Modeling of Tundish Steelmaking Process in Continuous Casting

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Lecture – 24
Turbulent Flow Calculations

Welcome to the lecture on Turbulent Flow Calculations. So, in this lecture we are going to have you know some background about you know these stress terms and how they are modeled. And what are those terminologies which are required to be known when we deal with the solution of the turbulence models. Also, we will have some introduction about the turbulence models.

So, what we have seen that in our you know earlier lectures in the in the past lecture even, that you have the eddies you know because of the fluctuating you know component. So, you will have. So, because of the turbulence you will have the eddies of different you know length scale you will have larger eddies you may have these smaller eddies.

So, you know they will they will have the different length scale and time scales also for their you know interaction. So, because they interact in a very complex manner and that needs to be modeled when we need to see the you know turbulence flow; you know modeling. So, and you have the numerical methods basically which are available to capture this, to predict, or to study these turbulence.

So, you know the methods which are used for studying these turbulence are broadly you know categorized or a broadly divided in you know three categories. So, one is the you know turbulence models for the RANS equations.

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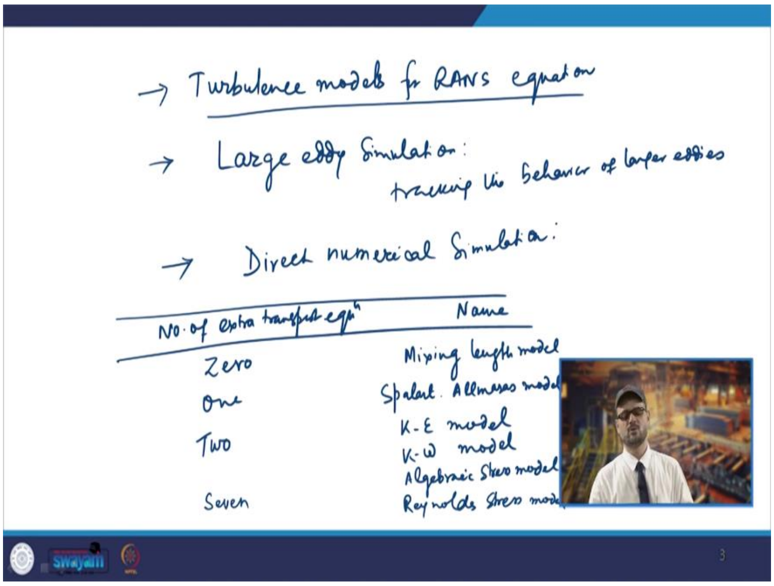

So, as we have seen that we focus on the mean flow and also the effect of turbulence on the mean flow properties. So, in that you are getting one you know extra term. So, and in this case what we do is that these extra terms which appear, which we have seen in the earlier lecture. So, they need to be modeled. And you know they will be modeled with the classical turbulence model and basically one of that is the k epsilon turbulence model; that is you know used. And also there is one model that is Reynolds stress model.

So, these models basically ; so they will be using they will be used for you know modeling these extra you know stress terms which are generated that is Reynolds stresses. And you know for them the computational time or the resource which is required will be normally reasonably. so in And in most of the cases it has been seen that it predicts somewhat form fine. So, you know. So, this approach is mostly used in most of the engineering flows.

Next, the second category of you know the second approach is the large eddy simulation you know. So, now this larger eddy simulation. So, is the intermediate form of the you know turbulence calculation. So, here you will have to have the calculation which will be tracking the behavior of the larger eddies. So, you will have, you know in the domain you will have the larger eddies as well as the smaller eddies. So, you will be doing that simulation based on or calculation based on the larger eddies, and the smaller eddies will be filtered so you will have the sub grade scale modeling for that.

So, what we do is, we will be tracking the behavior of larger you know eddies. So, what happens that you will have you know the space filtering of these unsteady Navier stroke equations. And prior to that computation you will have we doing these space filtration, you will be seeing the large eddies and the smaller eddies.

And the larger eddies will be passed and the lower the smaller eddies will be rejected. And then further you know these smaller eddies are also included using these sub unit scale modeling. So, what happens that in this case the computational resource you know; if you talk about the resources or volume of calculation its more than these you know what we do using these turbulence models like k epsilon or so.

So, this is the second you know category. And then when normally recently we also do the direct numerical simulation, that is also known as DNS. So, there will be computing, the mean flow as well as all the turbulent you know fluctuation component. So, you know this in that you have the unsteady Navier stokes equations and they will be you know solved in that special grids you will have the final grids to account for the fluctuations, and the you know timescales are also taken so that you know they can be even smaller time length scales; time scales are also taken into account.

So, that way you will have you know it will be taking large amount of time, but then they are they will be giving the most accurate results, but it will be taking basically very very you know large time. So, computationally it is very very costly. So, normally it is used for the industrial flow type of computations.

Now, what we do is; when we talk about the Reynolds averaged Navier stoke equations and about the classical turbulence models. So, you know in that what we have seen that normally we have the terms these stress terms. And also stress terms like $-\rho u'v',$ of $-\rho u'w'$, and of $-\rho u'^2$ that is averaging averaged part.

So, these are the stress terms which needs to be you know taken in two or if you have the scalar property also so $u'\varphi'$ my average part. So, these you know these parts needs to be modeled and you have the Reynolds averaged Navier stokes equations for that you have the turbulence models.

And you have, so turbulence models which we use normally is mixing length model as well as the k epsilon turbulence model where you have the two equation you know model;

you have one equation model, you have two equation model. So, these you know based on the additional number of transport equations which you need to solve for modeling these stress terms. So, based on that you have the you know different types of models which are used I mean they are known as the turbulence models.

So, you know what happens that; if you talk about you know the depending upon the number of transport equations which needs to be solved and that way you have those equations solved. So, you have you know if you take the number of extra transport equation which needs to be solved, and the name of the model will be.

So, suppose when you have a classical model that is mixing length model and that has nothing you have an expression, so then in that you get the expression for the any diffusivity. So, that we will see that what is that any diffusivity, so and that is in terms of the mixing length. So, you will that length will be a function of the characteristic length of the you know flow domain. So, you will have that is 0 equation, so since there is no any extra equation needs to be solved. So, that is the that is for the mixing length model.

Similarly, you have one equation; one extra equation needs to be solved and that is the Spalart-Allmaras models. So, in that you have one extra equation transport equation needs to be solved for modeling these for taking into account these extra stress terms. Similarly, you have to extra equations needs to be solved for you know in certain cases and that is for k and epsilon you know, and that is developed by spalding.

So, you have that model known as k epsilon model, so you have turbulent kinetic energy and you have the dissipation of kinetic energy equation. So, you have these two terms and you have these two terms they are solved and using the value of k and epsilon you get the you know value of those stress values. So, using the you know using the other parameters like ν_t or so; so that we will see.

Similarly, you have the k-ω model. So, that is also one where you have the two equations: one is k for k and another is for ω. Similarly, you have the algebraic model stress model. So, in these you know models you solve two extra equations, and then from there you try to find the value of these additional stresses which we need to compute when we are solving the turbulence equations. Similarly, you will have seven equations when you are using the Reynolds stress model.

So, as you see that normally in the commercial CFD software's you will have the provision of all these you know models and you can use take into account, any of the model and you can solve the problems. We will have some understanding about these models and what they are especially about the main models which are used. And we will try to see that how they are you know used for the computation of these stresses shear strain, shear stresses, you know for modeling of that.

So, coming to the you know concept of eddy viscosity and the eddy diffusivity. So, as we know that you have the eddy motion in the you know turbulence. So, you know you will have the stresses which are appearing on the right-hand side and you know as we see that you have the Newton's law of viscosity where we say that you will have the stresses. So, that in that stress you take as proportional to the rate of the deformation of fluid element. So, that is what is being told by the Newton's Law of Viscosity.

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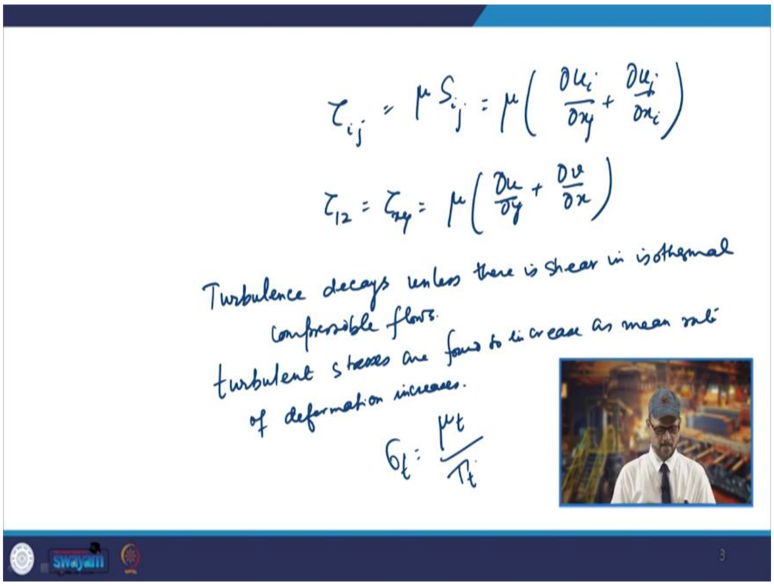

So, that you know for incompressible fluid what we write is τ_{ij} that we write as μS_{ij}. So, that is you know the Newton's law of viscosity and in that we talk about these viscous stresses. And it is nothing but the $\mu(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$. So, you know in this if i or j will be 1 in that case you would take it as x and if it is 2 then you can take it as the y or if you take the j equal to 3 in that case it will be for the third direction that is z direction.

So, if you talk about τ_{12} , so it will be τ_{xy}, so it will be $\mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$.. So, this is what normally we have understood. Now when we talk about the turbulence. So, in the turbulence you will have the decay of the turbulence also. So, you know. So, what is found that the turbulence decays unless there is shear isothermal incompressible flows.

So, if there is no shear than you know it will be you know decaying, so that is there. Now also it is found that the turbulent stresses are increasing you know at the mean rate of different deformation increases. So, these turbulent stresses are found to increase as mean rate of deformation increases. So, if the mean rate of deformation will be you know increasing in that case the turbulence is; stresses are found to you know increase. So, based on that these two you know you know findings.

What Boussinesq has proposed; he proposed in 1877. So, the Boussinesq is proposing that is Reynolds stresses might be proportional to the mean rates of deformation. So, that way based on these two you know findings it can be said that the Reynolds stress will be might be proportional to the mean rate of deformation.

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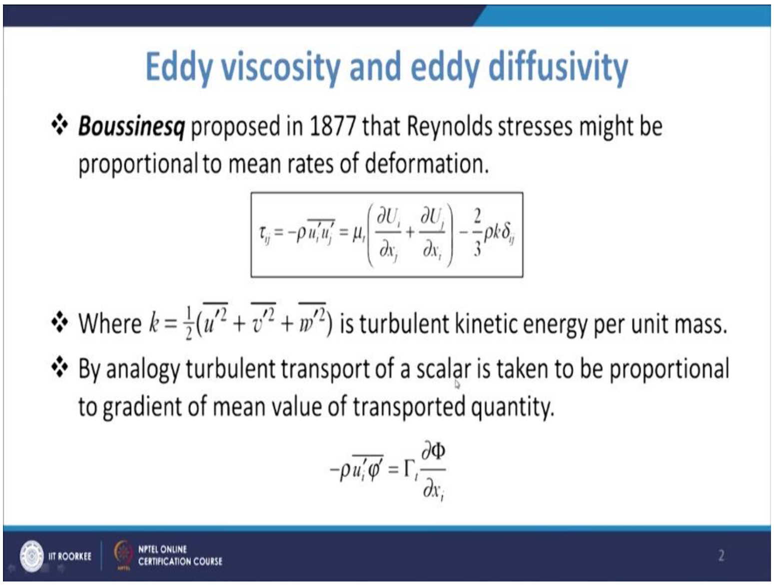

So, it will be τ_{ij} , that is mineral now stress will be $u_i{}'\ \ u_j{}'$ and you know every is part, so that will be you know this is your you know mean rates of deformation. So, this has been written as the equation $\mu_t(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}) - \frac{2}{3}\rho k \delta_{ij}$. This is you know δ_{ij} is the chronicle

δ and when i will be equal to j so that this term will be vanishing; δ_{ij} vanishes in those cases.

So, you know in that case you know this is expressed, so when you will have this τ_{ij} . So, now in this case the k it is the turbulent kinetic energy and it will be half of u prime. So, in this case k will be $\frac{1}{2}(u'^2 + v'^2 + w'^2)$. So, this is basically turbulent kinetic energy per unit mass.

Now, what you see in this case this μ_t . This is the turbulent viscosity or the eddy viscosity; you know this is known as the turbulent viscosity or eddy viscosity. And this is a based on that you will have the term ν_t; ν_t will be nothing but μ_t/ρ. So, that will be kinetic viscosity kinetic turbulent viscosity or visco turbulent viscosity. Now, Kronecker delta; δ_{ij} it will be 1 if i equal to j and it will be 0 if delta if i is not equal to j. So, its basically in the matrix wherever i equal to j along the diagonal it will have 1 part and otherwise it will have the 0 part.

So, that way you will have you know you know. So, it will be giving the correct result for the normal Reynolds stresses where i will be equal to j. So, for the τ_{xx} that is $-\rho u'^2$ you know average of the term. And similarly, you will have the you know τ_{yy} or τ_{zz}. So, you will have those terms coming into.

Now, if you try to have the scalar part, if you have the you know if you take the turbulent transport of something like heat, mass or other scalar properties. So, in that case what we take is we are taking this term and this is Γ_t. So, $-u_i'\varphi'$ so its average value it will be equal to Γ_t and the $\frac{\partial \Phi}{\partial x_i}$.

So basically, this term is the its known as the turbulent or eddy diffusivity. So, you know its you know its analogous to this term μ_t. So, what happens that you know for other a scalar quantity we use this tau t and you know what we do we have the Reynolds analogy. So, what we do is that, we introduce a number that is your Schmidt number. So, that is a turbulent Prandtl or the Schmidt number. So, that is defined as σ_t. So, we defined this sigma t that is turbulent Prandtl or Schmidt number. So, that will be basically the you know turbulent viscosity divided by the diffusivity term.

So, you know many a times we take its value as a constant and normally we take it as unity in most of the cases. And in most of the see of the solvers they use this you know turbulent the Prandtl number turbulent you know this Prandtl or Schmidt number that is taken as the value of unity.

Now, coming to the different turbulence. So, as we see we have; so this is about the eddy viscosity part. Now, we come to the different turbulence model. So, as we have discussed that we have different turbulence models which are used or the you know mixing length model, where we describe the stress by means of simple algebraic you know formula for μ_t that is and it will be as a function of the position. So, you will have some you know value of position that is lm; so length mixing length is taken. So, in the in that term we are expressing this μ_t value. So, that is why it is known as the mixing length model.

Then you will have the k epsilon model, where you have two equations for k and epsilon and based on that we find that these mu t. Then you will have other equations like Reynolds the stress equation models, advanced turbulence models will be there. And also you will have the you know rants turbulence model stuff, so this we will see.

Now, if we talk about the mixing length model. So, in this lecture we are going to have some idea about the mixing length model how you know these mixing length model works, and how we are using it for the you know prediction of these stress component turbulent stress components.

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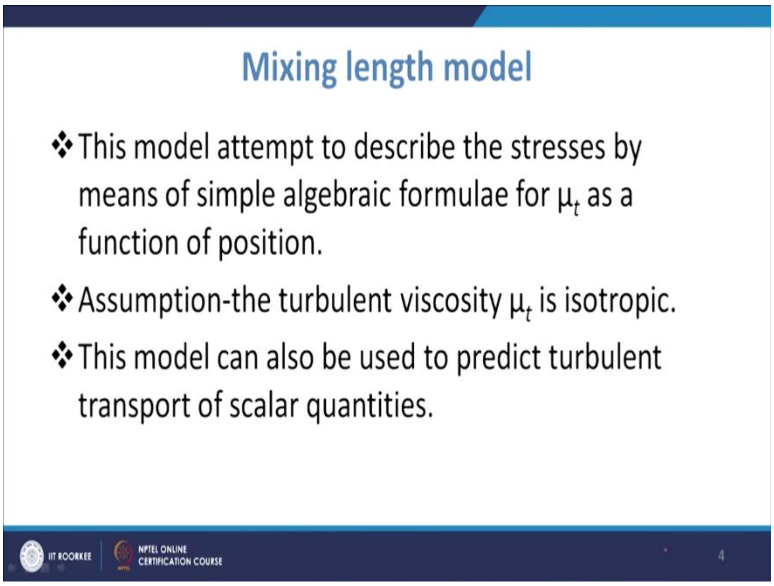

So, this model will be attempts to describe the stresses by means of simple algebraic formula for μ_t as a function of the position. And in this the assumption is that the turbulent viscosity mu t is isotropic and it can be used to predict the turbulent transport of scalar quantities.

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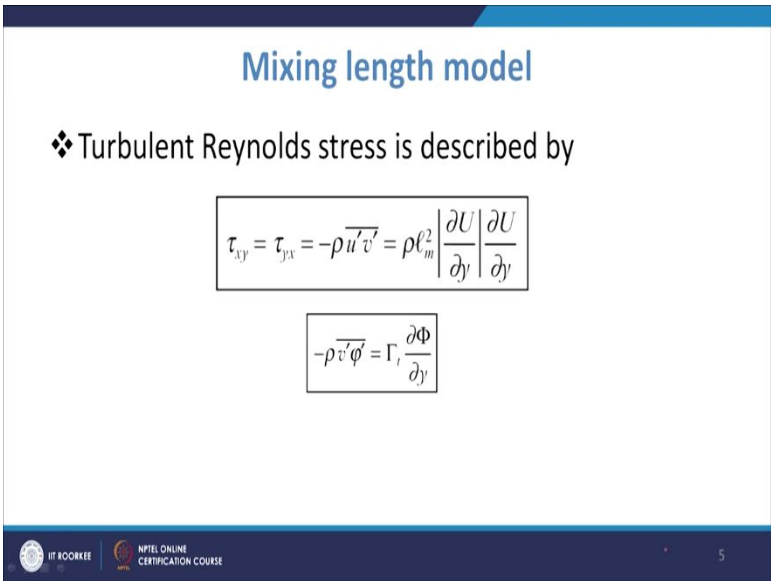

So, what happens you know in this case the turbulent stress I know. This is turbulent Reynolds stress what we do is we describe it in the form of this ${l_m}^2$, ${l_m}^2 \left|\frac{\partial U}{\partial y}\right| \frac{\partial U}{\partial y}$ here. So, basically what we see its expressed in terms of this l m, and then that is why you know we call it as a mixing length. Know its value also varies, so I mean there are certainly certain deviations for that.

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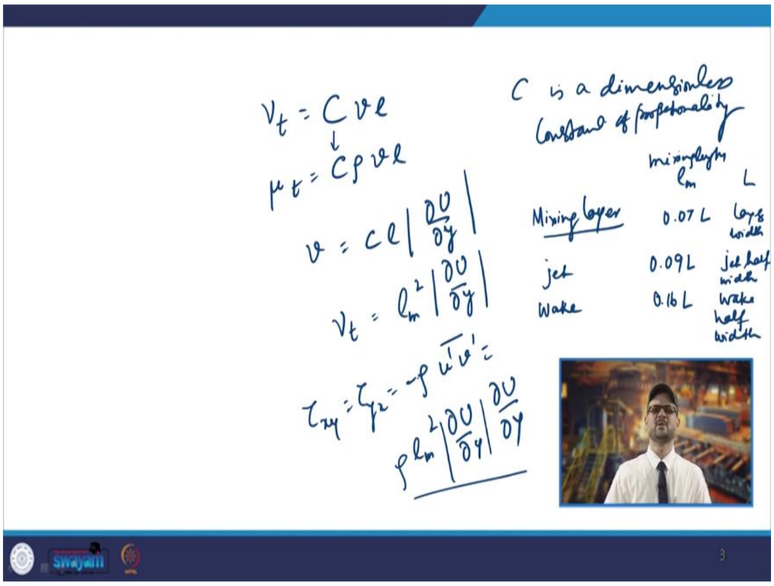

So, you know what is done is normally. What we see that ν_t; now in this you can write as the velocity scale and the length scale. So, because μ_t has the unit of meter square per second, so you know this is meter per second and meters so it will be meter square per second. So, if the one velocity scale and one length scale that is described the effect of turbulence. So, in that case the dimensional analysis will yield to this equation ν_t will be cvl.

Now, this C is basically a dimensionless you know constant of proportionality, because the ν_t had the unit of meter square per second now. So, we can write the μ_t as ρcvl; v is the velocity scale and l is the length scale. Now, most of the kinetic energy basically is contained in the largest eddies and the turbulent length scale l it is basically the you know the characteristic of these eddies, because they are containing most of the turbulent energies you know. And they are basically interacting with the mean flow.

So, if you talked about the; so what you can say there will be strong connection between the mean flow and the behavior of these larger eddies. And we can also think of attempting this link to the velocity scale of the eddies with the mean flow properties. So, what is seen is that you can have the velocity scale as a $cl|\frac{\partial u}{\partial y}|$.

So, if you take the dimensionally you know maybe it will be dimensionally correct, because if the eddies scale is eddies scale length is l, in that case if you correlate it with the velocity gradient. So, you know. So, in that case and because the turbulent stresses they are proportional to the you know they have also the relationship with the velocity gradient. So, you can have the velocity gradient that can be expressed in terms of $cl|\frac{\partial u}{\partial y}|$..

So basically, if you use them you can further write ν_t as the $l_m^2\left|\frac{\partial u}{\partial y}\right|$; should you take all these terms together and you have the constant so you can have the $\left|\frac{\partial u}{\partial y}\right|$. So, this is basically known as the Prandtl's mixing length model. So, this you know l m which is coming, this is because of this term this is known this is known as mixing length, so that is known as the mixing length model. And if the $\left|\frac{\partial u}{\partial y}\right|$ is basically the only significant we lost a gradient, so you can have the τ_{xy} or τ_{xy} which we write as $-\rho u'v'$.

So, it will be written as $l_m^2\left|\frac{\partial u}{\partial y}\right|$. So, it will be $\nu_t\left|\frac{\partial u}{\partial y}\right|$in fact. So, you can μ_t dou μ_t will be $\rho\nu_t$. So, it will be this term and then you will have the $\frac{\partial u}{\partial y}$, this $\frac{\partial u}{\partial y}$ has been kept in more, because the velocity has to have the positive values that is why it has been taken as the mod value. So, that way you know from this you try to have the modeling of operation of these stresses using this mixing length.

So, basically what has happened that if you has been found these for different types of flow you take these you know a mixing length as different value. Like you know for the for mixing layer, you take these mixing length. So, this mixing length l_m it will be $0.07\ l$ and l is the layer width. Similarly, if you have a jet. So, in the case of jet you will have $0.09\ l$ and where the l is the jet half width. So, for different types of flows you have the for the you know wake. So, for the wake suppose wake type of flow you will have $0.16\ l$ and this is again wake half width.

So, for different type of flows you have different values of these mixing length which is taken. And similarly for the boundary layer or even for the pipes and sandals you will have one expression; that is l with respect to y you know it will be varying have a functional value and you will have the l will be the pipe radius or the half of the you know channel width. So, those things are taken. So, you can have these values from the standard you know textbooks and that can be used. So, they are used basically in the turbulence models when you are trying to have the prediction of these turbulent stresses using these.

So, its use will be there in the tundish flow basically, when we do the tundish modeling in that case when we are to predict the quantities in that case which kind of model to be selected. So, being the simple in model simple model it takes minimum time to solve. So, you can start with this model and then go to other models and see how they are you know predicting the your you know different output parameters. So, this is about that. We will talk about more of the turbulence models which are used you know in commercial solvers or so. So that will be in the next classes.

Thank you very much.