Modeling of Tundish Steelmaking Process in Continuous Casting

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Lecture – 23
RANS Equations

Welcome to the lecture on RANS Equations. So, RANS is Reynolds Averaged Navier-Stokes Equations. So, this equation we are going to particularly discuss because when we deal with the turbulent flow, so, as we know that in the case of turbulent flow you have the mean component and also at the same time you have the fluctuating component. Now the presence of these fluctuating components they give rise to extra set of terms when we deal with you know or when we talk about the momentum you know conservation equations or momentum equations.

So, in this lecture we are going to see that how you know when we talk about the turbulent flow so what are those additional terms which are coming and then that is why that you know. So, we are averaging there are averaging principles, averaging rules are there because of that certain extra terms are coming. So, how then these Navier- Stokes equations you know change. So, that we will see here and that is why they are known as the Reynolds Averaged and Navier- Stoke Equations.

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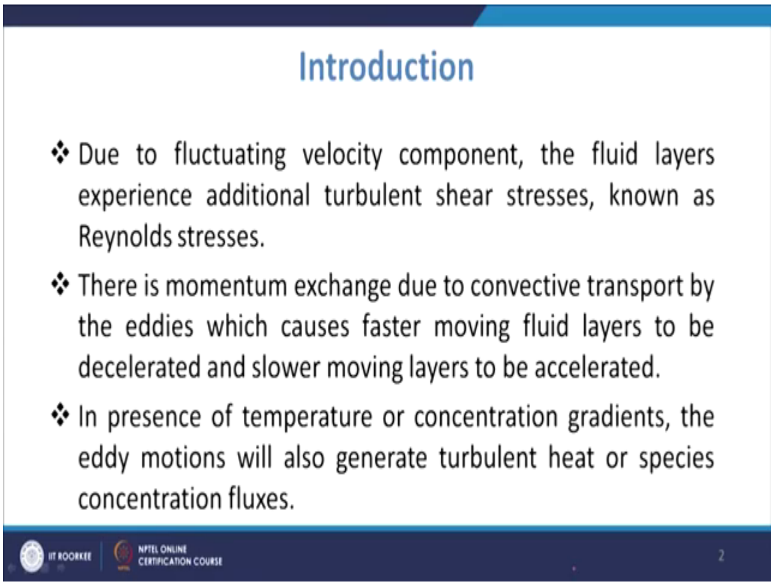

So, if you know if you talk about the fluctuating parts of the velocity, so, what happens that because of that you will have the fluid layers which will be experiencing the additional turbulence shear stress that is known as the Reynolds stress. So, you can understand it like what happens that when you have these fluctuating parts or you have you know if you take the turbulent shear type of flow.

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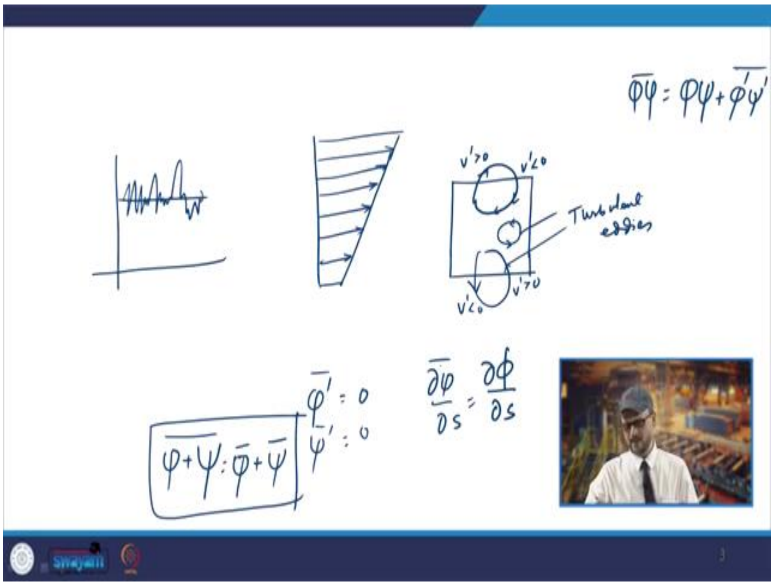

So, in those cases if you try to see the you know the velocity gradient in case of turbulent shear flow, so, you will have the velocity gradient like this. So, you know you have the velocity gradient in the y direction. So, there is a gradient in the velocity and if you talk about if you consider the a control volume. So, what happens due to these fluctuations?

So, what happens? You will have the formation of eddies and these eddies you know on the boundaries they will be looking like this. So, they will be moving like this. So, you will have increase in the y velocity in this direction and decrease in the y velocity on this side. So, this way due to these eddies so, in this side you have a fluctuating component will be more than 0 and in this side your fluctuating component will be less than 0.

So, this is the velocity fluctuation due to the eddy formation and you will have the turbulent eddies at many places which will be you know going on. So, what happens or if you talk about this surface suppose so, you will have again the eddies you know moving in this fashion. So, you will have on this side v' is less than 0 and this will be v'more than 0.

So, actually what is happening, now in this case what you see? So, these are the you know these are the turbulent eddies and these eddies are responsible for creating very strong mixing you know inside. So, and they are responsible also for the momentum exchange due to the convective transport. So, these are done by these eddies and which will be causing the faster fluid layers to be decelerated and slower moving layers to be accelerated. So, the faster moving layer which is their top. So, it will be you know there will be a breaking effect on that. So, that will be subjected to some deceleration and the slower moving layers will be there which will be subjected to some acceleration.

So, you know. So, that happens you know to the velocity component and even that is happening in the presence of these temperature or concentration gradients also. So, these eddy motions will generate the turbulent heat or specific concentration fluxes. So, basically as you see that you know you can we have seen that this is along the turbulent flow you know this control volume boundaries there will be you can see the eddies how they are going out or you know inside. So, they will be you know across that control volume boundaries they will be transport.

Now these things basically these. So, what happens that they will have the generally there is no generation of additional you know shear component and you will have the additional shear stresses will be generated this would do to this turbulent and that is why they are the turbulent shear stresses and these shear stresses are known as the Reynolds stresses and you know and there are many more you know we will see that how these fluctuating components.

So, these you know presence of these Reynolds stresses they need to be accounted when we deal with the you know conservation equations especially the momentum equation or even the continuity equation. So, we need to see that and these extra terms will be coming into the Navier- Stoke equations and they will be calling as the Reynolds Averaged Navier-Stoke Equation, because what happens that the velocity will have 2 component, one will be mean component, another will be the fluctuating component. So, ultimately when you are going to have the momentum equation so, you will have certain components coming in to additionally you know you know because of these fluctuating parts.

So, we will be talking about these you know Reynolds averaged you know Navier- Stoke equations, now before that we must you know we know that if you talk about the you know fluctuation components. So, you will have you know. So, it is averaged if you talk about the fluctuating part only. So, it is we have to consider the certain you know points that if you talk about the you know fluctuating and if you take the average of this. So, your average of the fluctuating properties, so that will be so, for the fluctuating part it has to be taken as 0. So, if you take another property like ψ. So, for that also if you take the average of the fluctuating component it has to be it is taken as 0.

So, as we know that you had the component. So, you have the mean and then you have the fluctuation. So, if you take the average of this on the on the time scale. So, it will be taken as 0, because you have the fluctuations above as well as the below. So, you will have this part we will be taking summation derivative that we are taking as 0. If you talk about the other things like if you have the $\frac{\overline{\partial\psi}}{\partial s}$. So, that will be nothing, but $\frac{\partial\varphi}{\partial s}$. So, that way we take it as these values. So, that will be mean part and then you have similarly if you have the part like you know this plus this. So, it is mean will be you know. So, you are taking the it is mean plus you know the mean separated.

So, there are certain you know properties which we will be using it while we discuss about it and similarly you will have you know. So, basically you need to know certain when we do the time averaging so, at that time that we will need to use like if you have the 2 properties and if you are taking the average of this product. So, it will be $\overline{\varphi\psi} = \phi\Psi + \varphi'\psi'$. So, it is average. So, because it is not going to be equal to 0 you can have other component also where one of the properties mean will be taken fluctuating parts mean will be taken. So, that will be anyway automatically 0. So, these are the you know concepts which are to be used for that.

Now if you go to the other properties like. So, that is known as the cumulative mean commutative property.

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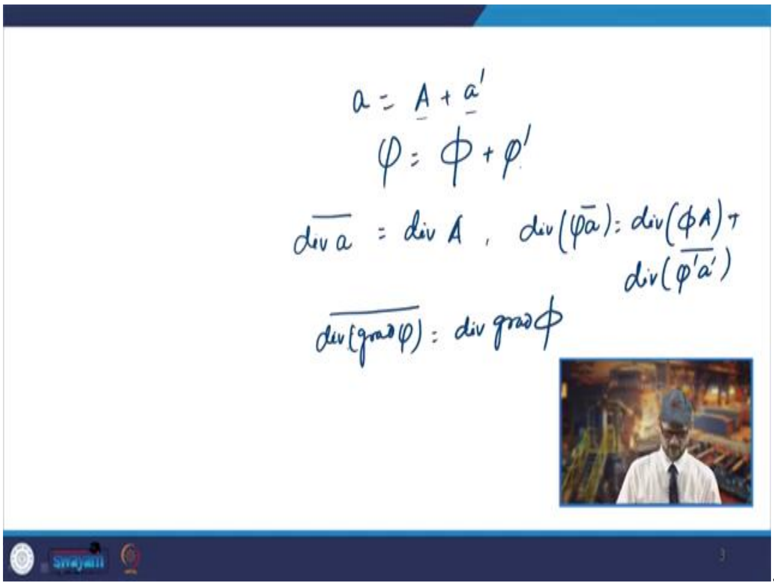

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Now, for any you know fluctuating vector quantity a, which is taken as it is mean part and also it is you know fluctuating part a'. So, similarly you take as the you have one is scalar fluctuating scalar and this will be taken as the mean part plus the you know fluctuating part of this scalar quantity. So, whatever happens that if you take the $\overline{div\ a}$.

So, it is mean will be $div\ \ A$. So, this will be it is mean part and this is the fluctuating parts if you take the div. So, this part will be anyway 0, because as we have seen that it is a change you know with time will be or with any you know if you do averaging it will be 0. Similarly if you take $div\ \overline{\varphi a}$ so, it will be $div\ \phi A + div(\overline{\varphi'\psi'})$. So, that is it is mean parts product and then it will be div of psi prime a prime and it is mean. So, that way you know these also need to be you know kept in mind to us. So, similarly $\overline{div.(grad\ \phi)}$ so, and it is a whole you know mean it is averaging that will be giving us $div. grad\ \phi$.

So, basically this is a mean part so, we are taking having the mean part. So, that is you know. So, because when we use these you know equations when we are going to take the you know any quantity as the summation of the mean quantity and the fluctuating part. So, at that time these you know expressions will be required. Now if you try to see the you know the Navier- Stoke equation. So, if you try to find the instantaneous you know Navier-Stoke instantaneous continuity as well as the Navier- Stoke equation for you know vector u.

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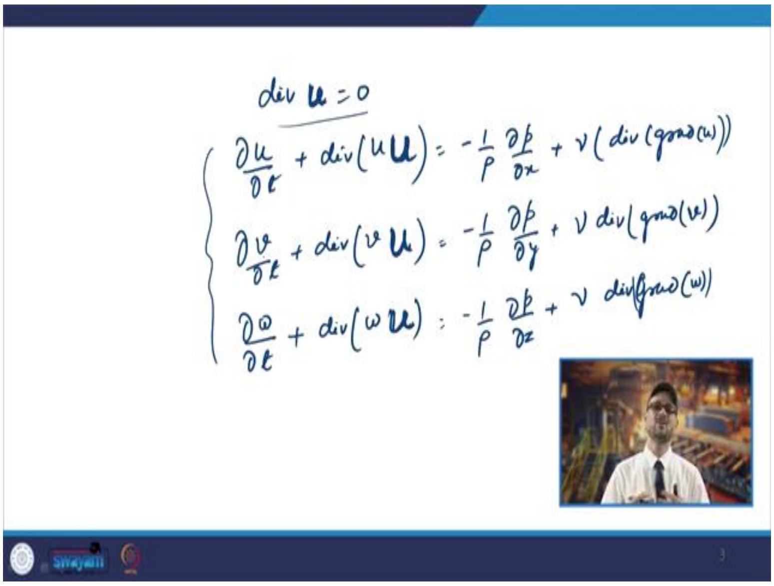

So, in that case your equation becomes $div\, u = 0$, similarly you will have this is your instantaneous you know continuity equation, similarly you will have Navier- Stoke equation that we have already seen it will be $\frac{\partial u}{\partial t} + div(u\boldsymbol{U})$ and this is basically bold one. So, bold one means this will be sum of the 2 quantities one is mean quantity as well as one is the fluctuating part. So, $div(u\boldsymbol{U})$ that will be you know $-\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu(div(grad\, u)$. So, that way you know what we see.

So, this is your x momentum equation that is what you get you know this x component equation similarly you have the you know y component and z component. So, you will have $\frac{\partial v}{\partial t} + div(v\boldsymbol{U}) = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu(div(grad\, v.$

So, then similarly if you take the w momentum equation so, that will be $\frac{\partial w}{\partial t} + div(w\boldsymbol{U}) = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu(div(grad\, w)$. So, that way you know we use these are the instantaneous you know this is the instantaneous continuous equation and this is the Navier- Stoke equation, these are the Navier- Stoke equations you know in the Cartesian coordinate system. So, that the velocity vector u that is this is the u it has 3 components u v and w. So, this u has 3 components that u v and w in that case your this will be your instantaneous those equations.

Now when we talk about the turbulent flow so, in that you will have the mean component as well as the fluctuating component.

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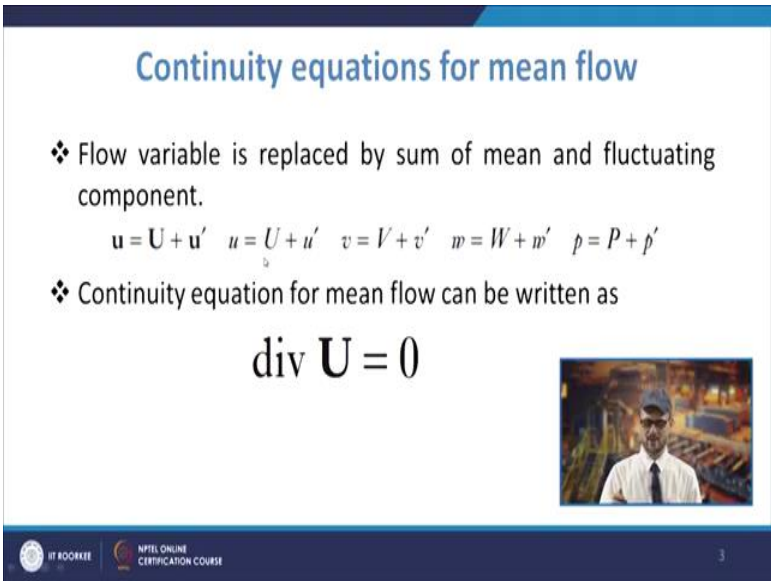

So, when we talk about these variables. So, what will happen, you will have the Reynolds decomposition. So, in that what is done is this **U** as you see. So, you will have the component u v and w and also it will have the you know fluctuating components. So, u will be the mean component plus this is the fluctuating component similarly it is u x component is u. So, that will be mean $\boldsymbol{U}$ plus fluctuating u that is u'we will be you know mean **V** plus a fluctuating v that is v'. So, that way you have to take these you know components taken into to my taken together. Now we have to use the time averaging and then we have to apply those rules.

So, if you go for the continuity equation for the mean flow. So, continuity equation if you now find and especially for the incompressible flow. So, in those case what is happening your $div\ u'$ will be basically 0. So, ultimately your equation of continuity for the mean flow becomes $div\ \boldsymbol{U}$ equal to 0. So, $div\ \boldsymbol{U}$ equal to 0; that means, that $div\ \boldsymbol{U}$ equal to 0. So, this is giving you the continuity equation for the mean flow. Now we have to find the equation for these x momentum y momentum and the z momentum you know and we have to do the time averaging of the individual terms.

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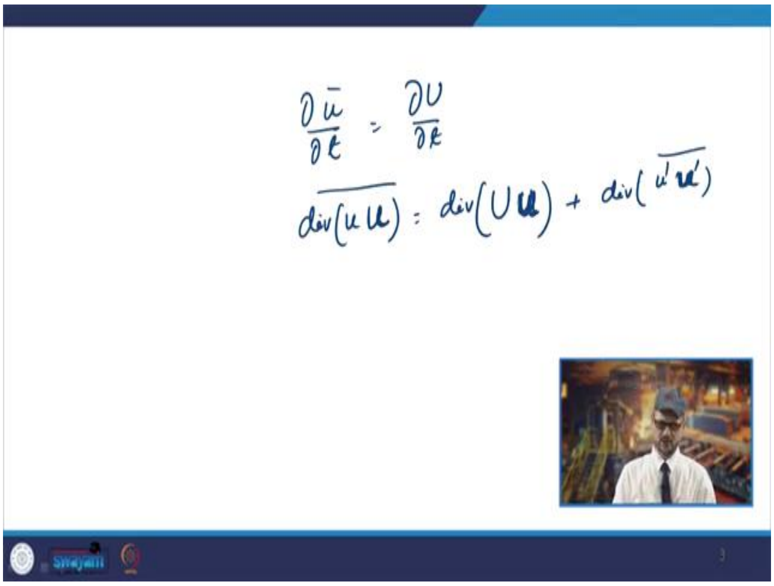

So, if you do some homework what we see that if you find the $\frac{\partial \bar{u}}{\partial t}$. So, as you know that you will be capital $U \ + u'$. So, it will be basically $\frac{\partial \boldsymbol{U}}{\partial t}$ because u prime $\frac{\partial \boldsymbol{u'}}{\partial t}$. So, that will be one, similarly if you find the $\overline{d\imath v\ uU}$ and this you know u that is you know averaging of this. So, it will be $div\ Uu$ and this will be you know bold one and then it will be plus $div\ u'\boldsymbol{u}'$ and this will be the bold component that is prime. So, that way whatever we have studied earlier similarly $-\frac{1}{\rho}\frac{\partial p}{\partial x}$ and it is average that will be again you know it will have the $-\frac{1}{\rho}\frac{\partial P}{\partial y}$. So, that is again mean component will be coming into it.

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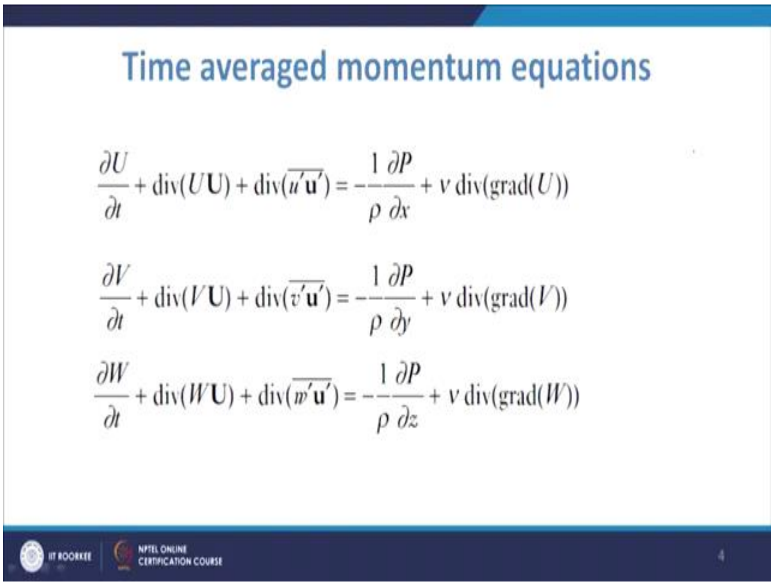

So, if you substitute you know these values what you see you know in that case these time averaged momentum equations if you substitute into it, your equations will become like $\frac{\partial U}{\partial t}$. So, you see what happens that you have we have seen these equations here now in this case this u will be $\frac{\partial u}{\partial t}$ will be. So, that you should $\frac{\partial u}{\partial t}$ will be it is averaging will be capital $\frac{\partial U}{\partial t}$. So, that will be seen here. So, $\frac{\partial u}{\partial t}$ is coming.

Now the $div(U\,\boldsymbol{U})$ that will be $div(U\boldsymbol{U}) + div(\overline{u'\boldsymbol{u}'})$. So, that is what you have seen here. So, from this you know line you can see that in place of the 1 term you are getting 2 terms in this case rest of the term comes like this you have $-\frac{1}{\rho}\frac{\partial P}{\partial x}$ this P is the mean pressure term coming here and then you have the term $\nu\ div(grad(U))$ this term is coming.

So, this is the x momentum equation which is coming basically when you are doing this time averaging and in that you know you see that you have otherwise if you look at these equations. So, what you see it has also it has 4 terms, these 4 terms are similar to it has some similarity these 4 terms; however, you have one extra term. So, that is there in all these cases now what is this. So, this basically term $u\,',\ u\,'$ this is basically the extra term which is coming this is the stress component that is because of the turbulence the fluctuating component and this is the turbulent shear stress which is you know coming into picture.

Now how if you if you try to you know if you try to expand it so, it will be you know and if you try to have the generalization. So, what we do normally that we keep these terms on this side and we take these terms on the you know on other side. So, what happens this will be negative term? So, what we do we take the plus terms and we take the negative sign. So, what we do normally, because it is the you know these components so, as we discussed that this is the you know additional term which is coming because of the turbulent fluctuation I mean a fluctuation component and this is because of these stress component.

So, what we do know customarily, we try to shift it towards the right hand side and if you try to put them towards the right hand side. So, you will have $\frac{\partial v}{\partial t} + div(v\boldsymbol{U}) + div(u'\boldsymbol{u}')$. Now if you find u' will be so, you will have the u' $v'$and w'component here and then you are getting the div part of it. So, you will have $u'u' + u'v' + u'w'$ and then we are getting the div of it. So, $\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$ will be coming.

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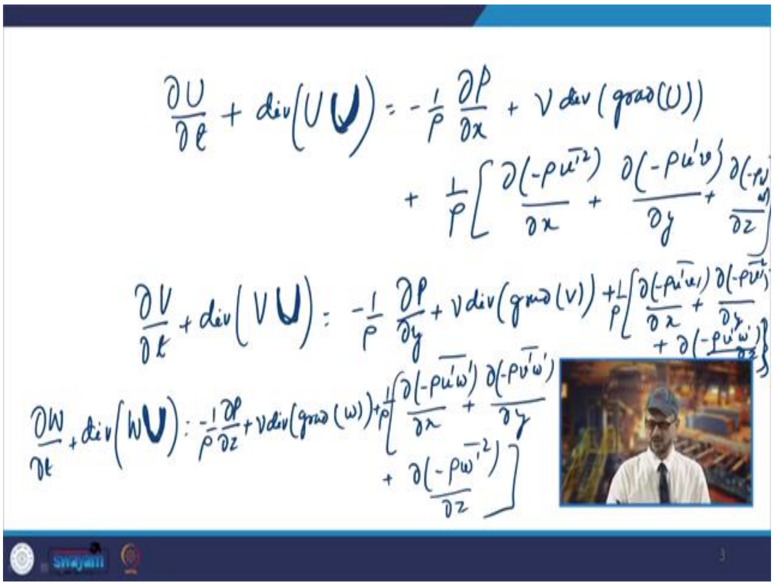

So, if you try to write you know now if you take these turbulent stresses on the right hand side then your equation becomes like $\frac{\partial U}{\partial t}$. So, that will be the first term second term is as usual. So, you have $div\ \boldsymbol{U}$ and this will be the bold U. So, that will be you know. So, you will have this is the bold part. So, then that will be the again $\frac{1}{\rho}\frac{\partial P}{\partial x}$ again I mean pressure has come here, then you have $\upsilon\ div.(grad\ U)$. So, that is you know coming further and then we have the another term that is $\frac{1}{\rho}$ of now this is your component which will be we are taking the div of those components.

So, that will be $\frac{\partial(-\rho\overline{u'^2})}{\partial x} + \frac{\partial(-\rho u'v')}{\partial y} + \frac{\partial(-\rho u'w')}{\partial z}$,. So, this basically these terminologies will come if you try to expand it. So, because this u' will be u'+$v' + w'$ this $\boldsymbol{U}$. So, that will be you know u'+$v' + w'$.

So, it will have $u'u' + u'v' + u'w'$and all the averaging will be done and it is div. So, you will have $\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$. So, that will be coming. So, that is what you are getting these 3 terminologies which is coming on the right hand side and this is basically the you know this is because of these turbulent shear stresses.

So, if you have the you know further if you try to write these equations. So, you will have for the V momentum you will have $\frac{\partial V}{\partial t} + div(V\boldsymbol{U}) = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\ div.gradV$. So, and then you will have again $\frac{1}{\rho}$ and now this time you will have you know $\frac{1}{\rho}[\frac{\partial(-\rho\overline{v'^2})}{\partial y} + \frac{\partial(-\rho u'v')}{\partial x} + \frac{\partial(-\rho u'w')]}{\partial z}$. So, that will be you know coming.

So, this way you will have 3 you know equations, these you know these extra stresses these terminologies which we have taken from the left hand side towards the right hand side your equation you know becomes like this. So, you will have $\frac{\partial w}{\partial t} + div(W\boldsymbol{U})$. So, that will be $-\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu\ div.grad\ W$+$\frac{1}{\rho}[\frac{\partial(-\rho\overline{w'^2})}{\partial z} + \frac{\partial(-\rho u'w')}{\partial x} + \frac{\partial(-\rho w'v')]}{\partial y}$. So, that is this way you are getting these same equations, but then these once you expand it and take towards the right hand side because all these stresses you are taking on the right hand side. So, that will give you these 3 you know equations.

Now if you look at the equations this $-\rho u'u'$ this is basically u'^2, now this $-\rho u'^2$. So, this is basically τ_{xx} similarly $-\rho u'u'$ it will be τ_{xy}.

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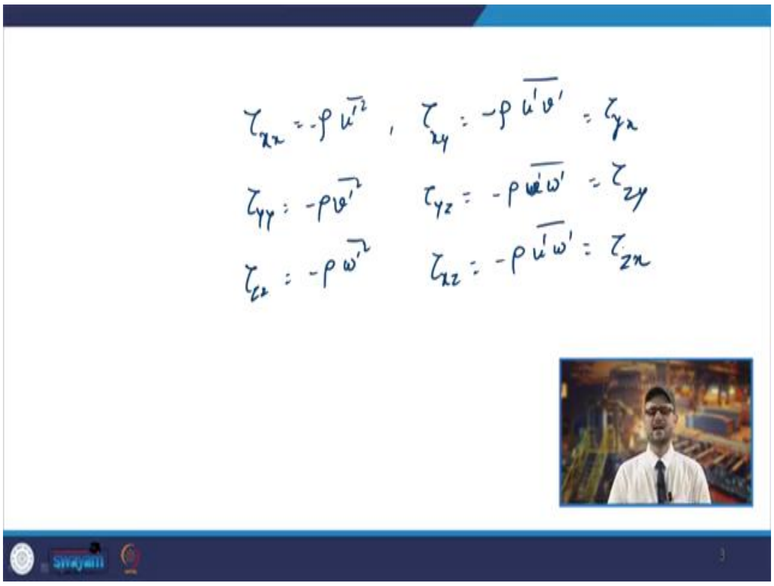

So, you know the, so because of that. So, that is why we can write τ_{xx} that we write as minus of rho and u prime square. So, that is your you know this is the turbulent stresses for that you have. So, and then you have τ_{xy} this will be $-\rho u'v$ similarly you have you know τ_{xy} is τ_{yx} and similarly τ_{yy}will be $-\rho v'^2$ everything of that and the τ_{zz} will be actually $\rho w'^2$.

So, that way and you have τ_{xy} similarly so that will be same as τ_{yx}, similarly τ_{yz}will be minus of rho you know v prime w prime actually v prime w prime so, that will be τ_{zy} and similarly τ_{xz} will be $-\rho u'w'$ and that will be you know τ_{zx}. So, this way these stresses term are you know appearing on the right hand side. And these are known as the Reynolds Averaged Navier- Stoke Equations.

Now if you try to find you know the you know for any extra turbulent term you know which will be because of the transportation of certain scalar quantity like temperature or so, in those cases you will have the time averaged transport equation.

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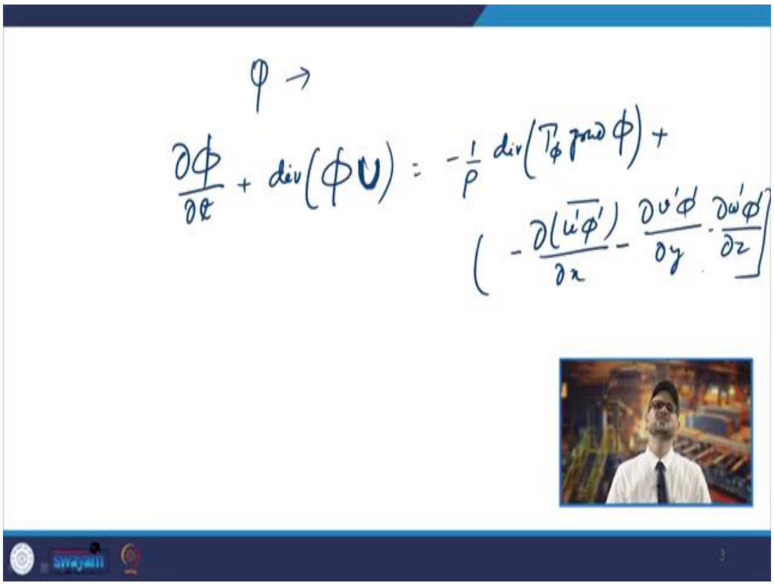

So, for you know that any scalar quantity transport scalar quantity you know φ So, for any this of the transport scalar quantity ψ you will have these time averaging equations. So, you will have the mean part of it will be $\frac{\partial \varphi}{\partial t} + div(\phi \boldsymbol{U}) = -\frac{\mathbf{1}}{\boldsymbol{\rho}} \boldsymbol{div}(\Gamma_{\varphi} grad\varphi) + (\frac{\partial(-\rho\overline{\varphi'^2})}{\partial z} + \frac{\partial(-\rho u'\varphi')}{\partial x} + \frac{\partial(-\rho\varphi' v')]}{\partial y}$.

So, this way you will have you know the transport equation for any scalar and normally we will have the temporary equation for the temperature or any other quantity when we have.

So, we normally have these equations also this is. So, this is what you get normally you know this is the extra term which you are you know getting. So, these are the equations these as known as the Reynolds Averaged Navier- Stoke equations, because we are doing the averaging and we are taking that fluctuating part in to component and this will be used while we try to study or try to model the you know turbulent equations you know or turbulent flows so that we will see in our coming lectures.

Thank you very much.