Elements of Ocean Engineering
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Lecture - 16
The Wave Spectra

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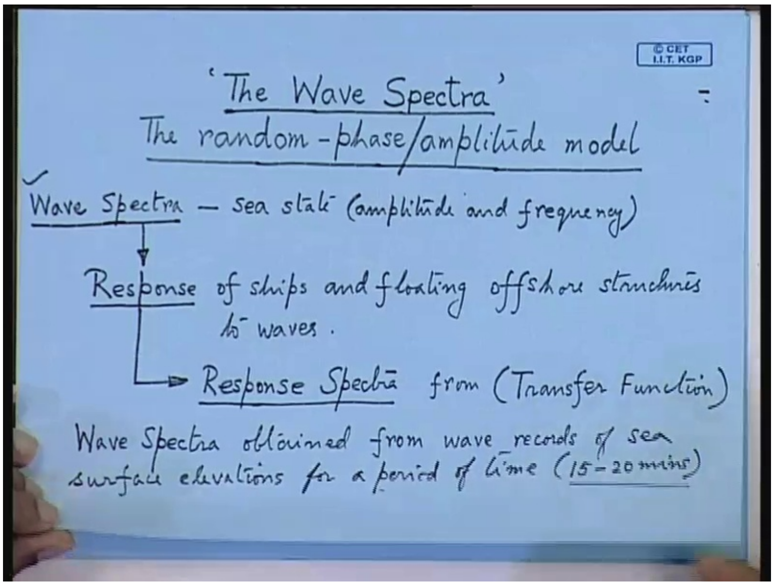

These few classes I will be talking about the Wave Spectra. So, previously I told you how to calculate the pressure from the waves, the hydro dynamic pressure. So, another important aspect of a ocean engineering is the wave spectra, so last class I also discussed about how to we started how to get this spectrum from the suffix elevation. Now, this spectra is important from the point of the view of calculations of spectra, you have to get what is called the sea state, so this actually describes your amplitude and frequency.

Now, unless you get this you will not be able to calculate, what is called the response what is going to be your response of ships and floating offshore structures. So, we have to know this how the behavior of ships and floating offshore structures are going to behave in waves. So, response basically the spectra is required to calculate response of ships, and floating offshore structures to waves.

Now, this we are discussing the wave spectra, now later on going to be finish with this then we have to calculate, what is called the from the response you have to obtain what is called the response spectra. Now, this I will tell you how to get this, but after not today in the subsequent class, so this is where we are going, so this response spectra you have two find out from what is called the wave spectra. So, wave spectra previous class I told you, this you have to get from the wave records. So, wave spectra basically obtained from wave records, wave records you write from wave records of sea surface elevation.

Sea surface elevations are recorded for a period of time, so this time period can be usually for a one record this last from say 15 to 20 minutes, this is the duration of one record. So, these our main object is to get this response spectra for different sea states, and this is obtained via a certain function from you write from what is called a transfer function. So, in vibrations also we come across this word what is called a transfer function anyway. So, this will come later, but in the art now the problem is to I have these wave spectra, so this normally duration is 15 to 20 minutes time record.

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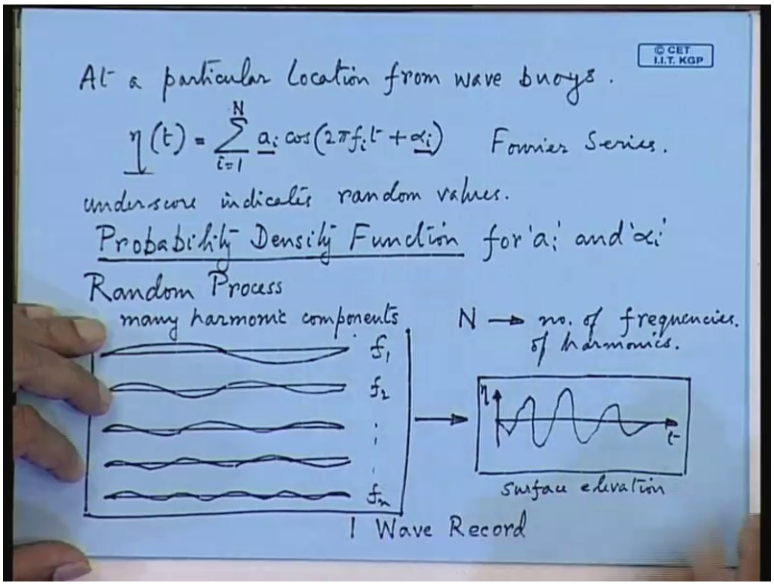

Now, this you have to obtained at a particular place or write at a particular location from wave buoys. Now, last class I told you that you can write the surface elevation as a Fourier series, say sigma i equal to 1 to N, so this N is a number of frequencies and a i cos 2 pie this is what we have written. Now, here there is a small change you simply put 1 underscore and a i and alpha i, so this the underscore actually says that they are random values.

Now, today class we will see how to get the surface elevation from these random values, now if you want to get this you have to do a little bit of statistics. And you have to take help or construct, what is called a probability density function, now in your wave analysis actually you will be dealing with, what is called random processes the wave records that you are getting those are actually random processes. And of course, in your vibration class this comes under random vibration, so there are certain parameters for this.

So, this is actually your random process and random process first you can analyze from a what is called a probability density function, you have to construct for what for this amplitude a i and alpha i. So, this probability density functions you have to see, now a wave record how you are going to build up or this surface elevations, so I told you, so this is actually your Fourier series. Now, your Fourier series are going to have number of frequencies, with various amplitudes and phases.

So, you write this are many harmonic components, so if you break this Fourier series into harmonic components, you get the number of frequencies. So, the first one you can get now remember the amplitude of these harmonics and all the same, so this is called the first harmonic or the fundamental frequency. So, this can also be a second harmonic you will get like this, so like this there will be number of harmonics you will get with different frequencies. So; that means, we have split up your surface elevation in to sum of all these elevations.

So, you will get something like this you keep on doing this, but the problem with this is that, you are getting the surface elevation from discrete number of frequencies, you are getting some discrete, what you have done is getting some discrete values listen it. So, this is many harmonic components, so you write this confirms to say frequency f 1, f 2 like this go on say frequency f r. They depends on N, the size of N, N is number of frequencies of harmonics.

So, you build up your surface elevation from this, so you add all these surface elevations for all the frequencies, then you get the exact nature of the wave surface. So, this is your surface profile, so your surface profile will not be harmonic here it will come something like this anyway. So, this is your timescale and the vertical axis is your surface elevation, so this is your surface elevation, and the whole thing is called what, this along with this is called especially this is called 1 wave record. So, you are analyzing a wave record, so 1 wave record has been obtained from, so many frequencies right.

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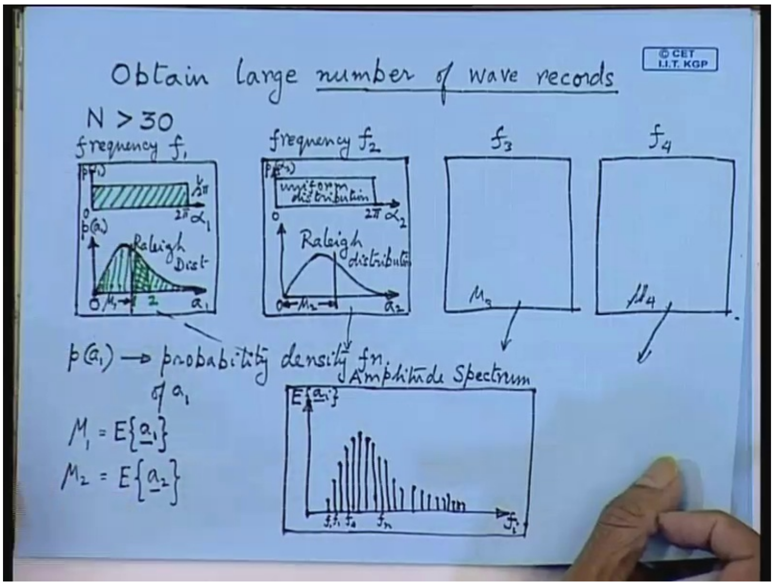

So, like this you have to obtain number of records obtain number of records, then only you will be getting the probability distributions or rather it is advisable to obtain large number of wave records. Now, if you obtain these large numbers of large in statistics actually, they say the large number means number the sample size should be greater than 30. So, this N actually this large means you go greater than 30, you then the error in the samples will be less, otherwise it will be more.

Now, what you do you plot your probability distribution for different frequencies, you make a frequency diagram like this. See this is for a particular frequency f 1, now this you have obtained from large number of wave records, so all wave records 1 wave record will give 1 surface elevation, other will give you ((Refer Time: 13:51)) for the same frequency in each records, you will get this kind of probability density function for the amplitude. Say this is your amplitude a 1, and you will get a this is called a probability density function this is called p of a 1.

And this will follow a Rayleigh distribution something like this you will get, now what is the essence of this distribution, how you calculate the probability you know how to calculate probability for probability density function. So, p a 1 is called a probability density function of a 1, and this substitute 1 denotes it is for a particular frequency f 1. So, what is the value of the area under this curve, so you are required to calculate at the how to tell me what is the area under this curve.

So, I think you should have done some simple statistics, now the probability of say if you want to calculate the probability from this diagram, see there are 2 values of a 1 say this is say this is 1 value and this is 2 values. If I tell you to calculate the probability that a 1 will lie between 1 and 2, so that will be given by the shaded portion that is this hast portion between 1 and 2. Similarly; that means, the probability of a line between 0 and say a point, which is here this is asymptotary the total probability of this will be one, so that is a offshore event, probability can never exceed value 1 listen it.

So, that you should remember, so the total area under this curve will always be 1, and probability that the amplitude now this is at the amplitude the harmonics remember that. So, line between 1 and 2 will be the shaded area between 1 and 2 given by the probability density function, so this is the thing that you have to plot, the other probability that is of interest is the phase angle. Now, phase angle is given by a uniform distribution.

So, this is your phase angle probability distribution for phase angle, so that is given by alpha 1, and here actually this equation I told you what was is this alpha 1, this is your phase angle. Phase angle of what, phase angle of this harmonics, not phase angle of the elevation, so what we have discussing is the probability density function for this a and alpha 1. And this f 1 is what, f 1 is already we given as f 1 if you want to calculate, you this is the frequency and how you get the frequency say f i is equals to i divided by D i is, so you from this diagram you can see, so that is 1 cycle this 1 cycle is listen it.

So, 1 cycle then you take the time duration, duration of time is this 1 D say 15 or 20 minutes, so the frequencies 1 divided by 15 or 1 cycle in 15 minutes. So, the first frequency you can say this is 1 by 15 like this, and if you have to say number of cycles out here it is say how many 1 2. So, will be 2 the duration of time is same, so this frequency for this harmonic is say this one is 1 by 15 this one is 2 by 15 like this you go on.

So, you calculate the frequencies in this manner right, so now, here, but the other point that we are discussing is this alpha i these are the a phase angles for the harmonics. Now, phase angles we are taking as uniform distribution, so my previous class I told you, you take the phase angle to be uniform the distribution. And you take an uniform distribution, then the height of the distribution will be 1 by 2 pie, so what is the meaning of this, so this will give as p alpha 1.

So, this varies from 0 to 2 pie and height will be; obviously, be 1 over 2 pie, so why because then what was the area of this diagram then only you will get 1. So, the total area under this diagram has to be 1 I told you total area under a probability distribution function should always be 1. So that means, your angle or your phase angle has definitely lie between 0 and 2; obviously, any angle will lie between 0 and 2 phi, so that is what this diagram will same.

Now, you make a frequency plot like this, now this is related to one particular frequency, now here actually what will be different. Now, for all this frequencies you will find the probability distribution of the phase angle to be the same, but what will be different is the amplitude. So, the amplitude is our main interest in our wave spectra, so you plot another frequency plot you take see this will be for frequency, say write f 2.

Now, here also the same distribution will be get, now the name of this distribution is a this is called uniform distribution. Now, you always take and the number of frequencies to the greater than 30, you can take less than 30, but there will be errors in your sample. So, this will be alpha 1 or alpha 2 this has to be alpha 2, so this will be p of alpha 2 and this again will be from 0 to 2 pie, so actually this distribution will be same for all the cases. Now, the only problem is with the amplitude which is going to be different.

So, here actually you plot this is your a 2, but you will find that most of this amplitude they are following a particular distribution, and that distribution you can call this as a Rayleigh distribution. So, this is called a Rayleigh distributions, now later on we will discuss that even this amplitude is also not sufficient for all data, now here actually this is also a Rayleigh distribution. But, with different mean see this is your mean, this is your mean value mu 1 is a mean.

And here you will get another mean see mu 2, now mu 1 is different from mu 2 it can be greater than or it can be less than. So, that you did not bother, but you keep on plotting now this you keep on here to make large number of records, see this is you plot for see this is you make have a plot I am not drawing any more diagrams here, see this is f 3 like this if you want. Now, after you have done this there is an average exercise that is to be done.

So, you have to plot what is called a amplitude spectrum, from this what are I shown, I shown only four frequencies, where actually there will be more than say 30 can be 100 also. Now, from these values you build up what is called a amplitude spectrum, so you can see they we have to do a lot of studies from waves, if you want to get this spectrum. So, this is called a amplitude spectrum, now what is the spectrum, so this is nothing but you plot the mean values that you obtained from this f 1, f 2, f 3, f 4 plots.

This mean values are mu 1, mu 2 from here you will get another mu values say mu 3, say mu 4, now this you plot. That is a mean value of your amplitudes for the different frequencies, now there is another term for mu say, so you get for discrete frequencies see this is 1 value, then you will get another value, then you will get another value. So, like this you have to make a plot, now the wanted is that you will find that all this distributions are mostly they are what is called lognormal in character or a stream values.

So, in wave mechanics and oceanography if you want to study or go deeper into this subject, then you have to know what is called extreme value statistics. So, here it will come now or again you may have another value like this, so let us suppose this is called a tail, now you are getting only discrete values this is a discrete points. What you have plotted is for different frequencies, so your horizontal axis is your frequency axis.

So, these are f i and this particulars are f 1, f 2 say f 4 like this you have to plotted, like this f n. So, you are plotting these mu values, now here mu instead of writing mu you write this as expected value, now mu of what expected value of these a i or a random values of a i. So, mu 1 is a expected value now expected value I am writing like this, the underscore signifies that it is a random parameter, otherwise some mu 1 larger a you always remember mean is your expected value in statistics you say it is the mean.

So, mu 2 will be expected value of a 2, so like these you go on, so from this plots you can make a amplitude spectrum, but your amplitude spectrum is descript you can see that you are plotted for discrete frequencies. Now, we are not bothered about the phase angle why because phase angle is uniformly distributed we are more bothered about what is called the amplitude spectrum.

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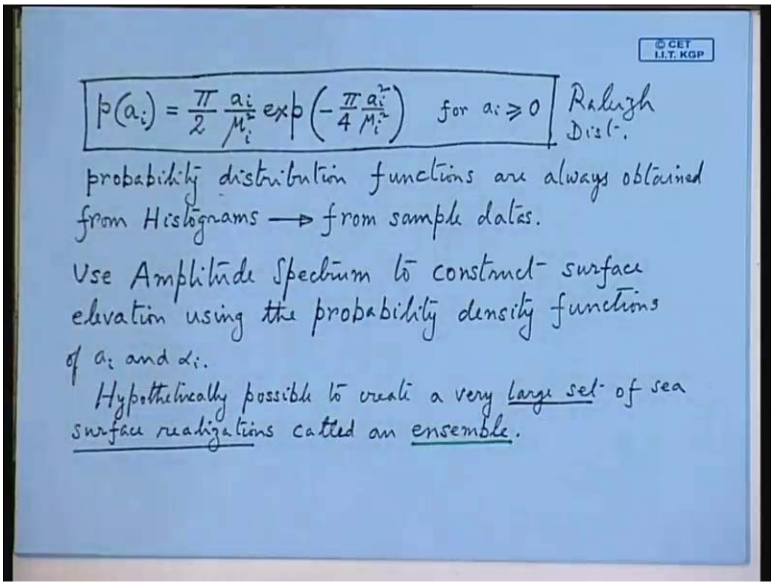

And these p is given the probability density function for a i is given by a Rayleigh function or a Rayleigh distribution. So, this is pie by 2 a i over, now that is why this mu I according to your various values of mu i your shape of the Rayleigh curve is going to change, there is a change in shape of course, my shape is mixed more with same. But, it is different this can be sharper or this can be flattered according to the these value you choose.

So, this actually it is says that it depends on the single parameter mu i, so exponential this e is to the power minus pie by 4 this is a i square over mu I square. Now, this you look up any statistics book, you will get this is for a i how is the amplitude greater than 0. So, this is probability distribution of amplitudes, so this expression as come this is all is the statistics obtain obtained from large number of the sample statistics of the surface elevations.

So, that is why you have to make originally these plots from the Fourier function you split up your waves and from there you correct the statistics. So, your probability density function that we are discussing, now you one thing you remember these are called this, Rayleigh distribution is also called a frequency distribution or if you make a distribution of any parameter, and in the p a's are nothing but the number of times that this a 1 occurs at a period during the class interval, so these are obtained from histograms.

So, in statistics if you go, so all this frequency probability density functions are actually obtained from probability density. Since, we do not your ask that question, we remember that the probability density functions are always obtained from histograms, and histograms from how do obtained this, these are obtained from sample data's anyway. So, this is from wave records and all these thing you have come we are not much bothered about that right now what you bothered about is a special for this, this is called a Rayleigh distribution.

Now, this is one important distribution we have got, now this Rayleigh distribution what we have got is for the amplitude. And you can also go the reverse way, you can calculate I am not showing you can go back from these, say you can use the amplitude spectrum to construct surface elevation. See use amplitude spectrum to construct surface elevations, you can go the reverse direction surface elevation using what, using the probability density functions of a i and alpha is, but you will get how many surface elevations.

So, from these see amplitude spectrum say for one particular frequency, now remember what was that equation that is started with are equation was basically for surface elevation was this one the Fourier series. Now, here if you want to construct eta i you have to know a i, f i and alpha i, you see the parameters, now from here what you do if you want to construct this eta. So, what you do for one particular frequency you read of this a i value, now a i you go back, now this a i is is your mean value a i is your the expected value is your mean value.

Now, according to your values of a i you refer to either this diagram or this diagram or this diagram, say f 4, f four frequency will constructs a you get the particular value of say a 4 out here, you do not mix up you go from say this and go any random. But, you have to because this actually tells us about the particular Rayleigh distribution, and now once you enter these diagram than it is, now from this diagram you read of any a value. And you have this draw particularly you read any alpha I that is why it is called a random distribution.

So, for one particular frequency, we are getting from this diagram what you are getting, you are getting a this a i and alpha i. So that means, for i equals to one frequency you are getting one value right, so now, you build this your eta from all this frequencies, and then you add, so you get 1 surface elevation. So, but you can see from all this record that is from one value from the probability distribution diagrams, for every value for these plots you will get one elevation.

So, how many elevations can you get from the frequency plots, from the probability distribution, you can get larger number of eta values as many as you like. So, literally you write hypothetically because your frequency distributions that you are plotted, these are continuous, you have not there are discrete values, but this are continuous distributions you have obtained. So, from hypothetically you write you can obtain possible to create a very large set larges set of sea surface realizations.

And this larger set this is called on ensemble, so this you remember later on we will come across this what is meant by ensemble average later on. So, this are the, so statistics, then we are coming into the picture, so right now I just told you how to obtain this surface elevations, from the probability density function. So, these are called frequency plots, so there you plot the n number of frequencies you will get this surface elevation, so actually here back calculative.

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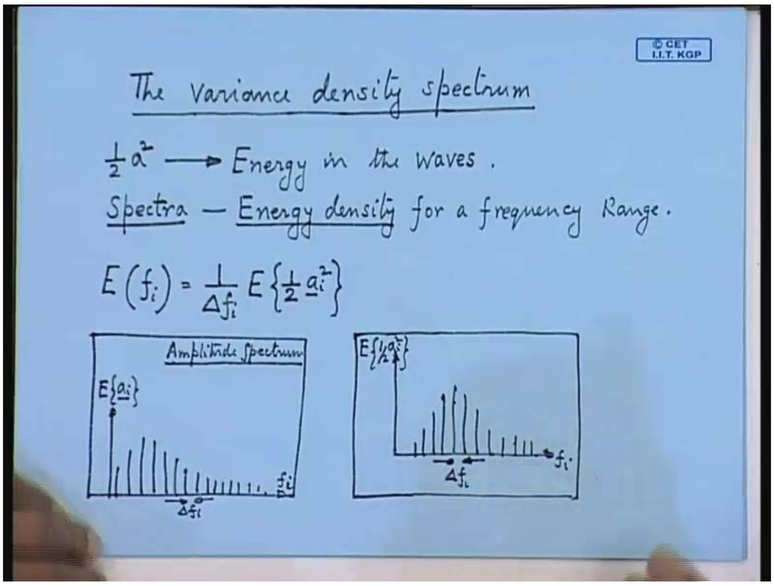

Now, so far, so good now what is more important rather than the amplitude what we want is the variance density spectrum, why I have told you last class. This is another spectrum which is called the variance density spectrum, now the amplitude is not of that much significance, as your half a square or the amplitude square why because this tells us the energy in the waves or energy density. So, your spectra that you are constructive should give some physical meaning, you should have some physical meaning.

So, normally when we talk about spectra; that means, we are actually talking about energy density or the energy spectra. Energy density for a certain frequency range, this in vibrations also we will come across these frequency range, normally we will find the earthquake, current all are described in terms of a spectrum. So that means, they all speak of a certain energy density for frequencies the number of frequency, so has an engineer we want to know how much is energy coming from the excitation.

Wave is nothing but an excitation, earthquake is nothing but an excitation force, so current, so how much energy we are getting from this based on that we have to design the structure. So, that is given by this a square or amplitude square, now amplitude square by itself is does not carries how much of importance, as this value as you divide this by you are the frequency or frequency range. So, this is called a density spectrum E f i is called a density spectra, if you divide by the frequency interval that is delta f i.

Now, what is this E, E is your the mean value this is half a i square underscore means it is a random value. Now, this you are not plotted listen it, so what you are plotted in from this other diagram that we have just now shown is only the amplitude, now similar plot you have to make for a i square, make it for a i square, and after you done this you divide by your delta f i, so you get plot like this from amplitude spectra.

So, this is your amplitude spectra that you have got, now those are all discrete values say normally you all this values they follow the extremer distributions. So, we are made a discrete plot, so this is what is called amplitude spectra or other you write amplitude spectra, now this is given by, actually I should made an axis, now your axis this somewhere here. Say let us write this as E, E is your mean or expected value, now here one thing you should know that your horizontal axis I should have drawn the diagram about these.

So, you write this as f i and this is your delta f i, so this is your you have seems your project a i values. So, these are the amplitudes of the harmonics, you get amplitude spectra, and now what you do, you plot amplitude square, so you make another plot, so amplitude model is a linear listen it. So, now, you are going to this plot now still we are getting discrete values of f i, you are still making a discrete plot remember that, but this is the plot is some more different from your amplitude anyway something you will get.

So, what is the vertical axis, so expected value of I say these are all random values, you make on underscore. So, these are half a i square values, expected value is the mean value, so again what is this, this is delta f i now if you see this two plots actually this diagram should be like this. So, this are discrete frequencies you are plotted, so you write discrete this is also discrete, now from the discrete value we have to go to continuous values.

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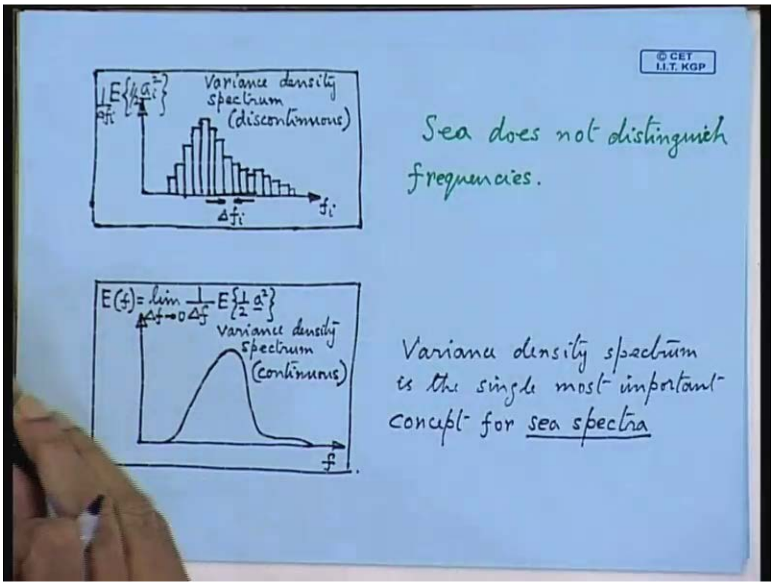

So, now, what you do you divide it by you make another plot, now the reason we are going from discrete values of frequency to a continuous spectra is that the see does not distinguish between frequencies. This thing you should know that sea does not distinguish frequencies, if you look at the waves you will find n number of frequencies that innumerable frequencies, you do not get a specific frequency like this one or this one if you get or do not get, but there will be a continuous distribution of frequencies.

So, that is why we should try to get read of this, then how you are going to from the discrete frequencies, you should come to the continuous frequency ridge that is not the done like this. Now, the delta f i is you will plot as a histogram, you get a histogram plot, now histogram you know that these values are called the class intervals that is, but still we are getting the plot like this, you look up any statistics book and see how you get a histogram from that you make your frequency plot. So, this are called class intervals and still this is your f i axis, so this is called delta f i, but what is your vertical axis.

Now, vertical axis the previous diagram we are plotted, if you can see that was E half a i square, now you divide these by delta f i, so E is your expected value. So, this is half a i again random these you write 1 over delta f i, so we are then you will get the variance density spectrum, but we have got variance density spectrum for discontinuous frequencies. So, variance density spectrum is the single most important factor or most right most important concept for sea spectra, so this is how we obtain the sea spectra. So, next class we will see what is this sea spectra.