Modeling of Tundish Steelmaking Process in Continuous Casting

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Lecture - 07
Concept of similarity in Physical Modeling

Welcome, to the lecture on Concept of similarity in Physical Modeling. So, in the last lecture we talked about the physical modelling principles. So, as we discussed that there the similarity principle, or its concept is important to be kept in mind when we are making the physical models.

Which is normally a reduced scale model and because the model most in most of the cases the system involves the flow of species, flow of the fluid you have chemical reactions taking place heat transfer taking place. So, how you know I mean to predict you know properly you need to have the similarity principle you know similarity satisfied.

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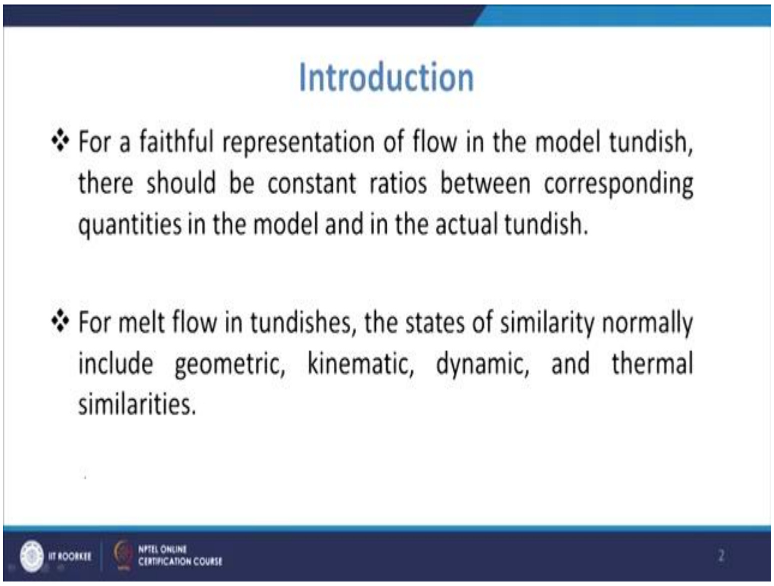

So, you have as you have seen that, for the faithful representation of flow in the model tundish, there should be constant ratios between corresponding quantities in the model in the actual and in the actual tundish. So, you will have the you know quantities like you have the velocity or the geometric dimensions or the velocity or the heat transfer. Values suppose which is there in the actual tundish and the in the model tundish.

All needs to have a ratio so, that is that should be you know maintained for the model and the actual tundish. So, for melt flow in the tundish is the states of similarity which we normally include is the geometric similarity, kinematic similarity, dynamic similarity and thermal similarity.

So, many a times we call it as a geometric similarity or even mechanical similarity thermal similarity apart from that also we have the chemical similarity that may also be there wherever the chemical reaction is taking place. So, but in normal case when we talk about these similarities, we normally talk about geometric, kinematic dynamic and the thermal similarities so, we will talk about these similarities. Now the geometric similarity is you know it tells that the actual tundish and its model must be geometrically similar.

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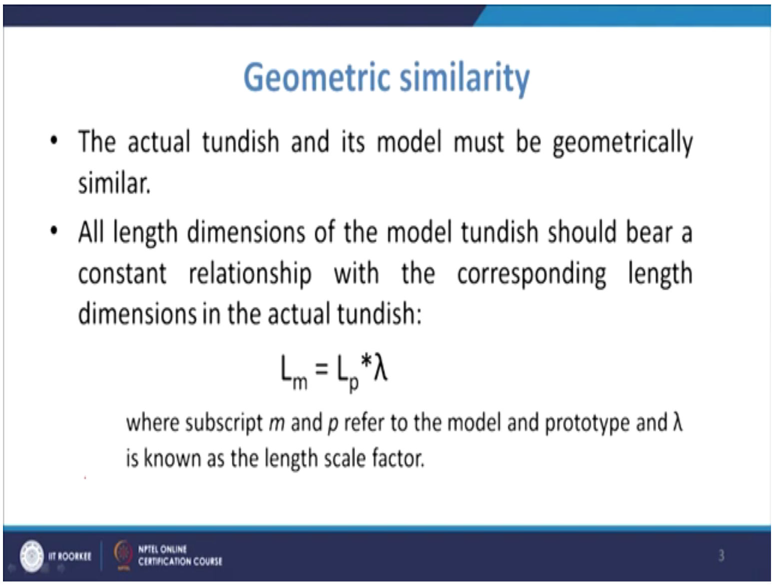

So, when we are making the model tundish in which the flow has to be there. So, this would be geometrically similar, means; that the ratio of their geometrical parameters this would remain constant. Like, if you have a tundish of 3 meter length and if you are making the model tundish of 1 meter length it means the ratio of the 2, you know will be you know 3.

If you take the ratio of the prototype to the model or if you take the ratio of the model to prototype it will be less than 1 that is 1/3. Now, this 1 by 3 is basically so, it is a 1/3 model. So, we can say like that so, 1/3 will be that scale factor length scale factor. So, this has to be maintained in all the geometric dimensions like your width also should be accordingly you have to adjust.

Which should have a ratio of $1/3$. Similarly for the height also so, it is telling that so, this is how all the length dimension of the model tundish should be are a constant relationship with the corresponding length dimension in the actual tundish. So, that is how we try to you know represent it, like L_m will be $L_p * \lambda$. So, m refers to the model and p refers to the I mean prototype. So, you know λ will be $\frac{L_m}{L_p}$.

So, λ is known as the scale factor length scale factor and you know it is if you are taking a reduced scale model in that case it will be less than 1, if you are going for the scale up model it will be more than 1. So, that way you know in most of the cases in tundish flow or so, in the steelmaking we normally make the reduced scale model. So, the value of λ will be less than 1.

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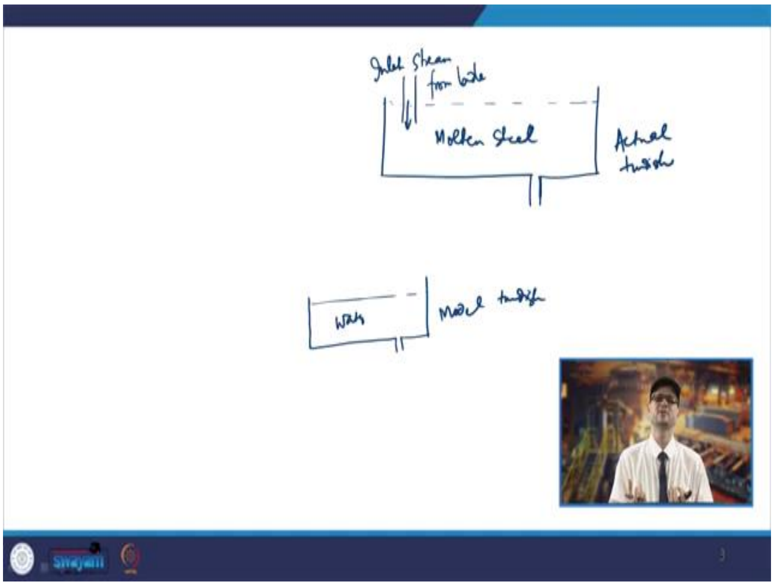

So, if you try to see that you will have suppose, in the case of your actual tundish may be like this. So, you will have in this case you have the molten steel here and you will have the level of steel suppose here and then inlet is from this side so, your molten steel is flowing. So, you will have inlet stream from ladle.

Now, this is your you know normal, you know as a normal scale you can have a physical scale so, you will make suppose about 60 percent or so, it will go like this so, this way you can have a physical model here. So, you can have. So, this dimension to this so, this ratio to this ratio dimension or this ratio to this ratio dimension like that. So, that should be a constant value that is your so, this length by this length or its width or so.

So, these values have to be similar so, you will have this is your model tundish, this is your actual tundish that is your prototype tundish. So, you will have molten steel and here you will have water. So, but the ratio has to be same so, that is what is the meaning of the you know geometric similarity. So, similarly is on the dimensions like you have the diameter in case of suppose your outlet is in cylindrical, you know in the circular type cross section. So, there the diameter will come into picture.

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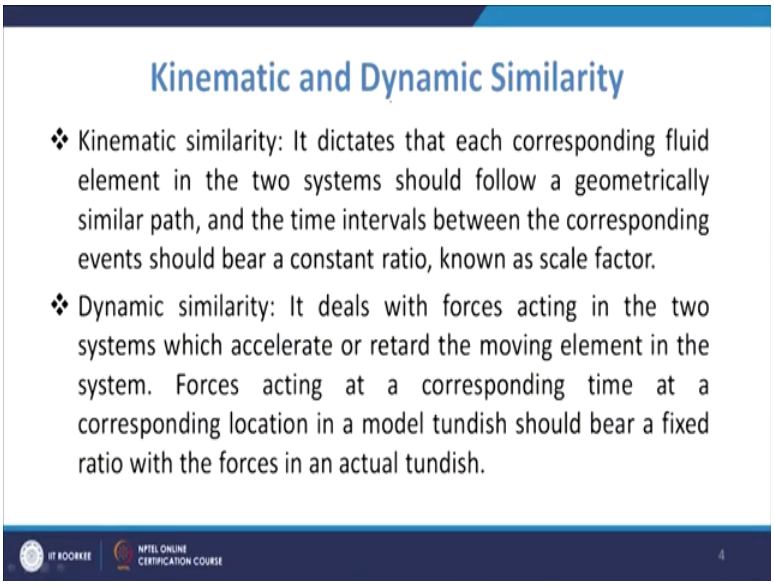

Now, the next you know type of similarity next is the kinematic similarity. Now kinematic similarity will be indicating that each corresponding fluid element in the two system, should follow a geometrically similar path and the time interval between the corresponding events should bear a constant ratio that is your you know scale factor.

So, that is your you know kinematic similarity. So, then, so here they should have the similar you know time interval velocity and all that they will be taking into account in these cases and they should have the you know same value if you talk about the x component velocity ratio or y component velocity ratio or z component velocity ratio.

So, there you know you will have a constant ratio so, that will be talking about the kinematic similarity. Then comes the dynamic similarity. Now dynamic similarity will be talking about the forces acting in the two system so, it will in kinematic similarity we are not talking about the forces; however, in the dynamic similarity we will be talking about the forces.

So, it will be dealing with the forces which act in the two system which accelerate or retard the moving element in the system. So, as we talk about any system we will have different kind of forces which will be acting inside the system, and forces acting at a corresponding time at a corresponding location in a model tundish should bear a fixed ratio with the forces in an actual tundish. So, this will be the dynamics similarity

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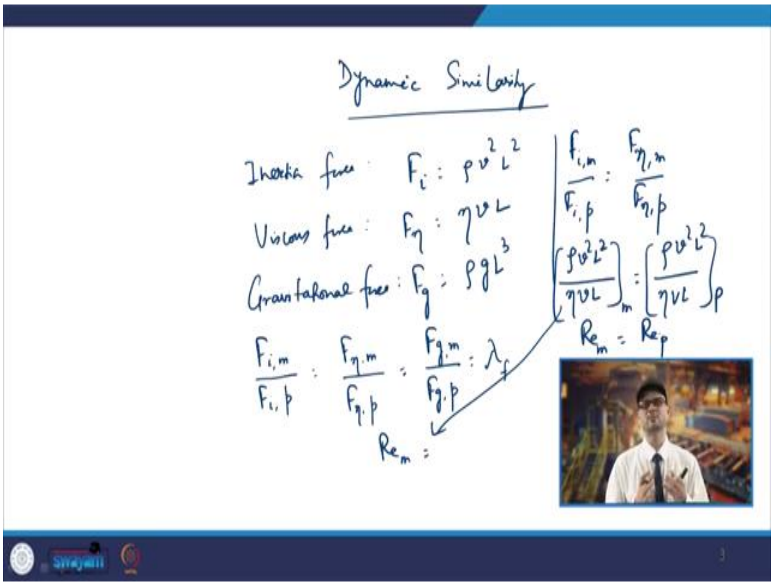

So, if you talk about the different type of forces if you go to the you know dynamic similarity. So, when we talk about the flow in the steel making. So, you will have different, normally you have some special you know some common type of forces. So, suppose you have the inertia force so, inertia force will be normally denoted by F_i and it will be $\rho * v^2 * L^2$.

So, that is your Inertia force, and similarly if you talk about the viscous force. So, normally you have the three kind of forces which are you know commonly seen inertia force, viscous force and gravitational force. So, viscous force will be you know F_η so, we call it as the you know ηvl. So, that is your viscous force, similarly you have gravitational force. So, gravitational force is the F_g and it is defined as the $\rho * g * L^3$.

So, these are the different forces which are you know commonly you know they are coming to the analysis, in the case of the flow in the steelmaking applications like in tundish or so. So, what, we this dynamic similarity condition says, that $(F_i)_m \text{ to the ratio of } (F_i)_p$ so, there should be same as the $(F_\eta)_m \text{ to the ratio of } (F_\eta)_p$. Or the $(F_g)_m \text{ to the ratio of } (F_g)_p$, and there should be some constant that should be λ_F so, that is for force.

So, the similarity condition tells of the for the dynamic similarity, it tells that these values should be you know same. So, if you talk about the you know this condition $(F_i)_m \text{ to the ratio of } (F_i)_p$ and that should be equal to $(F_\eta)_m \text{ to the ratio of } (F_\eta)_p$. So, if you know try to go further into it so, what you did see is $(F_i)_m$ is $\rho * v^2 * L^2$, and that is for the model so, if $(F_\eta)_m$ will be ηvl.

So, it will be and that is ηvl so, this is ηvl. So, if you take this for the model it will be same as you know $\rho * v^2 * L^2/\eta vl$. So, from here what you see, if you further go you know to from this point. So, what you see you can cut these terms so, it will be you know rho.

So, this will be coming to a term you know you must have come across this term that is your Reynolds number term. So, it will be Reynolds number of the model this should be. So, here so, you need not go there so, it will be Reynolds number of the, model should be same as the Reynolds number of the prototype.

So, this is the ratio of you know basically the two kind of forces that tells you the value of the Reynolds number for the model, it should be same as the that of the prototype. So, that you know talks about these you know conditions which is governed by this dynamic similarity, and the Reynolds number which is found in the you know model physical model it should be same as that which is there inside the actual prototype model.

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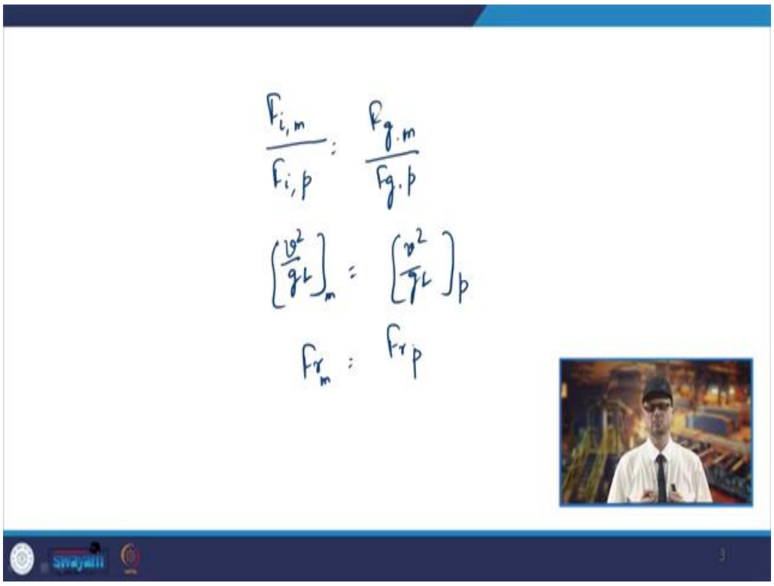

Similarly, if you talk about the ratio of the inertial to gravitational force, so, if you go to you know this $(F_i)_m/(F_i)_p$, for the prototype and if you take the ratio for the $(F_g)_m/(F_g)_p$. So, they will lead to you know another kind of the non dimensional number.

So, what you see here is you get $(\frac{V^2}{gL})_m = (\frac{V^2}{gL})_p$. So, you get the another number that is your Froude number and so, you get the Froude number for the model should be equal to Froude number for the prototype. So, these conditions emerge out you know when we talk about these you know similarity conditions, as you know especially maintaining the dynamic similarity in the case of fluid flow.

So, if you so, what we see that these conditions need to be checked, when we are you know further predicting the values and then we say that it is dynamically similar the model as well as the prototype is dynamically similar. Then comes the you know thermal similarity now thermal similarity, you know is something where we deal with the heat losses.

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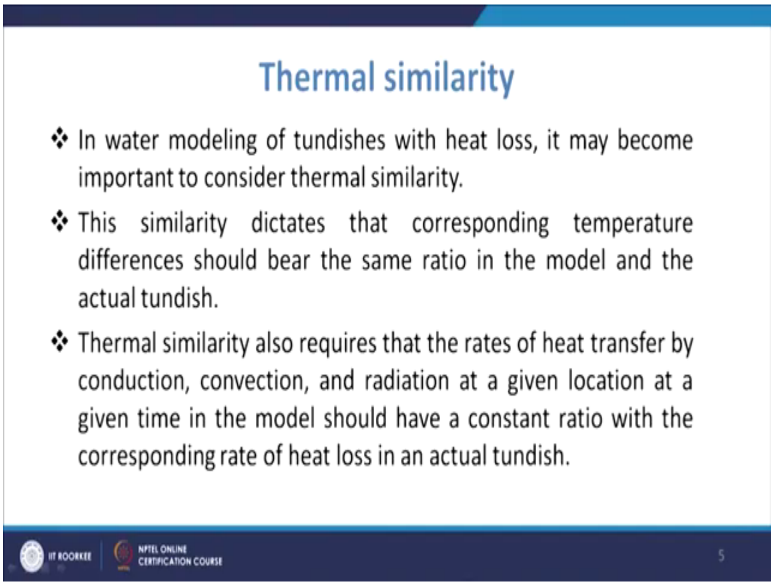

So, when we talk about the water modeling so, normally in water modeling you know when we do for the tundish. So, with that the heat loss you know which is taking place at that high temperature so, its very difficult to correlate. But then we will have we can have this thermal similarity and what we can do is here also whatever the temperature difference is there suppose, if they are in the same ratio then we can say that the systems are thermally similar.

So, the similarity indicates that dictates that corresponding temperature differences should bear the same ratio of the model and the actual tundish. So, suppose at some point you have the temperature in the model tundish and same point you have temperature in the you know in the actual tundish. So, if the ratio if you are taking at different point this would be similar. So, that is what the thermal similarity means.

Thermal similarity also requires that the rates of heat transfer by conduction convection and radiation at a given location at a given time in the model, should have a constant ratio with the corresponding rate of heat loss in actual tundish. So, similar to the temperature it also applies for the you know heat transfer which is taking place.

So, heat transfer is taking place by the conduction convection or radiation. So, they also should have the you know same ratio as in the case of model or in the case of prototype. So, that is what the you know thermal similarity means.

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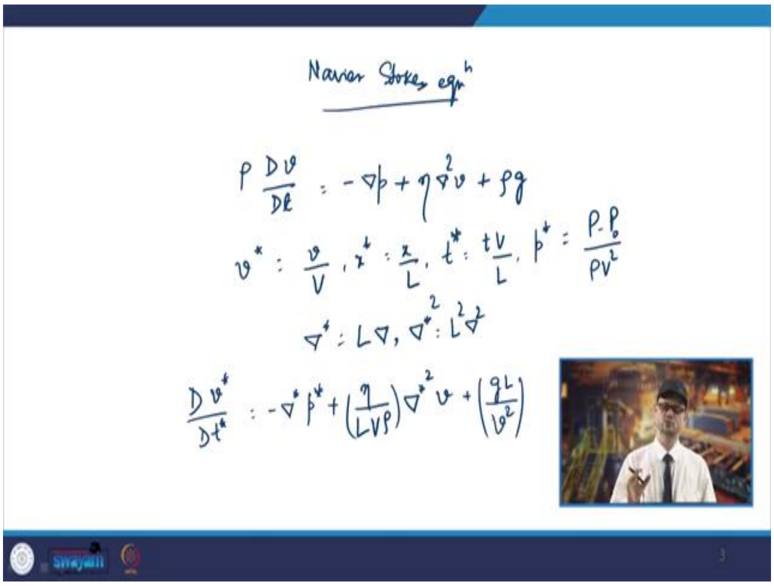

So, we can have the discussion about the similarity criteria by even working with the equations and that is another way of looking at it. So, suppose we are talking about the you know Navier Stokes equation. So, you know I mean there also you can have the you know terms we can go for individual terms and we can have with the help of the dimensionless numbers we do the analysis.

So, the Navier Stokes equation, which is normally you can you express it $\rho \frac{Dv}{Dt} = -\nabla p + \eta \nabla^2 v + \rho g$. So, this is your another term that is a source term

So, now in this case what we do is we non dimensionalize these terms. So, using a Non dimensional quantities we are making the v^*. So, suppose $v^* = v/V$ or we take $x^* = x/L$. So, a statistic length of the tundish you further non dimensionalize $t^* = tv/L$.

So, it will be t with a certain time we are dividing, then you $P^* = \frac{P-P_0}{\rho v^2}$ So, this is how P^* is defined, you define ∇^* as L*∇ and then you have . So, ∇^* square term will also come which is here so, that will be you know $L^2\nabla^2$.

So, that way if you use these non dimensional terms, so, you know if you substitute these non dimensional numbers into this equation you get, you know the further the term that is $\frac{Dv^*}{Dt^*} = -\nabla^* p^* + (\frac{\eta}{LV\rho})\nabla^{*2} v + \frac{gl}{v^2}$.

So, what you see in these cases you have you know in this term under the in the parentheses you have two terms which are coming up, and they are basically this is the inverse of the Reynolds number this is $\frac{\eta}{L\rho v}$. So, and then you have this is the inverse of the you know Froude number so, these terms you know are coming and basically you know these are the important forces which are governing inside the tundish.

So, this is coming because of these inertial you know gravitational and viscous forces. So, there has to be a constant ratio of these forces. Now, if you imply the inertial forces which should be you know the if you talk about the Reynolds similarity. So, that is why you have the Reynolds similarity as well as the Froude similarity which is coming up and you have to maintain that.

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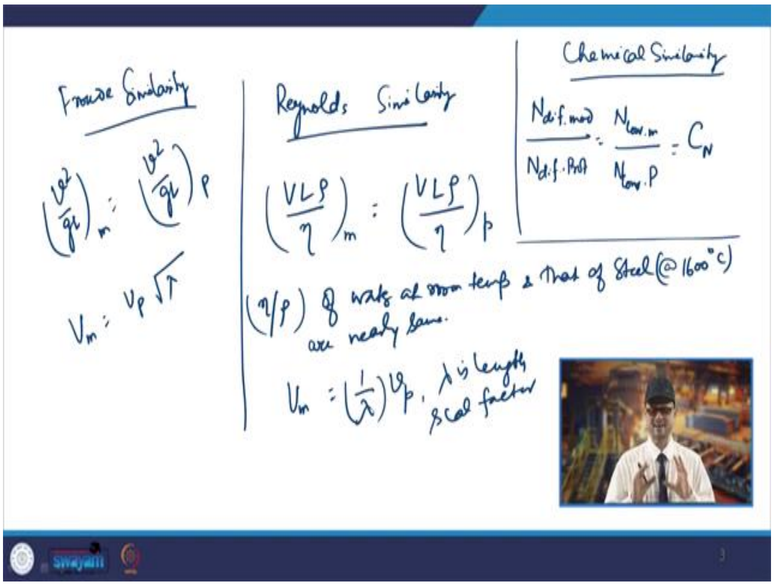

So, if you try to you know see the Reynolds similarity. So, in that case you have $(\frac{vl\rho}{\eta})_m = (\frac{vl\rho}{\eta})_p$. So, you know what we see that normally, the kinematic viscosity of the molten steel and the water at room temperature they are somewhat similar. So, they are somewhat equal so, eta by so, this ratio becomes 1. So, in that case what you get is so, your $\frac{\eta}{\rho}$ of water, at room temperature and that of steel at 1600 degree centigrade. So, they are nearly same.

So, what we get out of this similarity principle that this $v_m$and you know this is your $v_p$and similarly this $L_m$and L_P. So, if you lambda eta by rho that will be you know cancelled. So, v_m/v_p will be L_p/L_m. So, basically you will have the inverse of the scale factor. So, what you see is $v_m = v_p * (\frac{1}{\lambda})$ that is what you get. So, because its ratio will be $\frac{1}{\lambda}$that is your geometric scale factor. So, lambda is the length scale factor. So, the Reynolds similarity gives you this condition.

Now, if you go to the Froude similarity, if you further use the Froude similarity. So, using the Froude similarity what you see. So, you can further do the same you know calculation. So, you have $(\frac{v^2}{gl})_m = (\frac{v^2}{gl})_p$.

Now, from here as g will be same so, what you see is that v square of v of model by v of the prototype so, basically you will have a square term. So, you will have $v_m = v_p\sqrt{\lambda}$ because it is $L_m$upon L_P it is going on this side so, $L_m/L_P = \lambda$. So, $v_m = v_p\sqrt{\lambda}$. So, that is you can see that this is V m and this g term will be cancelling.

Now, what you see that you are getting $v_m = v_p\sqrt{\lambda}$. in this case using the Froude similarity, and using the Reynolds similarity you are getting $v_m = v_p * (\frac{1}{\lambda})$. So, you know you are getting the different values you know different you know proportionality factor in these two cases.

Now, if you try to have you know the , you know the try to ensure that both these you know condition, similarity conditions are satisfied that is only possible when you have lambda that is equal to 1. So, that is you know when you are making the model of the same scale as that of the prototype then only these two can be satisfied.

So, you know, both these conditions can be only met when you have lambda equal to 1 so, that is your scale factor is one itself so, you are having the model as well as the prototype of these same you know dimension. So, that is of that cannot be you know said to be a feasible solution. So, now if you look at this Reynolds similarity. So, in that case what you see is that the velocity of the model so, when you are making a you know scaled down model in that case lambda value is less than 1. So, in that case the 1 by lambda value will be more than 1. So, in that case your velocity in the model has to be more than that is what it is there in the case of prototype.

Whereas, if you take the Froude similarity criteria here the velocity in the model will be less than the velocity in the prototype so, it is because it is the under root term. So, normally what we use. So, basically that is this value will be less than 1. So, that way model will this value will be lesser. So, normally the whenever we do the physical modeling we do this by assuming the Froude similarity criteria.

So, that is the normal practice in the case of you know physical modeling. Apart from this as we had discussed that apart from that this geometric similarities, dynamic similarity or kinematic similarity and thermal similarity, many a times we also come across the term known as the chemical similarity. So, when we talk about, because many a times these this there are many type of chemical reactions which are taking place.

So, you will have to deal with the concentration profiles you know in the full scale system. So, you will have the discussion about the chemical you know similarity in which you will be talking about the concentration differences you know at the corresponding locations in the model and in the tundish.

So, if you have the concentration difference between two positions in the model certain value, and similarly the same value in the tundish I mean in the actual case in the prototype. So, this would be a you know the same value so, this may be because of the diffusion you will have the mass transfer taking place, you will have the mass transfer by convection maybe by diffusion or maybe many a times you have the generation of the species or dissipation there through the chemical reaction.

So, these quantities also need to be bearing a certain ratio and that is taking into considerations by taking this chemical similarity condition. So, in this case what we tell is that you have the, suppose you are talking about the diffusion so, and diffusion in the you know model and if you talk we take the ratio of diffusion that is in prototype.

So, that ratio should be suppose you are talking about the, using the convection in the model and ratio by for convection in the prototype. So, similarly if you are talking about the you know radiation quantity. So, these all these values this should be having a constant value that ratio should be same. So, this means the there is chemical similarity. So, these are the you know different kinds of you know similarity principles which are need to be you know addressed into and which must be satisfied when we are talking about the physical modeling.

And altogether when we talk about the static or dynamic similarity they are also known as the mechanical similarity so, many a times we talk about the mechanical similarity as a whole. So, these different kind of you know similarities are there which needs to be you know seen and many a times this chemical similarity is very difficult to be achieved, because of their strong dependence on the temperature.

So, in many cases it will not be you know feasible to have these similarities; however, you we must know that what the chemical similarity means, because when we do the you know suppose we are talking about the chemical similarities in two systems. Now in those cases the temperature being higher many things you know change by orders of magnitude quite high you know because of the temperature.

So, that may not have the similar ratios, but then many parameters may be you know checked like the diffusion of certain things which is not temperature dependent or when we do the cold modelling studies in those cases the chemical similarity holds certain meaning. So, we will talk about you know other aspects of the physical modeling like use of dimensional numbers or so in our coming lectures.

Thank you very much.