Elements of Ocean Engineering

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Lecture - 11
Waves – II

Good morning, so today we begin continue our discussion on Waves. And that dispersion relationship that we already found out that we examine for deep water.

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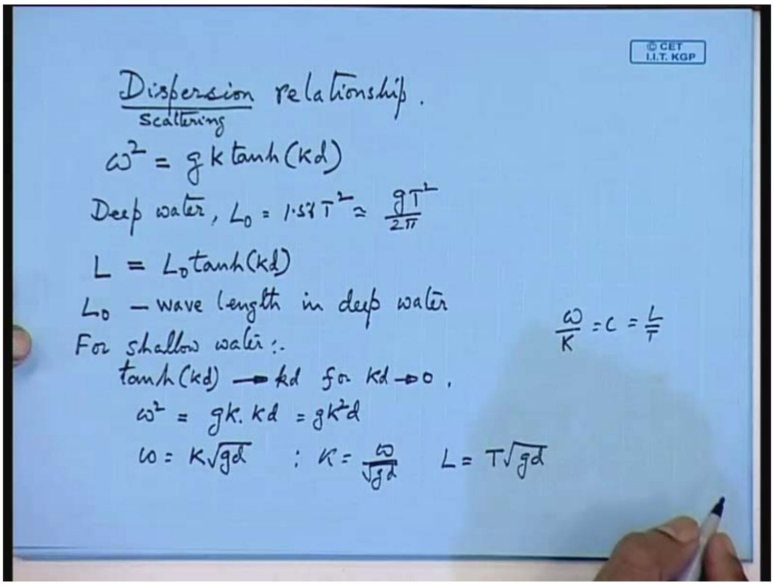

So, the relationship was now, in deep water we got the value of as 0 as 1.56 of t square. So, this was from this equation that is omega square, this equals to g k tan hyperbolic k d, this is your dispersion relationship. And today, we will see that in the wave group, in the individual waves are different frequency, and they are disperse; that means, the scatter on the sea in waves in dispersion comes from scattering, so this we will see today's class. Now, from here we get the deep water relationship that is l 0 equals to, now there are two conditions, one is deep water the other is shallow water.

So, in deep water we have obtained in the last class, the deep water wavelength to be 1.56 times the time period square, now from this equation, so you can express air, the ordinary wavelength is L 0 multiplied by tan hyperbolic k d. So, that L 0 is the wavelength in deep water, so this we have already found out, because they are L 0 1.56

we have obtained from what was the expression was g T square over 2 pi, so you substitute this in the equation for your omega square, there is you will get this.

Now, waves for shallow water, let us see what happens in shallow water, this tan hyperbolic k d, so this approaches k d for k d tending to 0, so you are dispersion relationship becomes omega square is equal to g k multiplied by k d. So, this is equal to g k square d, so omega is equal to k multiplied by route over g d, so this is your shallow water expression. So, from this you can get the value of wavelength, so two parameters are important, one is your wavelength and other is the frequency.

So, the value of k is omega over route over g d from this you can get, length is equals to T route over g d, so this we have got from omega, omega by k is equal to C is it not, omega by k is equal to c, this is equal to L by T from this you can get route over g d. So, these are the values of frequency and wavelength.

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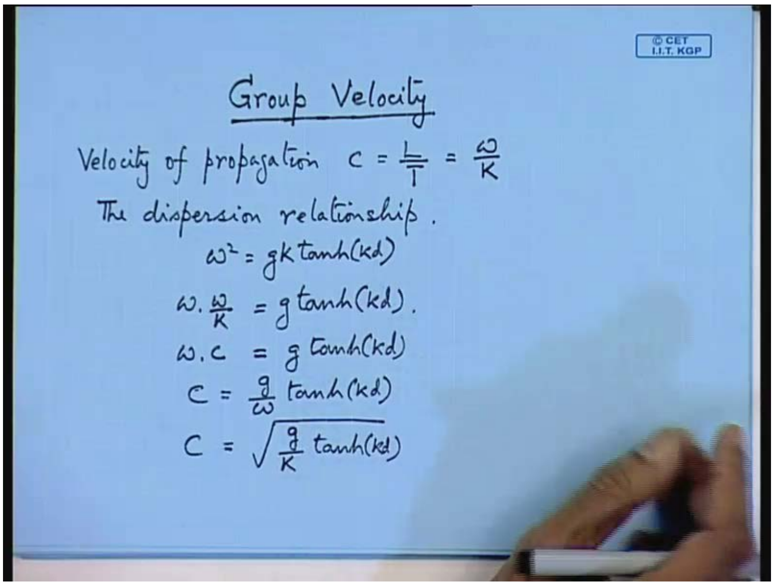

So, now let us see there as come to having important questions, which is called group velocity, now what is this I told you that, the dispersion relationship that is your omega is coming, basically it come the dispersion of scuttling you will find, it comes in wave group. Where there is a lead frequency and the trial frequency, there are number of waves in a particular wave groups, so in a sea you will find it is not composed single wave, but there number of waves.

So, let us start from velocity of propagation, so your velocity propagation is, so this we have already founded to be how much, so this is your C value for particular waves. So, this is simply the wave length over there time period and this we are said this is equal to your omega over k, k is even number. Now, from dispersion relationship, so this is your dispersion relationship, so let us see what happens here, so omega square is equal to g k tan hyperbolic k d, so this is for load any depth.

Now, you can split this have you can write this as omega multiplied by omega by k, so this is g tan hyperbolic k d, now what is this omega by k, so omega by k is nothing but your velocity of propagation that is C, C is equal to g tan h hyperbolic k d. So, C we are getting as g by omega tan hyperbolic, so this is the velocity of propagation of a single wave. Now, from here this will come to if you substitute value of omega, will get C is equal to route over g by k tan hyperbolic k d, if substitute value of omega from omega square, omega will be route over this, so you bring is down, so you get this equation.

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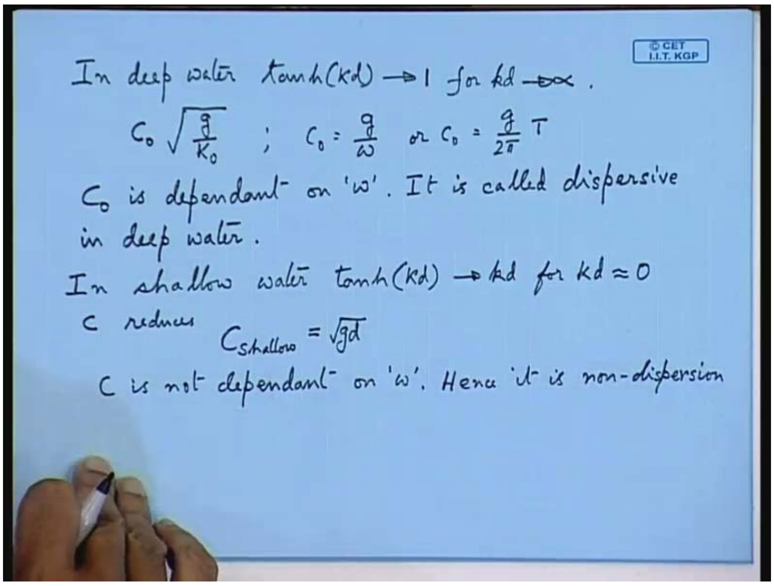

Now, in deep water in deep water what as tan hyperbolic k d approaches 1, so this approaches 1 for k d 10 into infinity, so then what happened for the velocity of propagation. So, C 0 is simply route over g over k naught, now this substitute not signifies it is in deep water, so from here we get the value of C naught to be g over omega or C naught equals to g over 2 pi multiplied by T. So, omega is how much, omega is T is to pi over omega time period, so from this you can get.

Now, in this expression you find that is C 0, C 0 is depended on what from this equation, C 0 is depended on omega, since it is depended on omega it is called dispersion; now this phenomenon is occurring in deep water. So, this is your deep water phenomenon, that is waves are dispersion that is been getting scattered, according to the frequency. Now, in shallow water, now let us see whether in shallow water you get in omega square term or not, now in this omega term is present it will be called dispersion otherwise, it will be in non dispersion, now in shallow water tan hyperbolic k d is how much.

So, this tan hyperbolic k d will approach for k d near the equal approaching 0, so this C reduces to what are the expression for C route over g by k tan hyperbolic k d, now tan hyperbolic k d is approaching k d. So, C writ, C shallow, so this is simply become equal to route over g d, so from ((Refer Time: 11:54)) this expression you can write tan hyperbolic k d, because multiplied by k d, so k k is simply have route over g d. Now, in this you find C is not depended on omega, reduce only wearing with respect to root over g, root over g is the constant.

So, it is not depended on omega, hence it is non dispersive, that is no dispersion of the waves in shallow water, so waves are not getting to the dispersion according to their frequency. So, this is called group velocity and if you want to have look at the physical phenomenon what is happening.

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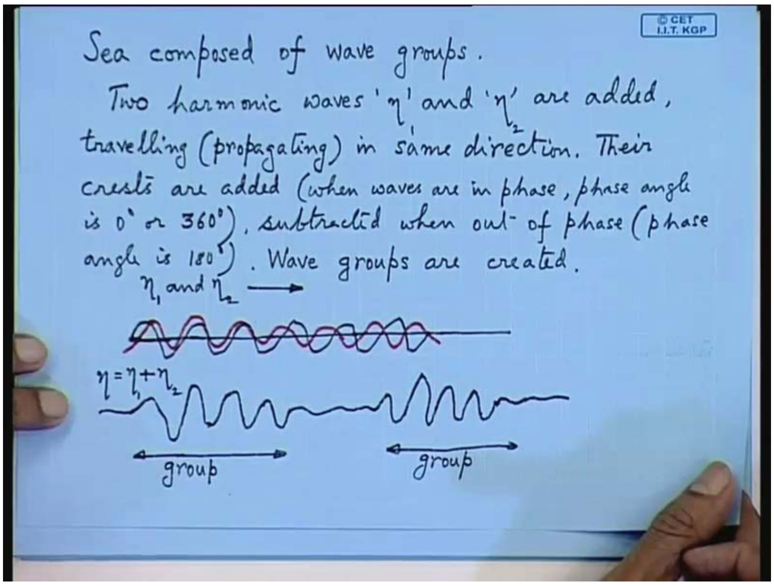

You will find in the sea, there is not a single wave what rather there are in sea is composed on group of waves, or rather you write of wave groups that is why the analysis is particularly difficult. Now, if you want to study this, let us take the example of two harmonic waves, let us take the simple example instead of having single waves, that are have two waves. So, two harmonics are rather you write two harmonic waves, now the elevation of the waves is write as eta 1 and the other one you write as eta 2.

And just look of the expression for eta 1 on eta 2, now two harmonic waves are added and they are traveling in the same direction, traveling or you can write propagating, this is propagating in same direction, then what happens their crests are added. Now, this will be added when waves are in phase, so that means your phase angle this either 0 degree, phase angle is how much, waves are in phase winds your phase angle this is 0 degree or 360 degree, now this is the situation.

Now, these are subtracted when out of phase, so out of phase 0 phase angle is 180 degree, when phase angle 180 degree we say out of phase, now here you find wave groups are created. So, here actually draw in the figure is that will difficult, now you can see a wave groups, so here we have two harmonic waves are rather you can write two harmonics in this simplest case, we have harmonic like this, and you super impose another harmonic, so you write eta 1 and eta 2. Now, both should be propagating and traveling in this direction.

Now, on top of this you add another wave, so another wave will be there is a comes short of here there is frees back, my fear is you add like this you will get the near the two waves eta 1 and eta 2. Now, here the actual picture that you will get something like this, it is just add the wave height at the particular instant, so you can see this sea is not your exact sin wave is it not, or the cosign wave something like this, so this is other confused single is it not. So, if you want study waves, it is your actually studying is signal, so this is situation of eta equals to eta 1 plus eta 2.

Now, you can see there are two distinct figures, that is there is a series of elevation and there is no elevation, there is something near about the wind sea level flat. So, this is one wave group, so this is what will be study and here you get another group, so this is your physical phenomenon. Now, this is the example we have taken with only two waves, eta

1 and eta 2, you remember there may be many waves, this for all simple study we have done this, so now you at the surface elevation.

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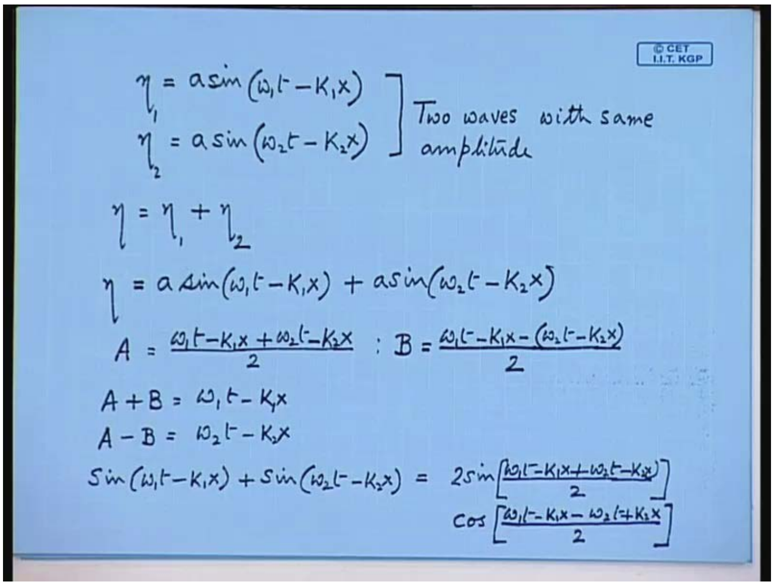

So, your eta 1, let us say the these two are two harmonic waves, so first harmonic you write as eta 1 as you take same as amplitude, a sign omega 1 t minus k 1 x, so this is 1 harmonic. And the other harmonic you write as eta 2 you take same average a sign, but you have frequency and wave number will be different, so this will be omega 2 t minus k 2 x, so these are actually two waves having same amplitude. Now, you add this two same amplitude, so the result in it will be you write eta equals to eta 1 plus eta 2, now you composed the same as single wave from this two different waves.

You find out single wave having these two component waves, so how can you do that, so what is the value of eta, so eta is simply a sin omega 1 t minus k 1 x plus a sin omega 2 t minus k 2 x, may add this two, can you add. Now, if you want add you find out trigonometry expression for these two sin, you write this thing, the trigonometry use spilt this frequency in, you write A equals to omega 1 t minus k 1 x in simply add this plus omega 2 t minus k 2 x, what will this is divided by 2 tell me what will be B. So, this is at expression sin A plus B plus sin A minus B, so from that new employee, so this will be omega 1 t minus k 1 x minus this things, this is omega 2 t minus k 2 x for now you can do this.

Now, if you add A plus B your getting sin omega 1 t minus k 1 x, so A plus B you are getting omega 1 t minus k 1 x, and what is the value of A minus B, omega 2 t minus k 2 x. So, now you write down the expression, so therefore what you are getting sin omega 1 t minus k 1 x, you add this to plus sign omega 2 t minus k 2 x, so this will be simply 2 A, so this will be twice sin what, so you are getting large number listened. So, what is the value of A, so this is omega 1 t minus k 1 x plus omega 2 t minus k 2 x divided by 2, they will be another term cos, cosine of what open this brocket this will be omega 2 t plus k 2 x. For now are we getting a single wave, so our expression that we started was we have to get eta.

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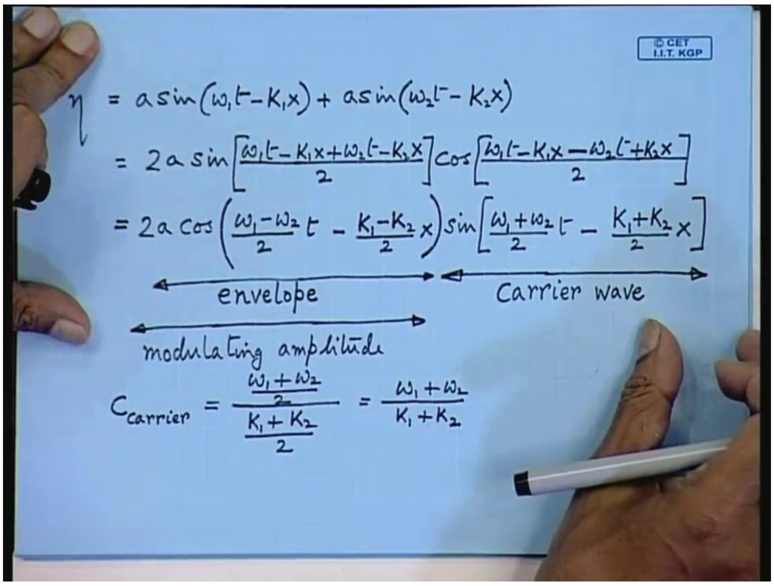

So, eta you multiplied the whole thing by A, so then we will be getting eta, so what is the value of eta, eta is a sin, so this is the single trigonometry expression which we are employed more complex math's still now. So, this become twice a, the other term will be cos, now from this expression if you see we are getting a single wave, then what is the amplitude, now for an analysis you bring the cost term forward, sin term we are getting this as this t we are having t 2 and cos is minus 1 here.

So, now you write down this expression as 2 a, now cos you split it up, cos expression is coming as omega 1 minus omega 2 over 2 this is t, and other expression will be minus or plus minus k 1 minus k 2 you just look here, minus k 1 you take a minus sign, so this is the minus k 2 over 2. So, this will be x the other term also used eta in dispersion, so this is sin omega 1 minus omega 2 plus omega 2, now here this will be omega 1, so there is no minus, so omega 1 plus omega 2, so still over 2. Now, outside the bracket you write this as t and other expression will be minus k 1 plus k 2, so this will also be over 2.

So, this is k 1 x and this is k 1 x you can minus, so this will be your distance that is x now your manage to get a single wave like this now in this expression, this is your what this sort of equation you will also coming vibration, if you go deeper down into this, this is called a envelop. This is your amplitude is it not, but your amplitude confuse of a multiplied by cos omega 1 minus omega 2 t minus k 1 minus x, so amplitude is wearing with respect to t and x, the other one it is called a this wave, it is called a carrier wave, so remembers this. So, there is an enveloping wave and carrier wave, so in vibration in sometimes you will find, you come across the carrier frequency.

So, you can see one wave carrying another wave and if you multiplied this by 2, the whole thing you write this as modulating amplitude, so this is a simple we have got terms which only two waves, we are getting one carrier wave and a envelop. Now, what is your carrier wave, carrier velocity are find out C carrier, expression for C is what simply omega by k, so you look at the carrier wave, what is the carrier frequency, carrier frequency is omega 1 plus omega 2 over 2. And carrier wave number is k 1 plus k 2 over 2 you divide it these two expressions you will get C carrier, that is omega 1 plus omega 2 over k 1 plus k 2, now next you find out C envelop.

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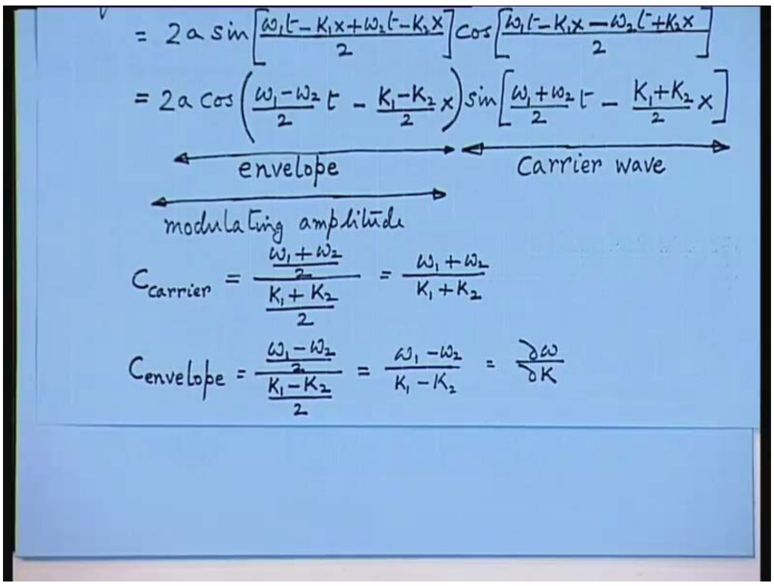

So, there is carrier wave, enveloping wave, so you will see this C envelop will be omega 1 minus omega 2, the other 1 is k 1 minus k 2, now this expression you can write as delta omega over delta k. But, you cannot write delta omega by delta k on C carrier, c carrier is simply the two frequency are added and wave numbers are added, now there are some interesting conclusion from this. We are trying to aim at what is called your group velocity, we trying to see this significant what this C g, g significant is it not, now from c envelop get we have written that is you have got delta omega by delta k.

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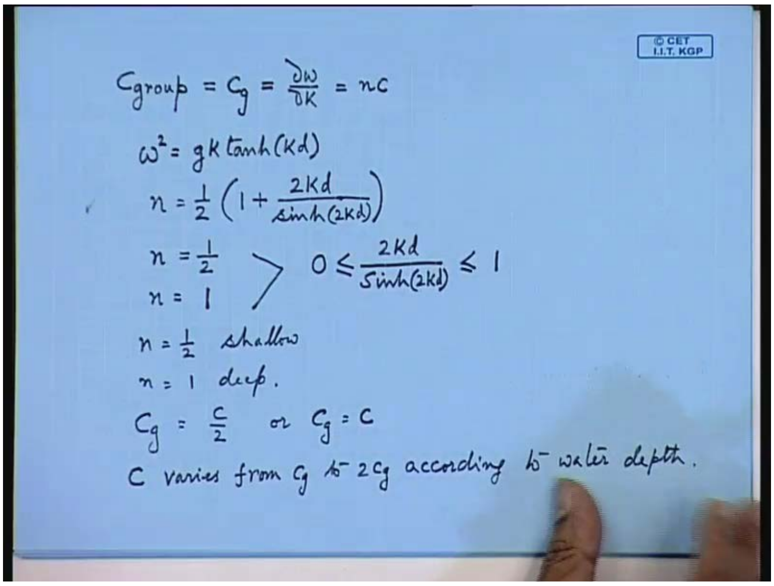

So, C group, so if you consider a group you write this as C g, which one you will take carrier on envelop carrier is one hidden wave, you take the physical aspect, it is a hidden wave within the envelop. So, C g will be your, you want consider this group, it is a envelop what you have seeing is the total picture, that is a total outcome of the two waves, so that is the angle. So, C g will be your C envelop, so this will be delta omega over delta k, now this expression you can write this as n times C, C of single wave, now this expression you derive from the dispersion relationship.

What is the your dispersion relationship, omega square equals to C k tan hyperbolic k d is it not, your g k tan hyperbolic k d, now from this expression you simply differentiate you will get this expression has n C, and your aim will come to be half of 1 plus 2 k d over sign hyperbolic 2 k d, at this you can find out from your differentiation. Now, what should be the expression for n, if this value is 0, k d is 0 then what is this, n equals to half, so this d is 0 then that means, d is near to is 0.

So, in varies from half and another value n will be how much, so there will be two values of n, this is minimum value of and maximum of 1 and depending on the value of k d, so half are we getting for deep water, so this 0 less than. So, this expression will be what is the value, minimum is 0 and maximum of this will be 1, so it is 0, so this actually at this middle mistake this should be shallow water. So, half we are getting deep water or shallow water, so n equals to half for shallow water is it not, d will be equal to 0, but here it is a written off the opposite.

Now, in all this cases what is the value of C, so accordingly your C g would be how much, either this will be C by 2 or almost C g will be equals C, so C varies from C g to 2 C g, according to the depth a water. So, it is never a fraction of series, it varies from 1 C g to 2 C g, it is always greater than C g, so that is what we C g are, is that the individual wave velocity, what is are inference, individual wave velocity is always greater than moved velocity. And these value of c is depended on this value of n, now if you take a, I told you in this wave group, there are the carrier wave and the enveloping wave.

So, if you have number of waves, if you have number of values k, according to the values of k your n waving, so that whether of n you substitute here, you get the corresponding. But, C g will be the same, so you will get a corresponding value of the particular wave velocity.

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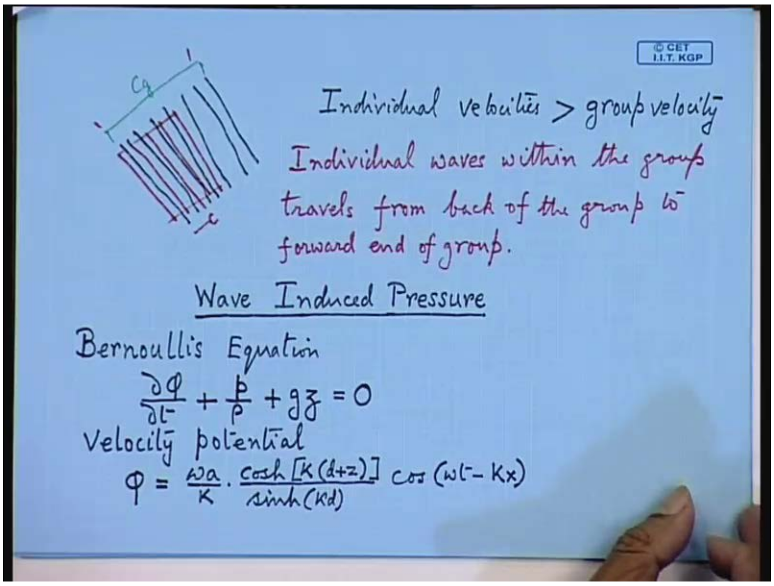

So, from this expression we see that, if you look at particular wave group, so this is the your wave group that is traveling, now your individual velocities is greater than wave group, all this wave group greater than group velocity. So, in physical cyanide, so these are the wave trend, so you will find a single wave going like this, now since the velocity is greater than C g what it will do, so this single wave this is velocity is C and this velocity is C g. So, individual waves within the group will travel from behind to forward, individual waves within the group travels from back of the group to forward end of group, so this quit interesting phenomenon.

So, you will find this red wave groups traveling from this end to this end within the group, because of this greater than C g, so this is the significance of wave group, so normally you have see study group of waves we do not study single waves. Now, since we have some time, the next item is wave induced pressure, now if you want calculate the pressure, you look into the constitute equation that we founded in last class. What was the equations, there are diagram have given you, when your studying the dispersion relationship, now in this equation there are two important.

So, this is your diagram from this diagram there are two important equation that we have found out, one is the Laplace equation and the other one is the ((Refer Time: 48:16)) what is this equation, in feed mechanics hydrodynamics is also come across this equation quiet frequently. So, Laplace equation is coming from the equation of continuity and this is the momentous balance equation which is called the Bernoulli equation, now if you want to calculate pressure you have to use this Bernoulli equation.

We use a Bernoulli's we equation, but you have to know what is your velocity potential, this five value is to be know, then only we calculate this, so two equations which we will be giving at. So, the first equation is the Bernoulli's equation why are written this, so waving induced pressure you calculate from Bernoulli's equation, now you look of the equation for velocity potential. So, Bernoulli's is expression is at the sea surface are, so Bernoulli's getting from the momentum balance equation.

What is the momentum balance equation, that is law of conservation of momentum, so that is momentum cannot be created at destroyed there is no force right or wrong, so this is coming from this equation. Now, what is the expression for velocity potential, now at the beginning I told you what is your surface elevation equation, and there is another important relationship that is 0 velocity potential. So, what was that, so this is omega a over k the wave number cos hyperbolic of, this is check whether this is or not, now how do you found velocity from velocity potential, this partial differentiate to this to what. Del phi del x or del phi del t, what was the expression for u x, del phi your, now you tell me what is this, so in the Bernoulli's is equation we got this term.

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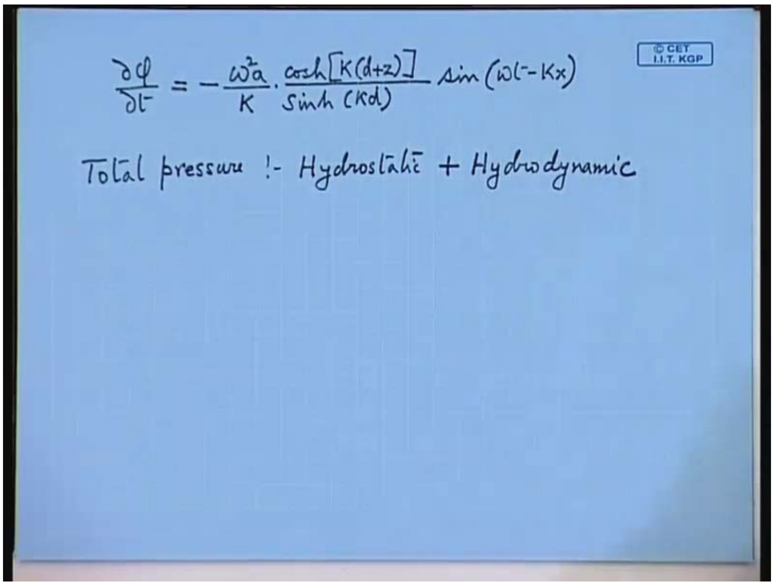

So, this is differentiate partial with respect to time, so the time parameter is omega t minus k x, so other one will be same, this term will divided as it is, so we are getting this as minus omega square a over k and cos hyperbolic of k into d plus z divided by sin hyperbolic of k d, and instead of cos you will have sin omega t minus k x. So, one omega as come out, so that is why it is square; now you substitute, so now you are getting the one of this equation, but not in terms of del phi by del t, but in terms of a expression in sin omega t minus k x, so you substitute this simplify.

And then you find out d over deep water and shallow water relationship, so this will do next class and after this we will, so we come across in important relationship. So, we have studied how to calculate hydrostatic pressure, the Bernoulli's equation will also give you the expression for a hydrostatic pressure. So, hydrostatic pressure will come from this term g z, the other term that will come this will equal the hydrodynamic pressure. So, at the end you come the total pressure will constitute two terms, one is hydrostatic plus hydrodynamic, so this is have.

So, whenever you calculating pressure below the sea surface, if you have com water of course, the hydrodynamics pressure will not be there. But, in near the sea surface pressure will constitute of this two terms, they have to add them, anyway thank you, so that been research to the end of this.