

**Modeling of Tundish Steelmaking Process in Continuous Casting**  
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**Lecture – 38**  
**Mathematical Modeling of Inclusion Removal in Tundish- II**

Welcome to the lecture on Mathematical Modeling of Inclusion Removal in Tundish. So, in the last lecture, we discussed about the similarity considerations and especially, while doing the modeling for the inclusion removal. And, even in the water modeling case, you know what should be the density of the inclusion that should be selected, based on you know the other similarity criteria's and also you know other things taken into account like the density of the inclusions in the steel or size of the inclusion in the steel.

So, based on that you can take those; so, from there you can have the idea that when you model, at that time how you have to select those sizes properly so that you can have a proper simulation. Now, we will discuss about the aspects which needs to be taken into consideration when we do the mathematical modeling or numerical modeling of the inclusion removal processes in tundish.

So, as we know we already discussed that the inclusions are basically going to be very harmful, if they are coming inside I mean out of the tundish. So, we need to have the removal of the tundish; but removal outside or it should float you know that way and you know apart from that, there may be other conditions that may be put in so that inclusions are trapped or inclusions are stuck to the walls or source. They are the part of the boundary conditions.

Now, we will talk about the modeling you know issues mathematical modeling when we talked about. So, that is normally handled using the Eulerian Lagrangian Approach, where you have this is the discrete phase particle. So, inclusions being the particles; so, they are the discrete you know particles. So, basically you need also to know that where these inclusions are going. So, we need to track them and that is known as the you know tracking of their path. So, tracking of their trajectory so, that is the aim basically when we do the mathematical modeling.

So, as I told that this is normally a Eulerian Lagrangian type of approach, you have the fluid in which you are injecting the inclusion particles and this inclusion particles are basically having. So, initially they will be given certain kind of velocity with which they will be going inside and once they go into the fluid domain; then further, they are subjected to the conditions which is prevalent in that fluid domain.

So, when you are going for studying the doing the flow analysis with by taking the turbulent flow in consideration. So, you may have even you know either you can have the mean velocity component into consideration or you can have even the stochastic component also like you can have the fluctuating component also, you know that can its effect can be seen.

So, basically what we do is that we will discuss that how that stochastic you know element can be incorporated to see the trajectory path and all that. So, what we are talking that that is what we do normally you have a Eulerian Lagrangian type of approach, where the inclusions will be interacting with the fluid and fluid flow field.

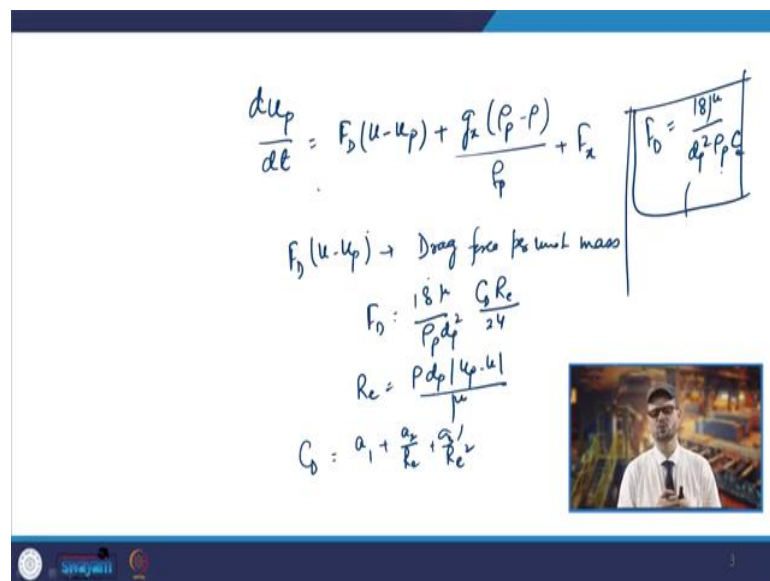
As we know we are solving the Eulerian way and this is done in the Lagrangian manner and we have a particular geometry we make and in that, we are injecting the you know inclusion particles and then, you know we do that force balance. So, that is that equation is there that is that needs to be solved.

So, basically as we have seen that the inclusions will be also experienced in the buoyancy. So, they will be rising with the stokes rise velocity and in that case, we are tracking them; we are wherever they are going. So, we try to have their motion and we see that how they are being trapped or so. So, keeping you know. So, what we do is normally we make the geometry, we you know we provide the boundary conditions.

And you know you have the normal governing equations like initially for the Eulerian you know phase, you have the governing equation like you have the continuity equation; you have the Navier Stoke equation. And you can have the equations of for the turbulent kinetic energy and the rate of dissipation of turbulent kinetic energy that is  $k$  and  $\epsilon$  if you are using the standard  $k$   $\epsilon$  model. So, in that case you are solving that and then, you are injecting the inclusion also and.

So, in this case you are going to solve the extra equation that is you know that will be your the force balance you know particle force balance equation in this case. So, in the normal way, we will be creating the geometry. We will be having all these conditions. And apart from that you need to solve the extra equation and that equation will be your particle, you know you know force balance and that will be you know based on the force which is acting on the particle. So, if suppose we are taking for the x direction.

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The image shows a slide with handwritten equations for the force balance on a particle in the x-direction. The main equation is:

$$\frac{du_p}{dt} = F_D(u - u_p) + \frac{g_x(\rho_p - \rho)}{\rho_p} + F_x$$

Below this, the drag force term is defined as:

$$F_D(u - u_p) \rightarrow \text{Drag force per unit mass}$$

$$F_D = \frac{18\mu}{\rho_p d_p^2} \frac{C_D Re}{24}$$

$$Re = \frac{\rho_p d_p |u_p - u|}{\mu}$$

$$C_D = a_1 + \frac{a_2}{Re} + \frac{a_3}{Re^2}$$

There is also a boxed equation on the right side of the slide:

$$F_D = \frac{18\mu}{d_p^2 \rho_p} \frac{C_D Re}{24}$$

A small video inset in the bottom right corner shows a person in a white shirt and tie, likely the presenter.

So, in the x direction cartesian coordinate, we have these equation that needs to be solved will be  $\frac{du_p}{dt}$ . So, that will be you know that is standard equation which is even used by the even commercial course. So,  $F_D(u - u_p) + \frac{g_x(\rho_p - \rho)}{\rho_p} + F_x$ .

So, basically we solve this equation and in this case you know this  $F_D(u - u_p)$ . So, this is basically the drag force per unit mass. So, this is that quantity  $F_D(u - u_p)$  and you know you have the  $F_x$ . So, this so, now, this  $F_D$  basically will be in terms of the Reynolds number and the  $C_D$  is its coefficient. So, normally this  $F_D(u - u_p)$ ; so, this is known as the Drag force per unit mass and that is defined as  $\frac{18\mu}{\rho_p d_p^2} \frac{C_D Re}{24}$ .

Now, the  $C_D$ ; so, in this case  $\rho$  is the density of the particle;  $D$  is the diameter of the particle;  $d_p$  and your  $Re$  is the Reynolds number. So, you know that Reynold number you

know based on that you this value will be taken and if you so. So, you can have the value of this Reynolds number as  $\frac{\rho d_p (u_p - u)}{\mu}$ . So, that will be your Reynolds number in this case.

And though the  $C_D$  this is the drag coefficient. So, its value also you know. So, it can be taken you, it will be basically a function of the Reynolds number and so, you may have different values. So, people can take these you know different values and that will be a function of you know Reynolds number. So, some people may take it like  $a_1 + \frac{a_2}{Re} + \frac{a_3}{Re^2}$ .

So, you will have these you know  $a_1$ ,  $a_2$  and  $a_3$ , they are the constants that are you know for the smooth spherical particles and you know there are they are also expressed in terms of you know other one another parameter like that will be  $\phi$ . So, that phi again will be you know depending upon the surface area of the spherical particle taken into account.

So,. So, you may have the you know different of these values. So, this thing can be taken and that can be incorporated into your model and you can have this that you know being solved you can also have for the submicron particles, you have the Stokes law that is Drag law is applicable.

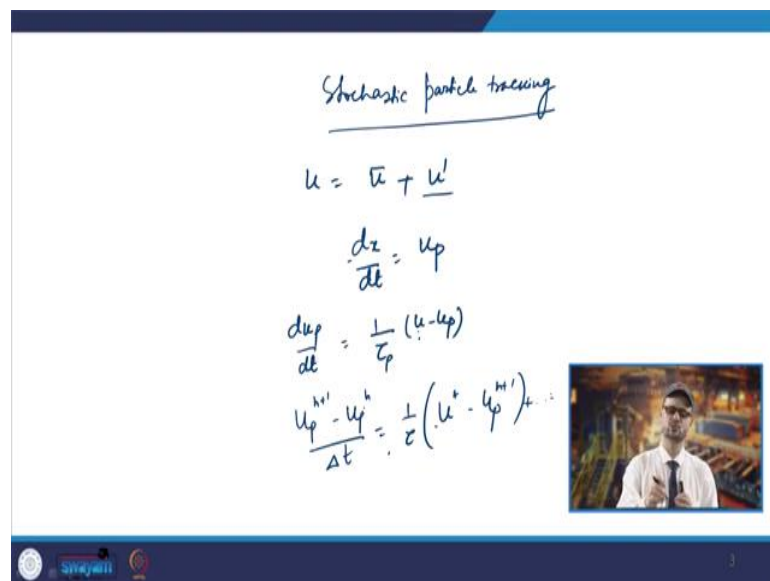
And for the for those submicron particles, you have this  $F_D$  as  $\frac{18\mu}{\rho_p d_p^2 C_e}$ . So, that again you have different kind of expressions you may have so that you can take from the respective literatures and you can use them depending upon the different type of you know particles. And further, that can be incorporated. So, once you do that, then also comes that how to take the stochastic you know component into picture.

So, apart from that you know before going for that you can consider or other kind of also forces lift forces or so; you know that may be considered for the force which is acting on the particle. What is you know seen in the case of the inclusion you know modeling in most of the codes is the use of the stochastic element and that will be you know that will be known as the stochastic particle tracking, especially in the case of turbulent flow.

So, as you know that in the turbulent flow you have in the in the flow component, you will have one is the you know one is the fluctuating component. So, that part needs to be you know incorporated on the for the effect to be seen on the particle and that will basically be taken from the you know flow value.

So, when you are having the flow field, the Eulerian flow field, in that case you will have the values of the turbulent kinetic energy and from that you know value, you can have the calculation of the fluctuating component part. And, then the its effect you know so that that will be affecting the particle trajectory you know in that case. So, you can see that if you take that random you know component and if you do not take the random component, there is there is a visible difference which is observed when the modeling is carried out.

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Stochastic particle tracking

$$u = U + \underline{u'}$$

$$\frac{dz}{dt} = u_p$$

$$\frac{du_p}{dt} = \frac{1}{\tau_p} (u - u_p)$$

$$\frac{u_p^{n+1} - u_p^n}{\Delta t} = \frac{1}{\tau_p} (u^n - u_p^n)$$

So, in the case of the stochastic particle tracking; so, you will have the trajectory tracking that is required in the case of the turbulent flow, especially and as we know that in the case of turbulent flow your instantaneous value you know that will be; so, value will be the mean average and then, you have this is the fluctuating part.

So, you know for you know for knowing the effect of this turbulence. So, you take, you try to have the value of this being calculated from the turbulent kinetic energy part and then, its you know its net effect is seen when we draw the trajectories. So, what we do is normally when we get the value of  $u_p$ . So, then what we do is when we have seen the first equation that is your this equation. So, that is integrated and we find the value of the velocity of the particle and then, with the help of that we can have the trajectory you know estimation.

So, that can be we that we can have by having the solution of the equation that  $\frac{dx}{dt} = u_p$ . So, you can. So, one wants to integrate it. So, you will get the value of x. So, then you understand you know the what is the, where is that x. So, that is how you try to have the tracking of the path of the inclusions.

So, what happens that you may have you know the you know approaches by which you solve. So, you will have the trajectory equation that can be even simplified and you know you should linearize these source terms and all that. So, what you see the trajectory equation which we have written earlier  $\frac{du_p}{dt}$ . So, basically it is a function of  $u - u_p$  and that is it is written as  $\frac{1}{\tau_p}(u - u_p)$ .

So, you know this tau we can p we can call it as. Now, this is basically so, that is for particle tau p and this  $\tau_p$  is basically that is known as the particle relaxation time. So, it is nothing but this unit of time and then, you can you know solve it. So, it will be you know you can use these differencing schemes and discretization schemes and then, you can have the solution like  $\frac{u_p^{n-1} - u_p^n}{\Delta t}$ . So, that will be  $\frac{du_p}{dt}$ .

So, it will be  $\frac{1}{\tau}(u^* - u_p^{n+1})$ . So, that way it will be going. So, that way you know this is being solved and your  $u^*$  will be nothing but the average of  $u^n$  and  $u^{n+1}$ . So, that way you know you have and then you have again  $u^{n+1}$  will be for the you know that will be again expressed in terms of  $u^n$  and you know and grad of the  $u^n$  part. So, that way you know this is the normal way of solving the equations.

So, you can solve it and your ultimate aim is to have the position. So, I mean you must know that how this x value is calculated once you know that  $u_p$ . So, that is by integration of this equation. Now, there are also you know considerations for the size distribution. So, what should be that particle size?

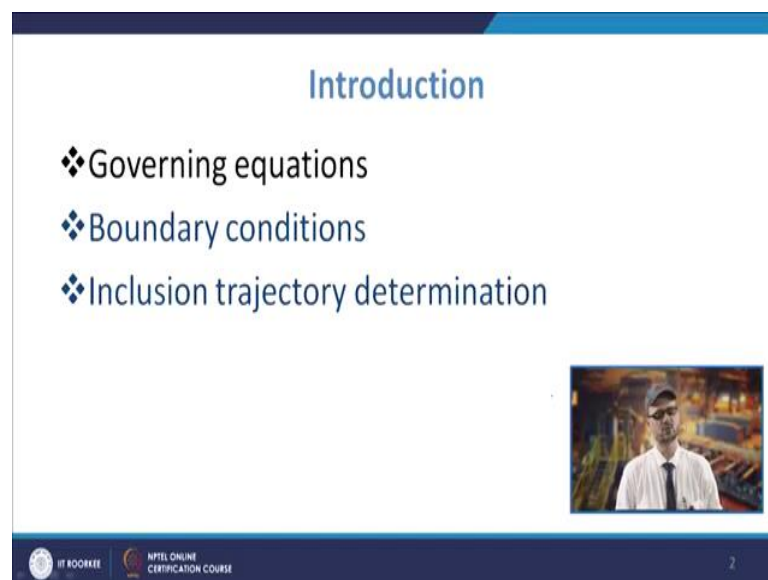
So, there has been you know mentioned about different type of distribution of these sizes and one of the you know distribution which is being used by some of the course are the Rosin-Rammler type of distribution and other kind of distributions also are there.

So, that can be taken which talks about the you know percentage of fraction of the particle mass which will be off certain size, if you take certain size into this. So, that particle

distribution is you know that also can be taken into account. Now, as I was telling you that, when we talk about these stochastic models; so, normally what we do is you find these  $u'$  or you find this  $v'$  or  $w'$  so that so they are  $\sqrt{u'^2}$ . So, that will be expressed in terms of the kinetic energy that will be  $\sqrt{\frac{2k}{3}}$ .

So, from that  $k$  value which is there in the flow field, you try to have these  $u'$  or  $v'$  or  $w'$  that will be you know so that particle will subjected to this and then, accordingly their movement will be controlled. So, that for that you will be again solving those you know velocity equations. You know you must know also the different kind of boundary conditions.

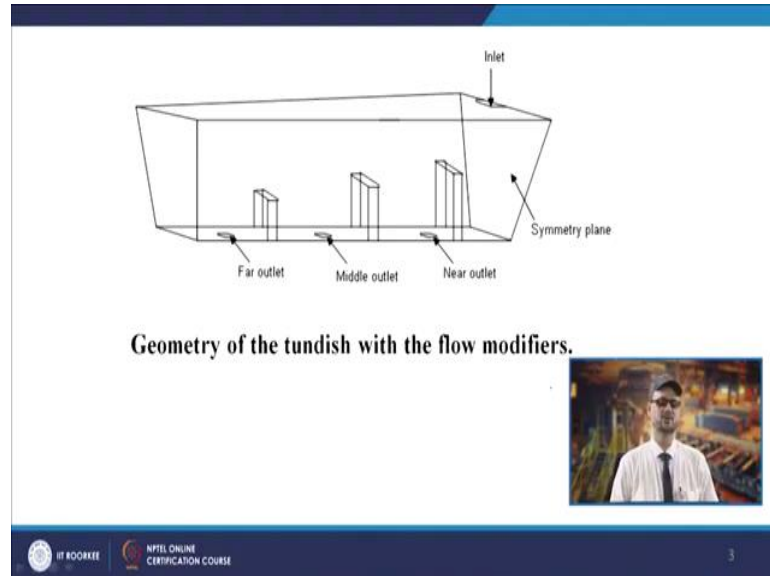
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So, what we saw that we have the governing equations you have also the boundary conditions. So, you have different type of boundary conditions also there are on these discrete phases which we use. So, you will have the boundary condition for trapping or you may have the reflection boundary condition. So, once they you know you may have the depending upon the you know elastic or inelastic collision, they will be reflected. So, that conditions may be given like they may escape through the boundaries or they may trap at the walls. So, all these conditions may be incorporated as the boundary conditions and then, we try to solve the equation and we get the results.

So, we try to see that how these work are done. I mean we can see one work which is done by the group which worked with us.

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So, we and that is also a published work. So, in that how we make the geometry. So, in the geometry you have inlet, you have a symmetry plane. So, it is basically a six-strand; you know tundish and we also have used these flow modifiers. And this is the near, middle and far outlet from the inlet. So, that is why we have named it like that and this is the symmetry boundary condition and this is the inlet through which the inclusions are also you know may insert it or allowed.



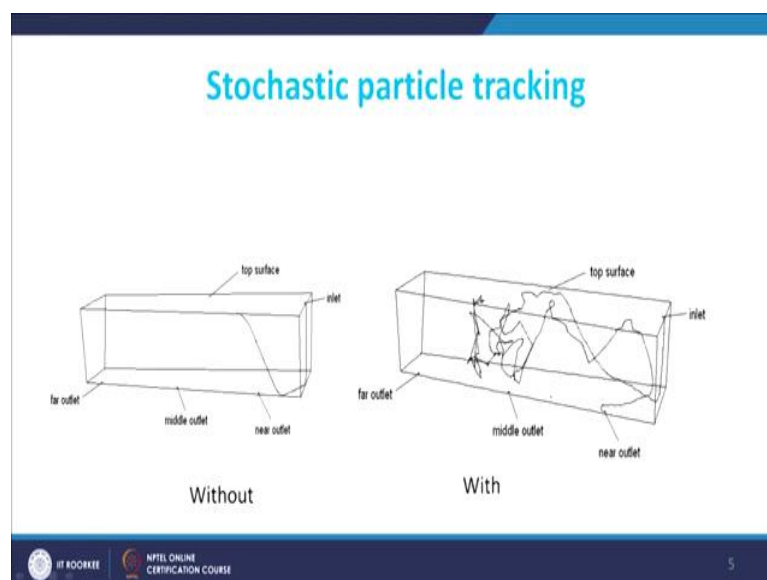
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Height	Dams from the inlet	Height of the Dam (in mm)	Distance of the Dams from the left corner of the Geometry		
			Position-1	Position-2	Position-3
0.	Near Dam	300	2700	2650	2600
	Middle Dam	250	2000	1850	1700
	Far Dam	200	1000	800	600
1.	Near Dam	250	2700	2650	2600
	Middle Dam	200	2000	1850	1700
	Far Dam	150	1000	800	600
2.	Near Dam	200	2700	2650	2600
	Middle Dam	150	2000	1850	1700
	Far Dam	100	1000	800	600

Table 2.2.1: Variation in dam heights and positions (PS01-PS03, PS11-PS13, and PS21-PS23)

And, we have tried to have the you know use of these dams. So, we have given the different kind of nomenclature or the case name when we have put the dam height like 300, 250 and 200. So, that will be you know PS01. So, 203 like that. So, you have the different you know position 1, 2 and 3 so that way your different you know height of dam at different positions have been used and you have different names for that.

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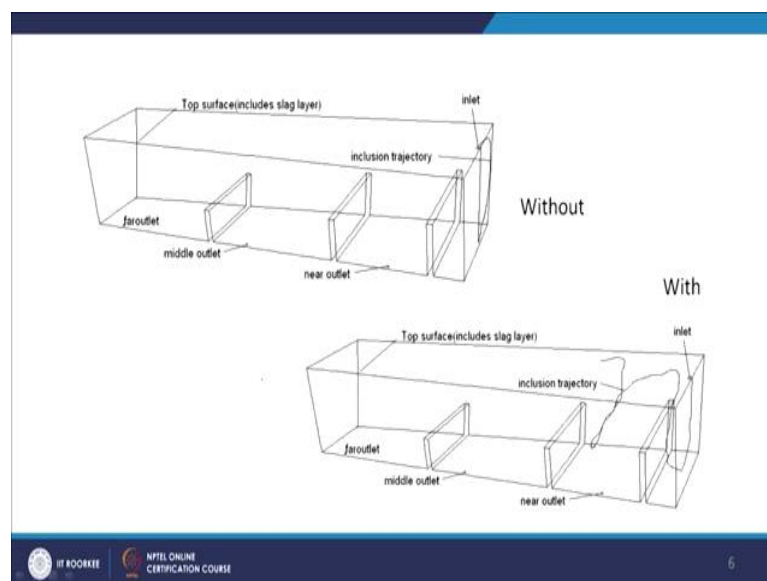
So, that simply creating the geometry. So, what I was telling that when you use these stochastic particle tracking, in that case what has been seen was that when we did not use

without the stochastic particle tracking you know method that is it was seen that the particle has come and it has hit the bottom wall and then, it went you know to the top surface, where the condition is that it will go and it will get stuck.

So, that way that condition because of that it has stuck there. Now, if you go to the other condition where we have used these stochastic particle tracking, in that case what is seen is that the particle has come and then, it has move to the different you know places in size. It has a large you know movement inside and then, it is getting trapped and that is what actually happens.

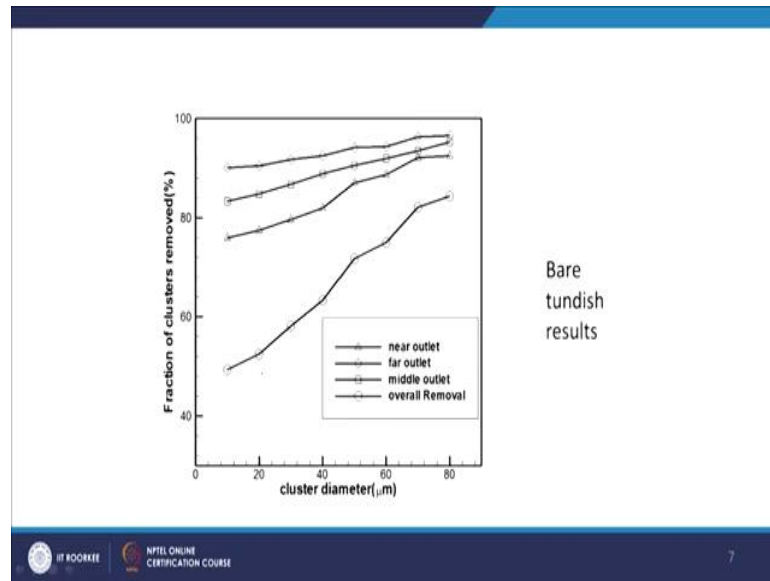
In practical case, this is more like that is what is telling that it will be coming and because of that you know the implementation of that stochastic algorithm, that will be moving inside for longer duration and then, it will have the chances to get floated. You know in this case if it is on the bottom in that case, it may have the chance to go through the outlet so that is normally a lesson in this case.

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So, you know top surface is slag layer and we have kept. So, once we use these you know the flow modifiers, how this transact this with and without you know how these trajectories; I mean the inclusion trajectory can be plotted. So, that is one example. So, you once you will do, you will have your own geometry and you can use this algorithm to have the trajectory of the inclusion.

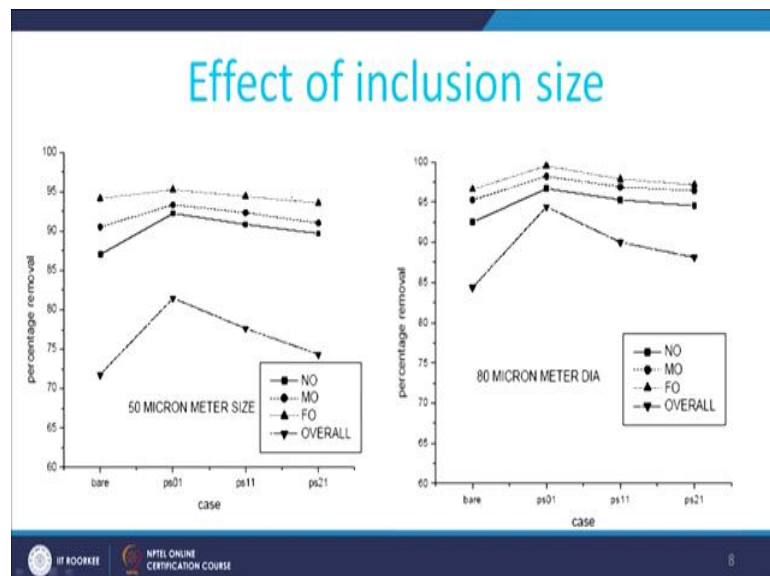
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Then, these are some of the results. So, if you do with the different sizes of the inclusion particle. In that case, you are you will be having certain kind of graph you know for the bare tundish and you see that you have the fraction of clusters that is removed, that will normally be you know very high in the case of the near outlets.

When you are not using the flow modifiers, in those cases and then lesser than that will be the you know the middle outlet and then, you will have the far outlet. So, that is the inclusion removal you know that will be there from the tundish.

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Now, there has been different other parametric studies like you do with the you know the studies for the different sizes and what has been seen that if you increase the size removal will be higher. So, in those cases you will have the larger fraction of the inclusions getting removed. So, this basically tells that you know if you do that, if you change the you know operating parameters, if you change those variables, you are going to have the different kind of results.

And that you need to be you know inter you need to interpret that as per the you know configurations of the tundish, as per the operating conditions of the tundish. And you know and in normal case, in normal water modeling you can have those results also, you can see that how many have come through which of the you know outlets and based on that you can have also the you know calculation of their residence times and then, where they are you know getting stuck or where they are getting you know removed.

So, apart from that you can have the many parameters being studied maybe you can take the different sizes that so that is what has been taken. You can have different densities, even the work has been done with the different inclusion density and then, their effect also can be seen you know that what is the removal rate when you take the different density lower or higher value. So, these are normally this is how you know you have to do the modeling studies on the inclusions inside the tundish. So, we will have few case studies also a studied in our you know final lectures you know in the coming time.

Thank you very much.