**Modeling of Tundish Steelmaking Process in Continuous Casting**
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**Lecture - 08**
**Dimensional Analysis**

Welcome to the lecture on Dimensional Analysis. So, we talked about the similarities. Now, what we do normally in the case of dimensional analysis. It is frequently used in the process engineering to represent the physical phenomena in terms of mathematical equation that is essentially dimensionless.

So, basically, if we have these equations we talked if you know the equations governing the fluid flow or heat transfer or. So, in those cases what we do is normally we try to you know have this equation in the dimensionless form for example, if you talk about the fluid flow equation. So, what we do normally is you have v.

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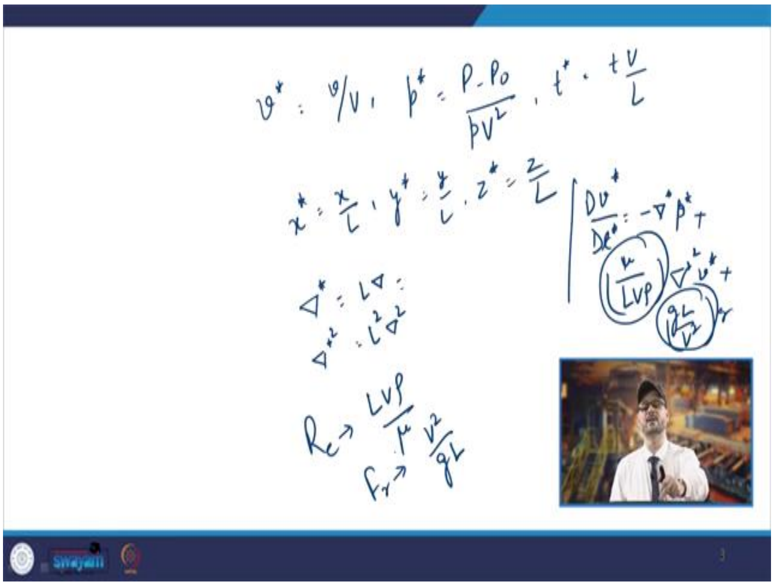

So, we are expressing it by dividing with a characteristic velocity. Similarly, $p^*$will be; you know that is $\frac{P-p_0}{pv^2}$. So, this way you get you know you have again $t^*$. So, $t^*$will be $\frac{tv}{L}$. So, you have a characteristic length, you have a reference velocity.

So, similarly, you know $x^*$ will be, $x/L$, then you have $y^*$ again, $y/L$ or $z^*$will be $z/L$. Then you get you have you know the operators you know this they also need to be; you know express. So, that $\nabla^*$ that will be you know $L\nabla$. So, that will be again you will be using. So, you will have delta 1 times.

Similarly, you will have $\nabla^{*2}$. So, that will be $L^2\nabla^2$. So, you will have $\frac{\partial}{\partial x^*}$ like that. So, you will have $\frac{\partial^2}{\partial x^{*2}}$; in this case, you will have $\frac{\partial^2}{\partial x^{*2}}$and delta 1 multiplied by that.

So, that way, you do and then once you put them into the equation for the continuity and also the momentum conservation. So, in that case also if you must have done that and in that case; you get you know the two kind of you know number, dimensionless numbers and that is one you are getting as Reynolds number that is $\frac{LV\rho}{\mu}$ and then you are also getting the Froude number, that is $\frac{v^2}{gL}$.

So, this is how you know; that is used for that is the use of these non dimensionalizing and then on both these sides when you do. So, you get, you know you will be having the expression coming up like $\frac{Dv^*}{Dt^*}$. So, that will be coming as $-\nabla^* p^* + \frac{\mu}{Lv\rho}\nabla^2 v^* + \frac{gL}{v^2}$.

So, $g^*$is coming. So, basically what you see you are getting these two terms. So, that is what it is coming you get these non dimensional quantities that is what you are getting.

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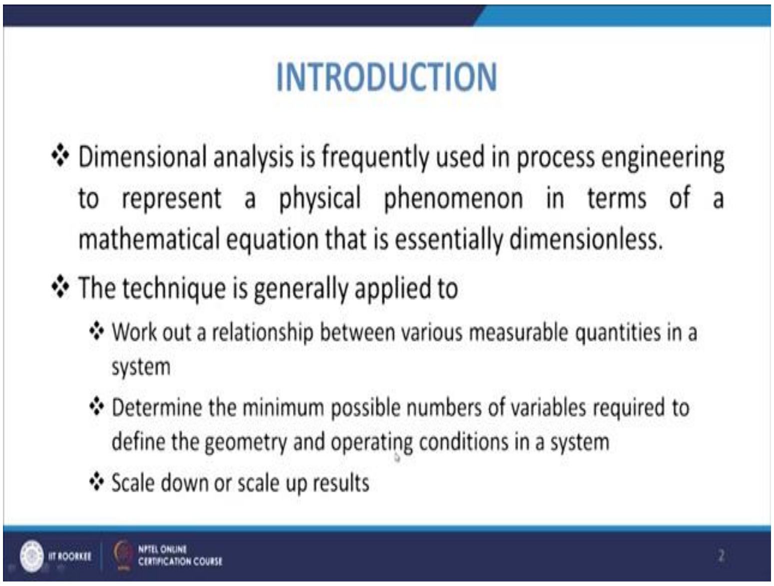

What I mean to say that; normally, we try to convert these equations that is essentially dimensionless. Now, this technique which is generally applied it is to work out relationship between the various measurable quantities in a system. So, what we is there that if you talk about any system. So, you will have one variable that will be represented in terms of other, you know; parameters or the variables.

So, you will have to have a particular relationship and you determine the minimum possible number of variables required to define the geometry and also operating conditions in the system and then you go for scaling down or scaling up of the system. So, that is the you know; the way you that there is the technique which is normally applied. So, the first method which is used for such approach is the you know, Raleigh's method of indices.

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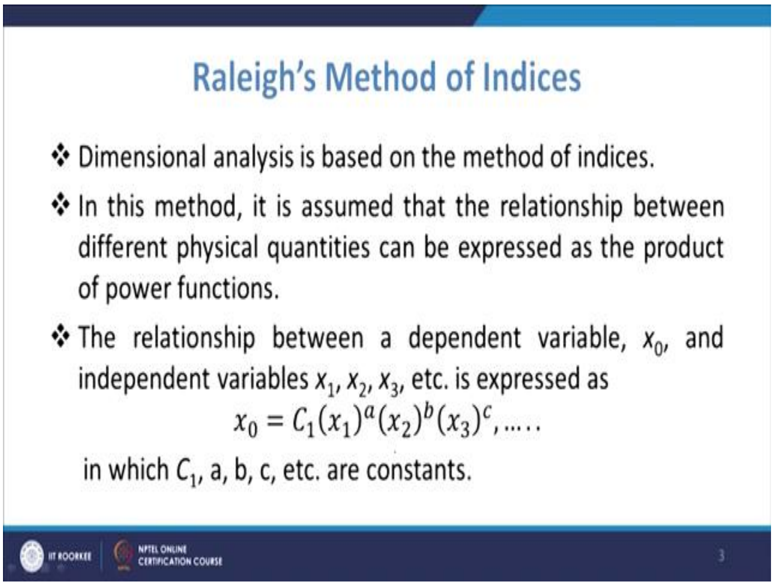


Now, what Raleigh's method of indices say that; in this case the dimensional analysis will be based on the method of indices. So, you know what happens that the relationship between different physical quantities can be expressed as the product of power functions. So, what you do here that you have $x_0$, suppose; this is a dependent variable and that is expressed in terms of $C_1(x_1)^a(x_2)^b(x_3)^c$……….and.

So, suppose $C_1$ is expressed you know; $C_1(a, b, c)$, all these are the constraints. So, you know; if you try to have the dimensionless groups. So, if you see further; if you have the you know, from this place onwards again you can have the $x_1$, $x_2$ and a $x_3$. So, they can even be expressed, you know you can have these you know; dimensionless groups defined that is $x_0$.

Similarly, $x_1$, $x_2$and a $x_3$. So, they also can have with the different constants A,B,C , you can express them. So, you can further you know express like if you have non dimensional group.

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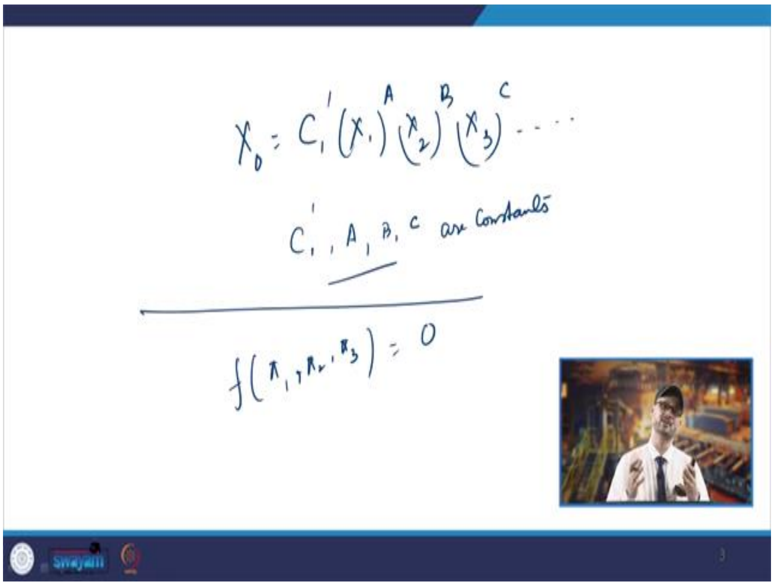

So, that is $X_0 = C_1{}'(X_1)^A(X_2)^B(X_3)^C$ and you have the you know $(X_1)^A$and similarly, $(X_2)^B$ similarly $(X_3)^C$. So, these are the non dimensional groups that can also be expressed you know, in terms of these indices and that will be going on.

So, here again $C_1{}'$ then A B and C they are again the constants. So, and then you are you know, we will see that; they will be further equated on both the sides and you can have the you the you know, evaluation of these constants and you can see that how they can be expressed. Another method which is also used is the you know Buckingham pi theorem.

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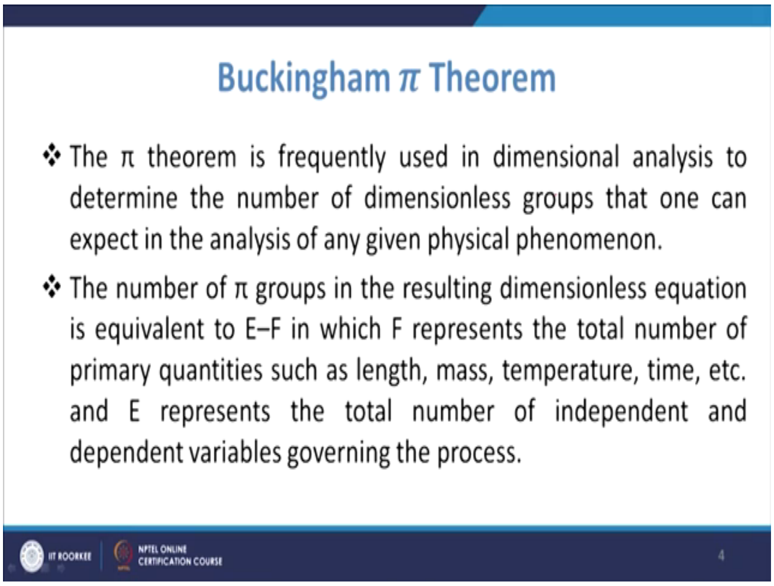


So, now in this case, it is normally used in the dimensional analysis to determine the number of dimensionless groups that one can expect in the analysis of any given physical phenomena. So, here what you do is normally, you are having the number of the dimensionless groups you are trying to determine and that will be depending upon you know the value that will be equivalent to the $E - F$.

So, you have basically, F; that will be representing the total number of primary quantities the which is required for you know, the expressing. And then the E value is will be representing the total number of independent and dependent variables which will be governing the process. So, $E - F$ will be basically telling you the number of those pi you know groups.

And then this way, you can have those dimensionless you know; numbers coming into existence. So, that can be found out. So, what you do in this case you have $F(\pi_1, \pi_2, \pi_3)$. So, in the case of pi theorem you will have the $\pi_1, \pi_2, \pi_3$. So, they will be equal to 0.

So, they will be basically, they are the independent you know, products of the you know arguments that is there which is dimensionless. And you know, you find these you know; common numbers will be there that will be equal to $E - F$. So, this way, you try to have you know the calculation of these you know pi groups.

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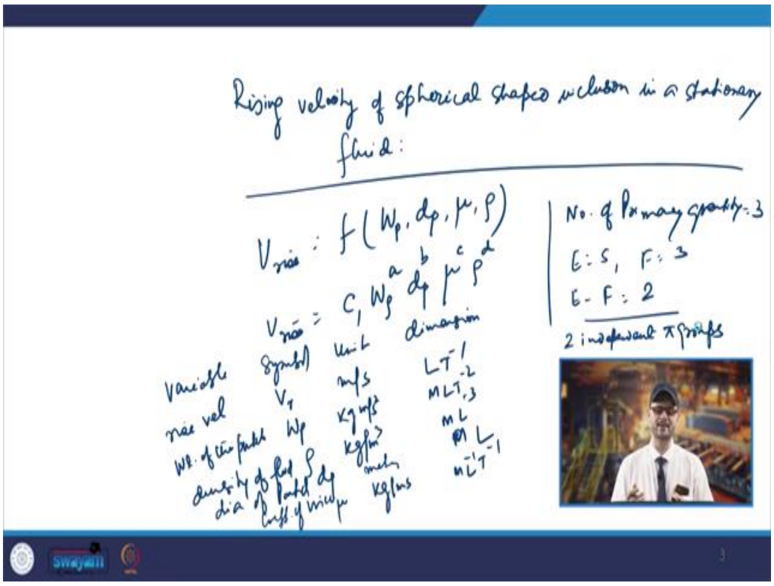


So, for example, if you look at one of the problem like you have to have the you know rising velocity you have to have this for this problem rising velocity of spherical shaped particle, a spherical shaped inclusion in a stationary fluid. So, suppose you have to have the you know calculation of the dimensionless, you know these groups.

So, what you do, you can have the use of this Buckingham pi theorem and, what we see that; normally, if you talk about the rising velocity which is there because the this is a problem in the case of steelmaking where the inclusion, if it there they are there they are normally lighter and they have the tendency to float you know towards the upper side

So, normally, your V rise, so this V rise will be function of the weight of the particle, then you have the density of the particle, you have the coefficient of viscosity of the medium and also the density of the medium. So, normally, it is a function of you know $v_{rise} = f(w_p, d_p\ \mu, \rho)$..

Now, if you use this Raleigh's method of indices; so you can write a functional relationship of the type. So, you will have $v_{rise} = C_1({w_p}^a, {d_p}^b\ \mu^c, \rho^d)$..

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So, that way you can have you know; you can these are the indices which are you know these. So, you will have the to know the what are the dimension of these you know variables. Suppose, your variable is your raise velocity. So, you are you know symbol is you have $v_r$ and its unit is as it is velocity. So, it will be meter per second and if you have the to know the dimension. So, it will be $L\,T^{-1}$.

Similarly, you have the weight of the particle. And weight of the particle you do it by $w_p$. So, it will be you know normally, $kg\ meter/s^2$ and you will have $ML\,T^{-2}$. Then you have the density. So, if you talk about the density of fluid. So, you have that is $\rho$. So, you will have kg you know per meter cube and it will be $M\,L^{-3}$.

So, then comes your after density you have the diameter of the particle. So, you have $d_{\,P}$ so, diameter will be in terms of meter only. So, you will have M. I know this is L and then last is your $\mu$. So, that is coefficient of viscosity. And you know, that is $\mu$ and that will be you know kg you know; that is if you look at that will be $\frac{Kg}{m\,s}$and you will have $ML^{-1}T^{-1}$.

So, once you know you must have the idea about these dimensions and you have to you know equate or known on this side you have to find the number of pi; which is required you know in that case, now in this case if you look at the number of you know primary quantity is 3. So, number of primary quantity will be 3. And, if you look at the independent and dependent variable that is $w_p\ \ ,d_p\ \ \ \mu\ \ ,\rho\ \ \ and\ v$ so, that is your 5. So, E is 5 and F is 3.

So, in that case, if you go for $E - F = 2$. So, you will have to have, you will have to you know independent pi groups that you can form. So, you will have two independent pi groups. And that will be required for the representation you know you know; that is in the dimensionless form.

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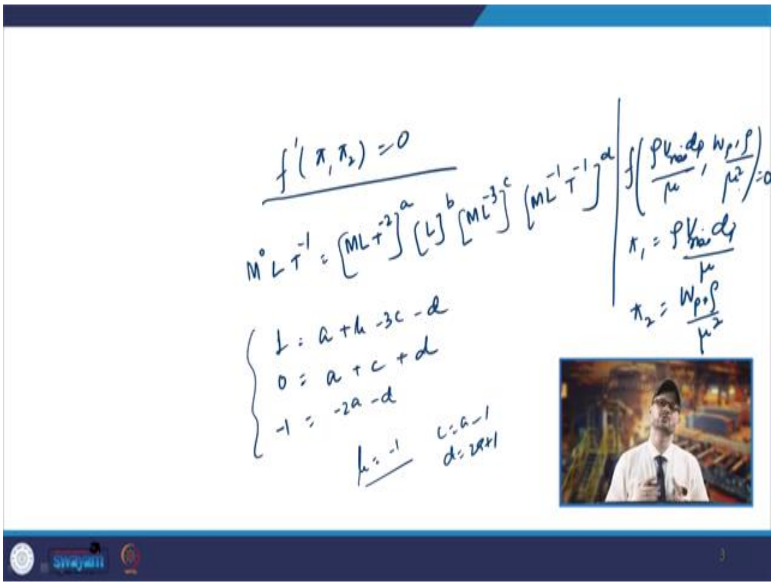


So, you can write you know that; $f'(\pi_1, \pi_2) = 0$. Now, what you see is that; if you equate on both these sides. So, you have to have the dimensions on both the sides. So, you have you know in earlier k this is $LT^{-1}$. So, you will have $M^0 LT^{-1}$. So, that is and that will be that we have seen that; you will have the four parameters and that to that have the indices a b c and d.

So, those values are ${w_p}^a, {d_p}^b, \mu^c\ \rho^d$ . So, you will have those expressions. So, you will have $[MLT^{-2}]^a$.. Similarly, you will have $[L]^b$, similarly you will have $ML^{-1}$. So, you will have those values. So, you will have then $\mu$; $\mu$ will be $ML^{-1}T^{-1}$. So, on that so, we have taken $[\rho]^d$. So, $\rho$ is your $ML^{-3}$.. So, you will have $ML^{-3}$and you will have the $ML^{-1}T^{-1}$.

So, $ML^{-1}\ ML^{-3}$. So, $\frac{kg}{m^3}$and then you will have $ML^{-1}T^{-1}$and that rise to be c and d. So, basically, you can have you know you can do the equation can be equated. So, this side it is 1 and if you look at here it will be 1. So, that is $a + b - 3c + d$ . So, that will be 1. So, 1 will be $a + b - 3c + d$. So, similarly, you have T is you know M is 0. So, 0 will be equal to if you look at the M. So, it will be $a + c + d$.

So, this way and if you go for T. So, it will be T is minus 1 on this side and on this side if you look at it be you know T is here it is minus 2 a and similarly, you will have T as minus d. So, you know ML. So, that way you will have the equation. So, that you can come and if you equate you know these equations. So, you will have that is minus 2 a and you will have further minus d.

So, you can have you know you can find these a b c and you may have d. So, if you manipulate these equations, then you can have you can find the value of b as minus 1 and that you can have the expression of, you know; c and d are expressly in terms of a because your three equations and four unknowns. So, you c and d can be expressed in terms of a.

So, you have c as a minus 1 and d as $2a + 1$. So, we can have that minus 1 as 2 minus 2 a minus d. So, you get b as minus 1 that by some manipulation and then you get the c and d expression in terms of a. And you can have you know further; you can express, you know the in terms of the two quantities; that will be $f(\frac{\rho v_{rise} d_p}{\mu}, \frac{w_p, \rho}{\mu^2}) = 0$

So, basically, you are in a position to devise two kind of you know non dimensional numbers that is for these inclusions when you do the inclusion analysis. And the two groups which you get will be that will be $\frac{\rho v_{rise} d_p}{\mu}$and the another group which you get $\pi_2$ it will be $\frac{w_p, \rho}{\mu^2}$.

So, what you see; now, if you look at these numbers what you see this is nothing but, this is the number which is an analogous to the Reynolds number which we have studied $\frac{\rho v d}{\mu}$. So, and this is another you know dimensionless group which you are having in that case.

So, this way you try to have the solution of these you know equations. So, you have to first write down these two equations on both the sides, you have to have the indices you know being equated. You have to solve and then you get these expressions and further; you are having, you know these values of these two pi groups.

So, that is how, you get these non dimensional you know numbers and they have their significance when you are trying to you know analyze that system that time you try to have the use of these numbers. So, you may have different types of you know you know dimensionless groups which are used in the case of tundish modeling and you know like when you go for the fluid flow type of you know cases.

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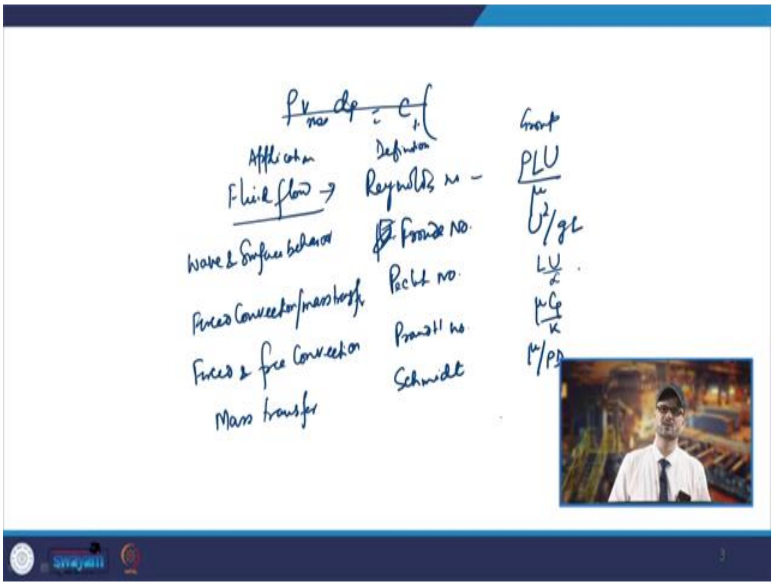


So, in the case of fluid flow; you have the group is your Reynolds number and that is by $\frac{\rho UL}{\mu}$. So, that is how you have these you know dimensionless number group that is. So, this is your application and that is how you define. So, this is your definition and this is the group.

Similarly, if you go for the wave and surface behaviour. So, in that case you know what you do or when you are going to use for the pouring stream. So, in that case, you have that value is $\frac{U^2}{gL}$and that is your Froude number. So, we have seen this number you know we have talked about, we have dealt with this number earlier. So, that is your Froude number.

Similarly, you go for you know the force convection of mass transfer. So, in those cases you have a number that is your Peclet number and the Peclet number will be you know $\frac{UL}{\alpha}$ or $\frac{UL}{d}$. So, that way you know it is basically the ratio between convection and diffusion. So, that way you get these Peclet number defined.

Similarly, you know when you have the atomization of liquid jets in those cases you come across the number that is your Weber number. So, there you have the surface tension term coming into picture. Then many a times in the case of force and free convection. You come across the group and that is your Prandtl number and the Prandtl number is defined as $\frac{\rho c_p}{k}$. And in a case of mass transfer when you will deal with the you know concentration equation or solving for the scalar you know transport equations. In the those cases you have Schmidt number.

So, that is Schmidt number and you know that is the group is $\frac{\mu}{\rho D}$. So, that is you know that is also uses. So, normally you have certain value assigned to it we assume. So, basically, these are the different type of you know groups which you normally come across. So, you also will also come across other numbers like you come to for the tundish Richardson number, when we talk about the convection or natural convection specially when we deal with the heat transfer cases.

So, in those cases, those numbers also or you know coming into picture and these are you know the groups. So, that is you are you know they are going to be dealt with, when we you know deal with these cases. Then you know their you know that is normally used now, these values also it has been reported that.

When you use these various dimensionless numbers or groups. So, normally, you go for the water modeling in the case of the tundish system. So, the steel is basically replaced with water because, their kinematic viscosity is same at the melting temperature of steel and at the normal temperatures in case of water. So, normally, you have when you go for the full minus scale model.

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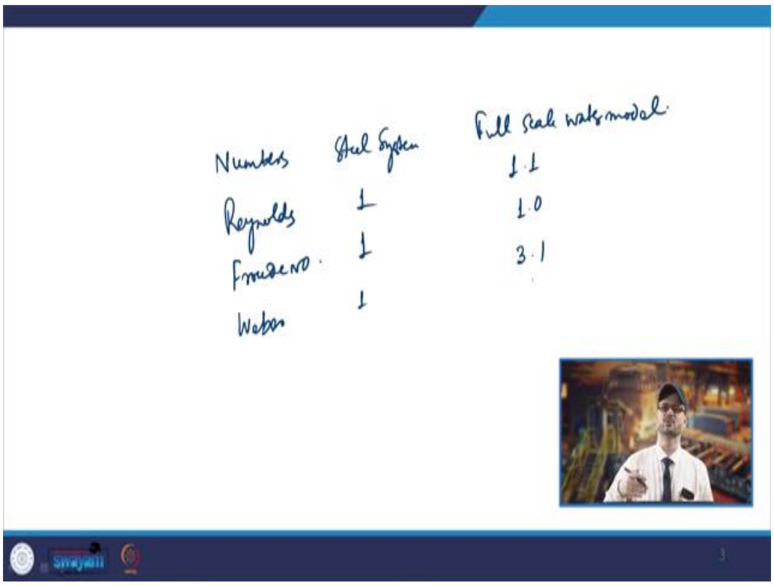

So, when you have this if you talk about these numbers and you have a you have the steel system. So, and if you go for the full scale you know; water model. So, if you see that when you talk about this Reynolds number. So, you do the Reynolds similarity. So, in that case if it is 1. So, normally, it is reported you know in the you know by exactly that normally, we should report it to be 1.1 for the Reynolds number. In steel system it is 1, so full scale water modeling you can have 1.1.

Similarly, you can have for the Froude number. So, if it is 1. So, you here also you keep 1. So, that Froude you know Froude similarity is normally you know it kept in mind. So, that is you know in the steel system which is 1, then full scale water model also you keep it as 1. So, similarly, for the you know Weber number; if you talk for Weber you know it is if it is 1. So, in that case it is will be 3.1.

So, similar to that you have you know; other numbers like, you have sometimes the motor number. So, motor number is the 1; which is used for the velocity of bubbles in the liquids and that is normally, used for the two phase flows a multi phase flows when we do in the tundish.

So, in those cases, for the this is the dimensionless group is the motor number. And that is you know g mu you know. So, you have expression for that and for that the for the full scale water model that ratio becomes close to if it is 1 in steel system; it is coming close to 45 in the water model. So, these, you know these numbers. So, that needs to be kept in mind; when you are doing the water modeling and how you know that full scale water model.

How these you know numbers now; how these for the steel as well as full scale model, what numbers you know should be there. So, that you need to know. So, that can be taken as a reference from the literature also and this is how typically the vary. So, this is about you know; the dimensionless you know, doing the dimensional analysis in the case of this tundish system.

And we will, when we will be doing that discussing about these water modeling or this scaling. So, at that time we will have the introduction to these numbers specially Reynolds and the Froude. So, you will have the similarity based on these two that is what we have already discussed and also their you know significance in those cases.

Thank you very much.