Modeling of Tundish Steelmaking Process in Continuous Casting
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Lecture – 25
Turbulence Modeling Using k-e Model

Welcome to the lecture on Turbulence Modeling Using k -epsilon Model. So, in the last lecture we discussed about the mixing length model and it is a simple model which is in which you have in an algebraic expression that is used for finding the expression for μ_t. So, which is used for you know modeling of the term related to Reynolds stresses.

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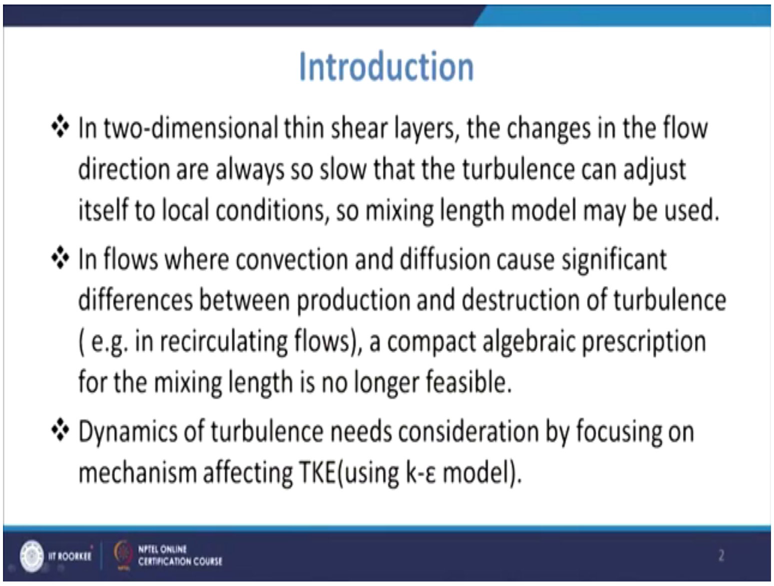

Now, that is normally you know applicable law that is more useful when you have a very simple type of flows or in 2 dimensional things here you know layers, where the changes in the flow direction is slow and that ensures that the turbulence can adjust itself to local conditions. So, in those cases those mixing length models are useful.

However, when there will be complicated kind of flows or when you have in the flows where convection and diffusion will be causing the difference between the production and dissipation of the you know turbulence. So, my production and destruction of turbulence mostly it is found in the case of recirculating flows for example. So, in those cases you know these compact algebraic expression you know that is for the mixing length which is prescribed that will not that will no longer be you know suitable.

So, what we need that, we need to understand, we need to take into account these dynamics of turbulence and we have to know the mechanism you know by which there is a effect on the turbulent kinetic energy and for that these k and epsilon model is used. So, where k is the turbulent kinetic energy and epsilon is the dissipation rate of dissipation of this turbulent kinetic energy. So, based on that you know we tried to have the prediction of the effect of turbulence. So, in this lecture we are going to have the discussion about the k epsilon model.

Now, before that you know we need to understand because we have studied about the instantaneous equations for the you know continuity or even the x y and z momentum equations then we had the equation for the mean you know flow mean parameters like for the continuity equation and for also the you know velocity component also.

So, that we have already seen and those x equations will be made use for finding the expression for the turbulent kinetic energy and also we will have the governing equation for. So, we will have another equation for the dissipation of you know once we will have the expression for the mean you know turbulent kinetic energy and then you will have you know mean kinetic energy and then you will have for the kinetic in turbulent kinetic energy.

So, now, from there onwards we will talk about the k epsilon model which was devised by which was suggested by researcher Spalding. So, he has given these 2 equations for k and epsilon and with the value of k and epsilon we tried to have the expression for μ_t. So, that is what the you know m becomes.

So, if we try to see you know we have seen that we got when we try to have you know for these any parameter we have one mean component for any property. So, we will have one mean component and another is the fluctuating component. So, similarly if you have the kinetic energy also so, you will have a mean kinetic energy and then you have the you know turbulent kinetic energy.

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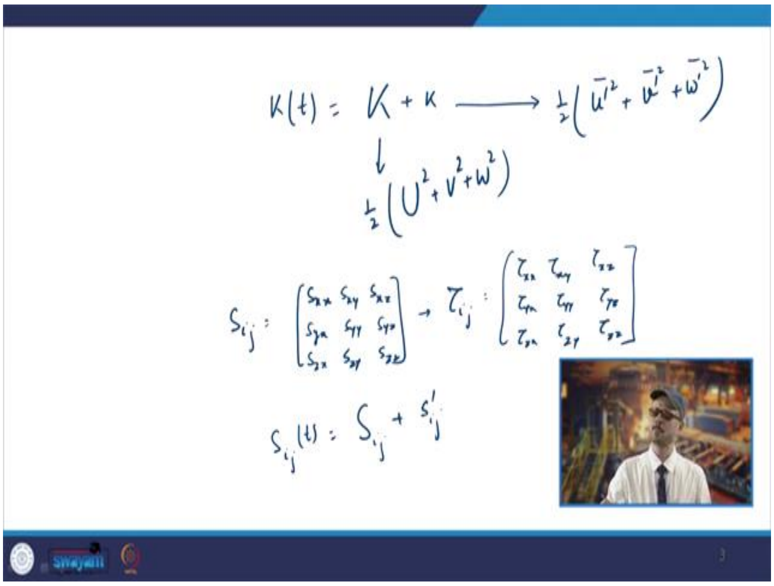

So, we write the instantaneous you know values of K(t). So, that will be you know the mean kinetic energy plus you will have the instantaneous you know that is the turbulent kinetic energy. So, that will be you know this will be $\frac{1}{2}(u^2 + v^2 + w^2)$. So, this will be half of u. So, that will be $U^2 + V^2 + W^2$and this will be $u'^2 + v'^2 + w'^2$. So, there will be bars on those values. So, this will be basically $\frac{1}{2}(U^2 + V^2 + W^2)$ and this will be $\frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$. So, that will be you know that is what we have normally seen.

Similarly, now if we try to express these stresses and the mean rate of deformation as the tensor is. So, you try to express them in tonsorial form. So, you will have the you know rate of deformation S_{ij}. So, that will be you know that will be represented by matrix in the tensor form. So, it will be you know $\begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix}$. So, that will be you know you know in the similar line if you try to denote for the stresses.

So, you will have τ_{xy} or τ_{ij}. So, that will be again $\begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$. So, you will have this way you have the rate of deformation also in the fluid element you will be you know if you talk about this instantaneous value. So, you will have the mean component as well as the you know a fluctuating component. So, you know this gives. So, that will be used.

So, what we do? We normally go for suppose S_{xx}. So, it will be $\boldsymbol{S}_{xx} + \overline{S_{xx}}$you know bar so, that way you know you will have.

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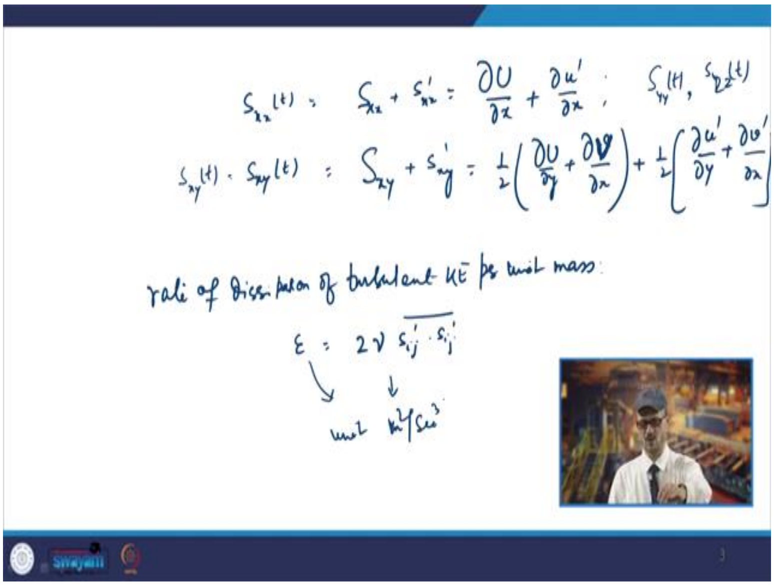

So, if you take the value of suppose $S_{xx}(t)$. So, it will be you know $S_{xx}{}'$. So, that will be. So, as we know that S_{xx} will be $\frac{\partial U}{\partial x} + \frac{\partial u'}{\partial x}$. So, that way you will have the $S_{yy}(t)$. and $S_{zz}(t)$. , similarly if you have to find the $S_{yx}(t)$.. So, that will be also you know meaning part plus fluctuating part. So, it will be you know. So, mean part S_{xy}. So, it will be you know $\frac{1}{2}(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}) + \frac{1}{2}(\frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x})$

So, that way you will have the values of further you can have the value of $S_{yy}(t)$. and similarly you can have the value of $S_{zz}(t)$. . So, in this way also you will have you know it will be same as $S_{xy}(t)$. . So, you can have the expression for $S_{yz}(t)$. or as $S_{zy}(t)$. or so all these you know they can be taken in that form. Now, what we need to know? That, we need to find the turbulent you know the governing equation or the you know the equation for the mean turbulent kinetic energy.

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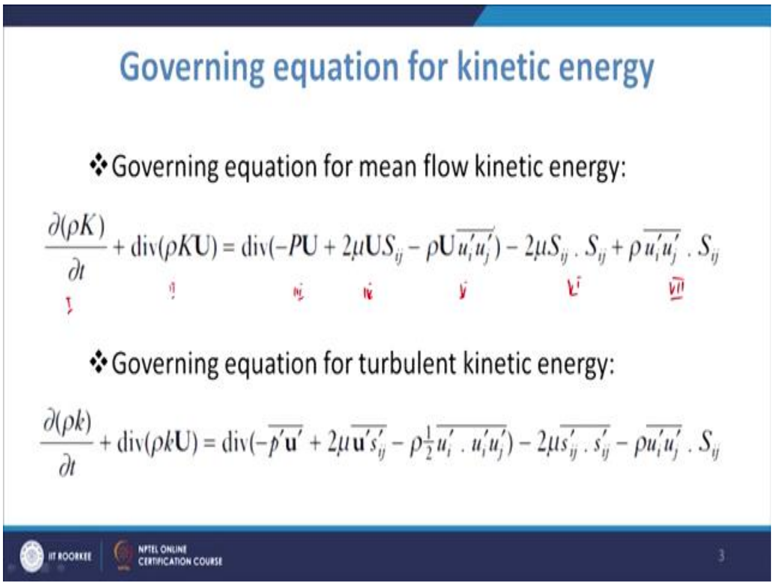

Now, for you know finding the equation for the mean turbulent kinetic energy you we had the equation you know for the you know for this mean component of velocity. So, what you can do is in that particular equation you multiply that with the mean component of velocity. So, that capital U and capital U will be U square. So, you will have 3 equations for that you know ah mean you know x component y component and that component momentum equation if you recall.

So, in that though you row of U by dt by dou U U there that will be so that you had those expressions. Now, what you do normally? You try to multiply them with the you and then you so, that x momentum equation will be multiplied with capital U, y momentum which we will be multiplied with capital V and z momentum will be multiplied with a capital W.

So, you will have the u^2 term will be coming in the you know in that x momentum equation, in the v^2 term will be coming in the y momentum equation and z and w^2 comma will be coming in the z momentum equation and if you do certain rearrangement in that case you are getting the expression for the mean flow kinetic energy.

So, that is for the mean flow. So, as you know that you will have one is mean flow part and another is the fluctuating part. So, there you get this equation, if you now look at this equation you have 1, 2, 3, 4, 5, 6 and 7 terms now if you look at this term this is the rate of change. So, this will be rate of change of the mean you know kinetic energy you know that is with respect to time. So, this is that rate of change term then this will be the term which is $div(\rho KU)$.

So, this is for the mean you know kinetic energy and this is by the convection. So, this is the you know transport of K by convection. So, the second term if you take this term as the first one, this as the second one, this as the third one, this as the fourth one, this as the fifth one, this as the sixth one and this as the 7 th one. So, same thing here also. Now, in this case if you look at the first term it is the you know change. So, that is rate of change of mean kinetic energy this is the transport due to you know convection of the mean you know kinetic energy.

Similarly if you look at this -PU so, this is again mean pressure. So, this is the transport of K because of the you know pressure so, this is because of the pressure you will have the transport. Now, if you look at this term $2\mu US_{ij}$; so it uses this μ. So, it will be with the because of the viscous stresses you have here this is the μ and then you have S_{ij} . So, that is your rate of deformation. So, that leads to the viscous stress. So, basically this is the transport of K because of the viscous stresses.

Now, if you look at this term; this term contains this fluctuating velocity component and then bar over it. So, that leads to u rho u i use a you know prime $\overline{u_i'u_i'}$. So, that is the indication of having the turbulent you know shear stresses. So, this is your that is we call it as the Reynolds stresses. So, this is the you know transport of K by the Reynolds stress then come to this term this is basically $-2\mu S_{ij}S_{ij}$.

So, this is basically the rate of viscous dissipation term. So, what you see this will be dissipation this is a negative term. So, this will be the rate of viscous dissipation you have the μ term and this is. So, that will be S_{ij}. So, that will be viscous dissipation is there going on.

So, this is that term and this term again you if you look at this is again $\rho\overline{u_i'u_i'}.S_{ij}$. So, this is because of the turbulence production. So, this is again this is a negative term. So, that is why we again call it as a you know destruction because this term minus of this term is normally positive. So, this will be destruction of the you know turbulence production. So, this is that term. So, these 7 terms you are getting they are the governing equation for the mean for the kinetic energy now what we have to do is.

Now, you have you recall in our earlier lecture you had one equation for the instantaneous you know velocity. So, for that you had the 3 equations that nearest to equation. So, you got for the small u that is comprising of a mean part plus fluctuating part. So, again you know that will be multiplied you know and then you have the multiplication to the mean you know mean velocity component equations. So, that momentum equations and further you are when you are taking the difference.

So, in that case you are getting the equation for the turbulent kinetic energy k. So, turbulent kinetic energy k will be nothing, but if you are getting having the expression for the k(t) that is your you know turbulent kinetic energy and if you take the difference that is of the mean part of the kinetic energy. So, you will get that for the fluctuating part.

So, in that if you do those you know some kind of computation, in that case you are getting the governing equation for the turbulent kinetic energy and this turbulent kinetic energy for that you know that. So, there are certainly some of the there are many you know computations on that and this is done by Tanaka San . So, they have devised.

So, with that you know with some of the reframing of the equations and all you get further the equation for the turbulent kinetic energy k. Now, for that again if this equation comes out to be $\frac{\partial(\rho k)}{\partial t}$so, again this is fist. So, this is again the change of rate of change of the turbulent kinetic energy here. So, this is rate of change with respect to time, then you have $div(\rho kU)$.

Now, here it is the mean you know kinetic energy mean flow kinetic energy capital K and this is a small k. So, this is for the turbulent kinetic energy part. So, this will be ρkU. So, again this is because of the convection. So, it will be the transport you know of turbulent kinetic energy by the convection. So, that is $div(\rho kU)$..

Then the third term similarly here if you have the capital P that is mean pressure and this is the U. So, in that place you have the p' that is your fluctuating you know is that part of pressure as well as the velocity. So, you have again the transport of turbulent kinetic energy. So, that is because of the pressure.

So, here it was also the because of pressure. So, here also it will be because of the pressure, then you will have the again this is the $2\mu\overline{u's'_{ij}}$. So, this will be the because of the viscous stresses. So, that is the $\mu\overline{s_{ij}'}$. So, you will have the viscous stresses. So, because of that so, this is transport of k b by the viscous stresses similarly this will be transport of k because of the Reynolds stresses and now here you will have the rate of dissipation of the k and this is the rate of production of k.

So, now in this case this is minus and this is plus. So, this will be the rate of production in that case it was the destruction because it was plus and this value is overall becoming negative. So, you will have this term as the rate of production of the turbulent kinetic energy.

So, this way you are getting these 2 governing equations for the mean flow kinetic energy and the turbulent kinetic energy and we will define we will see that there are 2 governing I mean 2 equations for k and epsilon which is which has been given by the Spalding and with that value of k epsilon how we find these you know mu t value.

So, you know we see few terms and the term that is 6 one this is $2\mu S_{ij}' \, S_{ij}'$ over, here this is $S_{ij}' \, S_{ij}'$ this is the you know viscous dissipation term and this will be nothing, $2\mu(\overline{S_{11}'^{\,2} + S_{12}'^{\,2} + S_{22}'^{\,2} + S_{33}'^{\,2}} + 2(\overline{S_1' + S_2'}) + (\overline{S_2' + S_3'}) + \ldots\ldots$ So, that way that term can be defined as and then it was you know.

So, this is the you know this has it is it has a negative contribution. So, you know this is because of the you know this square of these fluctuating terms. So, you know. So, you have the dissipation of these turbulent kinetic energy which is you know caused by the work done by the smaller eddies. So, against these viscous stresses and for that there is a parameter which is defined. So, you know that is a rate of dissipation you know per unit volume and then rate of dissipation per unit mass.

So, this rate of dissipation of this turbulent kinetic energy so per unit mass if you take as, if you multiply that with rho so that will be for the per unit volume and for per unit mass so, we call it as the epsilon. So, this epsilon is further you know used for finding the μ_t. So, this epsilon is basically defined as $2\nu\overline{S_{ij}' \, S_{ij}'}$ and then this is what is defined as the rate of dissipation of turbulent kinetic energy from this point so, it will be divided by ρ.

So, you will have $2\mu S_{ij}' \, S_{ij}'$ and you know so, that basically will be this parameter is this is a term which is known as the rate of dissipation of turbulent kinetic energy per unit mass. Now, if you look at it is you know unit it will it is unit is basically for this the unit is meter square per second cube. So, this is very important in the case of turbulence dynamics and we have to see that how you know we try to have the it is significance when we get the capsule and turbulence model. So, what we do? We try to find the you know equation for the k and epsilon and that is given by the Launder and Spalding.

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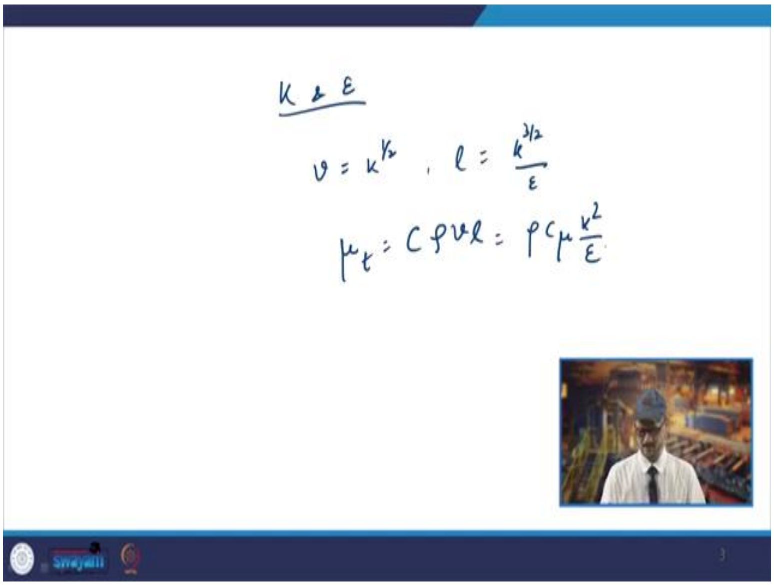

So, what they have given initially, what they have suggested that you have you know the k and epsilon. So, that will be used to define the velocity scale and the length scale. So, the velocity scale will be defined as the $k^{1/2}$because as we know that k will be half of. So, it will be power proportional to v^2. So, we will be k it is to the power half. Similarly the length scale can be taken as $\frac{k^{3/2}}{\varepsilon}$.

So, that there are a certain you know you know conditions or there are certain assumptions why they have they have been taken. So, why we are taking this length scale and when you know small eddy variable is there that is epsilon. So, we are taking the, you know we are taking that to define the largest scale length l. So, certainly because they are extracting the energy so, they have the interaction so based on that you know they have been defined this length scale as in terms of epsilon.

Now, using the dimensional analysis what we can get the eddy viscosity μ_t as the $c\rho vl$ and that way we can get $\rho c_\mu \frac{k^2}{\varepsilon}$. So, basically what we do, we try to have the value of k and epsilon and then you are getting this value of μ_t using that value of k and epsilon and this k and epsilon you are getting from the 2 you know governing equations for the k and epsilon. So, the 2 equations for one for k and one for epsilon that has been given by the Spalding and they are like this.

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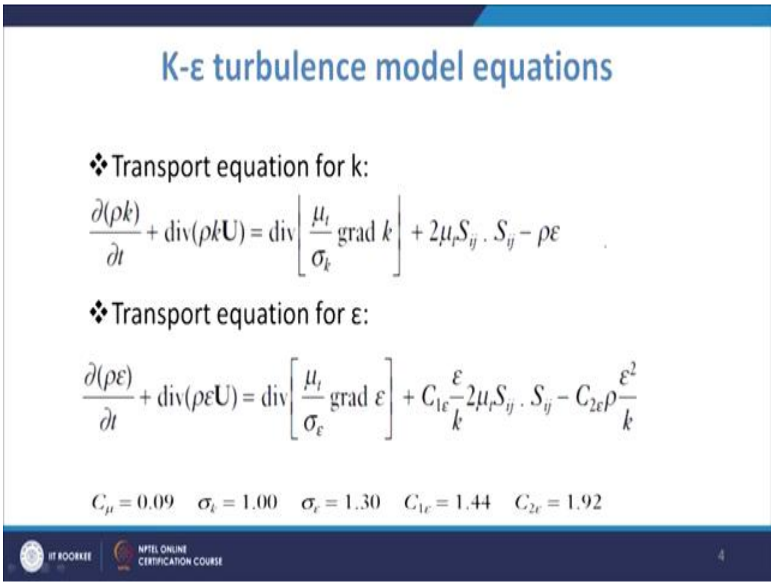

So, this equation for the turbulent kinetic energy k has been given as the $\frac{\partial(\rho k)}{\partial t} + \nabla.(\rho kU) = \nabla.\left[\frac{\mu_t}{\sigma_k}\nabla k\right] + 2\mu_t S_{ij}.S_{ij} - \rho\varepsilon$. Now, if you look at this term so, this term again this is the rate of change of k or epsilon. So, if you look at these 2 terms they go you know one and one. So, here $\frac{\partial(\rho k)}{\partial t}$ and this is $\frac{\partial(\rho\varepsilon)}{\partial t}$. So, these 2 terms are the rate of change of either k or epsilon. Next will be the transport of k epsilon because of the convection. So, this is for the k equation and this is for the epsilon equation.

Now, coming to right hand side we are taking this parameter sigma k and this div. $\left[\frac{\mu_t}{\sigma_k}\text{grad } k\right]$ and similarly div. $\left[\frac{\mu_t}{\sigma_\varepsilon}\text{grad } \varepsilon\right]$. So, this is the transport of k or epsilon because of the diffusion. So, you have diffusion and that is because of the new terms and sigma terms. So, this is because of the diffusion terms, then come to the $2\mu_t S_{ij}.S_{ij}$ this is in the case of you know k and this is here we are taking $C_{1\varepsilon}\frac{\varepsilon}{k}2\mu_t S_{ij}.S_{ij}$. So, now in these 2 terms they relate they are basically talking about the rate of production of the you know either k or epsilon in this case.

And then the last term is the rate of destraction, I mean destraction so that is dissipation and here also it is the dissipation. So, you are getting these values. So, you are having these 2 governing equations are used for finding the value of k and epsilon and then in this case you are having these constants C_μ. So, this C_μ which is used you know that C_μ will be it is value will be 0.09 and C mu is used for finding the mu t. So, where we have seen that earlier so, mu t will be C_μ and then you apply this you know other parameters.

You have the use of sigma k that is we are taking as one and see my epsilon we are taking as 1.3, C_1 epsilon and C_2 epsilon has been taken as 1.0 they are suggested by the researcher Spaulding. So, this is basically the these are the constants and they are used for finding the value of the k and epsilon and then further we use them for finding the μ_t once when we get the value of C mu there from and k and epsilon. Now, what we see in this case now we can have the computation of the Reynolds stress$\rho\overline{u_\iota{}'u_\iota{}'}$. So, that we can get using the Boss disk approximation so, their form also you get it.

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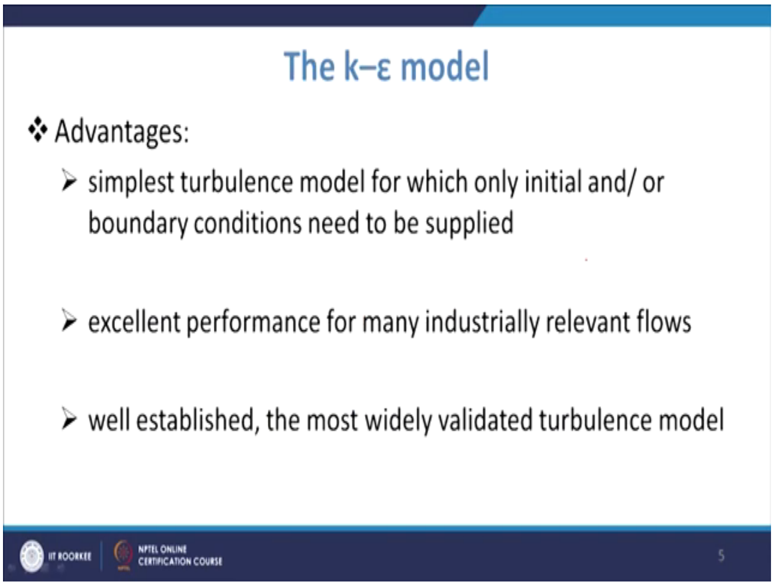

Now, the thing is that if you talk about this model. So, this model is basically a very you know simple turbulence model which is used for which only initial and order boundary conditions need to be supplied. So, we have certain name initial and boundary conditions suppose at inlet we give the distribution for k and epsilon.

So, those values need to be supplied, then you know at the outlet you are or the symmetry line you may have the you know the dou k by dou n equal to 0 or 2 epsilon by doing equal to 0. So, that way you are giving this boundary conditions and you are solving at the walls basically you are giving the condition based on the Reynolds number. So, you have already seen that what are those law of walls. So, based on that we are giving the boundary conditions, this is a very you know model which has a very excellent performance for many industrial relevant flows and this is well established and most widely validated the turbulence model.

Now, if you talk about this model so, but this model has also certain disadvantages and those disadvantages will be now you know that it is more expensive as compared to the mixing length model and then you have also some poor performances in the variety of important cases. So, in some of the unconfined flows or flow with large extra strains where, I mean in the case of curved boundary layers or swirling flows or rotating flows or even flow is driven by anisotropy of normal Reynolds stresses in these cases this model does not work.

So, in our lectures to come sometimes when we talk about the boundary and also other models at that time we will also try to have the discussion about you know certain you know aspects of this model and there are certain modifications to this you know k epsilon model, because that does not work good you know in that reason where the turbulence more Reynolds number is low. So, you have certain damping you know coefficients are damping constants are also supplied.

So, that the that decays it gives you proper you know it predicts proper parameters for the turbulence is you know very small in those reasons. And so there will be certain modifications that we will see that what modification we do in this model. So, that it can be used even for certain cases like where you have the low Reynolds number flows also near the walls or, so that we will see in our long run, but for the time being we should be you know knowing the terms of this k epsilon turbulence model. So, that when the mostly when we use them for predicting the you know turbulent flow. So, we should feel confident about all these terminologies.

Thank you very much.