

**Modeling of Tundish Steelmaking Process in Continuous Casting**  
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**Lecture - 35**  
**Non- Isothermal Flow Consideration in Tundish**

Welcome to the lecture on Non-Isothermal Flow Considerations in Tundish. So, we talked about so far the mixing and you know related phenomena inside the tundish and in that we did not consider the you know isothermal flow considerations.

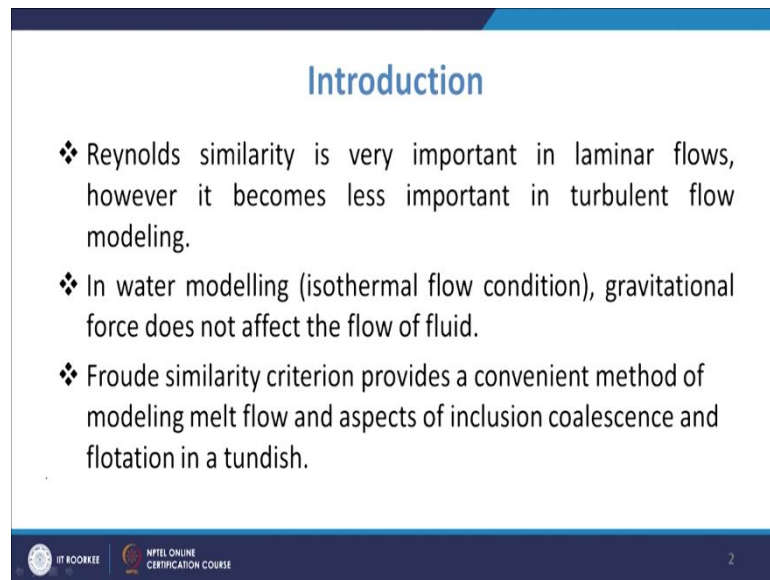
Especially that happens when there will be change in the density and because of that you know you will have the change in that source term. So, especially it will be we will be talking about those conditions when you will have the natural convection type of conditions will be occurring, you know when we solve the energy equations.

So, how those things need to be taken into account and which are those dimensionless numbers which are required to be kept in mind while we talk about the similarity conditions. So, in this lecture we are going to discuss about that.

Now before that we have already discussed that we try to maintain the you know similarities. We have the you know geometric similarity, we have kinematic similarity, we have dynamic similarity.

So, we come across you know the different conditions we have you know we get so especially we get the Reynolds similarity as well as the Froude similarity. Because of the you know different type of forces which whose ratios we try to keep same you know between the model and the prototype. So, based on that we you know come across few you know dimensionless numbers: one is Reynolds number another is Froude number.

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The slide is titled "Introduction" in blue text. It contains three bullet points, each preceded by a blue diamond symbol. The first bullet point discusses Reynolds similarity in laminar and turbulent flows. The second bullet point discusses water modeling under isothermal conditions and the effect of gravitational force. The third bullet point discusses the Froude similarity criterion for modeling melt flow and inclusion coalescence/flotation. At the bottom of the slide, there are logos for IIT ROORKEE and NPTEL ONLINE CERTIFICATION COURSE, along with a small number "2" in the bottom right corner.

- ❖ Reynolds similarity is very important in laminar flows, however it becomes less important in turbulent flow modeling.
- ❖ In water modelling (isothermal flow condition), gravitational force does not affect the flow of fluid.
- ❖ Froude similarity criterion provides a convenient method of modeling melt flow and aspects of inclusion coalescence and flotation in a tundish.

So, Reynolds similarity, it will be important in laminar flows; however, it becomes less important when there is turbulence. So, and also what happens that in the water modeling when we do the isothermal flow conditions, In those cases you know the that three forces which are mainly there; inertial and gravitational and viscous forces.

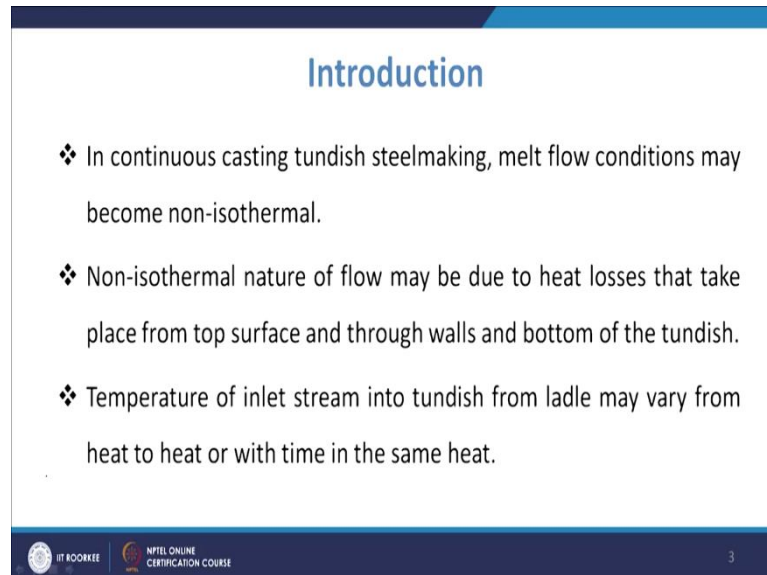
So, that gravitational force will be not affecting that you know flow field. So, what happens we have seen that using that two similarities, if you maintain the Reynolds similarity, you have you know if you try to maintain both of them then you have to keep the you know the model and prototype size as the same, so that is what we have studied earlier.

But Froude similarity criteria what is you know to be noted is that it will be giving a convenient method of modeling the melt flow and aspects of inclusion coalescence and flotation in the tundish. Because you know whenever you have those considerations, where you have the buoyant effects into considerations in those cases this so, the effect of gravitation will be there.

So, in those cases when you do the inclusion you know flotation kind of modeling and all that in those cases these Froude similarity criteria will be providing a convenient method, because the particle inclusion particle which is going inside, now, that you know is subjected to these buoyant forces and also when we talk about the temperature conditions, when we talk about the you know difference in the temperature because of the either the

heat transfer, or the you know like you know or the different temperature of the inlet stream from the ladle.

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The slide is titled "Introduction" in blue text. It contains three bullet points, each preceded by a blue diamond symbol. The first bullet point states that in continuous casting tundish steelmaking, melt flow conditions may become non-isothermal. The second bullet point explains that the non-isothermal nature of flow may be due to heat losses from the top surface, walls, and bottom of the tundish. The third bullet point notes that the temperature of the inlet stream into the tundish from the ladle may vary from heat to heat or with time in the same heat. At the bottom of the slide, there are logos for IIT ROORKEE and NPTEL ONLINE CERTIFICATION COURSE, along with the number 3.

**Introduction**

- ❖ In continuous casting tundish steelmaking, melt flow conditions may become non-isothermal.
- ❖ Non-isothermal nature of flow may be due to heat losses that take place from top surface and through walls and bottom of the tundish.
- ❖ Temperature of inlet stream into tundish from ladle may vary from heat to heat or with time in the same heat.

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So, when that comes, so in the continuous casting tundish steelmaking many a times your conditions may become you know non isothermal. So, because the temperature may not remain, the constant temperature may change. And these non isothermal you know nature of that flow it may be due to the heat losses that is taking place from the top surface through the walls, through the bottoms.

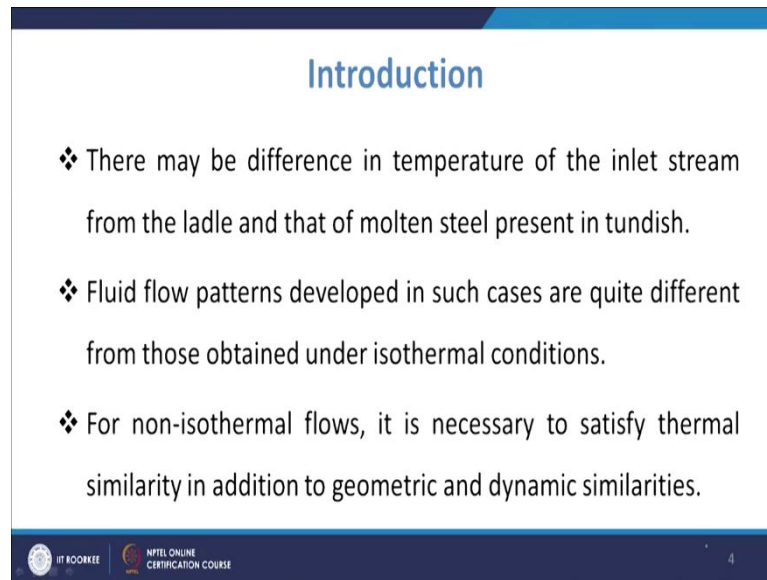
So, from all these places you have these heat transfer taking place heat losses being taken place. And also in certain cases what we see that the temperature of the inlet stream which is coming into the tundish. So, even that may be you know that may vary from the from heat to heat.

So, suppose in one case the ladle has emptied and in that case the ladle is moving out and another ladle will be coming. So, the ladle which is coming now that is the new ladle that may have the different temperature. And you know or maybe with time in the same heat also the temperature may change because the ladle is continuously you know giving the liquid metal and slowly in the end there may be change in the temperature.

Now, because of these change in temperature there are certain changes, there are certain considerations need to be taken, certain similarities need to be maintained and what are

those you know extra terms which come into picture when we non dimensionalize these you know governing equations especially the Navier-Stokes equation, or the temperature equation. So, that will be basically interesting to know.

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The slide is titled "Introduction" in blue text. It contains three bullet points, each preceded by a blue diamond symbol. The first bullet point discusses the temperature difference between the inlet stream from the ladle and the molten steel in the tundish. The second bullet point states that fluid flow patterns developed in such cases are quite different from those obtained under isothermal conditions. The third bullet point mentions that for non-isothermal flows, it is necessary to satisfy thermal similarity in addition to geometric and dynamic similarities. At the bottom of the slide, there are logos for "IIT ROORKEE" and "NPTEL ONLINE CERTIFICATION COURSE", along with a small number "4" in the bottom right corner.

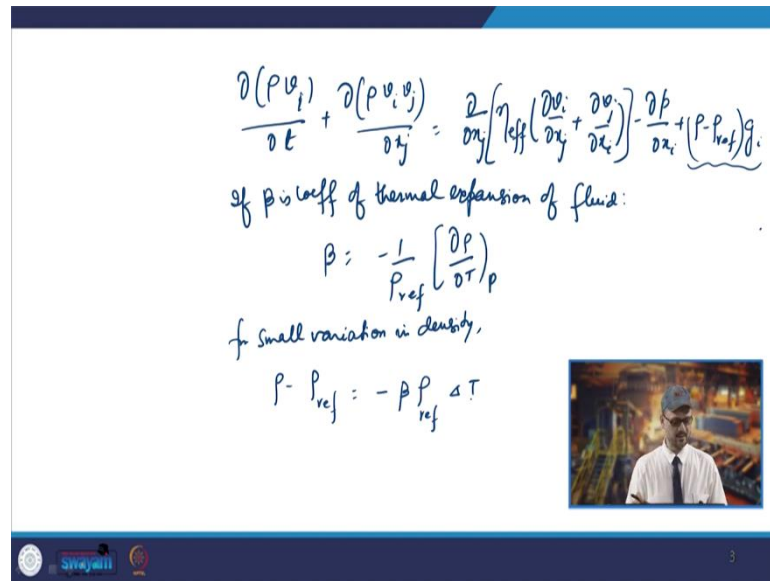
- ❖ There may be difference in temperature of the inlet stream from the ladle and that of molten steel present in tundish.
- ❖ Fluid flow patterns developed in such cases are quite different from those obtained under isothermal conditions.
- ❖ For non-isothermal flows, it is necessary to satisfy thermal similarity in addition to geometric and dynamic similarities.

So, there may be difference in the temperature of the inlet stream from the ladle and that of the molten steel which is present in the tundish. So, that is what I told you that there may be difference in that temperature, which is already there in the tundish and that in you know when the metal is coming from the new ladle.

So, there will be changes in the temperature and this change in temperature is basically you know that will be affecting these flow structures, flow patterns inside the tundish. And they will be different than the isothermal conditions and that may be you know this is quite possible during that continuous casting process because you know in that you have continuous movement of liquid steel and you have continuous changing and changing of the ladles.

So, you know so in that case, you need to you know satisfy these thermal similarity in addition to the geometric and the dynamic similarity. And for that basically you need to you know see those criteria's that how you will manage that. So, what happens that if you go for the momentum balance equation for the you know turbulent flow you know conditions.

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$$\frac{\partial(\rho v_i)}{\partial t} + \frac{\partial(\rho v_i v_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \eta_{eff} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \right] - \frac{\partial p}{\partial x_i} + (\rho - \rho_{ref}) g_i.$$

if  $\beta$  is coeff of thermal expansion of fluid:

$$\beta = -\frac{1}{\rho_{ref}} \left( \frac{\partial \rho}{\partial T} \right)_p$$

for small variation in density,

$$\rho - \rho_{ref} = -\beta \rho_{ref} \Delta T$$

So, for that your equation becomes like you have  $\frac{\partial(\rho v_i)}{\partial t}$  and then you have the convective term. So, you have  $\frac{\partial(\rho v_i v_j)}{\partial x_j}$ .

So, that is your convective term then on this side other side you have the diffusive term.

So, you will have  $\frac{\partial}{\partial x_j}$  and then you have the being the you know turbulent, you know flow

you get the effective value of the  $\eta_{eff}$ . And then you have  $\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}$ .

And then you get the pressure term that is  $\frac{\partial p}{\partial x_i}$ . and then an extra term which you get will be because of the change in the density you know as the temperature will change. So, that will be  $\rho - \rho_{ref}$  and then it will be multiplied with  $g_i$ .

So, this is the extra term which is basically this will be accounting for the buoyancy term, buoyancy force that will be per unit volume. So, you have all these you know terms we have already seen that these are you know these what are these different terms which what they indicate and all that.

So, you know so  $\rho - \rho_{ref}$  that will be your change in the density. Now if for the fluid if the beta is the coefficient of thermal expansion, so, if  $\beta$  is coefficient of thermal expansion of the fluid, in that case what happens that you can express you know  $\beta$  will be you know nothing but you know the change of the density.

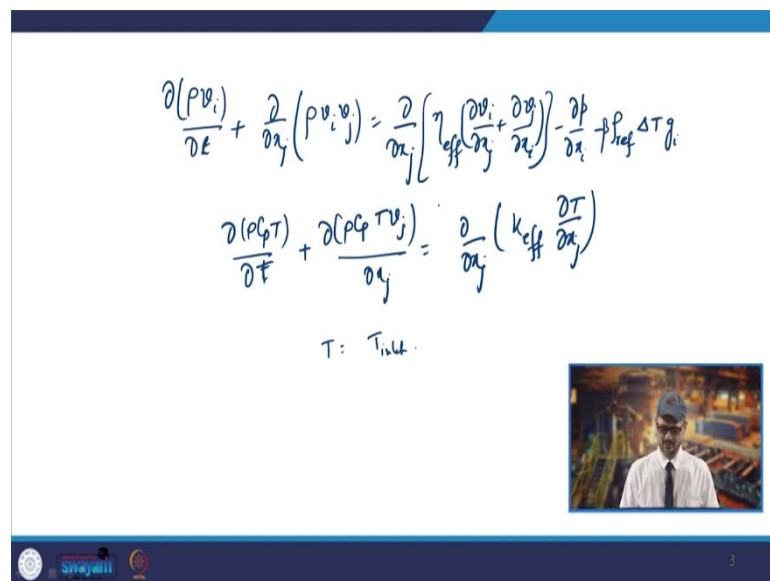
So, you will have  $-\frac{1}{\rho_{ref}} \left( \frac{\partial \rho}{\partial T} \right)_p$ , so, at constant pressure. So that is what you know the expression for beta will be thermal expansion of the fluids that is at constant pressure.

Now, if you have very small variation in the density, so then you can write because that way you will have  $\rho - \rho_{ref}$  here. So, this  $\rho - \rho_{ref}$  will be  $-\beta(\rho - \rho_{ref})\Delta T$ .

So, for small variation in density you can write this is  $\rho - \rho_{ref}$  and this will be basically  $-\beta\rho_{ref}\Delta T$ . So, that is what you can write when you have the very small variation in the density.

Now, we can write in place of the  $\rho - \rho_{ref}$  we can write this  $-\beta\rho_{ref}\Delta T g_i$ . So, that term will come here and.

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$$\frac{\partial(\rho v_i)}{\partial t} + \frac{\partial(\rho v_i v_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \eta_{eff} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \right] - \frac{\partial p}{\partial x_i} + \rho_{ref} \Delta T g_i$$

$$\frac{\partial(\rho c_p T)}{\partial t} + \frac{\partial(\rho c_p T v_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( k_{eff} \frac{\partial T}{\partial x_j} \right)$$

$T: T_{inlet}$

So, accordingly we will write the equation  $\frac{\partial(\rho v_i)}{\partial t} + \frac{\partial(\rho v_i v_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \eta_{eff} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \right] - \frac{\partial p}{\partial x_i} - \beta\rho_{ref}\Delta T g_i$ .

So, that is what you get when you incorporate this term because of the change in density that buoyancy term.

Now, if you write this equation for the temperature, so, for the conservation of thermal energy our equation will be becoming  $\frac{\partial(\rho c_p T)}{\partial t} + \frac{\partial(\rho c_p T v_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( k_{eff} \frac{\partial T}{\partial x_j} \right)$ . S

So, this is again the effective thermal conductivity it will be for the molecular as well as for the eddy component in the case of turbulent flow. So, that is what these two together they will be governing the dynamics of the flow inside the tundish.

Now if we try to you know write, so before that we will have the also the boundary condition and the boundary condition at the inlet will be  $T$ . So, that is  $T$  is a  $T_{inlet}$  that will be your boundary condition. Now, if we try to write the dimensionless form of this equation, so, your both the equations can be dimensionally in the dimensionless manner it will be written, and few non dimensional parameters will be you know propping up in that case. So, if you try to write the non dimensional form of this equation, so you will have again the equation coming up.

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$$\frac{\partial(\rho^* v_i^*)}{\partial t^*} + \frac{\partial(\rho^* v_i^* v_j^*)}{\partial x_j^*} = \frac{\partial}{\partial x_j^*} \left[ \frac{\eta_{eff}}{\rho_{ref} \nu L} \left( \frac{\partial v_i^*}{\partial x_j^*} + \frac{\partial v_j^*}{\partial x_i^*} \right) \right] - \frac{\partial p^*}{\partial x_i^*} - \frac{\beta \Delta T L}{\nu^2} g_i$$

$$\frac{\partial(\rho^* T^*)}{\partial t^*} + \frac{\partial(\rho^* T^* v_j^*)}{\partial x_j^*} = \frac{\partial}{\partial x_j^*} \left[ \frac{k_{eff}}{\rho_{ref} c_p \nu L} \frac{\partial T^*}{\partial x_j^*} \right]$$

$$\frac{\eta_{eff}}{\rho_{ref} \nu L}, \quad \frac{\beta \Delta T L}{\nu^2}, \quad \frac{k_{eff}}{\rho_{ref} c_p \nu L}$$

So, 
$$\frac{\partial(\rho^* v_i^*)}{\partial t^*} + \frac{\partial(\rho^* v_i^* v_j^*)}{\partial x_j^*} = \frac{\partial}{\partial x_j^*} \left[ \frac{\eta_{eff}}{\rho_{ref} \nu L} \left( \frac{\partial v_i^*}{\partial x_j^*} + \frac{\partial v_j^*}{\partial x_i^*} \right) \right] - \frac{\partial p^*}{\partial x_i^*} - \frac{\beta \rho_{ref} L \Delta T g_i}{\nu^2}.$$

.So, if you non-dimensionalize these equations, so, this  $\nu^2$  term will be coming here and then this is one of the another number that we will discuss that what is and that which is what are those things which are coming to picture. Then if you do the non dimensionalization of the thermal equation thermal energy equation, so, that will becoming 
$$\frac{\partial(\rho^* T^*)}{\partial t^*} + \frac{\partial(\rho^* T^* v_j^*)}{\partial x_j^*} = \frac{\partial}{\partial x_j^*} \left( \frac{k_{eff}}{\rho_{ref} c_p \nu L} \frac{\partial T}{\partial x_j} \right).$$

So, this way you will have these two equations as the one. So, we are basically non dimensionalizing the equation  $T^*$ . So, we are taking that as  $\frac{T-T_0}{T_{inlet}-T_0}$ . So, that is how you get these non dimensionalizing of the equation. So, you must have some idea about it.

So, we are getting you know non dimensional you know numbers and what that non dimensional group which we are getting from here? One is the  $\frac{\eta_{eff}}{\rho_{ref}.VL}$ . So, this is nothing but if you look at this  $\frac{\rho_{ref}.VL}{\eta_{eff}}$ , so, that is the reciprocal of the you know Reynolds number.

So, and also it is effective values this is for the turbulent flow. So, this is nothing but the reciprocal of the turbulent Reynolds number here. If you look at so you will have  $\frac{\eta_{eff}}{\rho_{ref}.VL}$ .

So, this is you know indicative of the turbulent Reynolds number. Similarly, if you look at the further you know a number what you see, this number  $\frac{\beta\rho_{ref}\Delta T L g_i}{V^2}$ .

So, if you look at this number, this is another number which we are getting this dimensionless group and this is suggested by the researcher Damle and Sahai. So, they got you know they have invented this number and that number is known as the you know Richardson number and this is basically known as the tundish Richardson number. So, that is indicated by  $T_u$ , then you if you look at this number  $\frac{k_{eff}}{\rho_{ref}.c_p VL}$ .

So, if you look at this, this is nothing but if you look at this is the reciprocal of so this will be one is  $\frac{\rho c_p}{k}$  and then you will have. So, it is basically it can be taken as the reciprocal of the product of the Prandtl, turbulent Prandtl number and turbulent Reynolds number.

So, basically you know what you see that these three numbers you know come into picture when we tried to you know study these non isothermal you know conditions in the case of the tundish flow.

Now, this number  $\frac{\beta\rho_{ref}L\Delta T g_i}{V^2}$ , so, as we discussed, so it is nothing but it is the ratio of the buoyancy force to the you know inertia force, and that is why this number is you know it is taken to be the number which whose which is very much considered for maintaining the similarity when we are talking about the you know non isothermal conditions in the tundish.

So, you know this number that ratio of the buoyancy force to the inertia force, so that needs to be you know that is tundish Richardson number so, Damle and Sahai they told that under these conditions, if you maintain that similarity if these are taken as that criteria and in that case you know the flow behavior which is predicted driven by the water modeling, so, it may be treated as the similar ones you know as you know will be done experimentally, or with the you know RTD investigations.

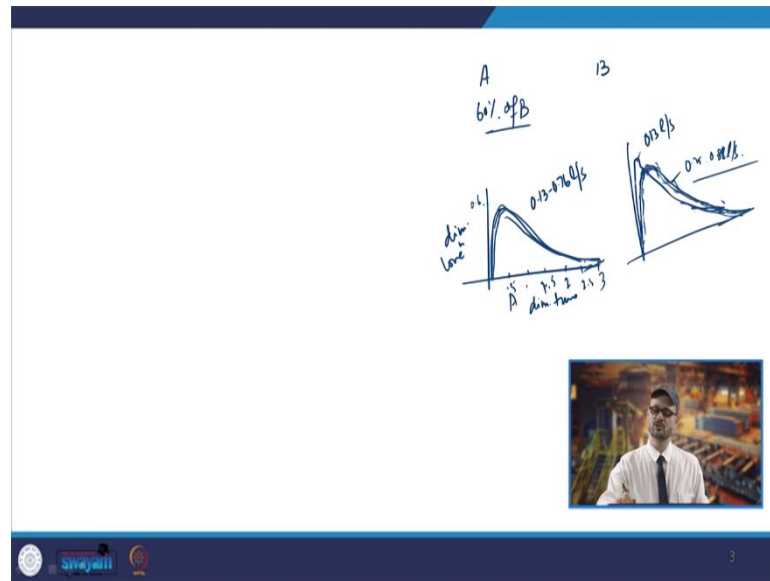
So, that is what you know that this is needs to be considered especially in the case of the natural convection, or the non isothermal case especially the you will see when you do the modeling. In those cases you will see that as compared to the non isothermal case you will have the changes in you know these flow fields, when you have you will see that there will be change in the temperature. So, when you do that you know case study, when you are doing the case of suppose little change over process, or you know even in during that process itself when you are doing the transient analysis in those cases as the time progresses and if that temperature will be changing.

So, because of that there will be change in the density and that may lead to the you know change in the flow field that needs to be you know considered.

Now as I was discussing that you know why what where those you know findings which were suggested by the Damle and Sahai basically. So, they have they investigated you know on the water mid modeling as well as on the you know numerical investigation also, and they found that why because why we are talking about these you know even turbulent conditions.

So, when they did the experimental studies on two kind of the tundish and the one of the tundish was taken bigger another was taken the smaller one, and then accordingly they did the you know water modeling studies. So, they did the pulse input.

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So they have shown that they took two cases, one was A, and another was B. And A was 60 percent of B in size. So, that is what was the case and they varied the you know flow rates in the two cases and B was, so as you see that B was larger. And they changed they did that pulse input study for a range of the you know flow rates which is in flow rate into the tundish. And then they found the studies and now what was seen that for A, and B you know normally the RTD curve was seen to be very close.

So, basically suppose if you talk about the work which is done by the Damle and Sahai, so, they have taken the suppose As length was supposed 0.8 meter, and B's length was 1.35 meter, so it was basically 60 percent. So, L is ratio of the length is 0.8.

Similarly you have the depth or the width they also have, so in all these length cases you have that you know  $\lambda$  is taken and then you have so, when they did in that case what they saw that as we discussed that you will have in both the cases they are getting the you know so, they have done for many cases and actually they got the flow fields you know very close like that.

So, when the flow rate you know or the you know it was changed from 0.13 to 0.76 liter per second. So, this was for A and the you know top height is going close to 0.58 or so this is 0.6. So, dimensionless concentration was measured and this is time, so this is dimensionless time and this is dimensionless concentration.

So, what they found that this is a time of supposed 0.5, then 1, 1.5, then you have 2. So, like that it may go up to 3, so 2.5 and all 3. So, for A they have got and for B also they got you know similar type of you know curve its going like that, except for so they have got that range in which all these 0.132.

So, in that case you know they have seen. So, it was varied from 0.13 to 0.88 liter per second, but only for the case of 0.13 liter per second, that they saw that this is for 0.13 liter per second. So, what they saw that you know when you are tundish is bigger and you are keeping the you know flow rate as the minimum one, in that case only otherwise these ranges were the same.

So, the on these you know RTD's here it was even closer to each other here they were somewhat spares, but only one of the line which is the for the minimum of the flow, so, that was seen to be you know having a larger peak little bit and the peak was observed to occur little early. And then it was you know going into so it was a decreasing and ultimately it has matched with the line later.

Now, this was only seen, and now the reason for this could be attributed to the prevalence of the non turbulent conditions. Because you have a larger tundish and this is for 0.25, 0.88 liter per second so that was the value. So, that may be because of the you know the laminar type of flow condition which might have been the cause for this changes. So, that is what was basically found by these researchers.

So, you know so thus to tell you that you know these water modeling as well as the mathematical modeling results have been proved to be matching with each other you know when the flow is turbulent, so in those cases they seem to be you know be validating each other.

So, that is basically the experimental work can be used as validation for the numerical investigation, if you are doing further you know on a particular geometry. So, that is you know these are the considerations that needs to be taken when you are going for the water modeling in the case of non isothermal you know conditions. So, we will talk about other aspects in the tundish flow in our coming lectures.

Thank you very much.