Modeling of Tundish Steelmaking Process in Continuous Casting
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Lecture – 17
Mass Conservation Equation

Welcome to the lecture on Mass Conservation Equation. So, we will talking about the governing equations especially the Conservation Equation of Mass in this lecture. And we talked about the certain points related to the fluid fundamentals. And now we will be talking about the conservation equation for mass.

So, there are certain assumptions in that we take the fluid as the continuum.

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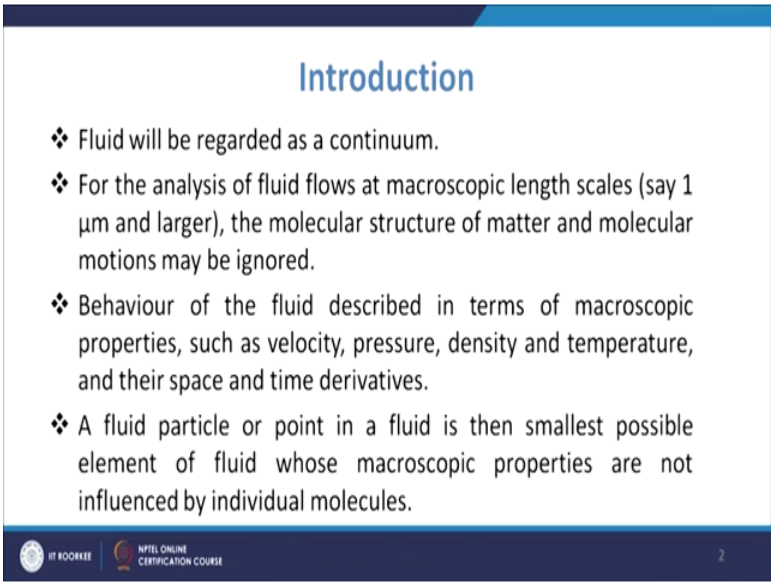

So, basically for the analysis of fluid flow at the microscopic length scale so, when it is 1 micrometer or larger, the molecular structure of matter and molecular motions we have to ignore for this analysis. Then we are also having the behavior of the fluid which we will try to describe in terms of the macroscopic properties. Like, velocity pressure, density, temperature, and they are space and time derivatives. So, we are going to have these properties and in that turn we are going to have the discussion about the flow behavior or fluid behavior.

Now, fluid particle or a point in a fluid will be the smallest possible element of fluid whose macroscopic properties are not influenced by individual molecules. So, the with these we are going to have the analysis of the flow behavior. And when we are going to talking about the conservative conservation properties you know, conservative principles for certain you know things like mass or momentum.

So, in this lecture we are going to talk about the mass.

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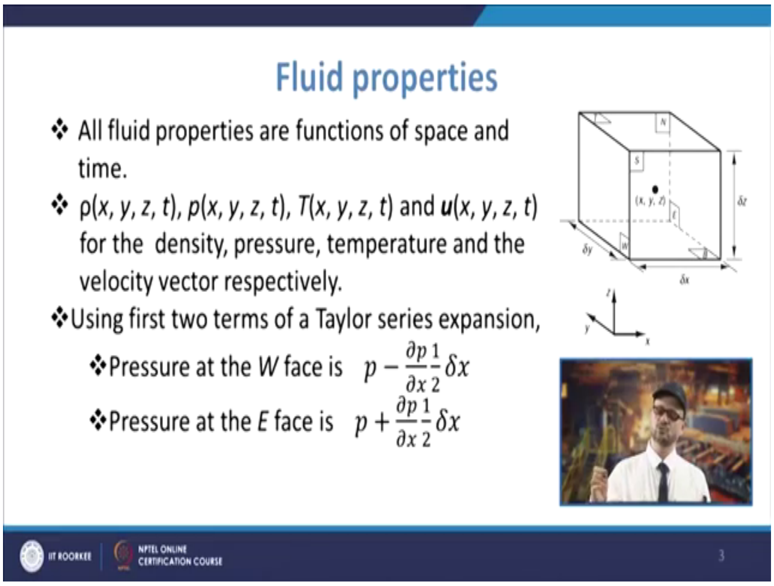

Now, if you talk about the properties of the fluids. So, suppose you have a point x, y, z where you are defining, now in that case the all the fluid properties are function of the space and time. So, you have properties like density, pressure, temperature, or velocity they are all said to be a function of a space as well as times. So, we define it as ρ, x, y, z, t. So, there will be a function of this space coordinate as well as the time co-ordinate. Similarly pressure of also functions of the space and time co-ordinate then you have the temperature that is also function of space and time and similarly the velocity.

So, if you try to have in the properties of to be found at the faces. So this is at the, suppose you are taking a cubical element now in that this x, y at x, y, z point you have these properties like you know these density or pressure or temperature or velocity. So, if you have define at the face, so that can be done by the Taylor series expansion and in that assuming it to be very small you can ignore the turns which are coming after the first two turns.

So, if you take this face, so that will be your West face on this side. So, on this face you will have if you take the pressure, so if the pressure p is at this point x, y, z. So, at this face it will be $p - \frac{\partial p}{\partial x}\frac{1}{2}\delta x$. So, δx is the, you know the length of this whole you know face; whole length is δx. And this point has the distance of $\delta x/2$ from this west face. So, it will be $p - \frac{\partial p}{\partial x}\frac{1}{2}\delta x$. Similarly pressure at the east face will be $p - \frac{\partial p}{\partial x}\frac{1}{2}\delta x$.

So, that way you are going to have the, you have to go to define these you know properties of the fluids at the different you know faces. So, similar when you go to the you know this north face so you will have a $p - \frac{\partial p}{\partial y}\frac{1}{2}\delta y$ y; so this direction is your z so this will be $p - \frac{\partial p}{\partial z}\frac{1}{2}\delta z$.

So, that way accordingly you will have the value of these you know properties across these faces. And that will be used when we talk about the conservation principle of the mass.

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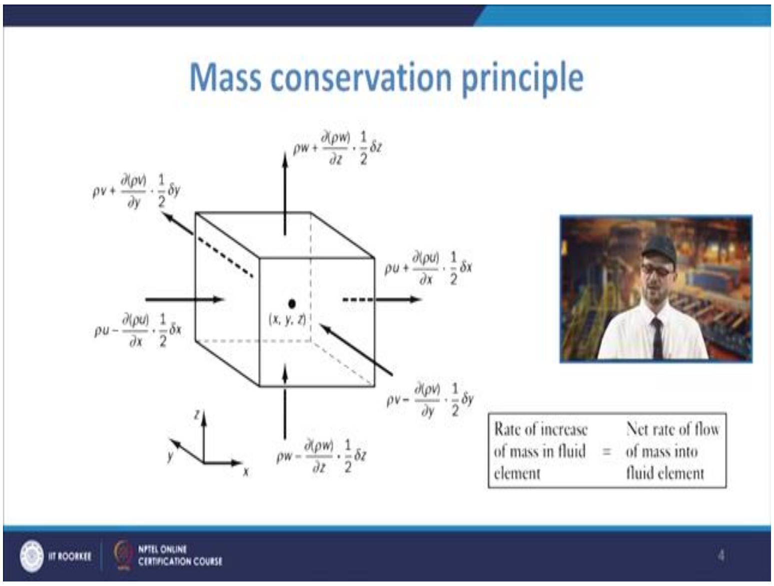

Now, coming to the mass conservation principle. Mass conservation principle tells that you will have the, you know rate of increase of mass in fluid element. It will be as same as the net rate of mass into the fluid elements. So, whatever net rate of mass flow there into the element it will be same as the rate of increase of mass in the fluid and any elements; so what we see.

As you have seen that you will have at this point if the these properties are ρ, t, u, v and all that, so p. So, that will be you will have the values on all these sides. And we will have the expression for this you know you know accordingly we will be trying to find so, the rate of increase of mass in the fluid element.

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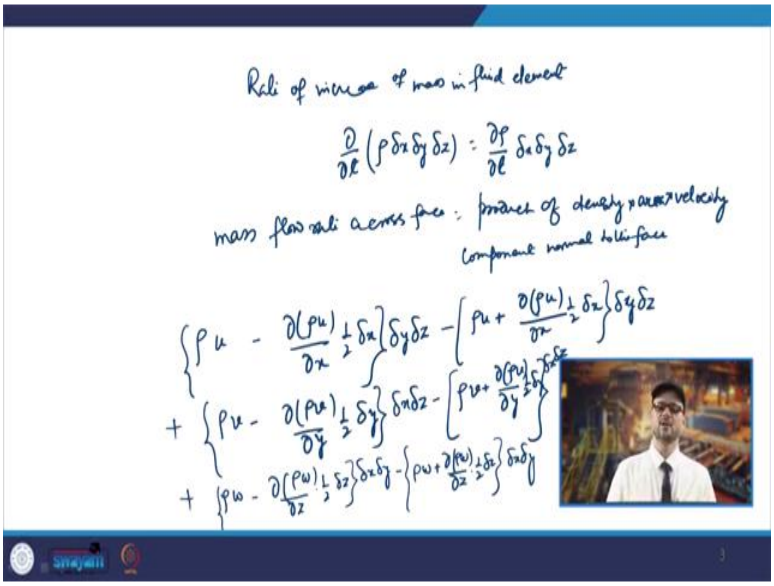

So, rate of increase of mass in fluid element. So, it will be $\frac{\partial}{\partial t}$; that is rate of increase with respect to time and mass will be density into volume. So, volume will be $\delta x, \delta y$ and δz that is cubical element. So, density will be ρ and then you will have $\delta x, \delta y$ and δz. So, it will be $\frac{\partial \rho}{\partial t}\delta x\delta y\delta z$, so that is being a constant so that will be coming out. So, $\frac{\partial \rho}{\partial t}\delta x\delta y\delta z$.

Now we need to have the value of the mass flow rate across the, you know faces of the element. So, you will have the six different faces. And we need to find out the mass flow rate across these faces. And it will be given by: so mass flow rate across the face. So, that will be the product of the you know density, so that will be ρ. Then you will have the area and that will be further multiplied with the velocity component normal to the face. So, you will you will have the velocity component which is normal to the face, and then you have density and the area.

So, if you take you know along the boundaries, so you will have the values and if you see it will be ρu. So, if you say you are taking the east face or so. So, it will be $\{\rho u - \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\}\delta y\delta z$. And then it will be, so one is this and another is on the right hand side. So, it will be $\{\rho u+\frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\}\delta y\delta z$. So, if you take that in the x directions so you have one flow from here another flow on this side. So, that is what you are getting this these two faces; taking into account this terminologies coming.

Similarly, you will have if you take the you know v components. So, you will have $\{\rho v-\frac{\partial(\rho v)}{\partial y}\frac{\delta y}{2}\}\delta x\delta z$. And that will be further minus of $\{\rho v+\frac{\partial(\rho v)}{\partial y}\frac{\delta y}{2}\}\delta x\delta z$. So, that will be for the y component.

And then you have the z component. So, that is z component will be again w will be coming. So, we will have $\{\rho w-\frac{\partial(\rho w)}{\partial z}\frac{\delta z}{2}\}\delta x\delta y$, similarly you have $\{\rho w+\frac{\partial(\rho w)}{\partial z}\frac{\delta z}{2}\}\delta x\delta y$. So, this is the basically net mass flow rate that is across the faces. So, you have the rate of that is net rate of mass flow into the fluid element. That is what you are seen from here.

So, one is coming and other is going. So, net rate of mass flow into the fluid element will be this particular you know value. So, if you are taking, so in that what you see you will be getting these terminologies cancelled and you will get certain terminology. So, here you are getting $\frac{\partial\rho}{\partial t}\delta x\delta y\delta z$.

Now in this case, what you say this is $-\frac{\partial(u)}{\partial x}\frac{\delta x}{2}\}\delta y\delta z$ and that will be $-\frac{\partial(u)}{\partial x}\frac{\delta x}{2}\}\delta y\delta z$. So, these two terms will be you know added and they will be negative term of $\{-\frac{\partial(u)}{\partial x}\frac{\delta x}{2}\}\delta y\delta z$. So, that is what you are getting. so And this will be term you know ultimately together and that will be equated to these term.

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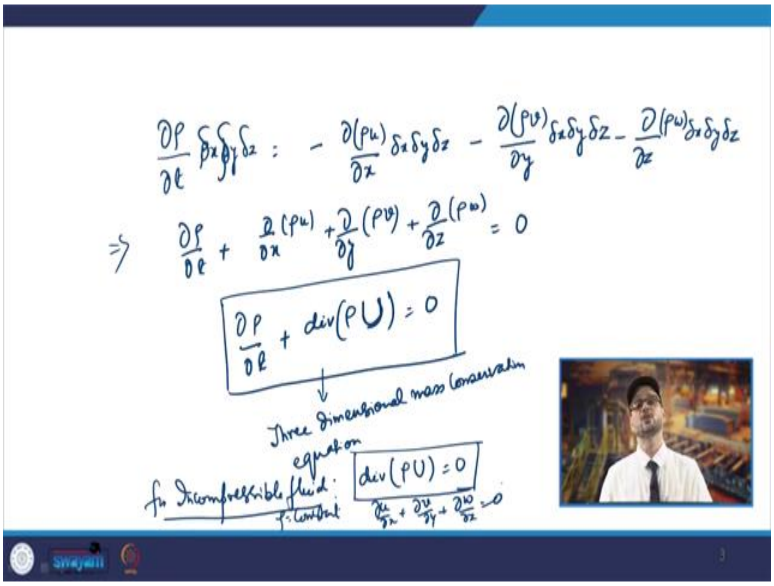

So, what you get is that you are getting this term $\frac{\partial\rho}{\partial t}\delta x\delta y\delta z$, that will be equal to this term $\frac{\partial\rho}{\partial t}\delta x\delta y\delta z$,. So, it will equal to a $-\frac{\partial(\rho u)}{\partial x}\delta x\delta y\delta z - -\frac{\partial(\rho v)}{\partial y}\delta x\delta y\delta z - -\frac{\partial(\rho w)}{\partial w}\delta x\delta y\delta z$. So, that is coming as the negative term when you add all these terms of this. So, it will be minus and minus, so it will be that $\frac{\delta x}{2}\delta y\delta z$.

Now, this with you can write. So, $\delta x\delta y\delta z$ can be cancelled on both the sides. So, you can write $\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$. So, this is what you get. So, this equation that is what you are getting that is known as the conservation of mass. This is known as the mass conservation equation.

Now what you see, so this if you try to you know right in a compact vectorial form. So, in upcoming vector notation you can write this as $\frac{\partial\rho}{\partial t} + div.(\rho U) = 0$. So, that is also this is also known as the mass conservation equation.

Now, this is known as the three dimensional mass conservation equation or we can also call it as the continuity equation. So, normally that is generally defined for the compressible fluid. However, if you have the incompressible fluid where density does not vary, so in that case ρ will be constant.

So, for incompressible fluid so, when we talk about the fluid flow like water or steel or so they are incompressible nature. So, in that case the ρ is basically constant. So, as the ρ becomes constant so your term become $div.(\rho U) = 0$. So, this is the, you know equation that is known as the continuity equation for the compressible flow.

So, you simply write in that case the rho becomes constant, so rho will come out so $div.(U) = 0$. So, it will be $\frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} + \frac{\partial(w)}{\partial z} = 0$. So, that is what the continuity equation you know is defined as.

Now, next thing what we will; so next will be studying about the momentum conservation. And before that we need to know something about the rate of change of the, you know following the fluid particle and for a fluid element. So, that is capital $\frac{D}{Dt}$ so, $D\varphi$ additive for any fluid property.

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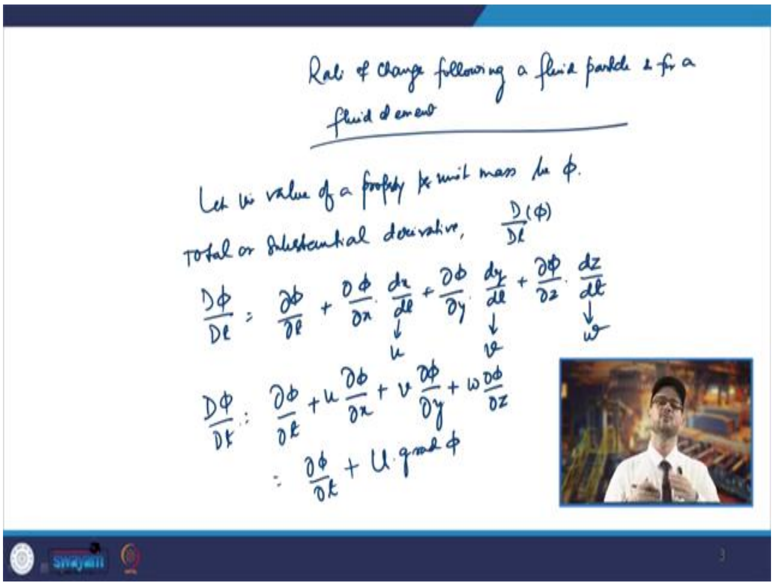

So, that will be the rate of change that is total derivative basically. So, that is the follow in a fluid particle and for a fluid element. So, you will have the changes in the properties of the fluid particle and for that Lagrangian approach is you know used. And each property will be the function of the position and the time so, the all the properties being the function of the position and time; so x, y, z and t.

So, we normally you have any property per unit mass which we define; so we define. So, supposed let we the value of a property per unit mass be φ. So, if you are seeing that φ, so it will be depending upon. So, its value will be depending value will be depending upon x, y, z and t.

And if you talk about the total derivative or the substantial derivative that is capital $\frac{D}{Dt}$, so total or substantial derivative. So, that is basically represented by $\frac{D\varphi}{Dt}$ of any property. So, that $\frac{D\varphi}{Dt}$ will be you know represented that will be equal to $\frac{\partial\varphi}{\partial t}+\frac{\partial\varphi}{\partial x}\frac{dx}{dt}+\frac{\partial\varphi}{\partial y}\frac{dy}{dt}+\frac{\partial\varphi}{\partial z}\frac{dz}{dt}$. So, this is the expression for the substantial derivative and this will be used when we will talk about the conservation of properties like momentum or temperature in that those cases.

Now, for a fluid particle you know which is in the flow. So, your $\frac{dx}{dt}$ will be u and $\frac{dy}{dt}$will v and $\frac{dz}{dt}$ will be you know w. So, this will be your u, this will be v, and this will be w. So, you can write you know $\frac{D\varphi}{Dt}=\frac{\partial\varphi}{\partial t}+\frac{\partial\varphi}{\partial x}u+\frac{\partial\varphi}{\partial y}v+\frac{\partial\varphi}{\partial z}w$.

So, you can further write this you know as: my first is the transient term that is $\frac{\partial\varphi}{\partial t}+U.grad\varphi$. so it will be the, you have three components u v w. So, accordingly you $\frac{\partial\varphi}{\partial x}u+\frac{\partial\varphi}{\partial y}v+\frac{\partial\varphi}{\partial z}w$. So, this basically $\frac{\partial\varphi}{\partial t}$ it will be defining the rate of change of the property φ per unit mass. And that way we are going to use it for you know for expressing you know when we are going to have the definition for the for the conservation of mass equation or for the conservation of momentum equation.

So, in that case we are going to have the use of these property φ in the different manner. So, if you talk about the mass conservation equation; so you if you talk about the mass conservation equation, you will have mass per unit volume is basically ρ. So, that is the conserved quantity. So, in that case you know if you talk about change of the densities. So some of the rate of change of the density you know in time and the convective term and you know in that mass consecutive term.

So, if you talk about the conservation equation for the mass which we have derived earlier that was the $\frac{\partial\rho}{\partial t}$. So, your this φ will be basically replaced by a ρ. So, it will be $\frac{\partial\varphi}{\partial t}+U.grad\varphi$. And since ρ will be will not be changing. So, so accordingly you can see here that was $div.(\rho U)$.

So, that is what you can get it you know from that particular equation itself.

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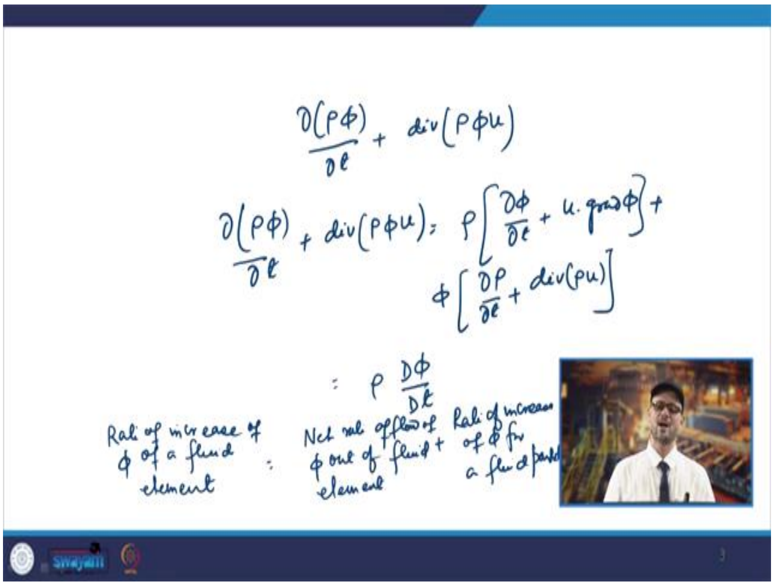

So, if you try to have the this you know for the for the you know arbitrary conserved property you can write the generalized you know term that will be: $\frac{\partial(\rho\varphi)}{\partial t} + div.(\rho\varphi U)$. So, you know if you try to derive the conservation equation for either the mass or the momentum you can directly get from here.

Now, if you see that, you know if you look at the expression the $\frac{\partial(\rho\varphi)}{\partial t} + div.(\rho\varphi U)$ So, that will be if you see it will be $\rho[\frac{\partial\varphi}{\partial t} + u.grad\varphi] + \varphi[\frac{\partial\rho}{\partial t} + div.(\rho u)]$So, that becomes equal to $\rho\frac{D\varphi}{Dt}$.

Now, in this case what you see that, the term that is $\varphi\frac{\partial\rho}{\partial t}$. Now this term is 0 because the density is constant. So, that term becomes 0 and also plus $div.(\rho u)$ so that this term becomes 0.

So, what is coming out of this expression is that you when you see the rate of increase of you know any property φ of a fluid element, so that is your $\frac{D\varphi}{Dt}$substantial derivative. That will be you know net rate of flow of φ out of the fluid element plus rate of increase of φ for a fluid particle. So, accordingly you can have. So, if you look at the momentum equation which we will try to you know derive in that you will have the term for x momentum y momentum and z momentum.

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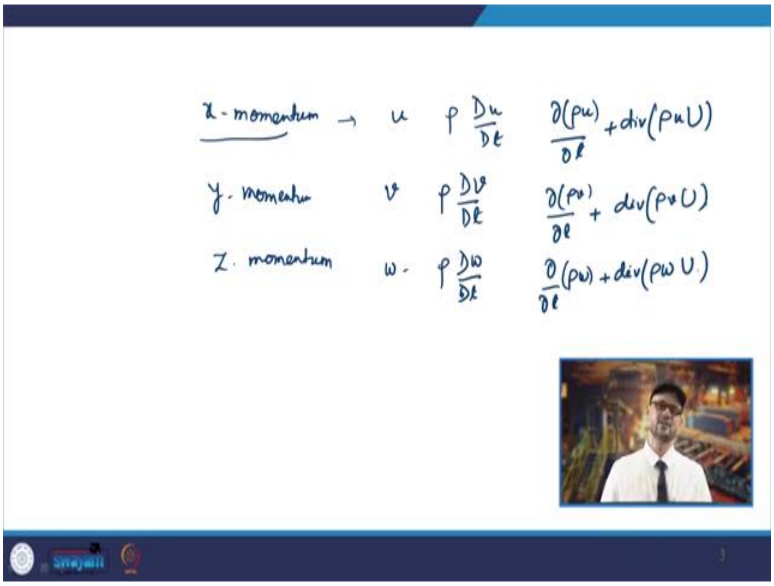

So, if you look at the x momentum term. Now in that case you will have a different terms. So, you will have the term u, you will have $\rho \frac{Du}{Dt}$. And accordingly you will get $\frac{\partial(\rho u)}{\partial t} + div.(\rho U u)$. So, that will be for the x momentum.

Similarly, if you go for the y momentum: y momentum you will have the velocity component is v. So, you will have the total derivative $\rho \frac{Dv}{Dt}$ and that comes as $\frac{\partial(\rho v)}{\partial t} + div.(\rho U v)$. So, so accordingly this way you will have the z momentum equation, and in that you have w so it becomes $\rho \frac{Dw}{Dt}$. And that becomes equal to $\frac{\partial(\rho w)}{\partial t} + div.(\rho U w)$

So, so accordingly you know if you use this total derivative term, this total derivative total or derivative or substantial derivative expression that will be used for finding these conservation equation for momentum also in the long in the coming lectures, where we will be having the derivation of these conservation equation for the momentum. So, in the coming will be having you know the expression for the momentum equation in three-dimension.

Thank you very much.