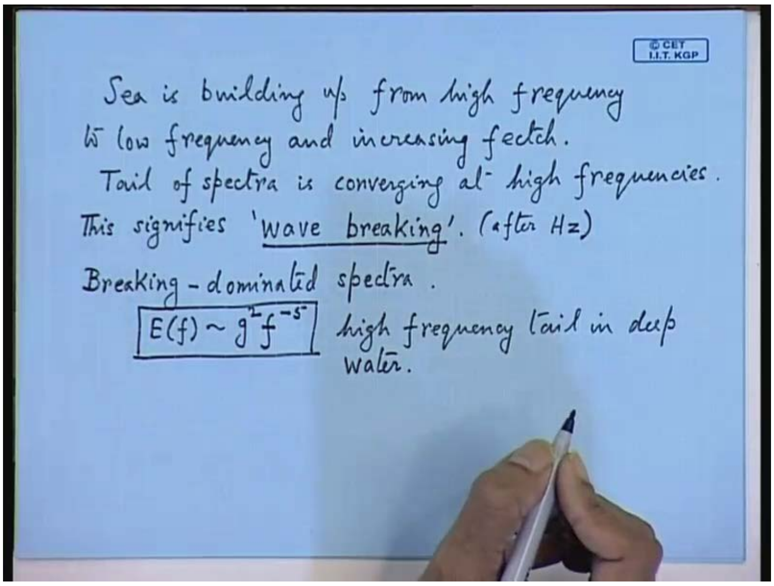

So, sea is building up from high frequency to low frequency and with what and increasing fetch, now why this is happening, because you are gradually transferring more and more energy to the waves from the wind actually. So, this is what is happening, but another problem that you are coming across is the tail end of the spectra, if you notice your tail end, why the tell end there is absolutely no various density. Tail of the spectra is converging at high frequencies you find in this ((Refer Time: 42:25)) all the tails are converging after certain frequencies.

So, in our case this is happening at 0.5 all the tail are converging, so that means, this signifies wave breaking, so waves are breaking up, so oceanographers ignores normally we analyze the scene terms of the spectra. So, that means, after this ((Refer Time: 43:08)) say 0.4 is ok, but after 0.5 that means, the waves are breaking up, so there is wave breaking that is the crust is no more what is called uniform, but it is breaking up. So, wave breaking is offering after 0.5 Hertz, now with this actually in order to incorporate this, lot of study has been made in Jonswap, otherwise Pierson-Moskowitz is ok.

Now, if the Pierson-Moskowitz, so this breaking dominated spectra is this, now Jonswap has actually incorporating both wave breaking as well as the fetch build up, so breaking dominated spectra. So, that is given by E f, that is your acceleration due to gravity, f is our frequency to the power minus 5, so this actually tells us about high frequency tail in deep water. Now, you have a this diagram ((Refer Time: 45:08)), if you have a look now this is the absolute value of f with respect to f, now if you normalize this and you are getting different peaks as you increase the fetch.

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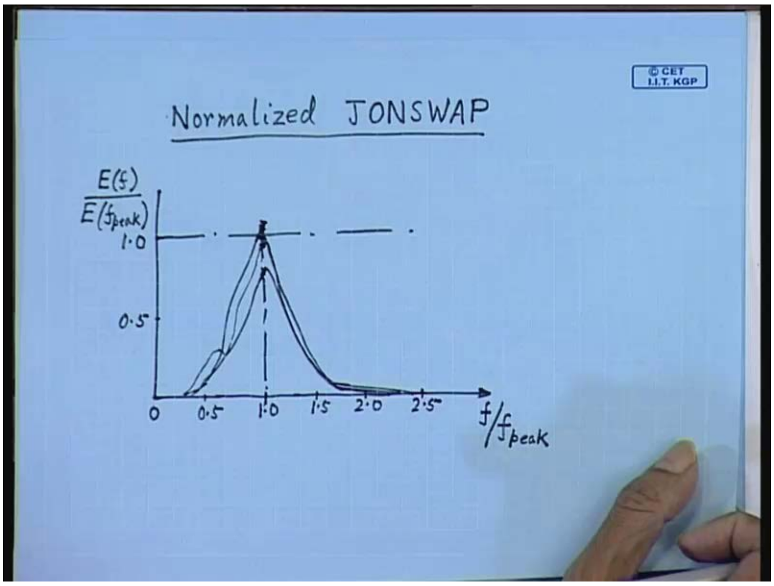

But, suppose now if you want to make any study without this fetch, then you normalized, what is called as normalized Jonswap, now normalized Jonswap to look more uniform in nature. Now, if you want to normalize you divide the frequency by f peak followed that means, you do not have any dimension, so you divide by E f peak. And here you get normalization the frequency E from 0.5 at 1.0 you get the maximum, and goes on like this, 1.5 this is 2.0, here 2.5, so this is normalized Jonswap. So, this is 0, then this 0.5 and maximum it will come obviously, it cannot be greater than 1, because E f will be equal to E f peak at 1.

So, one and one you get maximum somewhere here, you will get an number of curves ((Refer Time: 47:36)) this peak will come somewhere here, and then it will go by this, so this is your normalized to term. All the curves will behave like this, to follow this can do instead of this one, for different of course, different features, move on that I will drawn there, so if you contain this curve, normalized this curve you get a set of curve like this. So, it is having a peak value, this can normalize yourself now here the, so the I have given you what is the equation of that tail end. So, a lot of research has been gone out here, and if you look at the Pierson-Moskowitz formula, so Pierson-Moskowitz says this formula.

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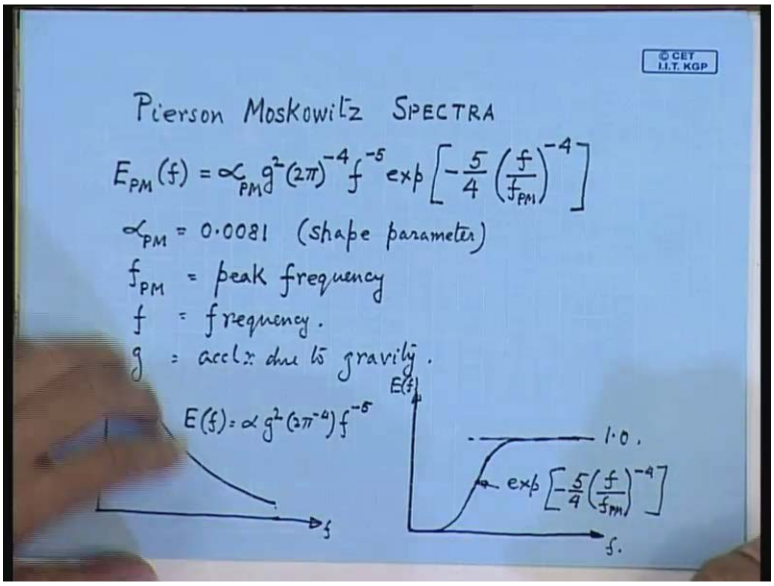

So, here if you want to build the spectra, it will compose of two parts, one is the peak end, the other one is the tail end, so Pierson-Moskowitz, so all of you are want to know the formula is it not E p m, p m stands for Pierson-Moskowitz. Now, alpha is equal to shape parameter alpha p m is called a shape parameter g is your square of the acceleration. So, this is giving us information about the tail end, this is minus 4 f to the, f is your frequency in Hertz minus 5 e x p that is exponential, exponential of minus 5 by 4, this is f divided by f Pierson-Moskowitz, this is power to the power minus 4; so this is Pierson-Moskowitz spectra.

Now, alpha p m you find this is equal to 0.0081, this is called a shape parameter, look at this term f by f p m, so this f p m is what, f p m is your peak frequency over. So, g is acceleration due to gravity, f is the normal frequencies given by f, now what is g is acceleration due to gravity. Now, this Pierson-Moskowitz you actually build up from two separate curves or two separate spectra, if you add this you get Pierson-Moskowitz. So, this is your E f, this is the first part of this equation that is alpha g square 2 pi to the power minus 4, what else is that f to the power of minus 5, so this tells us about the tail end of the Pierson-Moskowitz spectra this is your tail end.

And how do you build up the forward end, the forward end is something like this, this is asymptotic maximum value you get is 1, this also the E f, this is f. So, this is your exponential, the second term exponential of minus 5 by 4, this f by f p m, now you add this two you get the Pierson-Moskowitz.

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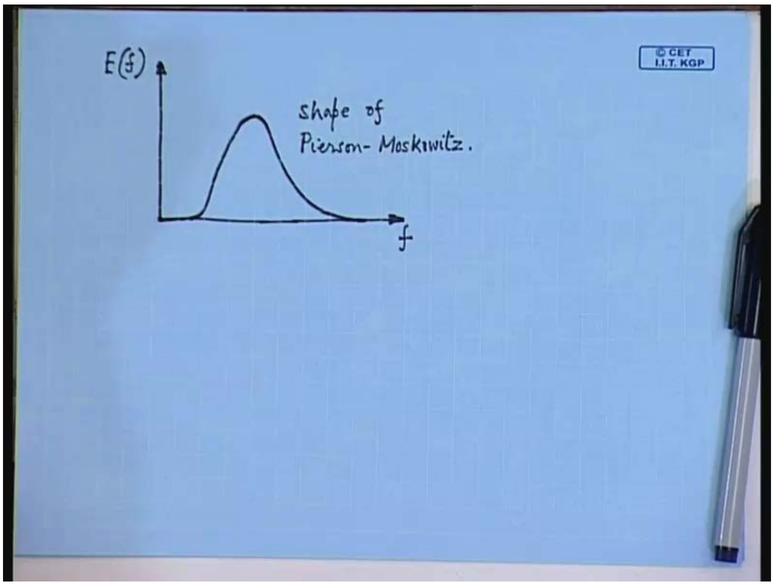

Now, the Jonswap they are not happy with this Pierson-Moskowitz, the reason is the nature of the peak, so Jonswap lot of study was done on the Norwegian sea they said that, the peak is not very sharp, so this is shape of Pierson-Moskowitz. So, next class I will tell you the Jonswap, so Jonswap is nothing but in the Pierson-Moskowitz you have add another term, into multiplied by another factor you will get Jonswap. Now, Jonswap if you plot you will find, you will get a sharper peak, so that is only the difference. Now, in the ocean engineering actually we make use of this two spectra, the other spectra I think that professor Baskaran will brief, which is not here it is called the Bretschneider.

So, thank you, so next class we will finish this on Monday, and will go to waves and structures.