Modeling of Tundish Steelmaking Process in Continuous Casting
Prof. Pradeep K. Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 22
Characteristics of Turbulent Flow

Welcome to the lecture on Characteristics of Turbulent Flow. So, we will have few more aspects about the turbulent flow and then how we start modeling you know what are the approaches towards modeling some of the parameters in power turbulent flow; so, that we will study in this lecture.

So, we will talk about initially some of the examples of the turbulent flows and especially with the constant imposed pressure there will be some kind of flow which is coming under the these examples like you have free turbulent flow.

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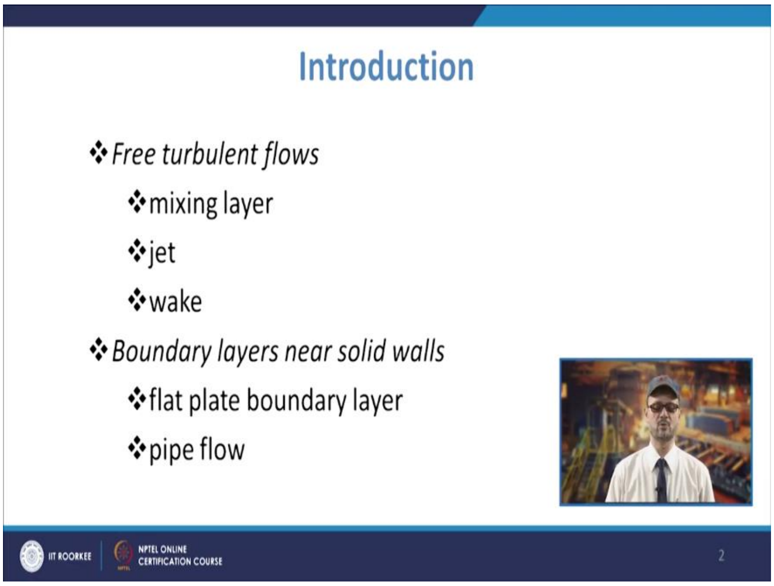

So, you will have the example of mixing layer or jet or wake. So, these are the examples of free turbulent flows whereas, if you talk about the boundary layers near the solid walls. So, in that you may have the flat plate boundary layer or the pipe flow. So, they are basically coming under the boundary layer near the solid walls.

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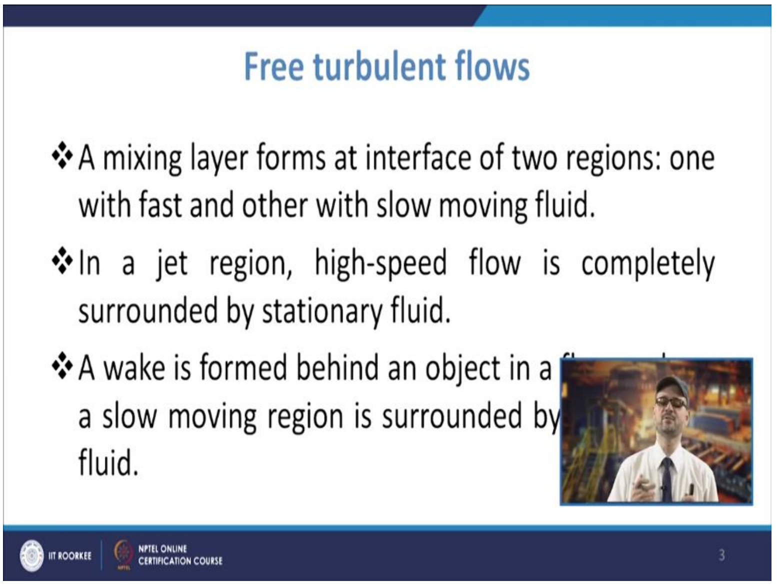

So, if you see the free turbulent flow so, in that case you will have 3 kind of you know flows. So, if you talk about the mixing layer. So, the in the mixing layer basically a mixing layer will be forming the interface of two regions: one with fast and one with the slow moving fluid. So, that is how they the mixing layer is defined.

So, you will have the interaction between the fast and the slow moving fluid similarly when you talk about a jet reason. So, there you have a high speed flow. So, you are leaving a jet. So, that is that has high speed flow and then when it goes. So, it is surrounded by the stationery fluid. So, then how you know the how it behaves how the you know there is interaction from the surrounding stationary fluid of the high speed flow. So, that is also the example of the free turbulent flow.

Then also there is a wake kind of situation where it is formed behind an object in a flow. So, here we will have a slow region moving region and that will be surrounded by the fast moving fluid. So, the first moving fluid is there and there is a behind the object you will have that interaction is there in the case of the wake. So, these are the examples of the you know some of the examples and we will talk about few of them and then we will talk about how the velocity profile or how you know you are going to have the calculation of the stresses and all that so that we will be discussing.

So, you know so, initially so, in these cases velocity changes which you know take place in the initial thin layer. So, that becomes very important in all these 3 flows and then you will have the transition to turbulence will be occurring after very short distance in the flow direction from the point where the different streams basically will be initially meeting.

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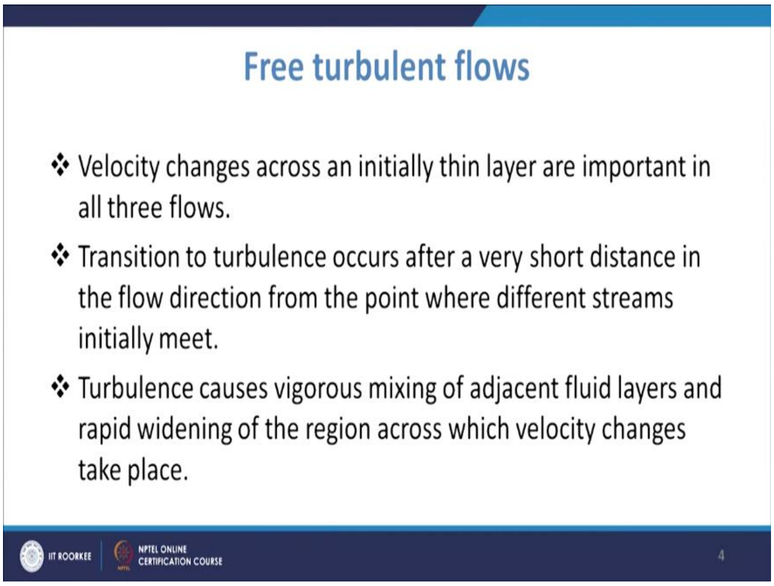

And turbulence will be causing the vigorous mixing of adjacent fluid layers and rapid widening of the reason across which were still changes take place.

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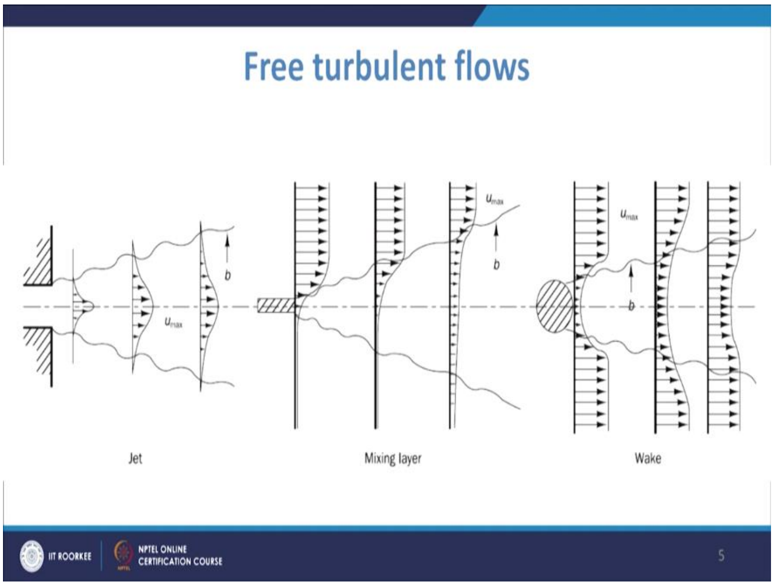

So, that can be understood by you know the example of these 3 kind of flows. So, that is normally for the case of jet flow where you see that you have a very high speed you know jet which is coming and it is, it has the interaction which is the stationary fluid. So, you will have a stationary on the sides and that is how the velocity profile come out to be.

Now in the case of mixing layer you will have a smaller moving your so, slow moving reason and that will be interacting with the fast moving one. So, that way you have mixing layer and development of these velocity profile you will see. Similarly this is the wake so, you will have a wake and in that when it will be velocities so that way, what you see that this is how the change in the velocity profile looks like. So, these are the examples of the free turbulent flows.

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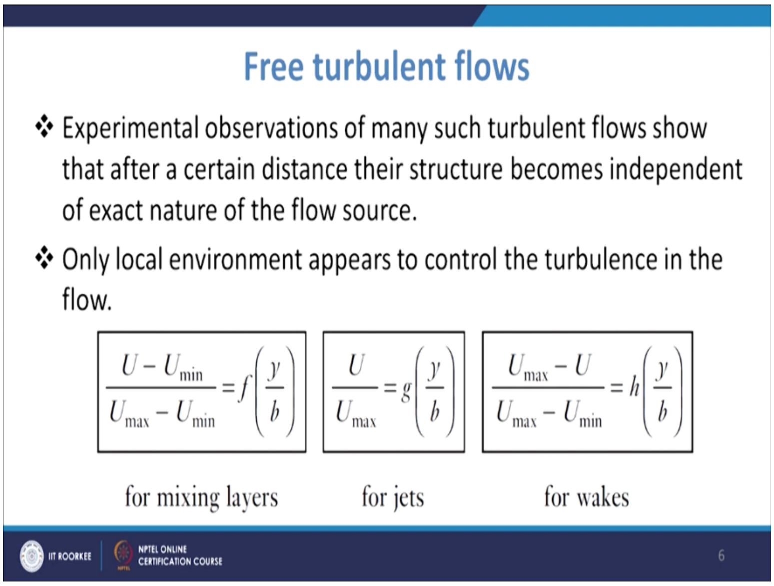

Now, when we talk about the experimental observations of the turbulent flows then what has happened that you know these for the for mixing layers. So, you will have these are the you know the equations which describe the mixing layers like you have $\frac{U-U_{min\cdot}}{U_{max}-U_{min}} = f(\frac{y}{b})$

So, basically the U_{max}, U_{min} they will be representing the maximum and minimum velocities. So, that will be at a distance y. So, that will be you know or at a distance x downstream. So, accordingly your, you will have the $\frac{U}{U_{max}}$. So, you know so, that you can see $\frac{U}{U_{max}} = g(\frac{y}{b})$; similarly you will have. So, this y is the distance in the cross stream direction and x is the distance in the, in that stream that stream direction.

So, you will have the $\frac{U}{U_{max}}$ it will be function of y by b similarly in this case $\frac{U-U_{min}.}{U_{max}-U_{min}}$ that will be for the as we a function of $\frac{y}{b}$. So, that will be y in the cross stream and then you will have similarly you will have for the and b is basically the cross stream layer width.

So, that is half width so that is how b you are taking and for the wakes what you see that this you see $\frac{U_{max}-U.}{U_{max}-U_{min}} = h(\frac{y}{b})$.. So, that is what is these formulas are there, you know which talked about these variations in the U which U_{max}, U_{min} and it will be a function of how it is for the $\frac{y}{b}$.

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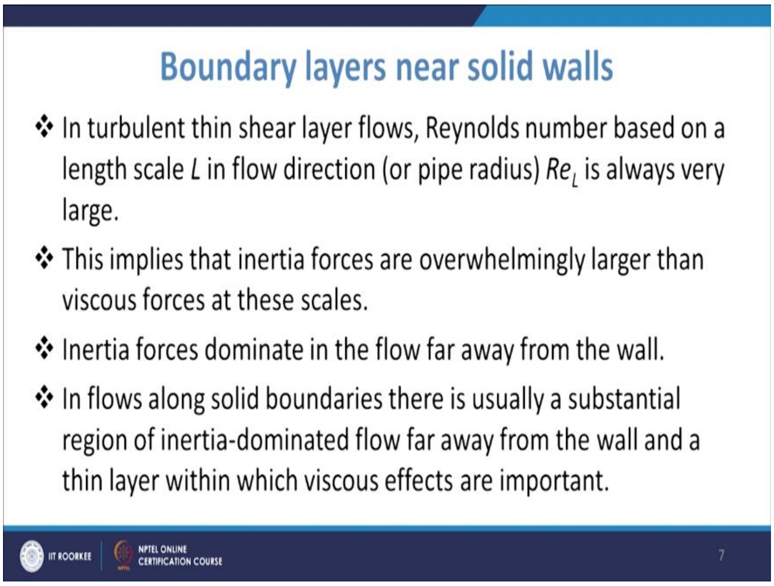

Now, if you go for the boundary layer near the solid walls.

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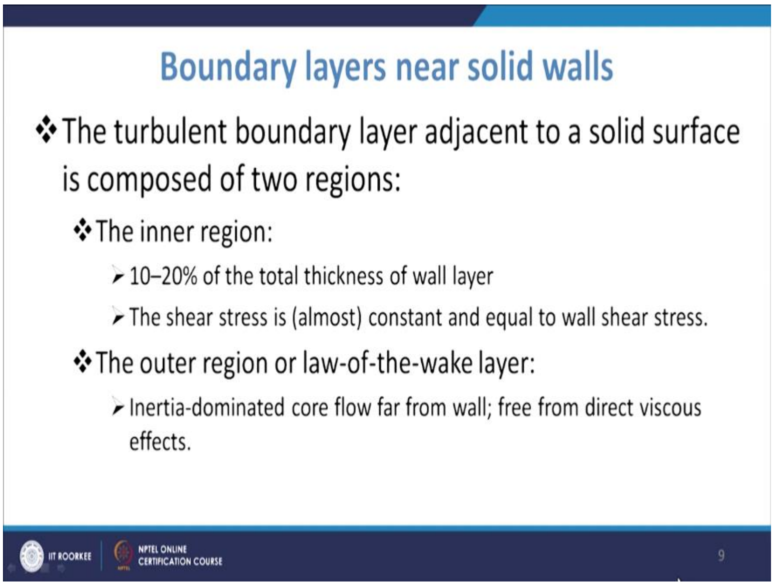

So, if suppose we are taking the case of boundary layer the near the solid walls. So, you know in the turbulent thin shear layer flows this Reynolds number based on length scale L in flow direction that is Re_L is always very very large. So, if you are taking that you know length scale L and if you are measuring that turbulence value this Reynolds number value that will be always very large and it that you know implies or; that means, that the flow is very much inertia dominated.

So, that is what it means that the flow is when the Reynolds number is very large and if you know that Reynolds number will be you know more than I mean the ratio of the inertia to the viscous forces. So, basically you will have very large the value of Reynolds number means that your Reynolds number is I mean inertia force is dominating over the viscous force.

Now if you talk about the inertia forces. So, that will be happening when your the flow is far away from the wall. So, there you will have this thing taking place in flows along the solid boundaries there is usually a substantial region of inertia dominated flow far away from the wall and a thin layer within which the viscous forces are important. So, you have basically a very thin layer where the viscous forces are important.

And, once you go to the you know region which is far away from the wall they are the inertia force I mean tries to be more important or more dominant than the viscous force. Now, if you talk about the close to the wall the flow will be influenced by the viscous effects and does not depend on the free stream parameters and the mean flow velocity will be only depending on the distance y from the wall fluid density rho and viscosity mu and the wall shear stress tau w.

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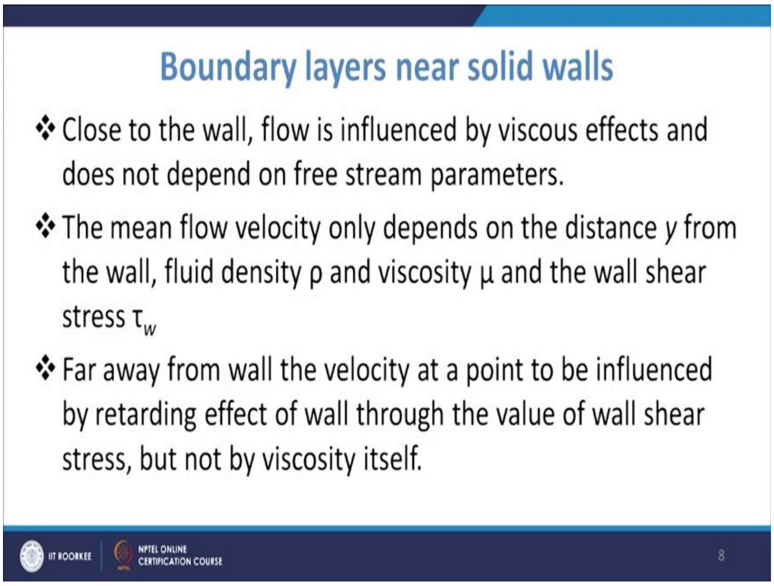

So, this point I mean this that can be represented by like if you are talking about this case like, where the at the distance y you will have this close to the wall it will be depending upon.

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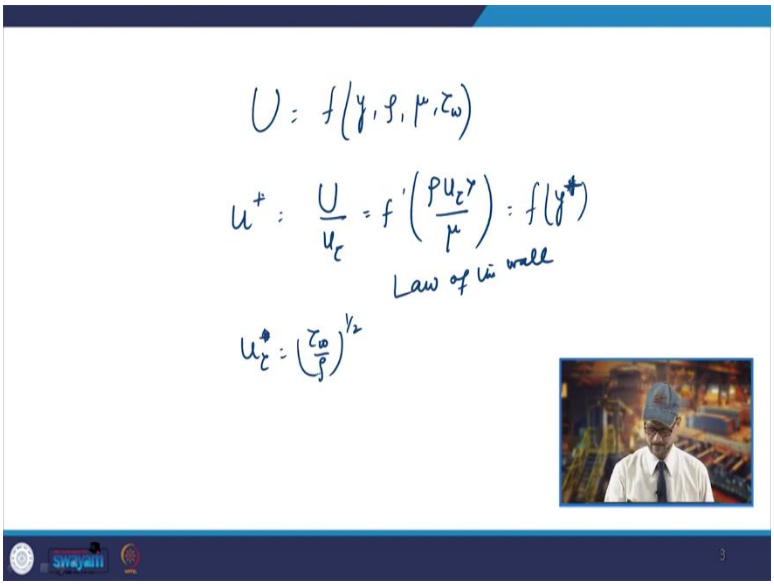

So, in that case your $U = f(y, \rho, \mu, \tau_w)$, now this$\tau_w$ is the wall shear stress in this case. So, this is basically for the close to the wall and when you do the dimensional analysis in this case then what you see is that the dimensional analysis will show that you get u^+that will be equal to $\frac{U}{u_\tau}$.

So, u_τ is the known as the friction velocity. So, and that will be basically $\frac{\sqrt{\tau_w}}{\rho}$ now that will be basically a function of you know $\frac{\rho u \tau_y}{\mu}$. So, this way you know you get another you know number I have been another parameter. So, that is $\frac{\rho u \tau_y}{\mu}$ and that is also known as $f(y^+)$.

So, this y^+no this is simply y^+. So, this y^+ will have certain values and certainly on the wall y is 0. So, it will be 0 and as you move from the wall y^+value will go on increasing. So, this formula u^+it will be a from you know $f(y^+)$. that is $\frac{U}{u_\tau}$. So, this equation is known as the law of the wall.

So, you will have you know two important dimensionless groups one is u^+ another is y^+. So, that way and this u^+ is basically defined as you know u_τ that is u_τ is defined as $\frac{\tau_w}{\rho^{1/2}}$. So, this is basically the friction velocity τ_w you know u_τ is known as and τ_w is the wall shear stress and ρ is the you know density. So, that is how you will have the 2 dimensionless groups you are getting you know in this case.

Now, when you go away from the wall now in that case you know the. So, if you go away from the wall the velocity at a point you know they can be considered to be influenced by the retarding effect of the wall you know through the value of the wall shear stress, but not by viscosity itself. So, as you know we move so far away from the wall so, there basically you will have the influence of the wall shear stress not because of the viscosity is itself. So, the length scale in this region will be the boundary layer thickness that is delta.

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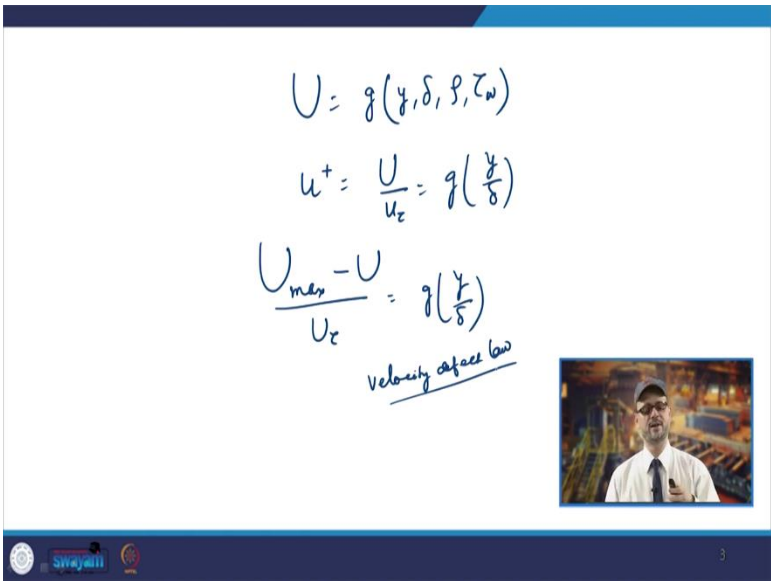

So, in that case when you move far away from the wall now in that case your U becomes another function $g(y, \delta, \rho, \tau_w)$,. So, you will have the y cross length you will have the boundary layer thickness you will have the density and also the wall shear stress and that is what is normally there when you are moving you know away from the wall. So, in this case if you do the dimensional analysis you get again u^+ that will be again $\frac{U}{u_\tau}$ that is your frictional velocity and that becomes you know $g(\frac{y}{\delta})$.

So, this you know this is the most useful form which you get and you know the wall shear stress basically you know what we seen it has the cause for the velocity deficit that is $U_{max} - U$ and it will be decreasing. So, as you go towards the boundary layer edge so, the pipe centerline. So, what we see in this case $U_{max} - U$. So, that will be and divided by u tau. So, that becomes the function of $g(\frac{y}{\delta})$.

So, that is what we had seen earlier also in the in those cases that you get such kind of relationship there. So, this what you get this is basically known as the velocity defect law. So, we are getting the velocity deficit in that case and that is why we call it as the velocity defect law. Now we move to the linear or viscous sub layer now that has to be for the you know linear or viscous sub layer.

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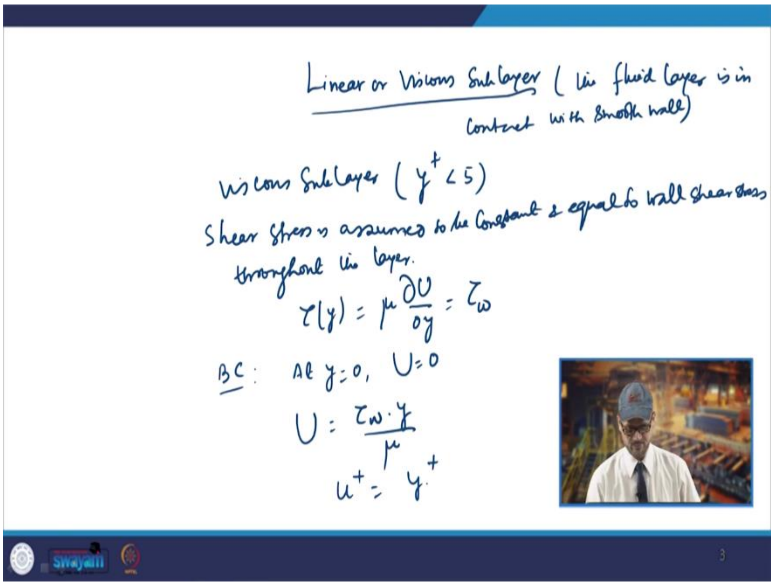

So, if you go to the linear or viscous sub layer. So, in that basically that the fluid layer is in contact with smooth wall. So, now, if you talk about that linear or viscous sub layer where the fluid layer is in contact with the smooth wall it means the since the wall is stationary. So, the fluid which is there that is also stationary, now the turbulent adding you know motions which are there. So, close to that will also be stopping and we will have you know. So, you will have the domination by the viscous effects there in that condition.

Now the viscous sub layer which is there so, that basically is you know very very thin. So, this viscous sub layer; so, being very thin it is basically confined to $y^+ < 5$ and it can be assumed there that the shear stress. So, you know shear stress will be constant approximately. So, shear stress is assumed to be constant and equal to the wall shear stress τ_w throughout the layer. So, in that layer as the layer is very very thin you can assume that the shear stress will be tau w that is constant throughout that layer. So, $\tau(y)$ will be $\mu \frac{\partial u}{\partial y}$ and that will be τ_w. So, that is what there in the case of you know a linear or viscous sub layer.

Now if you do the integration of this and if you apply the proper boundary condition the boundary condition will be that if you know at $y = 0, U = 0$. So, in this case what we see we get a linear relationship and after integrating what we get is $U = \frac{\tau_w y}{\mu}$ that is what you get from here once you get the integration constant then that you know. So, put in the boundary condition you get this value $U = \frac{\tau_w y}{\mu}$. So, you can have the further you can you know what you see earlier you have got the $\frac{U}{U_\tau} = u^+$ and y^+ also you know you see that is $\frac{\rho u_w y}{\mu}$. So, you can write here from U by $\frac{U}{U_\tau}$ and all that.

So, you can write here. So, with some algebra you can further define u^+ and y^+ and you get the u^+ is equal to y^+. So, you know this you have a linear relationship you know here between the u^+ and the y^+. So, that is between the velocity and the distance from the wall and that is you know in the fluid layer which is adjacent to the wall. So, you this is a linear sub layer that is why it is known as the linear sub layer because you have the linear relationship between u plus and y plus and that is why it is known as linear sub layer.

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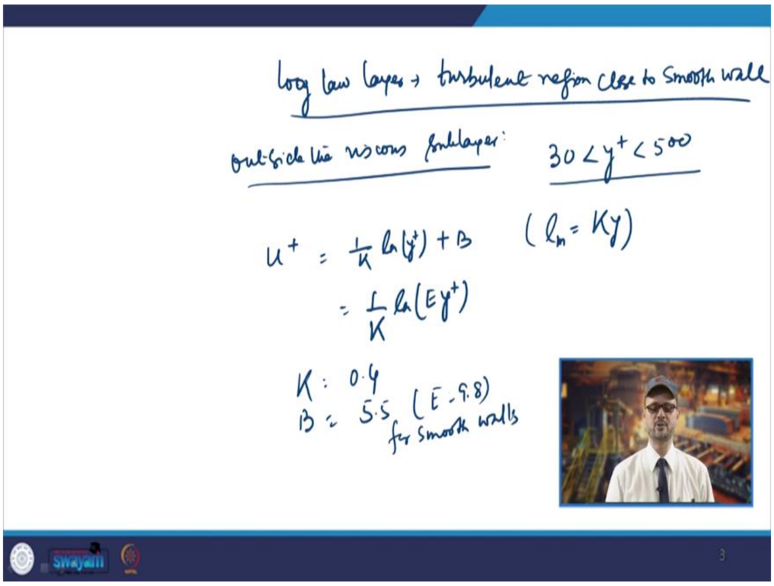

Now, if you go to the outside this viscous sub layer. So, you have a log law layer. So, that is basically the turbulent region close to smooth wall. Now, if you go to you know this region where you know in this region will be basically by region where the y^+ value will be more than 30 and less than 500.

So, here this is outside the viscous sub layer. So, in that region your y plus will be more than 30 and it will be less than 500. So, here these both these viscous stresses as well as the you know turbulent effects both are important in this region that is when you are going for the y^+ value more than 30.

Now, in this case the shear stress τ slowly will be you know varying with the distance from the wall and within the inner region it will be assumed that. So, what you see this y^+ for this value y^+ between 30 and 500. So, you are assuming you know these shear stress to be constant and equal to the wall shear stress in that thin region and in that basically what you see you get these u plus value you will be getting the $\frac{1}{K}ln(y^+) + B$. So, this is what you see what you get you know in the case of the that region which is outside the viscous sub layer.

So, here the shear stress tau will be varying slowly with distance from the wall and in the inner region it will be assumed to be constant and that will be equal to the wall shear stress and regarding the length of the turbulence mixing you know mixing length here $l_m = Ky$. So, see this you know u^+. So, you will have the relationship between u^+ and v^+in terms of this K that is you know. So, this will be $\frac{1}{K}ln(y^+) + B$ and we also take it as $\frac{1}{K}ln(Ey^+$ we are taking another you know constant. So, we call it as the $\frac{1}{K}ln(Ey^+)$.

So, basically because this is a logarithmic function in that u^+ and y^+ is varying in a logarithmic manner that is why we call it as a log law layer and in this case what has been found people have done the experiments and the numerical value of this kappa that value is taken to be about 0.4 and B is taken out to be 5.5 or even E is taken as 9.8. So, basically this is known as the Von Karman constant and this Von Karman constant is basically you know 0.4 and your this constant B it is 5.5 or you can have E value as 9.8 for smooth walls.

And, once you have the rough walls so, you will have the decrease in the value of B. So, your u^+ will decrease for the rough walls. So, your B will be decreasing and because of these you know relationship we call it as the log law in this case. So, that is why it is known as the log law layer because of this logarithmic you know function which is seen here. Now, we will come to another layer that will be your outer layer.

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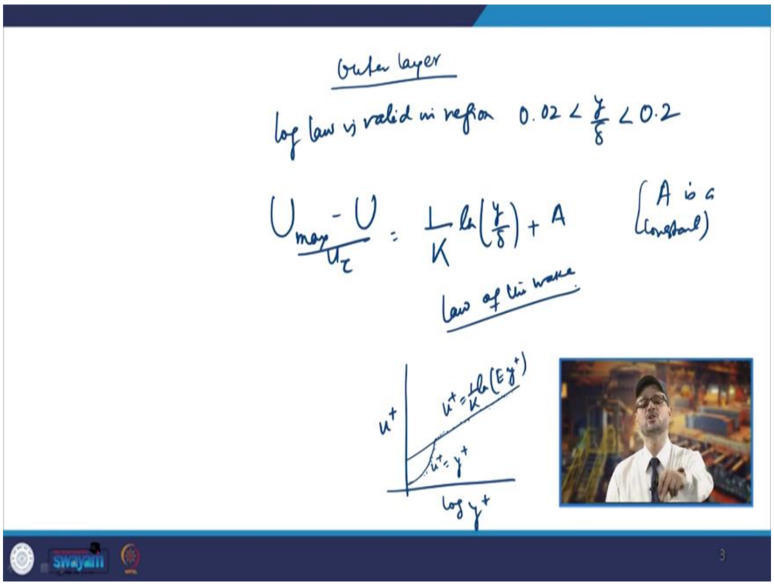

So, if you go to the outer layer. So, you know the log law of layer that is shown to be valid experimentally in the region. So, log law will be valid in region. So, $\frac{y}{\delta}$ so, in that value has to be more than 0.02 and less than 0.2. Now for the larger value of the y the velocity defect law which we have got that gives you the correct you know form and in the overlap region basically you have the log law and the velocity defect law that has to be equal.

So, based on that you can have the you know the relationship and the researcher you know Lumley. So, they have shown that there will be a matched overlap in that you know. So, you will have that function so, by over log if you make it equal. So, you will have $\frac{U_{max}-U}{u_\tau}$. So, that will be basically equated to this you know log. So, that will be $\frac{1}{K}\ln(\frac{y}{\delta})$ so.

So, that is basically you know applicable for the outer layer and here A is constant now the thing is that you know this law this is the velocity defect law only and this law is many a times now also called as the law of the wake. So, there has been closed you know findings by the researchers especially resisting was one of the researcher who has done large amount of work in the area of he has a standard book also on the boundary layer theory by resisting.

So, that we refer also. So, in that so, they have found that you have basically the if you go try to have the velocity profile. So, what you see that if you try to have the, velocity profile we have $u^+$and you have $\log(y^+)$. So, what you see you will have. So, this is your a linear profile and your profile looks would going like this.

So, in this case you have u^+ is y^+. So, that is that is this region and then otherwise you have here this reason is u plus is $\frac{1}{K}ln(Ey^+)$ this is your you know log law layer where are these logarithmic you know relationship holds good. And so, these are different points which are you know coming and it has been found to be you know here correct. So, you have this region as the you know log law layer so, here these log rule holds good in this case.

So, what we see that the turbulent boundary layer adjacent to the solid surface is composed of two regions you have the inner region which is 10 to 20 percent of total thickness of the wall layer. And, the shear stress will be almost constant equal to wall shear stress that is what we have seen outer region will be where it is the law of the wake layer. So, there you will have the inertia dominated core flow from the wall and it will be free from the direct viscous effects.

So, when you go from the wall away you will have inertia effects getting more and more dominated then if you go to boundary layer near the solid walls inner layer will have 3 zones. So, you will have a linear sub layer where the viscous stresses will be dominating the flow adjacent to the surface because there because of the wall also the flow becomes 0.

So, there the viscous stress is dominating you have a buffer layer where viscous and turbulent stresses are of similar magnitude and then you have a log law layer where the turbulent stresses dominate. So, these are you know so that basically now why we have studied this because they will be used when we go and solved it on this flow. When we talk about the flow near the wall so, how you have to model, how these you know quantities are to be computed, how the depending upon the y^+s value, how you have to take the different wall functions.

So, these things are basically you know very important to be understood that you know in which region which way these $u^+$and $y^+$is varying and what are the functions and accordingly you can have the calculation of the wall shear stress. And, then other related components that can be you know calculated in the case of any such kind of turbulent flows which we normally for the you know flat plate near representing the flat plate boundary layer or even other kind of you know flows in the industrial you know applications.

Thank you very much.