Modeling of Tundish Steelmaking Process in Continuous Casting

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Lecture – 18
Momentum Conservation Equation

Welcome to the lecture on Momentum Conservation Equations. So, in the last class we discussed about the conservation equation for mass. And, also we had the concept about the total derivative or the substantial derivative and we had seen that when we are talking about the terms like $\rho\frac{Du}{Dt}$ or $\rho\frac{Dv}{Dt}$ or $\rho\frac{Dw}{Dt}$.

So, there will be the terms for the x, y and z momentum and keeping that in mind we are going to have the expression for the equations of conservation of momentum.

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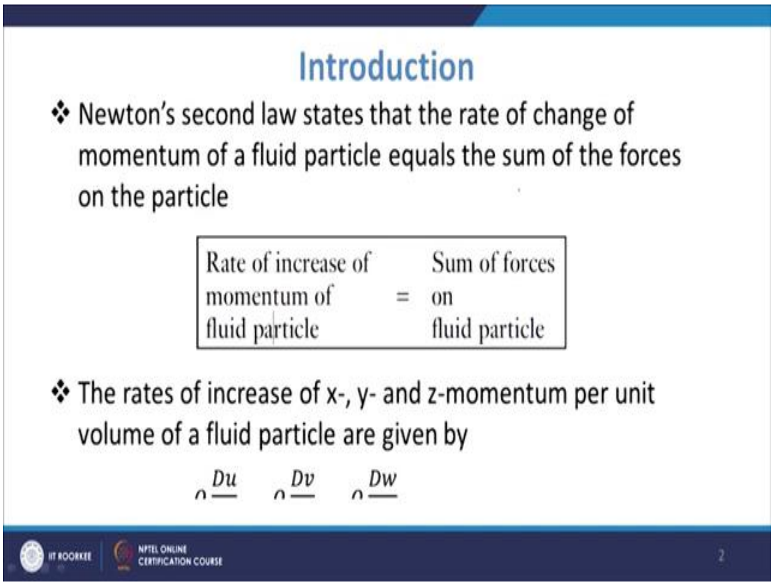

So, what we had seen you know if you talk about the momentum conservation equation. It is basically derived from the Newton's second law which tells that the rate of change of momentum of a fluid particle equals the sum of the forces on the particle. So, you will have that rate of increase of momentum that will be sum of forces. Now, rate of increase of momentum for the fluid particle so for the x y and z direction that momentum a per unit volume that will be you know represented by $\rho\frac{Du}{Dt}$ or $\rho\frac{Dv}{Dt}$ or $\rho\frac{Dw}{Dt}$ which is not appearing fully here.

So, that is the substantial derivative $\frac{Du}{Dt}$ or $\frac{Dv}{Dt}$ or $\frac{Dw}{Dt}$ and that also is you know we had already seen that you know for the component $\rho\frac{Du}{Dt}$.

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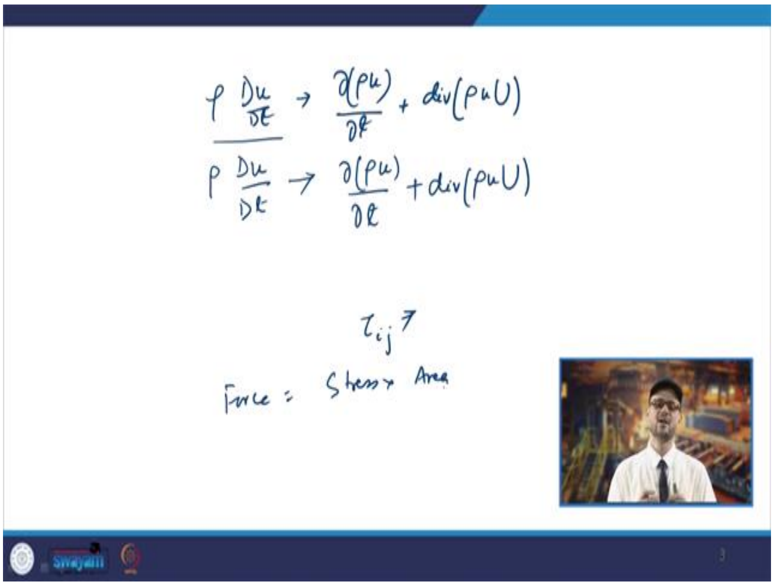

So, that will be you know $\frac{D\rho u}{Dt} + div.(\rho uU)$. So, while deriving that expression for the x momentum term and using that total derivative concept what we saw this $\rho\frac{Du}{Dt}$. So, that is you know that is for the x momentum and that is basically becoming equal to $\frac{D\rho u}{Dt}$.

So, this is the taking time into picture and then it will be $div.(\rho uU)$. So, that way you had a this x momentum. Now, similarly you will have the expression for the y momentum that is $\rho\frac{Du}{Dt}$. So, that will be $\frac{D\rho v}{Dt} + div.(\rho vU)$. So, that is then $\rho\frac{Dw}{Dt}$. So, like that we had seen these terminologies in our last class when we try to define this substantial derivative term.

Now, coming to the momentum conservation equation what we see that in this case you have the you know. So, once you have the rate of increase of momentum of fluid particle that is by this term and then we are going to have the discussion on the sum of forces on the fluid particle.

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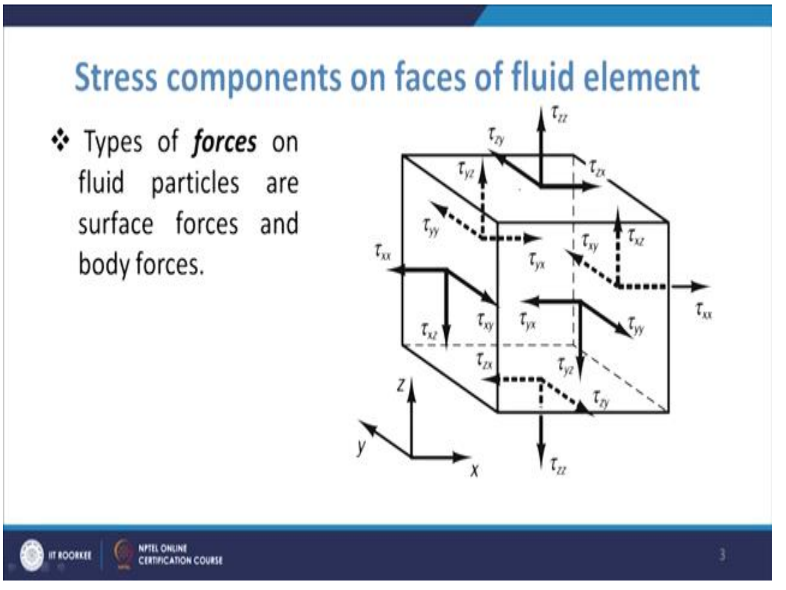

So, the forces we have to analyze and the forces which are typically you know there which are acting on the fluid particle they are the surface forces and the body forces. So, you have you know the pressure force or you have the viscous forces are there. Then also you have forces like; so, normally pressure and viscous forces they we used to take them in terms you know and we have also the gravity term.

So, that is normally taken as the source term; so, that we will see that when we derive with these equations then that time they are taking as the source terms. So, you know if you talk about the stresses which are acting. So, you have the surface forces which are taken as the separate terms in the momentum equation and the body force as we discussed that we take as the source term in the equation. Now, if you talk about the you know in component of forces to.

So, you will have these and we will talk about the x component of forces and then we will equate to the momentum term. So, we will have the rate of change of momentum that is equal to the force. So, we will have the you know forces that we have to see in the x direction. So, now, if you talk about these you know a fluid element. So, you have the different you know stresses which are acting on it and now you have as we have seen that you have the notation for the stress also like τ_{xx}, τ_{xy}, and τ_{xz},.

So, one will be the direction of the stress another will be we will be that and you know plane on which the normal direction is there. So, that way you have these two you know ij is there while we define these stress terms. So, what we see that we have the pressure term and also we have the 9 viscous stress components are there which are acting on the fluid and that is what is shown in this picture.

So, the pressure is the normal stress that is p and the viscous stress is denoted by the term τ. So, that is the you know the normal practice which we have and also we know that we have the direction τ_{ij},. So, you will have you know τ_{ij}. So, you know it will be indicating that direction of in which the stress will be acting and also you will have the i direction that is the surface normal to that i direction. So, that you have. So, accordingly you will have a τ_{xy} τ_{yx}, τ_{yz}, τ_{zy} τ_{xz}, τ_{zx} or so.

So, that way you will have these 9 stress components. Now what we need to know is that you have to analyze the forces which are resulting from these surface stresses and that will be force will be basically because of the surface stress and then multiplied by the area. So, force will be the stress times the area. So, that way you will have the, you have a finding the forces and you will have certainly positive and negative sign depending upon you know the place where it is applied.

So, on the coordinate axis if you are on the positive side you will have positive sign and otherwise you will have the negative sign and you will be finding the net force in the x direction and it will be the sum of the forces which will be acting in that direction on the fluid element.

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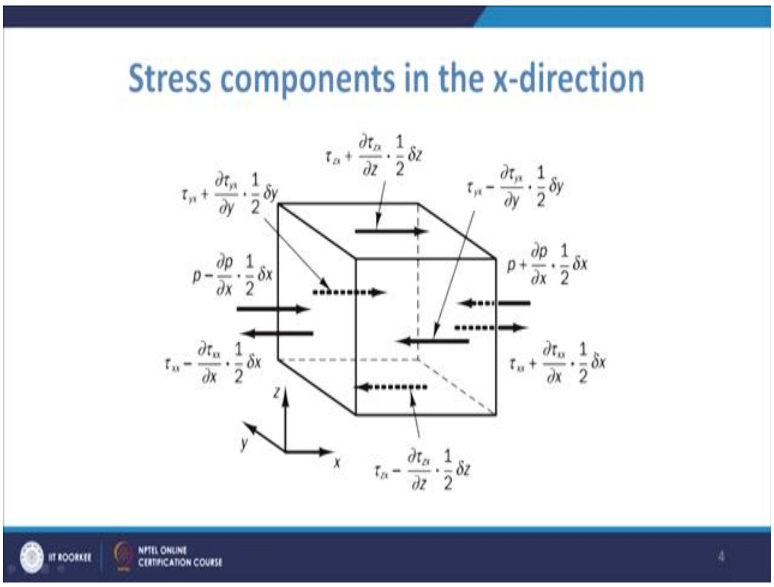

So, we will be talking about these forces and if you look at the you know the forces which are you know acting. So, you see you will have the pressure force or pressure being normal force you will have the element. So, on this side the negative x directions. So, you will have $p - \frac{\partial p}{\partial x}\frac{\delta x}{2}$ and similarly on our from this side you will have $p + \frac{\partial p}{\partial x}\frac{\delta x}{2}$

So, that is you know for the pressure term. Now if you come to the , τ_{xx} term. So, τ_{xx} term again you will have this for this direction. So, from here it is on the negative x direction. So, it will be , $\tau_{xx} - \frac{\partial \tau_{xx}}{\partial x}\frac{\delta x}{2}$ is the you know half of this length. Similarly on this side so we are basically neglecting the you know the terms which is coming after the second term in the Taylor's series expansion.

So, you will have $\tau_{xx} + \frac{\partial \tau_{xx}}{\partial x}\frac{\delta x}{2}$. So, this will be on these 2 sides. Now if you talk about other stress components like this term. So, you see this is direction is you know z and I mean x. So, this is tau the second component is x and the this is perpendicular to that z direction plane. So, you will have τ_z. So, this is τ_{zx} and since from the center this is half of δz above. So, on this phase the stress which will be acting will be $\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2}$..

Similarly, on this phase you will have $\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2}$... So, that way you will have these stress components and then they will be multiplied by these areas. So, that way in this side if you look at this will be $\delta x \delta y$. So, that way if you look at terminologies like you know τ_{yx}or you will have τ_{xx} and τ_{zx}. So, you will have you know different you know these value of a stresses and we will analyze the value of these stresses.

So, coming to the different faces if you come to the east and west face; so, if you come to this and this face. So, what you see. You have the stress $p - \frac{\partial p}{\partial x}\frac{\delta x}{2}$ here and this $p + \frac{\partial p}{\partial x}\frac{\delta x}{2}$. So, this is here and also similarly you have $\tau_{xx} - \frac{\partial \tau_{xx}}{\partial x}\frac{\delta x}{2}$ and this side you have a $\tau_{xx} + \frac{\partial \tau_{xx}}{\partial x}\frac{\delta x}{2}$.

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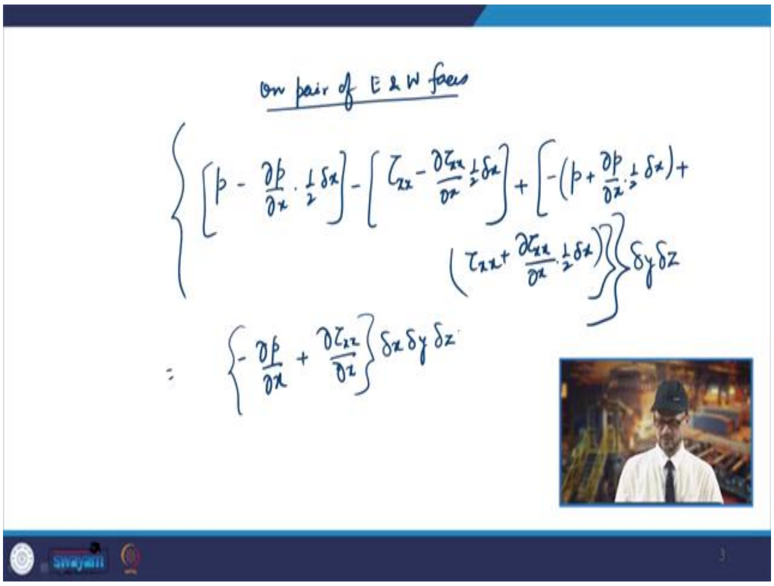

So, if you talk about the you know on the east and west faces. So, on pair of east and west faces if you know take the forces into consideration. So, what you see. You see the on the left hand side because of the pressure you get $p - \frac{\partial p}{\partial x}\frac{\delta x}{2}$. So, that will be there and then you have the negative of the $\tau_{xx} \; - \frac{\partial \tau_{xx}}{\partial x}\frac{\delta x}{2}$. So, that is the this τ_{xx} term is.

Now, in this case in the case of pressure since we take the compressive as the positive once we are taking this positive value. Now then you have the term you have if you go from the opposite side. So, now, this is basically because of the sign it is having this positive value. Now if you take the from the right hand side your pressure term is in the negative x direction. So, you will have -($p + \frac{\partial p}{\partial x}\frac{\delta x}{2}$).

And then you have the term that is $\tau_{xx} \; + \frac{\partial \tau_{xx}}{\partial x}\frac{\delta x}{2}$; so, this term. Now, these term all these terms so, this term as you. So, if you take whole term now this you know whole term if you take that will be multiplied by the area that area of that face. So, this face; so, this is your δz and this is your δy. So, it will be multiplied by $\delta y \delta z$. So, this is the you know forces which are on the E & W face in that you know x direction.

So, if you add these terms. So, you will have you know this p and this p will cancel and so you will get you know the- $\frac{\partial p}{\partial x}$ and you know and you have this δz term will come. So, this is $\frac{\delta x}{2}$ and this is $\frac{\delta x}{2}$. So, that it will be $\delta x\ \delta y$ and δz that will be going out and in this side you have you know τ_{xx} and here also you have $+\tau_{xx}$. So, this $-\tau_{xx}$ and $+\tau_{xx}$will cancel.

So, you will have the term $+\frac{\partial \tau_{xx}}{\partial x}$and here again you have $+\frac{\partial \tau_{xx}}{\partial x}$ and then you have δx term coming into picture. So, the term becomes $\frac{\partial \tau_{xx}}{\partial x}$ and in both the cases you have δx coming here and $\delta y \delta z$ is outside. So, you will have the term that is $\delta x \delta y \delta z$. So, this is you know on the pair of in form of east and west faces if you take that is how these term is coming a $(-\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x})\delta x \delta y \delta z$. Now, if you take the net force in x direction on the pair of north and south faces so.

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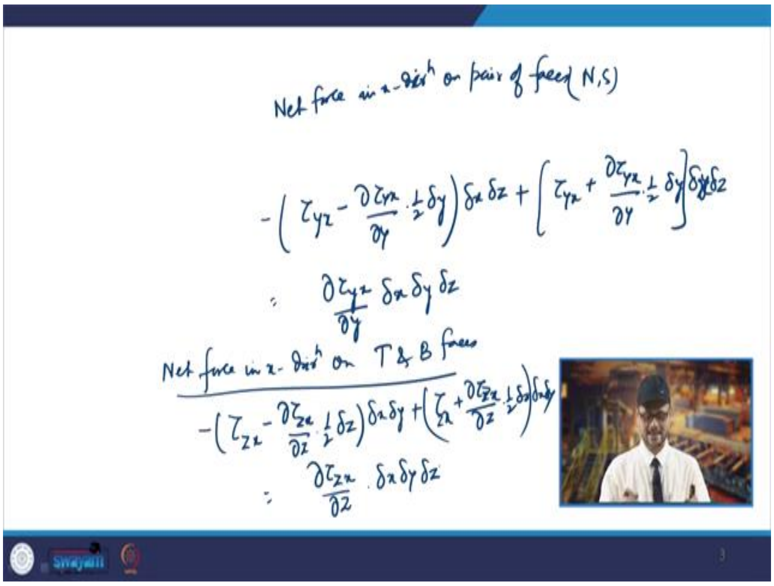

So, it will be the net force in x direction on pair of faces north and south. So, that can be seen this is your north face and this is your south face and the forces which are acting in the x direction. So, you will have τ_{zx} and this is τ_{zx} on this side. So, you will have accordingly you will have the you know. So, you can have; so now this is north and this is south face, this is your top and this is bottom face. So, you will have north face is $\tau_{yx} - \frac{\partial \tau_{yx}}{\partial y}\frac{\delta y}{2}$. So, this will be on that face that is north face.

And this is your south face. This is you know east and this is west face. So, you will have. So, once you see it is component in the x direction. So, that will be if you look at $-(\tau_{yx} - \frac{\partial \tau_{yx}}{\partial y}\frac{\delta y}{2})$. So, that will be your in the x directions.

So, that will be multiplied by $\delta x \delta z$ and then we are adding the another term. So, that will be you know $(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\frac{\delta y}{2})\delta x \delta z$. So, if you see that again $\partial \tau_{yx}$ is gone and you have these 2 plus terms. So, you will have $\frac{\partial \tau_{yx}}{\partial y}\ \delta x \delta y \delta z$.

So, this is we are taking that net force in the x direction on the pair north and south face. Similarly if you take the net force in x direction on top and bottom faces; so, on top and bottom face you will have $\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2}$. So, on the top face and on the bottom face you have $\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2}$. So, that is there on the bottom face and we multiplied by the area. So, that will be again $-(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2})\delta x \delta y$.

and then we are further adding $(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2})\delta x \delta y$. So, this will be again becoming same as $\frac{\partial \tau_{zx}}{\partial z}\delta x \delta y \delta z$. So, what you see that you have the total force which will be acting on per unit volume on the fluid. So, that will be equal to the sum of these forces net forces acting in the x direction. So, if you are having that you know net force in the x direction.

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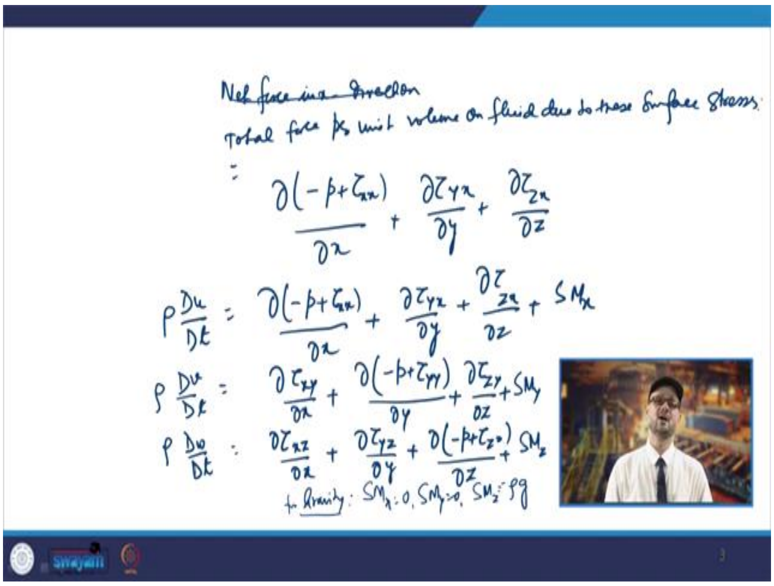

So, your net force in x direction; so, that will be summation of you know these forces which are acting on these different faces and their components in the x direction. So, that will be you know it will be sum. So, you know it will be total force. So, you know that is the, it is not the net force basically it is the you know total force. So, net force we have got in x direction for by the on the different faces.

Now, we are going to have the total force. So, total force per unit volume on fluid due to these surface stresses. So, it will be sum of these forces which you have derived in our. So, for the east and west face for the north and south face and for also the you know top and bottom face. So, that will be some of that. So, you are getting $\frac{\partial(-p+\tau_{zx})}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}$.

So, this is you know this is the force which is acting in the you know it is because of the x momentum. So, it is the total force in the x direction and so you can write you know that your x momentum you know equation that will be becoming. So, you will have to equate these to the x momentum what we you know rate of change of momentum. So, that is the so that is equated to that force and apart from that you will have other forces body forces or gravity forces.

So, by nature so you will have that term is taken as the source term. So, that you can have the source term in the you know x direction. So, if you take the you know x momentum equation. So, x component of the momentum equation that will be writing we will be writing as $\rho\frac{Du}{Dt}$. So, that is what we had seen that for the x component of the momentum that will be equated to the you know force which is acting in the x direction.

So, that will be $\frac{\partial(-p+\tau_{zx})}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}+S_{Mx}$. So, this is your source term in the x direction. So, that is a S_{Mx}. Similarly, if you try to have the you know y momentum equation in that case again you will have to do the analysis of the forces. So, similar only on the line if you try to have the equation for the y component though y component equation can be written as $\rho\frac{Dv}{Dt}$ so that u will be replaced with v.

So, that is you know rate of change of the momentum in the y direction and that will be the force that is in y direction. So, when we are talking about the y direction. So, in that case you will have $\frac{\partial\tau_{yx}}{\partial x}$ then you will have $\frac{\partial(-p+\tau_{yy})}{\partial y}+\frac{\partial\tau_{zy}}{\partial z}+S_{My}$.

So, this is the momentum equation of for the y component. In the in the same line we can also write for the you know for the z component. So, that will be $\rho \frac{Dw}{Dt}$. So, that is your in the z direction that will be $\frac{\partial \tau_{zx}}{\partial x}$. Similarly you will have $\frac{\partial \tau_{yz}}{\partial y}$and then here that pressure term will come in the you know this part. So, it will be $\frac{\partial(-p+\tau_{zz})}{\partial z}$ and then you will have the source term in the z direction. So, that way these are the momentum conservation equation which you get if you try to derive.

Now, in this case as you see that you will have you know the sign which is associated with the pressure. So, if you see that pressures. So, it is opposite to that which is associated with the viscous normal stresses. You know because the usual sign convention when you take now that will be taking the tensile as the positive and the pressure is normally you know which is by definition a compressive normal. So, that is that is why we take it as a negative sign. So, accordingly you know we have to have the concentration of the sign convention.

Now, if you talk about the you know we are taking the account of the surface forces surface stresses. Now this is source terms that is S_{Mx} or S_{My} or S_{Mz}you know they are because of the inclusion of the body forces. So, you know normally you know the gravity force which is there in many cases when you will have the suppose it when the flow in transition is there. So, and when there you will be gravity term coming into picture because of the natural convection or order of the buoyancy and all that.

So, in those cases you will have to induce these source terms and when you have to model this. So, what you can see is certainly when you take gravity into you know consideration in that case. So, for gravity taking into consideration when you try to model this gravity so your is S_{Mx} that will be 0, S_{My} will be 0 and S_{Mz}. So, that will be basically the force which is in the z direction and that is your gravitational force and since we are talking about the that force per unit volume. So, you will have mass per volume. So, that will be density times the g.

And then that will be with negative sign because it is acting in the you know negative direction. So, that term will be coming here. So, this S_{Mz} will be replaced by the ρg term $-\rho g$. So, this is the normal practice you will see that you will have source term. So, whenever we try to solve and we have extra force coming into picture magnetic forces or the gravity forces which are basically coming as the source terms only.

So, they will be given and their values will be provided and then the equation will be solved. So, that is the normal way of you know taking these momentum terms and these are these 3 are the momentum conservation equation which will be further used when we are going to deal with the equations which are derived like Navier-Stokes equations or when we deal with the turbulence modelling in our coming lectures.

Thank you very much.