Modeling of Tundish Steelmaking Process in Continuous Casting

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Lecture – 09
Physical Modeling of Isothermal and Non- Isothermal System

Welcome to the lecture on Physical Modeling of Isothermal and Non Isothermal System. So, we talked about the Reynolds as well as the Froude similarity. And in this class we will talk about the aspects of physical modeling, when we talk about the isothermal as well as a non isothermal system. So, when we do the aqueous modeling so, we will also discuss about it in our later classes.

So, in that case, we take the water as the common fluid. And if you talk about the modeling so, it can be done either in an isothermal manner or you have to take when the thermal considerations. So, in that case the thermal similarity has to be kept in mind. So, how you know which type of dimensionless numbers are going to come in these cases.

So, in this lecture we are going to have, the brief discussion about those aspects. So, as we know that you may have the counter with the either the isothermal system or the non isothermal system. Isothermal system means, when we are not taking into account the temperature difference we are just ignoring the effect of you know the buoyancy forces or we are ignoring the effect of any kind of temperature gradient in the melt. So, that kind of system is basically the isothermal system.

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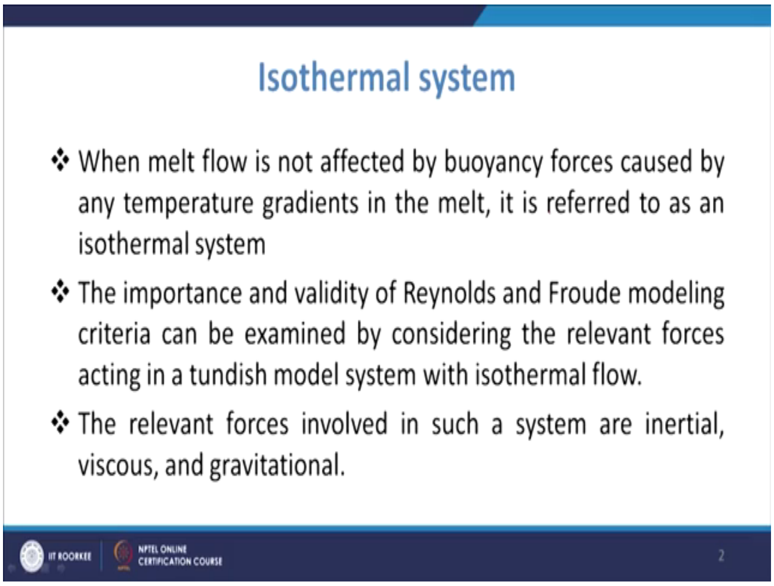

And in these cases, you know the importance and validity of Reynolds and Froude modeling criteria can be examined by considering the relevant forces acting in tundish model system with isothermal flow. So, basically you will have to examine, you know these Reynolds as well as the Froude modeling criteria.

So, you know as you know that the forces which are normally, you know taken into consideration in these you know cases or the inertial forces, viscous forces and the gravitational forces.

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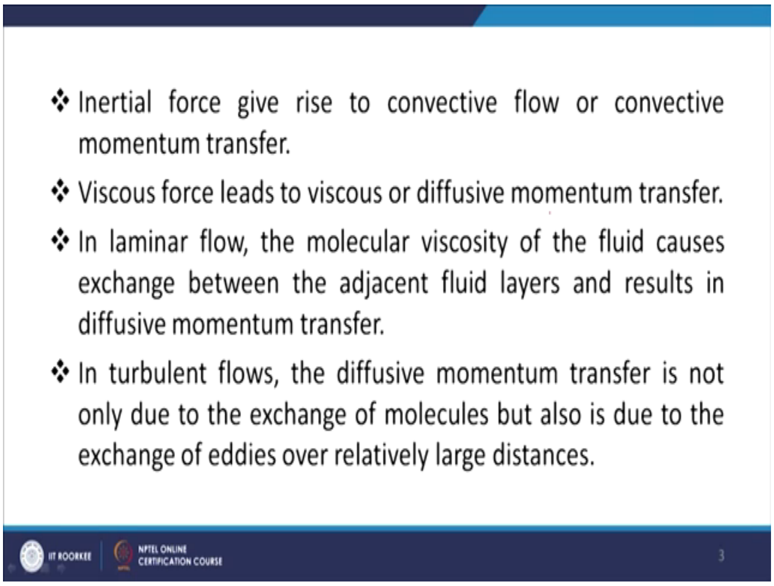

So, we know that we are basically also certainly ones must be knowing about these you know role of these forces. So, if you talk about the inertial force. So, they give rise to the convective flow or the convective momentum transfer. So, basically these things take place because of the inertial forces. Then you have the viscous force; so, viscous force which will lead to the viscous or diffusive momentum transfer.

So, that is how you know these viscous force role becomes. Now, what happens that you may have either the laminar flow or maybe the turbulent flow.

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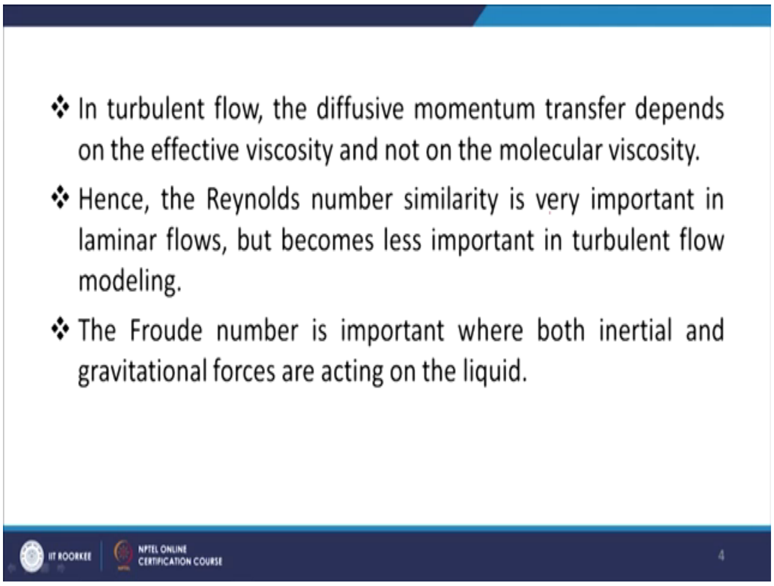

So, as you know that in the laminar flow, the molecular viscosity of the fluid causes the exchange between the adjacent fluid layers and result in the diffusive momentum transfer. So, that is what the trait off the laminar flow is and in that you talk about the molecular you know viscosity also so that comes into picture in the case of laminar flow.

Whereas, in the case of turbulent flows, if you recall then here the diffusive momentum transfer is not only because of the you know molecular at the molecular level or exchange of the molecules but, it will be also because of the exchange of eddies. So, eddy the eddies are of different length. And they are exchanging that over the relatively large distances known as the eddy lengths. So, that basically is of very very you know large value. So, that is what is you know happening in the case of the turbulent flows.

So, in the turbulent flow, the diffusive momentum transfer depends upon the effective viscosity and not on the molecular viscosity. So, basically what happens that, when we talk about the effective viscosity. So, it will be nothing but the summation of the molecular viscosity and the turbulent viscosity.

So, the diffusion momentum transfer which will be you know calculated, it will be basically based on these both these factors like you have molecular you know viscosity as well as the turbulent viscosity and both taking into account. So, the Reynolds number similarity is very important in the laminar flows but, becomes less important in the turbulent flow modeling; because as you know that in that case the well molecular viscosity which is there.

So, that is not only the sole criteria for having the determination of the parameters like for the of the calculation of the momentum transfer or. So, in that case it becomes less important in the case of the you know a turbulent flow. So, Froude number is important where both inertial and gravitational forces are acting on the fluid. So, if you look at the other you know number the Froude number; so, in that we know that its a ratio of inertial to gravitational force.

So, in that you have you know this is becoming important when both these you know forces are acting on the liquid so, that time your Froude number becomes important. And it becomes important in those cases especially it’s use is there in the case of inclusion floatation; because in the case of inclusion floatation the particle shape you know which we take it and the particle which is their inside the melt. So, that is you know that will be subjected to these forces and they will try to you know collide or they may try to float.

So, this Froude number that this a Froude number similarity that is to be taken into account. When you have both inertial as well as the gravitational forces are acting and mostly they are you know useful; when we talk about the inclusion flow or modeling. So, if you talk about the you know momentum balance equation if you recall. So, your momentum balance you know for the turbulent flow, in the tundish because, in the tundish normally the flow is considered to be turbulent itself.

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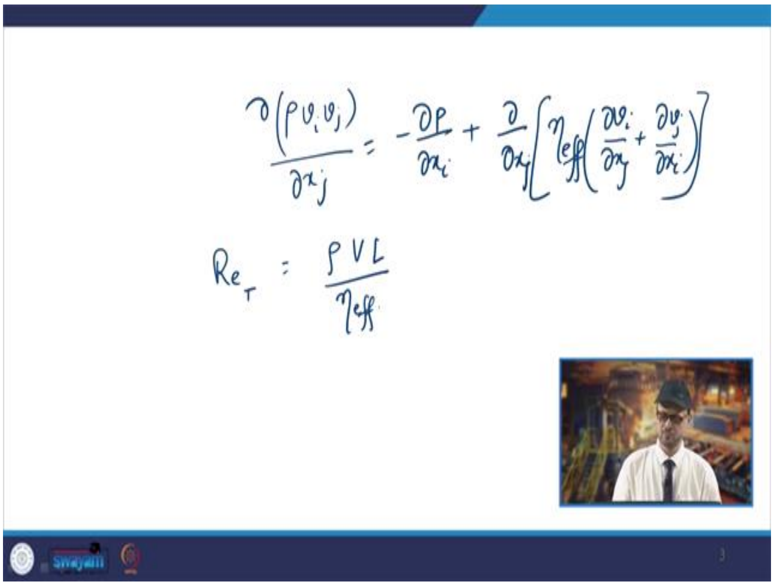

So, your momentum balance equation will be you know it will be like $\frac{\partial(\rho v_i v_j)}{\partial x_j}$. So, this is your steady state you know momentum balance equation. So, it takes this form so you will have the pressure term. So, you will have $\frac{\partial p}{\partial x_i}$ and then your term comes these convective terms; so that it will be your $\frac{\partial}{\partial x_j}$. And then you have you for if you talk about the turbulent flow situation, in that case you will have the effective viscosity coming to picture in place of the molecular viscosity. And then you have the term that is $\frac{\partial v_i}{\partial x_j} + .\frac{\partial v_j}{\partial x_i}$

So, that is how your momentum balance equation for the turbulent flow looks like. And if you try to analyze you know this you know if you go for the dimensionless form of this equation, then it will yield the dimensionless number and that will be you know in the laminar case you are getting the Reynolds number. And in the case of the turbulent flow, you have the origination of the turbulent Reynolds number and that is denoted by Re_T.

So, that is your turbulent Reynolds number that is found and it becomes $\frac{\rho v L}{\eta_{eff}}$. So, that is how you know this number is this dimensionless number ah, this dimensionless number is defined. And for you know for having the for maintaining that similarity you know between two tundishes which have the turbulent flows.

So, you will have to see that you know this they must have they are must be similarity you know, each term in the dimensionless form should have you know the same value in both the system. So, in the model as well as in the actual system each term must be same. And you know it is assumed that when you are talking about though thus that kind of system in that case apart from the you know geometric similarity your turbulent Reynolds number also should be same.

So, that has been you know found by many authors, it has been there has been you know modeling studies, water modeling studies, carried out by many researchers specially by Sahai and Emmi. And then apart from that there are many researchers who have worked on this and then they have found it that this Reynold number turbulent Reynolds number you know, that also should be you know the similarity in the value of this turbulent Reynolds number should also be maintained.

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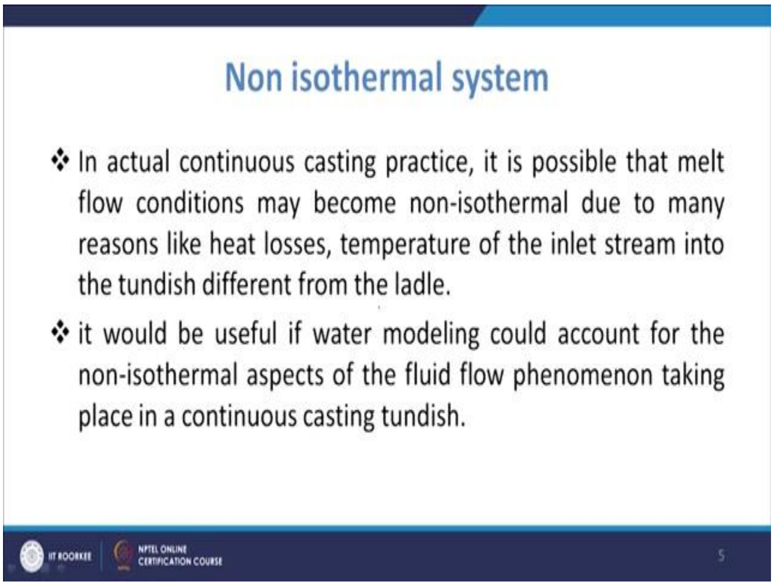

Coming to the non isothermal system; so, in most of the cases the system becomes non isothermal because the temperature cannot be said to be uniform. So, in actual practice the system is non isothermal because the temperature is varying. So, in actual continuous casting practice the melt flow conditions may become non isothermal because of many reasons like heat losses. So, heat loss may take place either from the you know top surface or from the walls.

So, there may be heat loss you know, from these you know places that will lead to the change in the temperature. So, the situation cannot be said to be completely an isothermal case. So, then other you know cases may be like temperature of the inlet stream into the tundish it is different from the ladle. So, you know you know whatever is you know entering so that is different from that ladle. So, that you know or there may be the temperature variation or the tundish inlet stream and at the at some of the corners inside the tundish.

So, because of these temperature differences, the buoyancy you know comes into picture and it has been seen that when you know when such is the case. So, when we talk about the such cases, in that case you know the non isothermal aspects are to be taken into account. So, you will have to see that you will have different you know kind of dimensionless groups will be coming whose you know considerations well be there for having the similarity analysis.

So, you know so it would be useful if water modeling could account for the non isothermal aspects of fluid flow phenomena. So, you will have to have the you know considerations of these non isothermal behaviors, you know which remains in practice.

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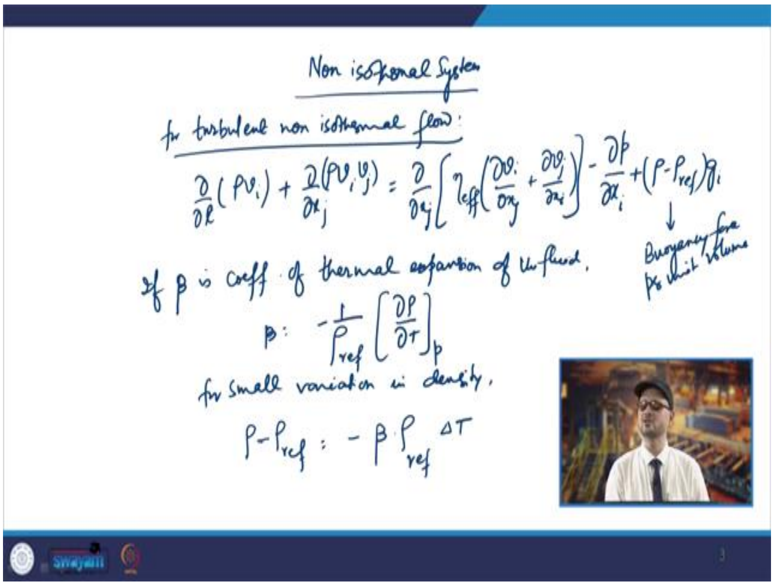

So, if you go to the you know non isothermal system, so, in that case if you talk about the you know momentum balance equation. So, the momentum balance equation for the turbulent for turbulent non isothermal flow, your you know your momentum balance equation will go like this.

So, it will be $\frac{\partial(\rho v_i)}{\partial t} + \frac{\partial(\rho v_i v_j)}{\partial x_j} = \frac{\partial}{\partial x_j}(\eta_{eff}(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}) - \frac{\partial p}{\partial x_i}$

And in here you will have another term is coming because of the difference in the density. So, it will be $(\rho - \rho_{ref})g_i$ So, this is you know taking that buoyancy force into account. So, this term is basically taken into account for the buoyancy force per unit volume.

So, now this is because of the difference in the density which is arising you know you know throughout the fluid body so that is why if it is coming. Now, if you take the beta as the coefficient of thermal expansion so in that case, so if β is coefficient of thermal expansion of the fluid; now in that case you know you can write that β is nothing, but minus $-\frac{1}{\rho_{ref}}[\frac{\partial p}{\partial T}]\ _p$ at pressure so at constant pressure.

So, this is how the β is defined. So, if you take very small you know variation in if you take that density variation to be very very small so, that will be $\rho - \rho_{ref}$. So, in that case this will be $\beta * (\rho - \rho_{ref})\Delta T$. . So, for small variation in density, you can write that is $\rho - \rho_{ref}$.

So, that will be equal to $-\beta * \rho_{ref} * \Delta T$. So, rho reference is the reference density and in that case the this Navier Stroke equation can be further rewritten. So, your r $\rho - \rho_{ref}$ is defined as $-\beta * \rho_{ref} * \Delta T$. So, that term will come here.

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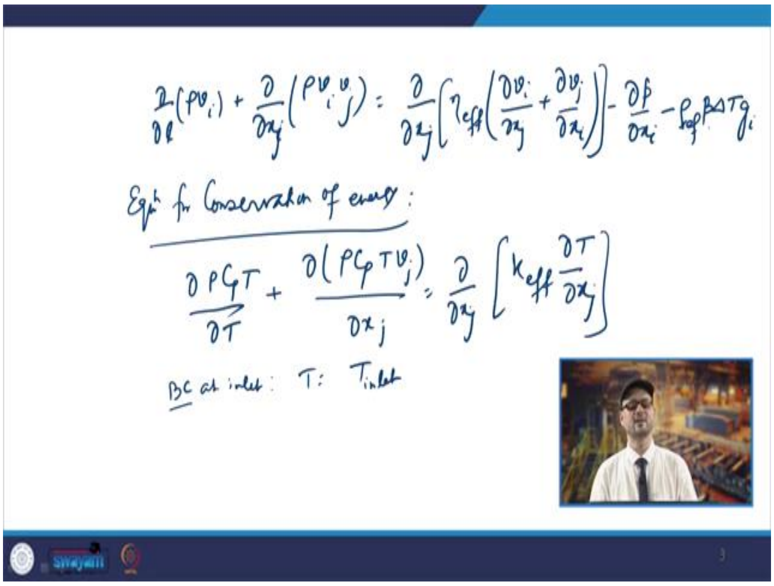

So, if you go to the you know to this Navier Stroke equation. So, that can be further written as $\frac{\partial(\rho v_i)}{\partial t} + \frac{\partial(\rho v_i v_j)}{\partial x_j} = \frac{\partial}{\partial x_j}(\eta_{eff}(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}) - \frac{\partial p}{\partial x_i} - \beta * \rho_{ref} * \Delta T g_i$.

So, this term is you know coming I am you know once you have the buoyancy which is there in the case of turbulent flow when you are taking the non isothermal conditions.

Now, if you talk about the when we talk about the non isothermal conditions then we also talk about the conservation of energy equation. So, your equation for conservation of energy so, that will have its own form and it will be like $\frac{\partial(\rho c_p T)}{\partial t} + \frac{\partial(\rho c_p T v_j)}{\partial x_j} = \frac{\partial}{\partial x_j}(k_{eff}(\frac{\partial T}{\partial x_j})$. .

So, if you look at these terms so, you will have a k_{eff} is nothing, but the effective thermal conductivity which is there in the case of the turbulent flow. So, in the case of turbulent flow, you will have the effective thermal conductivity that will be because of molecular as well the turbulent part.

So, that is how these you know it will be the summation of the molecular and the turbulent thermal conductivities. So, these together they will be governing these you know flow dynamics in the system. So, and also you will have the temperature at the inlets you will have boundary conditions at the inlet. So, if you talk about the boundary condition at inlet so, it will be T will be T_{inlet}. So, it will be; it will be the ladle stream temperature.

Now, now if you talk about this equation and if this equation is you know we try to write in the case of in a non dimensional form. So, you will have the non dimensional forms like rho v star rho star v star. So, you are a non dimensional analyzing it every term is non dimensionalized.

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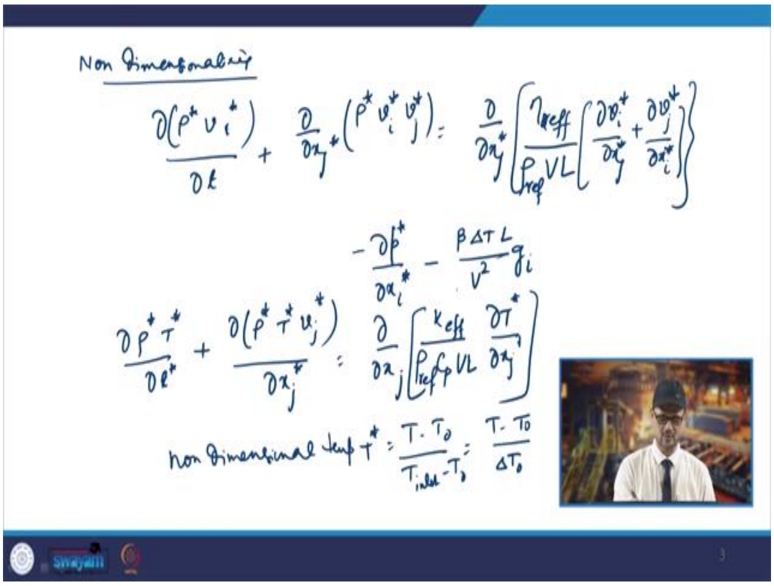

So, if you non dimensionalize these Navier Stroke equation. So, non dimensionalizing what you get is you will be getting these terms like you will have $\frac{\partial(\rho^* v_i^*)}{\partial t^*} + \frac{\partial(\rho^* v_i^* v_j^*)}{\partial x_j^*} = \frac{\partial}{\partial x_j^*}[\frac{\eta_{eff}}{\rho_{ref.}vL}(\frac{\partial v_i^*}{\partial x_j^*} + \frac{\partial v_j^*}{\partial x_i^*}] - \frac{\partial p^*}{\partial x_i^*} - \frac{\beta * \Delta T L g_i}{v^2}$.

So, this is the you know when you write the these this equation in the non dimensional form, then this comes into this shape. And if you write the non dimensional form of the thermal energy conservation equation, that can further be written as $\frac{\partial(\rho^* T^*)}{\partial t^*} + \frac{\partial(\rho^* T^* v_j^*)}{\partial x_j^*} = \frac{\partial}{\partial x_j}(\frac{k_{eff}}{\rho_{ref.c_p} vL}(\frac{\partial T^*}{\partial x_j^*})$..

So, your these terms are coming and in that case you will have these non dimensional number which is defined here. And your non dimensional temperature which is defined T^*. So, that is defined by $\frac{T - T_0}{T_{inlet} - T_0}$. So, it will be $\frac{T - T_0}{\Delta T_0}$. So, this is how you are you know having the definition and this is how you are getting those forms. Now, what you see this term if you may refer to this you know term what you see from here, you are finding certain dimensionless groups.

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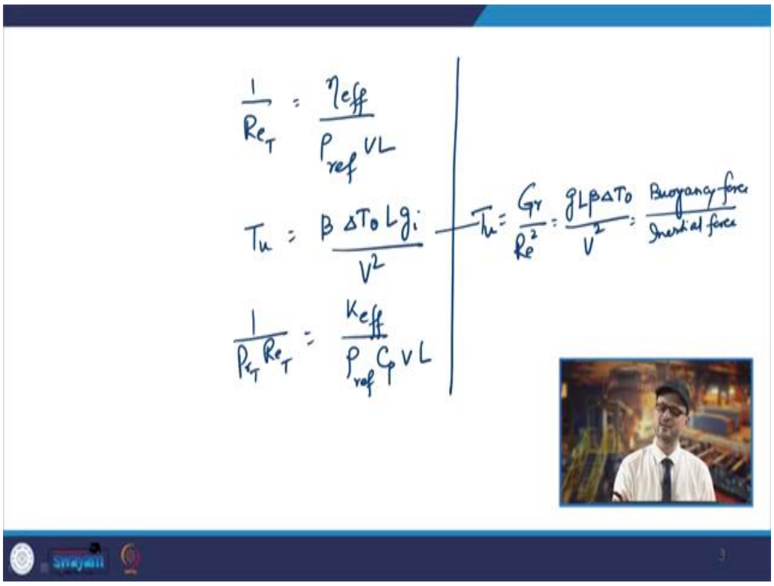

Now, that dimensionless group which is coming up that is $\frac{1}{Re_T}$ and that is becoming $\frac{\eta_{eff}}{\rho_{ref}.vL}$ that is what is you are getting $\frac{\eta_{eff}}{\rho_{ref}.vL}$. So, this number you are getting and this is nothing but the inverse of the turbulent Reynolds number. Then next is you are getting the you know one number which you are which is defined as the tundish Richardson number.

This is coming from here this is a you know this number which is coming this is known as the that the tundish Richardson number which is because of the buoyancy. So, we will talk about how this is defined. So, this is nothing but $\frac{\beta\Delta T_0 L g_i}{V^2}$. And one more you know number which is coming here. So, if you look at this is nothing but the inverse of the product of the Prandtl turbulent; Prandtl number and turbulent Reynolds number.

So, this is nothing but $\frac{1}{Pr_T Re_T}$. So, this comes as so, this is this term this is k effective upon rho effective. So, you will have $\frac{k_{eff}}{\rho_{ref.c_p}vL}$.

So, what you see is that, $\frac{\mu c_p}{k}$; that is your Prandtl. So, this is one this is the you know inverse of these product of these two numbers that is Prandtl turbulent Prandtl number and the turbulent Reynolds number. So, these are the you know dimensionless groups which are coming when we talk about the non isothermal system and they need to be you know constant.

And it is coming when we talk you know which we when we derive these thermal energy balance equation. So, you know already we have seen that the Reynold similarity has to be there. So, and if you see that if the Prandtl number is assumed to be constant in that case that similarities also you know maintained so that is also satisfied.

The only number which it needs to be satisfied is the this number T_U that is tundish Richardson number. Now, if you look at this number so, it is nothing but it can be related to somewhere it is related to the Froude numbers or Froude number is V square this you. So, if you take this term. So, you have only these term extra.

So, basically it has certain similarity, with the Froude number. So, you know if you look at this tundish number. So, if you see this tundish number is nothing but you can write it as $\frac{Gr}{Re^2}$. So, this is this number is you know given that Richardson number tundish Richardson number. So, if it is nothing but it is coming as $\frac{\beta \Delta T_0 L g}{V^2}$.

So, it is nothing but the ratio of the buoyancy force and that will be divided by the inertial force. So, you know this buoyancy force is resulting from the non uniform and temperature distribution inside the tundish and this basically causes the flow profiles to be changed many a times.

So, it has been seen that because of this the flow profile in the tundish will be changed as compared to what is being seen in the case of the isothermal system. And so, you know you know you it is said that if this is satisfied. So, this needs to be satisfied if the similarity has to be maintained.

So, it can be concluded that, if you maintain the constancy of these tundish Richardson number; then the you know it means the ratio of the buoyancy force and the inertial force is you know is satisfied. So, that will be you know of the two system will be same the ratio of these two forces. And it will be satisfying the dynamic similarity of the model as well as the prototype.

So, this when we talk about these non isothermal system, this is there is I mean additional this type of one more dimensionless form a number is you know coming into picture that is your Richardson number. So, we can see its effect how it affects. So, when we talk about the studies on the heat transfer studies inside the tundish, then we will have a view of its effect.

Thank you very much.