Elements of Ocean Engineering
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Lecture - 15
Waves- IV

So, we have already formulated this Stokes equation. So, now we will look at the stream function with theory.

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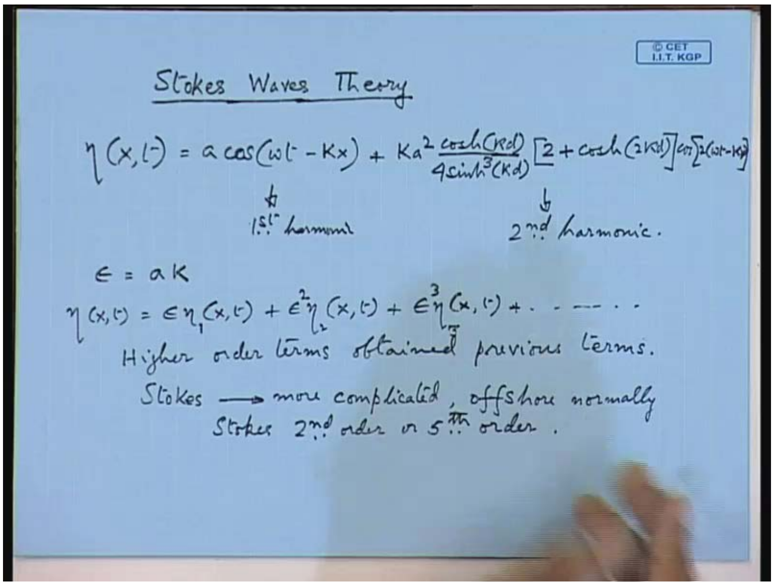

So, Stokes gives us this for non-linear waves. So, when you analyze offshore structures you look into the diagram that I have given; that is the applicability of all the theories, do not blindly apply your linear wave theory. Now in the Stokes with three I told you that you can split up into the first and the second and the number of harmonics. So, you get this kind of equation from the non-linear Bernoulli's equation or the other nonlinearities that is present. So, here I am just giving you the glimpse of what we have to do. So, this is coming like this.

Now that the problem that will come is in shallow water. So, 4 this is sine hyperbolic cube. So, this is your second-order term, and this actually consists of the first and second harmonic. Why harmonic you can see the second term is having a cos. So, this is your cos hyperbolic 2 k d, and this is cos of this is twice omega t, is it not, minus k x. So, that means your phase angle is this end. Now here actually this is your first harmonic, and this is your second harmonic, but you can split this up into as many harmonics you like.

So, here actually the Stokes waves it consists of two waves. The first is your linear waves, and the other is your second harmonic having the same phase angle. So, similarly you can go on doing this, and your epsilon is this is a multiplied by the k; k is your wave number, and a is the amplitude of the linear wave. Now here what you do? You can keep on doing this. So, your third-order would be epsilon like this etcetera plus epsilon square; this is beta 2 x, this is what I will do in x t, x t plus. So, like this you can move on. Now the each iteration actually consist of putting the first two terms of the previous terms into your nonlinear equation, and especially the Bernoulli’s equation get the higher order terms.

So, here actually higher-order term obtained from previous terms, because this is the essence of Stokes theory, but by this method actually you will find if you do a computer program, your Stokes theory takes lot of computer time. So, Stokes is actually more complicated, and in offshore normally we go for Stokes second-order or fifth-order; sometimes you also go for fifth-order term. So, the resume of application I have given you in the previous diagram. So, I am not going.

So, in the diagram actually you also have Stokes second, third and fifth-order terms. So, after the linear then Stokes, then you has cnoidal and solitary waves. Now here what you do? So, you can see in this Stokes there are number of waves composed of different nonlinear terms. Now Dean further said that when there is shallow water Stokes equation are more complicated.

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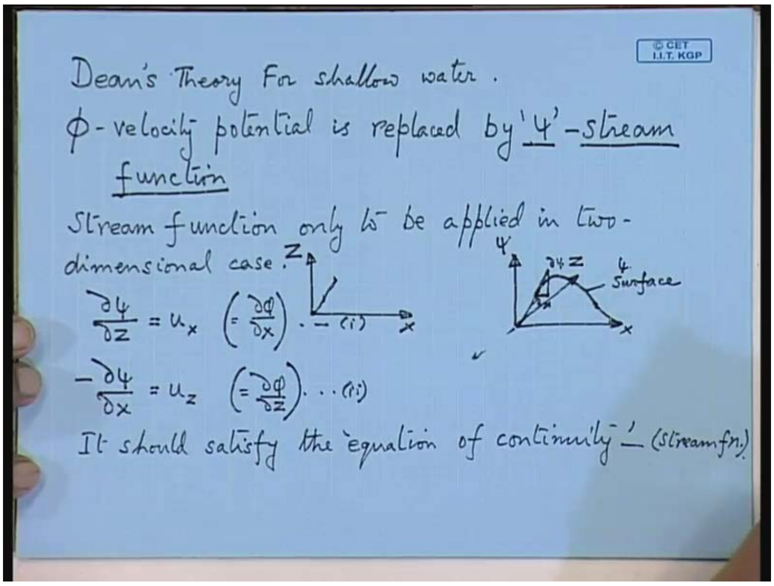

So, this is called Dean’s theory for shallow water. Now in this theory actually you do not use the velocity potential. So, phi velocity potential, now what is the meaning of this term velocity potential? So, that means the differentiation of phi will give you the velocity, is it not. So, velocity potential is replaced by another term by psi. This is called stream function. Now here actually the problem has come from applicability of Stokes theory, because in Stokes theory we will take up large computed time as you can see you can go to previous iterations will give you the next term. But here actually at one go you can do, but prior to that you have to define what is called a stream function.

And the steam function is defined in such a way that you can get the velocity potential. Now stream function you can apply only for the 2 dimensional cases. So, that is the limitation. Say, here you apply it say in the vertical excitement. So, in this plane you can apply it. Say, this is your x plane, and this is your z-axis. So, in this plane if you apply then you can get your velocity; that is your del psi del z. So, that will be equal to your velocity in the x direction, followed. Now what is the velocity in this x direction? So, this is nothing but del phi or del x. Similarly, we have minus. Now here actually the minus term is given for a definite reason which we will see just now.

So, minus del phi over del x. So, that will be equal to U z velocity in the z-direction. So, this is equal to del phi over del z, because this is the definition of the stream function. So, that means if you plot del phi over del z if you plot a stream function, say, this is your stream function; this is exit thing. Suppose on the horizontal x z plane, the three axis if you plot; this is your phi axis, then you have, say, this is your x-axis, and this is your z-axis. Then let us see what you will get. So, you will get a three-dimensional curve.

Now you look at these two equations. So, this is equation number one and equation number two, followed. So, you differentiate this with respect to z. So, actually you will be getting a contour; that is you will be getting a. So, this is no more a one-dimensional. So, you will get a surface like this. Now what is the meaning of this del phi over del z? Or rather you can look at this del phi over del x. So, that means you take from this curve a straight line, and say this is your del phi, and this distance is del x. So, this is actually the slope of the psi surface.

So, what we are getting is the slope of the psi surface. So, here actually you will get the velocity. Now velocity will be in what direction? You find out the slope from here from the psi surface contour. And slope you get the velocity not in this direction, or z-direction is going inside the blackboard. So, you will get in the opposite direction. So, this actually gives us both the magnitude and also the direction. So, actually you have to plot a psi surface with respect to x and z. So, now, you do the other term; that is now with respect to del psi del z, but that will give you velocity in the x-direction. So, this is the gist of the psi function.

Now the reason that we have taken del psi del x and minus del psi del x is it should satisfy some fundamental equation, what? Satisfy the equation of continuity; that is why this formulation you will find del psi over del z and minus del x del x. Now it means the psi equations. So, here you write your stream function.

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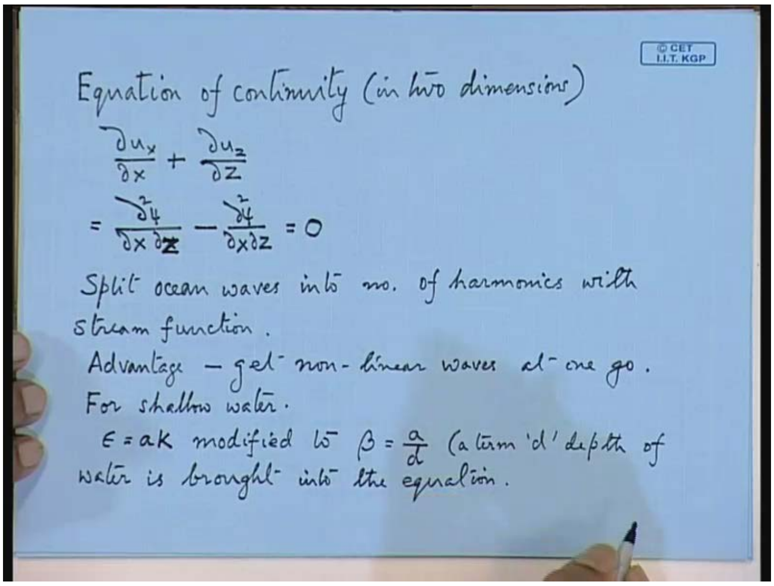

Now what is your equation of continuity? So, equation of continuity in 2 dimensions is you write the equation of continuity. So, this is del u x over del x plus x and z frame. So, you are getting plus del u del z. Now you will find out the value of this and see whether this satisfies your equation of continuity or not. So, what is your u x value? U x is del psi over del z. So, this will be how much? Del psi over del x del z, and similarly you write the other term. So, this will be minus; that is why we have given the minus sign. So, this will be del z, is it not; this is del psi over del z. Oh, sorry this I think this should be U x you are getting by del psi or del z? Sorry, this is your del z. So, there is a mistake; actually, this should be del z; this will be how much? Del square, yeah.

So, what will be this? You mean by this same thing unit. So, this is equal to 0. So, that means this satisfies your equation of continuity. So, that is why we have written del psi del z equal to u x, and this one equal to u z. Now with this you formulate this stream function. Now actually here also you split into number of wavelengths. So, number of harmonics; that is a split ocean wave into number of harmonics with stream function. But here actually you will find the Stokes example; that is sort of an iterative example, is it not. But here actually in one shot you can find out the nonlinear terms. So, advantage is get nonlinear waves at one go.

Now here for shallow water we have to make certain corrections; for shallow water you write this epsilon is equals to the amplitude of the harmonic wave multiplied by the wave number k. So, previously we had done this, is it not, epsilon into a k, but this has to be modified to this beta a divided by d; you bring another term d. So, here a term d that is depth of water is brought into the equation. So, this is what you have to modify that is all and now this surface elevation. So, that means you can see that the ocean waves are not cnoidal waves.

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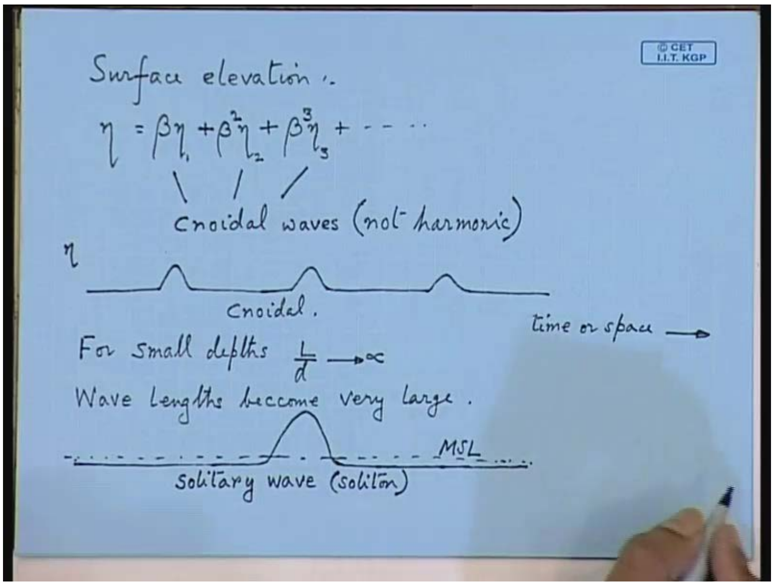

Then you write surface elevation equation in terms of beta. So, this will be. So, previously we had written in terms of epsilon. So, now you write this in terms of beta. So, these are your non-linear harmonics. So, like this you can go on. This is up to third harmonic I have given. Now in this case you find this of course these are called cnoidal waves. So, I will see to that this is not harmonic. So, you will not get a harmonic term as in the Stokes. So, you get cnoidal waves like this. Now here this cnoidal wave will come as a certain heel of water. So, you can see the ocean waves somewhat like this; that means when you are coming towards the coast. So, this is called cnoidal.

So, this is your surface elevation eta and your horizontal axis can be time or space whichever you can write. Now suppose this depth becomes equal to zero or very very small; for very small depths this L by d will tend to infinity. Now in this case wavelength becomes very large and you get a wave profile like this. So, actually when you are coming towards the coast from deep water to shallow water, the water actually piles up as you can see from this. This is your mean sea level. So, this is your mean sea level.

So, the amplitude of the wave is largely above the mean sea level. Now mean sea level actually also can come down here. So, this wave is called a, the wavelength is very large. So, another p q we will get somewhere here which I have not drawn. So, here you say that this is a solitary wave. See it is quite interesting, or sometimes this is called a soliton. So, this picture we are getting when you are coming from deep water to shallow water.

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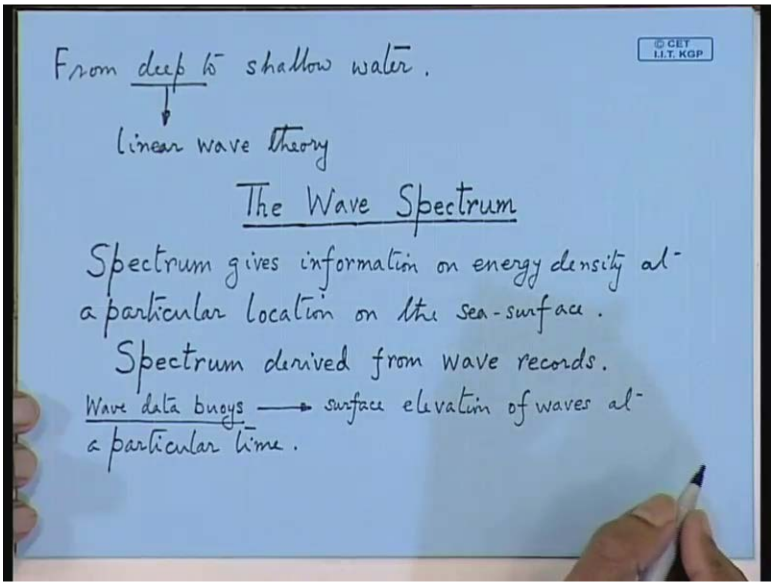

So, from deep to shallow water; so, that means in the shallow water our problems are more complicated, is it not. Deep water actually you can go for the linear wave theory, but your linear wave theory does not hold good for coastal waters as we can see from this diagram. See, actually I think we do not have much time here other than this class you try to derive these equations for the cnoidal and solitary waves. So, this is I wanted to give you the picture of the wave profile. Now there is another important theory which is called the waves spectrum. So, all of you are doing my vibration class.

So, waves and spectrum are analogous to your vibration. Wave is nothing but a vibration; it is disturbance of the surface profile by giving some energy of the waves spectrum. So, in ocean engineering you will come across this word spectrum. Now spectrum actually gives you the wave energy at a particular location. Spectrum gives information on what; on energy density at a particular location on the sea surface. Now how you have derived the spectrum? So, spectrum is always derived from what is called wave records.

So, on the sea surface or along the coast particularly everywhere you will find wave data buoys or data buoys. So, this will tell you the surface elevation of waves at a particular time. So, if you want to make your spectrum. So, you have to get data from wave buoys. So, wave buoys you will find on the sea, and they transmit data to the shore at specific time intervals. So, you record your surface equation.

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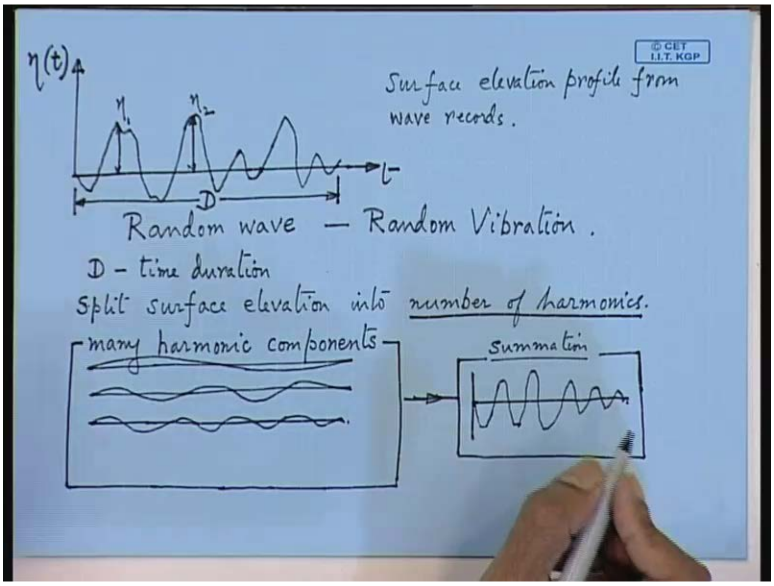

Now what you will get is not a simple harmonic wave, but you will get a wave record like this, but how to analyze this kind of wave record? So, this is a random signal, is it not? In vibration we call wave as a signal. So, this is a random wave. So, this is what you are getting from the wave buoys with respect to time you are getting a surface elevation, but can you make any analysis from this? So, this is called. So, those of you who are studying vibration, you know these random waves comes under random vibration. So, this is say now the wave buoys actually is giving you the surface elevation for a certain deviation of time; say, you call this d.

So, d you denote as time deviation. Now what is done actually? You have to split this up. So, split or rather surface elevation into a number of harmonics; rather you can split this as a sine wave out or a cosine wave, followed. Now how the picture is going to look like? Suppose this is your surface, say, a random you have got for, say, duration of time maybe one day or 12 hours or 1 hour, 2 hour. Whatever it is you have got the surface of the radiations. So, these are your eta values; now some scientific analysis has to be done, is it not.

So, this is from your means sea level you are getting this. Now if you want to do this, you split this up into, say, the harmonic components, but your harmonic components are going to have many frequencies. But there should be some definite methodology by which you do this. So, our main intention is to get information from this kind of a random signal. So, waves if you study actually you are studying signal processing; you have to do a lot of signal processing in order to get the required information. So, now you will find that this is, say, your mean sea level; you can have your first harmonic, say, this is one wavelength.

Then you can have a number of wavelengths, but the result of all these harmonics will give you this elevation; it has to give you this surface elevation. Suppose you are splitting up all this harmonic in some arbitrary fashion and then at the end you do not get this surface elevation profile. So, this is your surface elevation profile. In fact this is very important, because straightaway you are getting this from. So, surface elevation profile from wave records. So, like this you split this into number of components. So, this actually should give you generate this surface profile.

If it does not generate then you have to order the parameter of your harmonic components; it can generate or it may not generate. So, nowadays there are complex computer programs to solve your problem. So, instantaneously you will get the wave spectrum with which we are interested. So, you are getting this. So, this is summation. So, you are getting from here to this. Now the equation that you will apply is what is called a Fourier equation.

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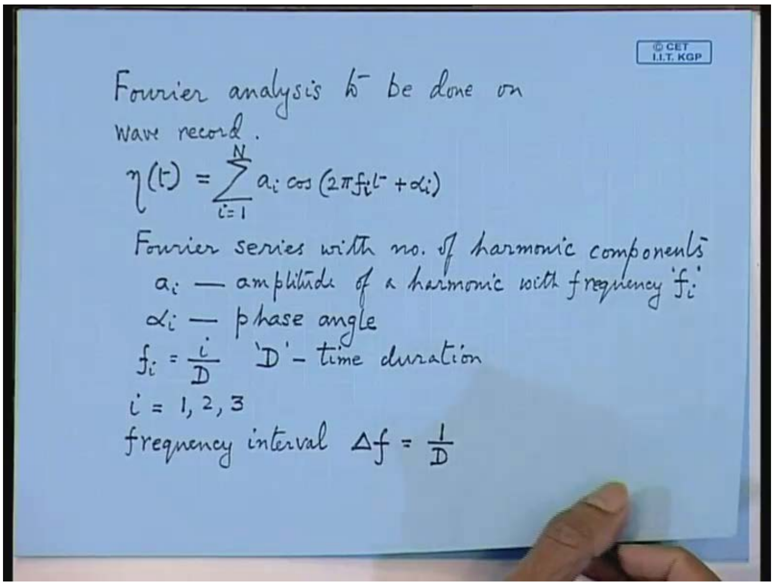

So, in your math class you must have come across what is called a Fourier analysis to be done on wave record. So, this particular thing actually you will study in dynamical question paper, and ultimately you have to get beta t; that is discard, followed, from what? From number of harmonic components; so, you write your equation in this form. So, this is in fact a Fourier series. So, this is what we should get, but you have to derive what? You have to find out a small a is the amplitude of a harmonic cannot be your surface elevation; this is amplitude of a harmonic, harmonic is other harmonic you give is sine curve or a cosine curve. So, this is the amplitude of a harmonic. What is your frequency? [FL], this is not f 1; you write this as f i.

So, your harmonic will have a certain frequency, and what else? And it should have a phase angle. Now this you should know. This three terms out here; that is a i is your amplitude, f i is your frequency, and alpha I this is the phase angle. Now f i you write this as i over d. Now d has already been defined as, what is d? This is time duration. Now we are getting number of frequencies from this time duration according to the values of i. Now i you can give 1, 2, 3, like this values. So, if you give 1, 2, 3, then what is your frequency interval? Frequency interval is delta f, and how much is that? So, delta f will always be difference between 1 and 2. So, this you will always get this as 1 by d. So, here we are getting from the wave record number of frequencies. So, that is the number of frequencies of your harmonics. So, now, what you do? In this equation you have to find out.

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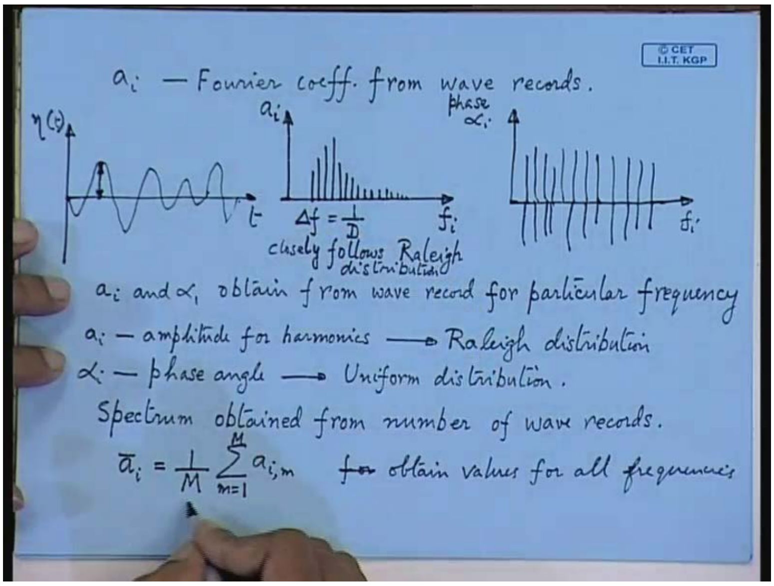

Find these are called Fourier coefficients means a i and alpha i. So, a i is called a Fourier coefficient. You have to find this out from wave records. Now the interesting part in the analyses you will find that you will get this kind of variation in the amplitude. So, this is the fundamental wave record that we have got. So, this is a random surface elevation. So, we are getting something like this, followed; of course, this is not decreasing. Now this if we graphs, you will find one will come for amplitude which we will shortly see that it is following a definite distribution. This is your amplitude profile will come a. So, rather you can write a i. Now what you are doing is plotting some discreet frequency values.

So, you get discreet values at particular frequencies. Now later on you will find that this is called whatever distribution. So, like this it will go on. So, these are your frequencies or rather you can write this as frequency interval. So, this is 1 by d. The horizontal axis will be frequency f i, a i is your air equation. Now from this you will get this distribution, and you will also get a phase distribution. Now you have to get the values of a I and alpha; otherwise, you cannot get your surface profile. Now i is for a particular frequency. Now in your phase distribution you will find that this is actually your longer curve. Now here actually you will get a phase distribution like this somewhat uniform but not exactly uniform and this also will be in the other side of your axis.

So, even this is your frequency, horizontal axis is frequency, vertical axis is what? It is phase. Now this a i and alpha i you have to obtain from wave records. So, this actually closely follows what you will find? This is called a Rayleigh distribution. Now you have the phase angle, alpha is your phase angle. So, a i is actually your amplitude for a harmonics; with different frequencies it follows what is called a Rayleigh distribution. So, these are all abnormal distributions, but surprisingly you find this angle alpha this is your phase angle. This is rather a uniform distribution.

Now actually if you want to have the spectrum, spectrum you have to get for number of wave records. The reason is that your wave elevations are random. So, you take wave records, the same time duration at a particular time of the day, consequent, say, four days, five days, seven days at a particular location. But you will find that you are not having the same elevation, and these elevations will be different for different time records. So, that means we are getting a random wave history or random elevation, followed. Now what is to be done is if you do a spectrum analysis, you have to find out the mean of a particular frequency, mean elevation for a particular frequency from all these records. So, you have taken m records. So, m is experiment has been done many times.

So, you have to have the average value m equals to 1 to m. Now this is done a i for a particular frequency. So, this i is for a particular frequency you do, and then you do this obtained value for all frequency. So, how this is done? So, you will obtain. So, I have told you this surface elevation you split into number of harmonics, but that will give you for one particular record which you have obtained. So, this is the first harmonic; for one record you have got. Then the next day also you do the same experiment. So, the experiment is repeated m equals to 1 to m number of times. Then you take the average value. So, this is giving you the average surface elevation. Now what is done is instead of taking the elevation you take the variants of the elevation.

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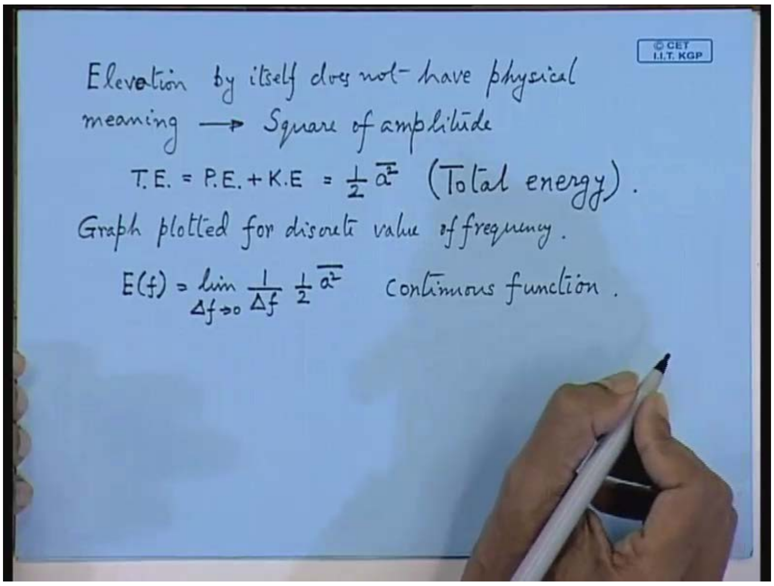

So, elevation of this surface by itself does not convey any physical meaning; it does not have any physical meaning, why? Because you find that if you take sum of the amplitude and the mean of the sum, they are not the same; they are not having equal. So, what you do is you go for square of the amplitude, why? Because the square of the amplitude you will find is directly proportional to the total energy of the wave. Total energy of the wave is combined of two parts. One is your potential energy plus kinetic energy is the total energy of the wave. So, what is the formula for potential energy? So, I think this is one-fourth and look at this and total energy will be half of the square of the amplitude; bar means I have taken the mean value. So, this is actually the total energy.

So, this has some physical meaning rather than simply your amplitude plotting. So, what we are getting, but these are what? In the graph that you are plotted is only for discreet values. So, graph that has been plotted for discreet values of frequency; now here you decrease the frequency range or frequency interval, then you will get a continuous function. So, in the limit you take delta f tending to zero, and this will be 1 by delta f and are half of the amplitude square. So, this is a continuous function instead of a discreet function. So, today we will stop here. Now in the next class we will have a look at what type of the frequency spectrum we are getting.