Modeling of Tundish Steelmaking Process in Continuous Casting

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Lecture - 30
Assessment of Discretisation Schemes

Welcome to the lecture on Assessment of Discretisation Schemes. So, in the last lecture we talked about the treats of the different type of discretisation schemes, we talked about the upwind differencing scheme, we talked about the you know hybrid differencing scheme, powered loss scheme and even the quick scheme.

So, in this lecture will try to understand by you know by looking into a problem and we will see that how they are you know solved and how we get you know the solution you know how this results which we get by using the differencing scheme for different cases, especially when you have the variation of the Peclet number because of the convection component. So, how the different you know differencing schemes you know they are able to predict the result and that how they match with the analytical result so that we will see in this lecture.

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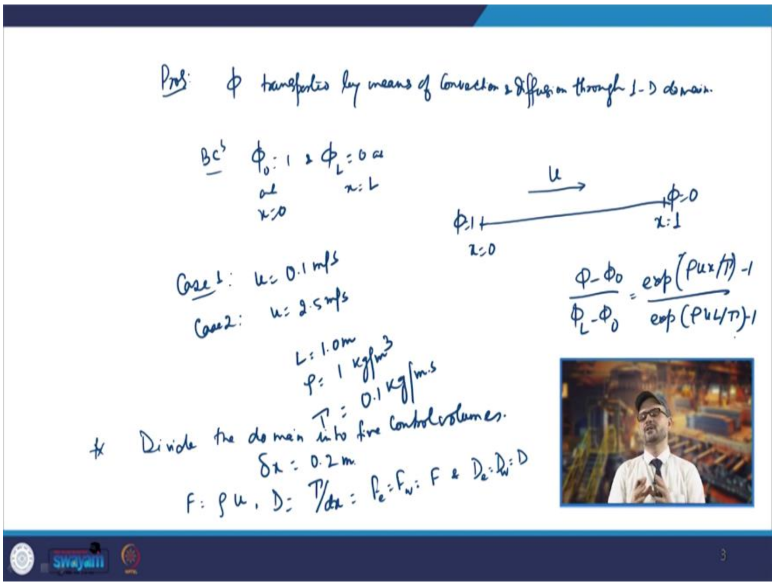

So, we will talk about a problem. So, that in a problem in that you know we have a property φ which are you know that is transported by means of the you know convection and diffusion. So, for this you know φ and that is for a one dimensional case. So, it is a problem through this. So, of the convection and diffusion through one dimensional domain and you know. So, you have you know this way φ is a transported and you have the conditions which is given. Now your u is you know in this direction. Now here this is your x is 0 and this is x is 1 and the value of φ it is taken as 1 at this end and it is taken as you know 0 at this end. So, these are the two you know boundary conditions.

So, you have. So, boundary conditions are a φ_0 is 1 and φ_1 is you know. So, this is at x equal to 0 and then you have φ_L will be 0 at x equal to L. So, you know this is the boundary condition. Now what happens that in this case we will see that how we find the control volume. So, we will have the number of control volumes in this domain and then we will try to assess that how you know the different schemes they are able to you know predict the result which is and how much it is close to the analytical result and we are going to have that. So, you are going to have the calculate the distribution of φ and as a function of x and for the two case, in one case we are taking u as 0.1 meter per second and in the another case we are taking the u as 2.5 meter per second.

So, these are you know the two cases and you know as you see that your length is given as 1 meter rho is a you know 1 kg per meter cube and tau is given as 0.1 kg per meter second. So, this way you have these values. So, that will be used for finding the f and d and the first job will be to have the you know to have the formation of the control volumes. So,. So, this is for case 1 u is 0.1 meter per second and for case 2 to 2.5 as you see that the velocity is very much increase. So, convection term will dominate into it in this case and then accordingly Peclet number will also change that also is a function of d x also.

So, if you change the number of points or number of control volumes that away also Peclet number is you know has the effect any way, but we will start with having the five control volume. So, we will divide this whole domain into five control volume. So, divide the domain. So, this domain is divided you know into five control volumes. So, if you divide these an length is 1 meter. So, in that case the δx becomes $1/5$. So, that is 0.2 meter.

So, once you have δx 0.2 meter, in that case you can have the if calculation of other things like $F = \rho u$, $D = \Gamma/dx$. So, you know based upon the value of Γ , you can have the value of D also being calculated. You will have the value of $F_e = F_w$ and similarly $D_e = D_w$ that will be equal to D that is you know everywhere.

Now, what we need to you know calculate in this case is that you have you know you will use these differencing schemes and you will be calculating. Now what we see that when we use the central differencing scheme, now in that case we know that you have you get one expression for phi p in terms of the phi w and phi e and also you have the source term also coming to picture. So, you know what we will do that next?

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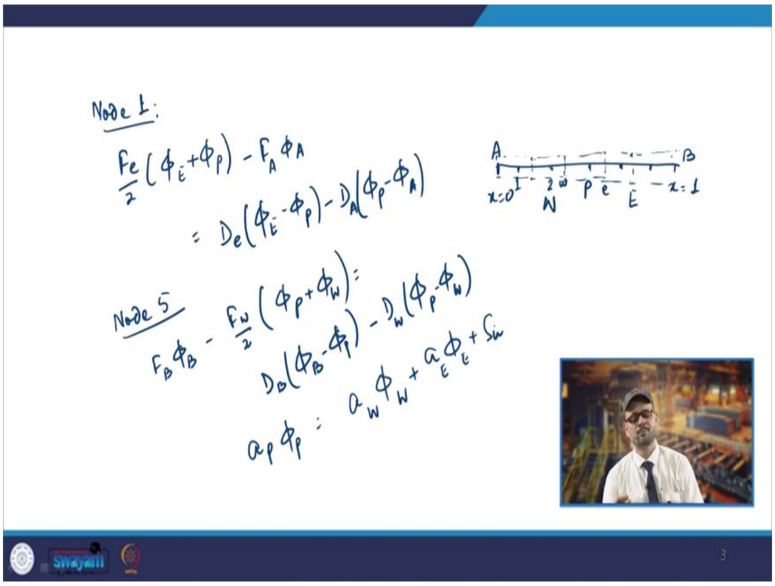

So, your domain will be looking like this. So, you will have this point on the side. So, this will be x equal to 0 and this will be x equal to 1, you have this point as A and this point as B.

Now, you have a five point; so, 1, 2, 3, 4 and 5. So, you will have a five control volumes will be found like this. So, control volume is found so that the boundary is matching with the you know the outer boundaries. So, that way you form these control volumes you are getting, you know this is how you are getting the control volume.

So, we get these you know five control volumes, now what we have already studied so far earlier that we will have the similar type of expression for you know for the nodes 2, 3 and 4 whereas for 1 and 5 we will have the somewhat different expression because of the you know finding the value at this point. So, if you take the node 1 you know at node 1 if you use, node 1 you will be having the you know expression that is $\frac{F_e}{2}(\varphi_E + \varphi_p) - F_A\varphi_A = D_e(\varphi_E - \varphi_P) - D_A(\varphi_p - \varphi_A)$ value is known here and that is any way given.

So, if you use those values, so, accordingly you will have the expression for this. Now similarly if you go for the node B you will have the you know B point that will be coming out. So, for node 5 you will have the $F_B\varphi_B - \frac{F_w}{2}(\varphi_P + \varphi_w)$. So, you know that because you are having this as suppose any anyone you this is one this is two or so. So you can have this as the west and then you will have this is the west phase and if you take you know this. So, this will be your P and this will be your east phase and this will be your east node or so.

So, F B phi B; so, once you go to the fifth node in that case you will have that F B. So, $F_B\varphi_B - \frac{F_w}{2}(\varphi_P + \varphi_w)$, it will be coming as you know D$D_B(\varphi_B - \varphi_P) - D_W(\varphi_p - \varphi_W)$. Source term, these are the two equations which you get you know at the extreme you know

1 and 5 and you know at other points you will have $a_p\varphi_p = a_e\varphi_e + a_w\varphi_w$ plus source term

So, what you see if you do the rearrangement you get the expressions for the or for every node, node y is you will have the value of a_w a_E a_p and then you have s_p and s_u. So, if you rearrange them you will be getting those value.

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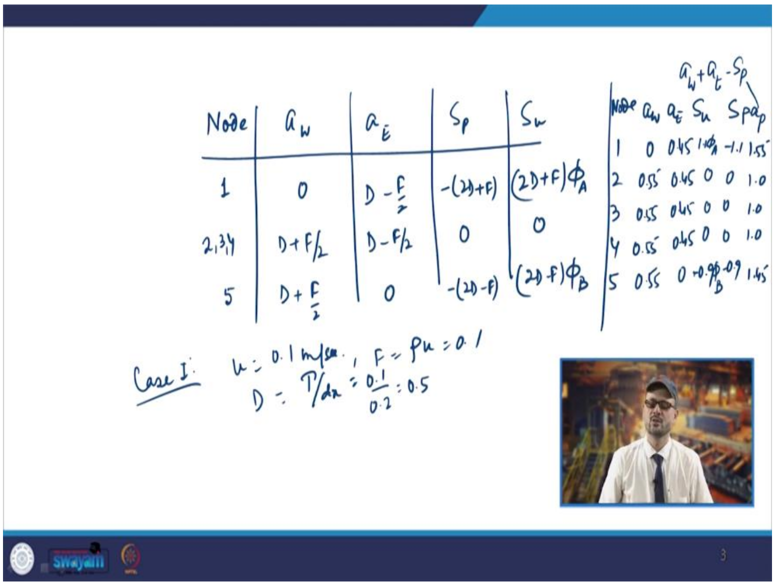

So, if you do the rearrangement you will have the node here and similarly you will have the a_W, then you have a_E and then you have S_p and S_U that is your once you linearize the source term. So, for node 1 your a_W becomes 0 a_E becomes $D - \frac{F}{2}$. Similarly you will have the S_p as –(2D+F) and then you will have the you know the a source term you know; so, S_U that will be $(2\text{D} - \text{F})\varphi_a$. So, that will be they are the node 1.

And similarly at node 5 you will have $D + \frac{F}{2}$ and this will be 0, this will be –(2D-F) and you will have $(2\text{D} - \text{F})\varphi_B$. And that is because of the boundary conditions which are imparted if you go for you know by go by you know working on that equation. So, you will get this expression and for 2, 3 and 4 you will have the expression that is $D + \frac{F}{2}$, $D - \frac{F}{2}$ and you will have this 0 and 0 that is what the common in that $a_p \varphi_p = a_E \varphi_E + a_w \varphi_w$

So, that is what you are getting $D + \frac{F}{2}$ and $D - \frac{F}{2}$ in these two cases. Now if you imply the boundary conditions and if you go for the case 1, so, suppose for case 1 where you have u as 0.1 meter per second and as you see that your F will be ρU. So, it will be 0.1, similarly D will be Γ/dx. So, it will be 0.1 by zero point you know 0.2.

So, dx. is anyway 0.2 because you have length is 1 and there are five control volume. So, it will be 0.2. So, you will have this is 0.5. So, in this case you can have you know the if you go for these nodes you will have the value of a_w a_E. So, you also know the value of D and F. So, from here you can get these you know value of these coefficients and then these coefficients will be put in terms of the matrix.

So, you will have. So, if you take the you know if you try to find the node and that will be node will be 1, 2, 3, 4, 5. So, for 1 you know a W is 0. So, anyway your a W becomes 0, similarly for 2, 3, 4 and 2 and 3 and 4 it is $D + \frac{F}{2}$. So, D is 0.5 and F by 2 is 0.05. So, it will be 0.55. So, 0.55 will be there for these three numbers and then for fifth it will be $D + \frac{F}{2}$ again. So, it will be 0.55, similarly if you go for a E so, you can have the calculation of a E again. So, this will be $D - \frac{F}{2}$. So, it will be 0.45 that will be 0.45 also 2, 3 4 it will be 0.45 and then last will be 0.

So, then if you go similarly you go for S_u. So, S_u also you can have $(2\,D + F)\varphi_A$ you know. So, accordingly you can have the values. So, you will have the values like 1.1 φ_A something like that and you know. So, that is S_u; so, 2 D plus F 2 into D; so, 1 plus 0.1 1.1 φ_A. So, that way it will go and then for the 2 3 and 4 it will be 0 0 0 and similarly in the and you will have minus 0.9 and similarly is you have S_P. So, S_P will be again. So, you can calculate these values and you will have the different values that will be you know for S_u it will be minus zero point. So, this will be 0.9 you know φ_B and for S_P you will have minus 0.9.

So, you can what I mean to say that a_P will be and you can get a_P a_P as you know $a_w - a_E - S_P$. So, that they can be calculated from here directly you can have the values of you know the a_W , a_E and the S_P. And S_P is -1.1. So, accordingly you can have the value of the a_P. Now what we mean to say that is. So, a P will be coming as 1.55, similarly you have 1 you have 1 here, you have 1 here and 1.45. So, that is you can you know calculate yourself and you have one know values in that table now this you know leads to the formation of a matrix.

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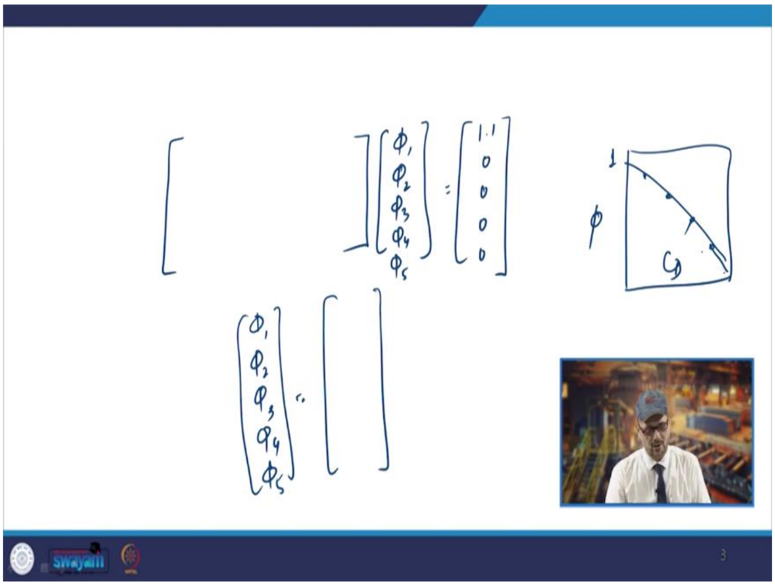

And then that matrix basically you have all these you know values. So, you will have the values accordingly you have as you go. And then that will be multiplied with the φ_1 φ_2 φ_3 φ_4 and φ_5 and accordingly you will have the you know values. So, if you take the you know φ_a as equal to 1, so, for φ_a equal to 1 and φ_b equal to 0 so, you will have again the expression you will have 1.1 0 0 0 and 0. So, accordingly you have to get the solution of this φ_1 φ_2 φ_3 φ_4 and φ_5 is you know computed and you get the values you know using the central differencing scheme.

Now, similarly now the thing is that the problem is that we have to know that how much it is getting closer to the analytical solution and when it was you know compared with the analytical solutions. So, analytical solution is anyway given for you know this, and when the it was compared with the analytical solution, so, it was seen that there is a match you know in this case. So, it comes like this.

So, you will have you know phi value 1 here and you know if you with so, if this is you are getting with a central differencing, so, with exact also it matches if you are taking the central differencing. Now what is the seen the problem is here is, you know the problem which can be seen later will be because of the case 2 which we will be dealing.

Now, if suppose we deal with the case 2, now in this case. So, if you go to the case 2 in that case u becomes you know a larger.

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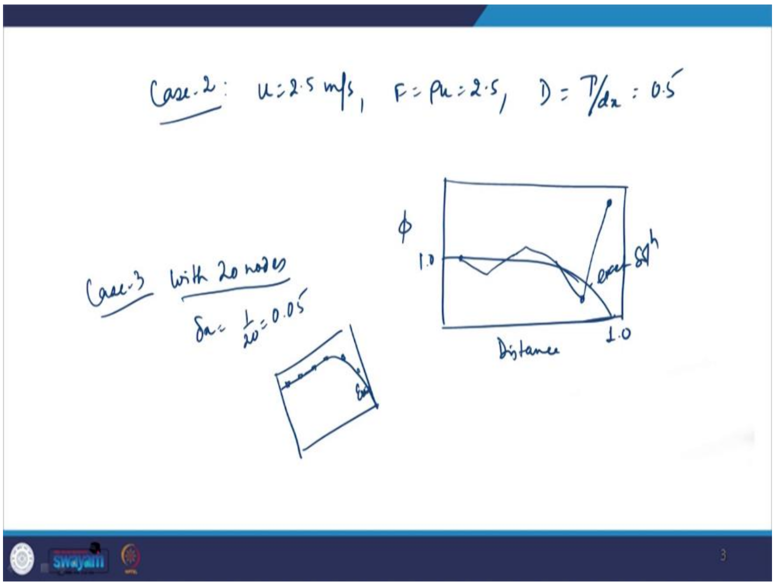

So, in that case you your u becomes 2.5 meter per second. So, your F becomes ρu that is 2.5 and $D = \frac{\Gamma}{dx}$. So, it will be 0.5. Now if you look at the Peclet number you know F by D. So, F/D will be 2.5 by 0.5 that is about 5. Now in this case again you will be getting the same you know matrix and if you try to further solve, so, in the in that case the and if you try to compare it with the analytical solution, the solution is seen to be the analytical solution is seen to be here. So, you have phi here. So, one is there and on this side you have distance. So, you on this side distance also it is 1.

Now, in this case this is your exact solution and if you try to do it through the central differencing for this condition, the result which is found is basically it comes like this, it is reported you know by in this manner. So, what you see that this result is you know it is very much far from the exact solution. So, the value is somewhere under predicted is over predicted like that. So, there is you know suiting of these results and this result is showing that when you have a convection dominated flow, in that case your you know this what you see is that it is not able to predict properly because of the flow.

So, its ability is there of the central differencing scheme to predict the flow in that case. Now what you can what has been seen that if you are solving this problem by taking δx to be smaller or if you take suppose you take the case 3 with 20 nodes, so, with 20 nodes you know what will happen that the width the δx control volume width will be smaller. So, the delta x will be you know, o, earlier you had the you had the δx_1 by 5 0.2, now you know delta x becomes you know less. So, it will be $1/20$. So, it will be 0.05.

Now, in this case if you do that analysis and if you can further find the value you write the matrix and solve it, what has been seen is that your you know equation in the same you know same result was seen to be. So, if it this is the exact, now in this case it was found to be very close, there will be there is not complete you know matching at all the point where this very much you know close it was a seen to be. Now that is because you see that the Peclet number that was F/D.

So, $F\ \frac{dx}{\Gamma}$ and if you see the you know the value of the Peclet number it becomes very small and in that case you know even so, even for the higher velocity case of 2.5 meter per second, you are getting a very close result to the you know the analytical results that is the you know effect of the you know number of nodes which you are increasing and that way you are getting a closer result.

Now, same problem when it is done with the other you know higher order schemes other schemes like the upwind scheme. Now in the case of you know upwind scheme, even with the smaller nodes and also with the you know for the you know for case 1 where the velocity is very small it is said to seen to be matching, but even if you know when we are where you have the large suiting seen in the case of larger velocity values, the upwind differencing scheme seems to be matching because you know matching with the analytical close match is seen in the case of the upwind differencing scheme.

You know because when the Peclet number is higher in that case you know the upwind differencing scheme will work better and with the lower number of grids also the closeness which is maintained will be you know smaller. So, what has been seen? So, basically what we have seen in this lecture that you can try with the other you know differencing schemes and just see that how your result is matching with the analytical value which is you know which has been you know given.

So, basically if the analytical value if you look at the you know problem there. So, anyway analytical value you know it has been you have to compare the results with that analytical value and that is basically given as $\frac{\varphi-\varphi_0}{\varphi_L-\varphi_0}$ that will be you know $\frac{e^{\rho ux/\Gamma}-1}{e^{\rho uL/\Gamma}-1}$. So, that way you know you have once you get these numerical results and then you compare with these analytical you know result and then you can compare and see that how the other differencing schemes work and how the effect of convection can be seen you know pre dominating and then in the you feel that need to have the use of these you know other differencing schemes.

Because your those conditions of the you know transportiveness, boundedness and conservativeness needs to be satisfied and that will be governed because of there are many you know as a factors like the coefficient value needs to be positive then there is has to be boundedness there has to be consistency in representing the conservativeness and all that. So, I hope that you will be you know having some more zeal to look into this aspect and go and study you know the different aspects of the other differencing schemes, which we will be you know mentioning while dealing with the flow you know analysis in the tundish.

Thank you very much.