Elements of Ocean Engineering
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Lecture - 9
Water and Waves

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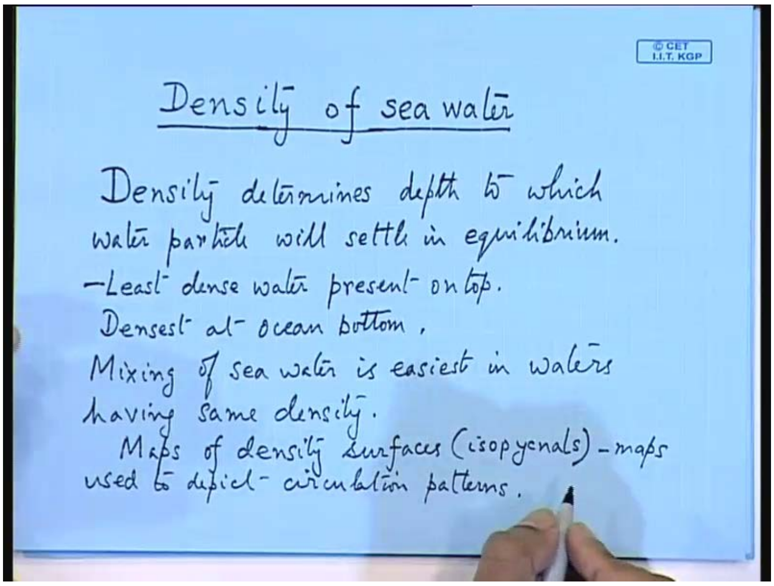

So, today will discuss about density of sea water. Now, density is important from the point of view of ocean circulation. So, primarily if you want to have look at density, so density determines depth to which the water will settle or depth to which water particle will settle in equilibrium, so this is the defining parameter for density. So, here you will find the least dense water, present on top, so this is called natural present on top, and densest at ocean bottom. Now, mixing is easiest, mixing of seawater is easiest in waters having same density.

So, this point is to be noted, because maps of density surfaces, so these are called isopycnals, so these are surfaces having the same density, iso means same, that means another term for density, the density are the same. So, these are mapped and these maps are used to depict circulation pattern, so density is a governing parameter for ocean circulations, so depicts of circulation patterns.

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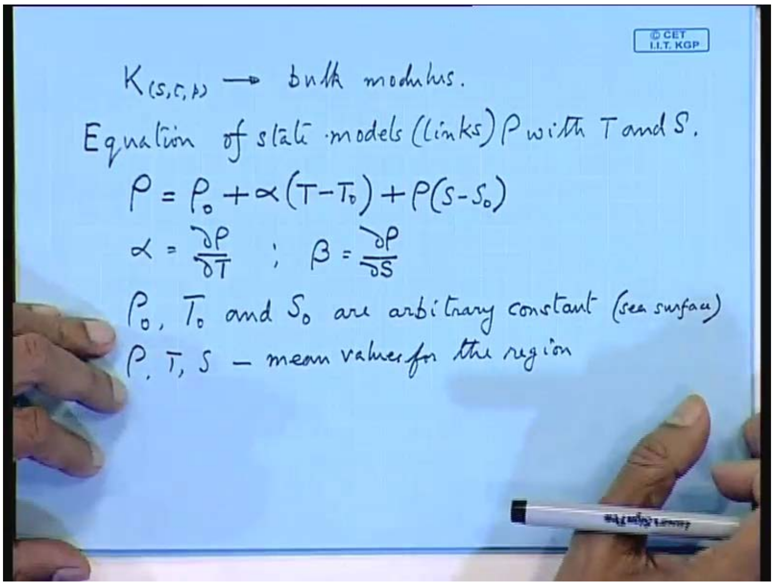

Now, how density is given, so density it is given in terms of this parameter called rho and rho for rho is equal to 1000 Kg per meter cube at 0 degree centigrade, and no salt. Now, see sea water density is 1021.00 Kg per meter cube, this is at sea surface and normally this sea surface is designated with p equal to 0, that pressure equal to 0 or atmospheric pressure. So, 1021 Kg at sea surface, and this becomes 1070 Kg per meter cube, this is at a pressure of 10000 decibar.

So, this is almost at 10 kilometers from the ocean surface, so density there is a variation in density, now common method of describing density is by this parameter called sigma. So, sigma s stands for salinity, t is temperature and p is pressure, so this is given in terms of density above 1000 Kg per meter, so in ocean graphic literature you find this parameter called sigma s t p. So, this is called in Situ density, in Situ density calculated at sea surface, so this is how the density is defined and there is another equation, which is to be noted that is density is also given as an equation of state for seawater.

So, this is given, this are all writing in Situ density you simply write rho, in terms of salinity temperature pressure and this will be rho in terms of salinity, temperature and pressure you take it 0. That means, at the sea surface divided by these factors 1 minus p divided by K, K is called the bulk modulus, so this is another expression for density. So, salinity temperature and pressure, at any pressure on the below the sea surface, it is linked with rho s, t, 0 at c surface 1 minus the pressure at that point K (s, t, p) transfer your bulk modulus.

Now, this equation also can be return in this form, now ((Refer Time: 09:19)) K hear it stands for bulk modulus, now here actually this is slightly different formula, but this is also nowadays is used. So, this stands for density at any point, so this is your rho 0 plus you can represent this in more mathematical form, so T minus T 0 I will give you what this T 0 rho 0 stands for and alpha and beta are constants, S minus S 0, so this is your equation. Now, alpha is the density gradient, so this is delta rho divided by delta T, so amount of change of density with respect to T stands for four temperatures.

And beta stands for what, this will stands for change of remember your parameter is your densities, so this is again delta rho divided by delta S that is change of density with respect to salinity. So, alphas stands for change of density with respect to temperature, beta stands for change of density with respect to salinity, and this rho 0, T 0 and S 0, so this represents the density temperature than salinity, you can take these are at any point are arbitrary constants. But, normally you can take them at the sea surface, and rho, T, S these are mean values for the region.

Now, if you take the special region in the ocean obviously, it will vary according to the your x and y values, where you just take the mean values. So, this gives you rho that is the density at that particular location with reference to some rho 0, T 0 and S 0 were you normally you can take this at this sea surface, if you write are at other any point.

So, this gives us the equation for state, linking density, temperature and salinity, so this equation of state actually models these three terms or links, so this is vital for defining your density links, density with temperature and salinity. So, that is why this equation is more preferable to the one that having that, bulk modulus K, so this is the equation of state and with this we finish our discussion on density, now there is a problem on this current.

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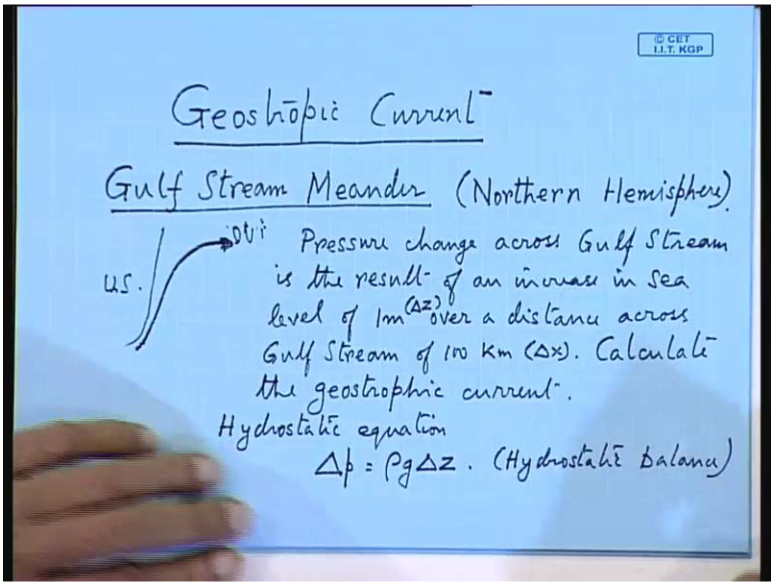

So, last time we discussed about geostropic current, now in the Northern Hemisphere you come across this huge current, which is called the gulf stream meander this is called gulf stream meander; so this is quite famous current, this is called gulf stream meander. Now, this current actually goes from the eastern coast of US to the united coast of United State kingdom, so UK is somewhere here, so this is called the gulf stream meander, it flows from the Northern Hemisphere like this and it goes towards the UK coast. So, this is one of the main reason for why you get on the eastern shores of UK you get the warm current, but whereas on the west coast you get a very cold climate.

So, this is the reason of called the gulf stream meander, so this occurs in the Northern Hemisphere, now let us try to calculate the velocity of this current. Now, in the problem this is given as pressure change across, you have to calculate this pressure change, so pressure change across gulf stream is the result of an increase in sea level value is 1 meter. Now, this take place over a distance, this distance across gulf stream of 100 kilometer, so pressure change results in a sea surface elevation of 1 meter.

So, in brackets you can write this as delta z, over a distance across 100 kilometers, so this is your delta x, now you calculate the geostropic current. So, first you get the hydrostatic balance equation or simply write down the hydrostatic equation, so what is the hydrostatic equation, so this is given by delta p is equal to rho g delta z. So, this is your hydrostatic also or sometimes this is called a hydrostatic balance, and this is actually supporting the weight that is why this is called hydrostatic balance. Now, in the Northern Hemisphere what is happening your flow, let us see have a look at it direction of the flow.

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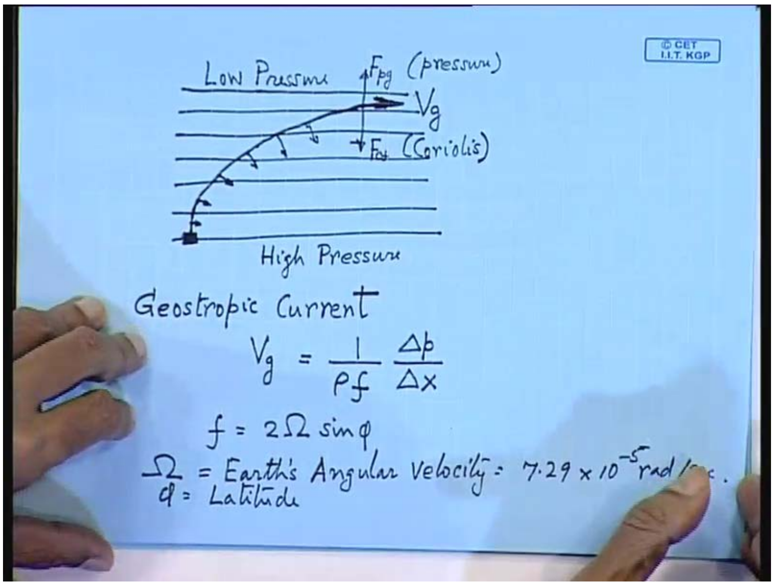

So, this is your low pressure region, so these the pressure of low pressure, so these are your pressure or lines of equi pressure and right has a bottom you have high pressure. So, in which direction their flow will be directed, so flow is always directed from high pressure, so this is a region of high pressure. So, flow is always directed from high pressure to low pressure region, so that is your driving force, your flow is going to go like this, so this is called the gulf stream meander.

And by towards a right, so your velocity vectors keep on increasing, now till at a certain magnitude you find the flow is becoming horizontal, now at this region you find your pressure that is given by F p g, your pressure force will balance the Coriolis force F c f. So, in bracket I am writing pressure, pressure force is going to balance your this is F c f is the Coriolis force, then you get the velocity of the flow this is given has V g, so pressure equation I have written.

Now, you write down the equation for geostropic current, this is called geostropic current; now in the Northern Hemisphere I have told you the flow is always directed towards the right, you are throwing a particle from the equator to the poll. So, that will have it same velocity plus vector will be add the rotation of the earth, which is the from left to right, so your particle will always get deflected towards the right. So, this is a result of the Coriolis force or Coriolis acceleration, now in this case you are the geostropic current, you have to calculate from V g this is equal to 1 by rho f.

Now, f is a this is a Coriolis factor multiplied by the pressure gradient, so del p over del x, so this is your pressure gradient and this value is given, and f you will take it has twice omega sign phi. Now, omega is your earth angular velocity, so this is a geostropic current or Coriolis it as to be linked to with the velocity of the earth or earths angular velocity. So, this is equals to 7.29 into 10 to the power minus 5 radius per second, so this is the angular velocity of the earth. Now, this phi is the latitude, anyway from this equation written calculate the value of f, now for calculation V g this value of f is already given.

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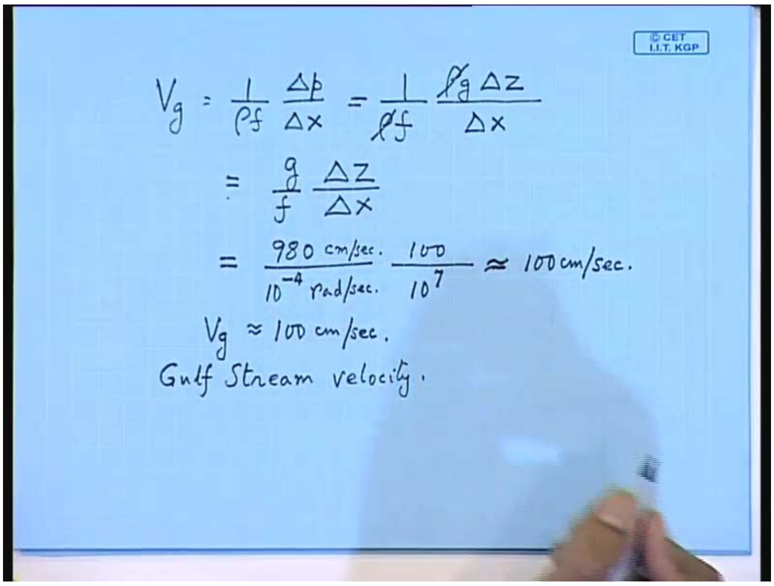

Now, V g is equals to 1 by rho f into del p over del x, now you substitute how much is the value of del p, we have find out del p from the hydrostatic balance equation and what is that value. So, this is equal to 1 by rho f del p is nothing but rho g delta z, now you divide this by rho f and what else you have got delta x, so ultimately you can see we are getting delta z over delta x, so rho will cancel out g divided by f. So, the final equation you will get has g over f multiplied by delta z over delta x, now value of g is in centimeters this is 980 centimeters per second.

And f is given as 10 to the power minus 4, so this is radians per second, so at that particular latitude what is the angular velocity, now what is the value of delta p, delta p we are given this is delta z, now delta z is given as 1 meter. So, 1 meter you are doing it in centimeters, so obviously, this would be 100 centimeters and this is the elimination over how much delta x value is 100 kilometers, so you convert that into centimeters. So, this will be 10 to the power 7, now you reduce this will be approximately this will give you 100 centimeter per second, (()) approximately coming here I am writing this.

So, this 100 centimeter per second, so geostropic current velocity V g is 100 centimeter per second, approximately you are although you are getting this has 98, so this is the gulfstream velocity 100 centimeter per second, you see the units right or wrong, so this is 100 square. So, this is 5 minus 4 is much you are getting almost in the this 98 centimeter per second, almost 100 centimeter per second, so that is your gulfstream velocity. So, with this let us finish about the properties of seawater.

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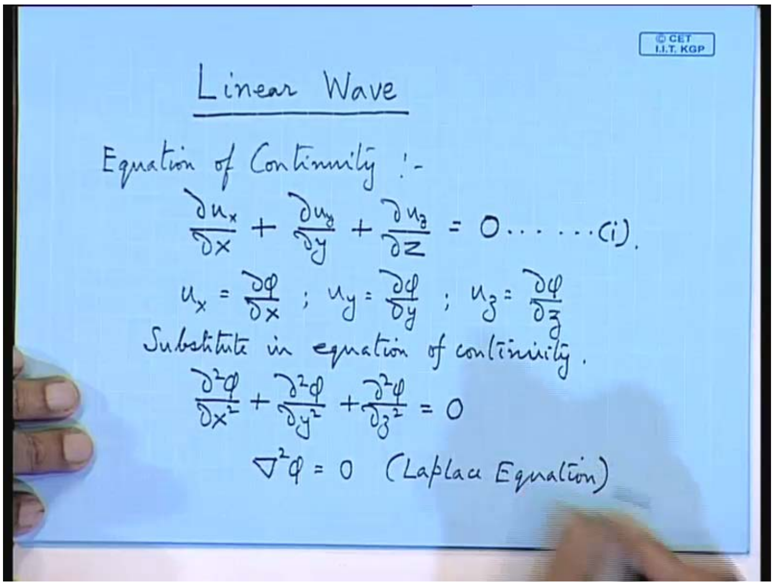

Now, next let me start about the linear wave, so how one gets this wave equation, so your wave equation, if you calculate the wave equation this comes from the equation of continuity. So, what is this equation of continuity, so this is given as del u or rather you can write it del u x over x plus del u y over del y plus del u z and del z equal to 0, so this equation continuity is what, what is the significance of equation of continuity. So, I am not going in detail this you ask your hydrodynamic teacher, how he has got the equation of continuity is coming from conversation of mass.

So, this regression of continuity, now you substitute this values of u x and u y and u z, a u x is how much it is del phi over del x, u y is del phi over del y and u z is del phi over del z, and you substitute these values in the equation of continuity and see what you get. Now, you substitute this you will get the Laplace equation are getting it, so you substitute this, so this will be del square phi del x square plus del square phi over del y square plus del square phi over del z square, so this is equal to 0. Now, this in mathematics that term this has number of square phi equal to 0, so this is your famous Laplace equation.

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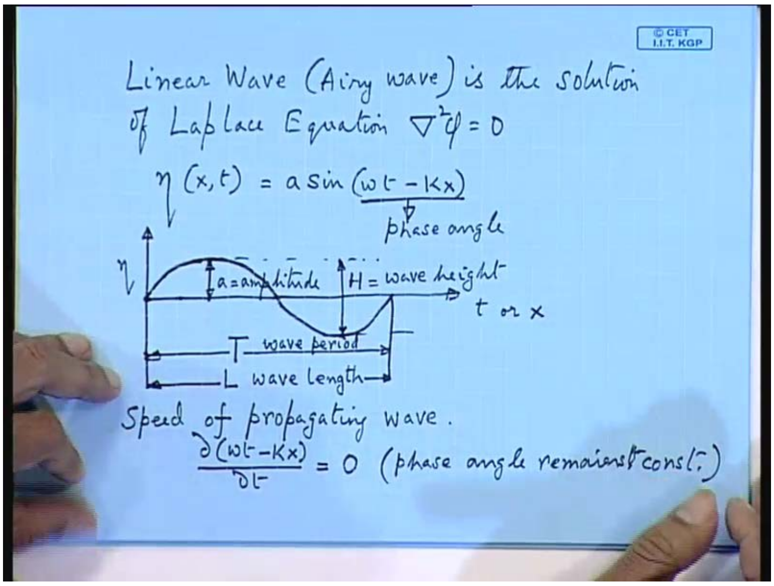

Now, the linear wave or sometimes this is called the airy wave is nothing but a solution of this equation, so a linear wave or an airy wave is the solution of Laplace equation. So, you remember this, solution of Laplace equation and what is this equation, equation in short is written as number of square phi equal to 0. Now, if you look at the solution, you find the solution is written in this form has beta, beta x t, beta is wearing with the position x and the time t. So, this is given as a sin of omega t minus K x, so this is your surface region four linear wave, now if you draw a graph this will look like this, now later on at the end of the class, I mean next class have look at non-linear waves.

But, before that whereas, expose this and what we had to be careful about is, the applications regime is of the linear non-linear waves, so this is what this is one wave period, now the horizontal axis can either the t or x, since your surface derivation beta, beta is called as surface derivation at any point of time. Now, what is your a, a is your amplitude and T is called wave period, L is the wavelength, now since I have selected one horizontal axis, you can debit as T and L, then what is wave weight and H is your wavelength.

So, that is the distance between the stuff and impressed, so these parameters are the physical parameters of the airy wave, now you calculate the speed of propagation. Now, this is not a standing wave, it is a propagating wave now propagating wave has certain velocity. Speed of propagating wave and if you want to calculate this, then this angle that is omega t minus K x is referred to as phase angle, now there is a simple method of calculating the propagating wave velocity.

You simply take the partial derivative of this phase angle to be 0, that is the phase angle remains constant, this is equal to partial derivative of omega t minus K x is equals to 0. So, this implies that phase angle remains constant obviously, the partial derivative of any remains constant, that is the derivative or a constant would always will give you 0.

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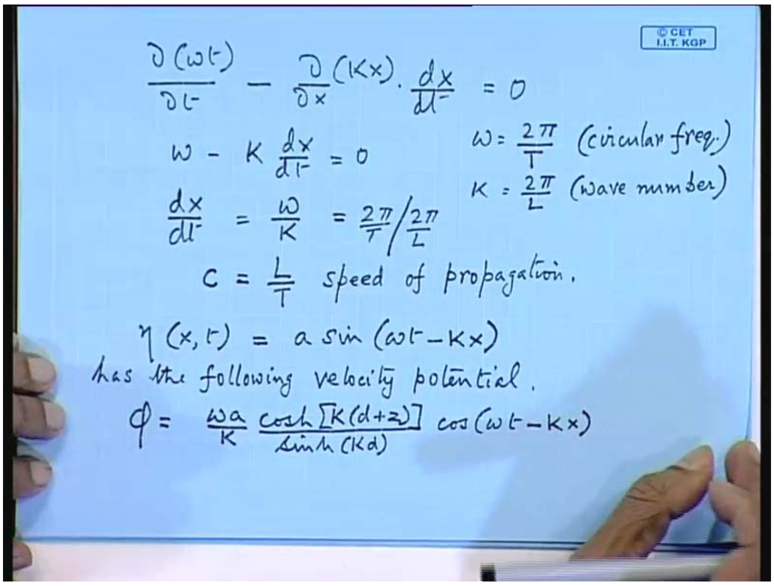

Now, what is you differentiate this with respect to time how much we get, so this will be del omega t over del t minus del del x, first you differentiate these two with this respect x, then you multiplied this by d x d t, so this is equal to 0. Now, from this we get omega minus k d x d t, so this is equal to 0, so now you can find out the velocity, so d x d t is how much, so that is how the linear velocity of the wave. So, this is we are getting this as omega by K, now omega is how much, omega is called the omega is equals to 2 phi, omega is a circular frequency and K is the wave number.

So, omega is the circular frequency fine, and K is 2 phi over in this case it is L, so this is called wave number. So, remember the expression for this omega and K, omega is 2 phi by T, T is your weight period and K is the wave number is 2 phi by L, L is the wave length, so omega is K and how much it comes. So, this is 2 phi by T over 2 phi by L, so this comes as a very neat relationship of L by T, and sometimes this is given as C is the propagating velocity, so this is called speed of propagation.

So, we have got the equation for this surface that is the airy wave, that is given by eta and we are got the, see there is a velocity of propagation, now you find out what is the particle velocity, now your surface relation equation is eta x, t. So, this is we are using a sin wave, a sin that is omega t minus K x, so actually there are two variables and one variable is t, and another variable is x. Now, you have to define velocity potential, now this velocity potential have to satisfy the Laplace equation for this surface profile. Now, ((Refer Time: 40:27)) this is your velocity potential, cos hyperbolic K d plus z divided by sin hyperbolic K d cos omega t minus K x, so this is the velocity potential that is to be used.

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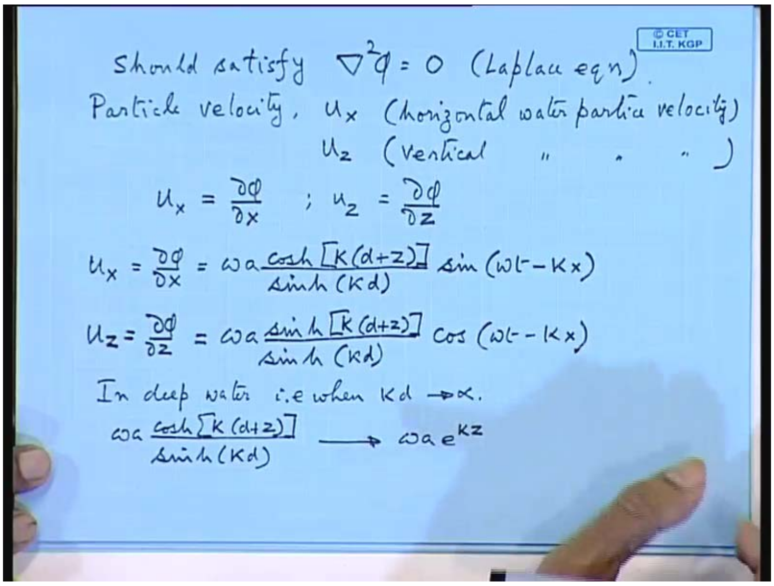

This should satisfy your Laplace equation, satisfy number of square phi equal to 0 the Laplace equation, now you find out particular velocity, now we are considering the wave in two dimensional that is x and z. So, particle velocity or rather you can write water particle velocity, u x horizontal water particle velocity and the other one is u z. So, this is your vertical, vertical water particle relationship, what is the expression for u x, now u x you will get if you differentiate the velocity (( )) phi with respect to x, and u z you calculate from del phi over del z.

Now, later on you find, if you want to calculate the pressure term, you will require u x and u z, your ultimate goal is to find out the pressure term has an a ocean engineer anyways, so that will come later on. Now, you find out what is this value of u x, u x is del phi over del x, and phi we have got the expression phi, phi is this is expression ((Refer Time: 43:55)), so you differentiate this with respect to x. And see how much will get, so u x will omega a into there will be two minus coming from that minus K u minus minus will cancel out, and K will also cancel out.

So, you get ((Refer Time: 44:23)) this expression cos hyperbolic of K d plus z and this is divided by sign hyperbolic of K d, and you differentiate cos you will get, so the differentiation will be over x, so cos will be sign, sign omega t minus K x, so this is your del phi over del x. Now, next to find out u z, so this is del phi over del z, so intermittent this expression, so this will be omega a sin hyperbolic of K multiplied by d plus z and denominator will be sin in hyperbolic of K d. And the other term will be you differentiate how much phi over del z, so phi is this differentiate with this is a cos, this will be u z, this term will be cosine of omega t minus K x.

So, this is your expression, now you find in deepwater, in deepwater what happens that is when K d is infinity, now this omega a cos hyperbolic this term, that is K d plus z over sin hyperbolic K d will a approaches certain value. So, this is omega a e raised to the power K z, now the another term was approaches the same value.

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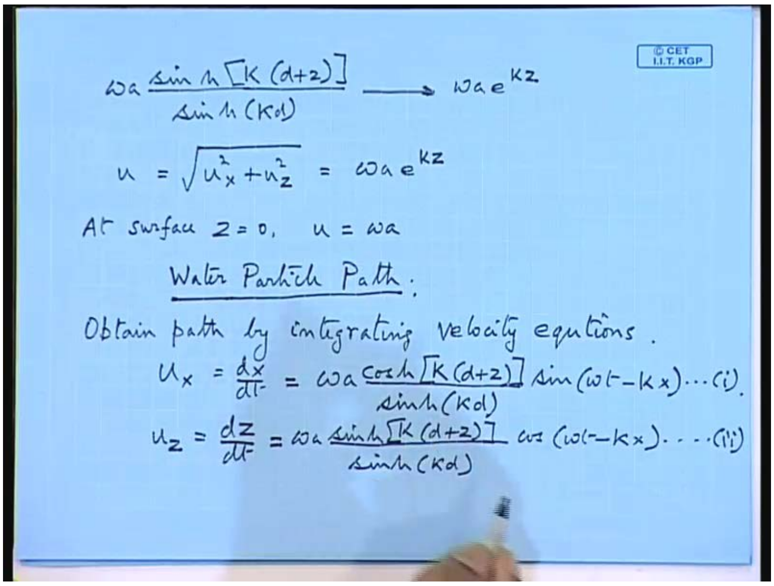

So, this you find out from hyperbolic expressions, so omega a sin hyperbolic K d plus z over sin hyperbolic K d, so this will also approached the same value, so omega a e raised to the power K z. So, now, you find out the resulting velocity u, u is root over of u squares x plus u squares z, so this is omega a e raise to the power K z, so at surface you put z equals to 0, velocity of water u equals to simply omega a. Now, this is your velocity equations, you find out water particle path, now once you get this part you will find, the shape of the or the trajectory of the water particles.

Now, how to find this you obtained path by integrating velocity questions, so what is actually velocity u x, u x is d x over d t, so you will get x if you integrate d x by d t. So, u x we have got us omega a cos hyperbolic K d plus z divided by sin hyperbolic, which is K d, and the other term is this sin expression omega t minus K x, now you try to integrate this, so this one equation. So, the equation number 1 and the other u z is velocity is how much d z over d t, so u z we have got the expression as omega a sign hyperbolic K d plus z over sin hyperbolic K d. And the other term your got is cosine, cos omega t minus K x, this is equation number 2, now integrate these two, you integrate these two you will get x and z; and if you integrate you find come across the interesting conclusion.

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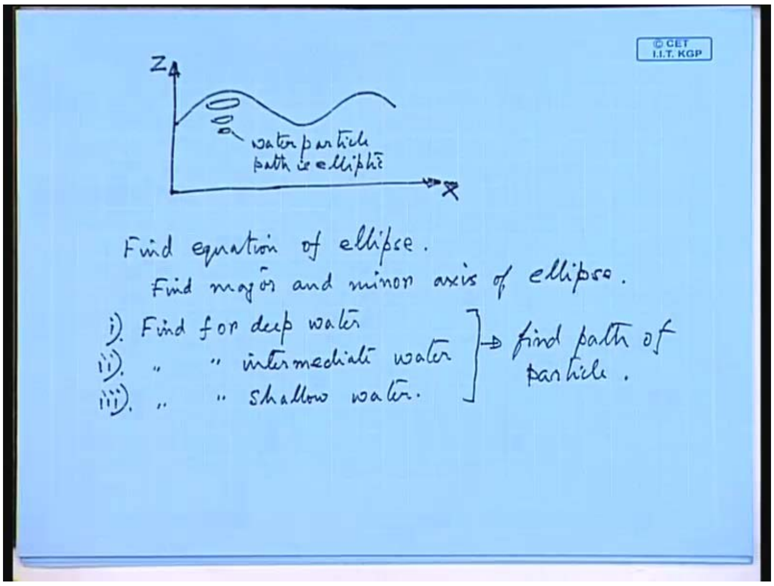

So, next class will do this, and you will find that the water particles part is elliptic, you will get an ellipse you try to do this, and this is your wave profile and you will find ellipse coming, so this is our z, this is our x. So, water particle path is elliptic, so we will explore this we try to find equation of ellipse, this is very easy from this expression you can find out the equation for ellipse. And you find out equation of ellipse you have to find major and minor axis minor axis of ellipse, now in shallow water you would be find, shallow water now there will be two distinct cases or rather create distinct cases.

Number 1 is the find for deepwater, next you find intermediate waters and the last category you will be shallow water, now these are three cases will come across, for all these you find path of particles. So, next class we will do this, but before this we find out the equation of the ellipse, so this is a discussion on the linear wave theory, and after this will go to the normal linear waves.